Frege's concept script and Peirce's existential graphs

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History of Logic
11 October 2019

Concept script (Begriffsschrift)

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- Primary aim to clarify arguments in mathematics, but Frege also believed his language would be useful for physics and philosophy.
- The notation was so cumbersome it was never widely adopted. However, Frege's language influenced the work of Russel and Whitehead, thus contributing to the development of first-order logic.

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- $(P \to (Q \to R)) \to ((P \to Q) \to (P \to R))$
- $(P \to (Q \to R)) \to (Q \to (P \to R))$
- $(P \to Q) \to (\neg Q \to \neg P)$
- $\forall x P(x) \to P(y)$
- $\neg\neg P \to P$
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- $(x = y) \to (P(x) \to P(y))$
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Inference rule: modus ponens.

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- This role was to be taken by existential graphs.
- There are three types of existential graphs: alpha, beta and gamma, which correspond to classical propositional logic, first-order logic and modal logic, although all these were introduced after Peirce.

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Semantics:

- SA is taken to be true.
- Letters are placeholders for names.
- A cut is a negation.
- Any collection enclosed by a cut is interpreted as a conjunction.

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We say that an existential graph is a *theorem* if there is a sequence of transformations reducing it to the empty SA. There is a one-to-one correspondence between alpha graphs and formulas in classical propositional logic (CPC). Moreover, it can be shown that an alpha graph is a theorem if and only if the corresponding formula is a theorem in CPC.

References

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- Charles Sanders Peirce. *Manuscript 514, with commentary by J. F. Sowa*. 1909. http://www.jfsowa.com/peirce/ms514.htm
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Questions

1. Can we say that someone who is doing the allowed manipulations in Frege's/Peirce's graphical language is thinking? If so, in what sense does it involve 'thinking in a language'?
2. Peirce makes a sharp distinction between a language that is aimed to help with logical inferences and a language that 'records' this reasoning in as much detail as possible. Frege, on the other hand believes that the same language will help to clarify reasoning in mathematics and in philosophy. Is either of these approaches justified?
3. Both existential graphs and the concept script (without quantification) are equivalent to classical propositional logic. Why were both graphical languages largely ignored? Could there be a reason why a one dimensional notation closely resembling the algebraic one prevailed?
4. One of Frege's arguments for the concept script is that it uses two dimensions. Why would two dimensions be advantageous for a language? Could an artificial (but not logical) language be written in two dimensions? Do you think the claim that the concept script is two-dimensional is justified?