Composite explanations

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- Lower level explains behaviour higher level (reduction assumption)
- Higher level allows for concise descriptions of complex behaviour

Syntax

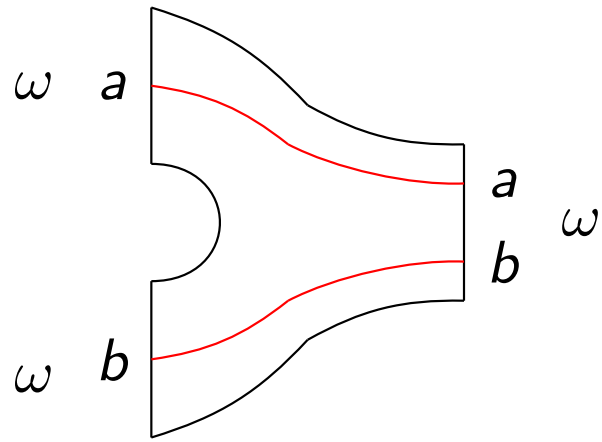
$$\frac{\sigma \in \Sigma \setminus \Sigma^i}{\sigma : (\mathtt{ar}(\sigma) \mid \mathtt{coar}(\sigma))} \qquad \frac{\sigma \in \Sigma^i}{\omega\,\alpha \;\boxed{\sigma}\; \alpha\,\omega : (\mathtt{ar}(\sigma) \mid \mathtt{coar}(\sigma))}$$

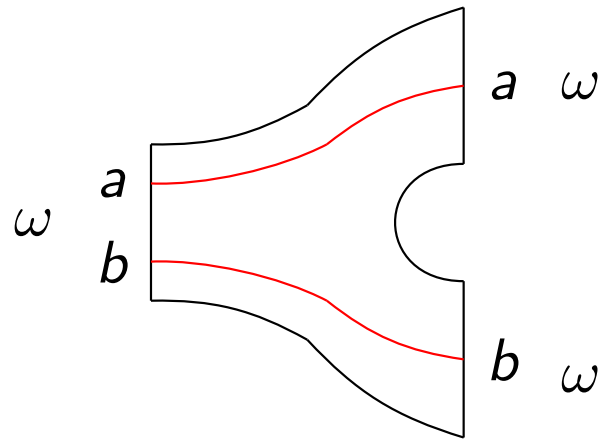
$$\frac{}{\square : (\varepsilon \mid \varepsilon)} \qquad \frac{}{\omega\,\alpha \;\boxed{}\; \alpha\,\omega : (\omega, \alpha \mid \omega, \alpha)}$$

$$\frac{}{\omega\,\alpha \quad \alpha\,\tau \\ \tau\,\beta \quad \beta\,\omega \;\text{(crossing)}\; : (\omega, \alpha; \tau, \beta \mid \tau, \beta; \omega, \alpha)} \qquad \frac{x : (t \mid s) \quad y : (s \mid u)}{x; y : (t, u)}$$

$$\frac{x : (\omega, \alpha \mid \omega, \gamma) \quad y : (\omega, \beta \mid \omega, \delta)}{x \oplus_\omega y : (\omega, \alpha\beta \mid \omega, \gamma\delta)} \qquad \frac{x : (t \mid s) \quad y : (u \mid w)}{x \oplus y : (t; u \mid s; w)}$$

Syntax 2





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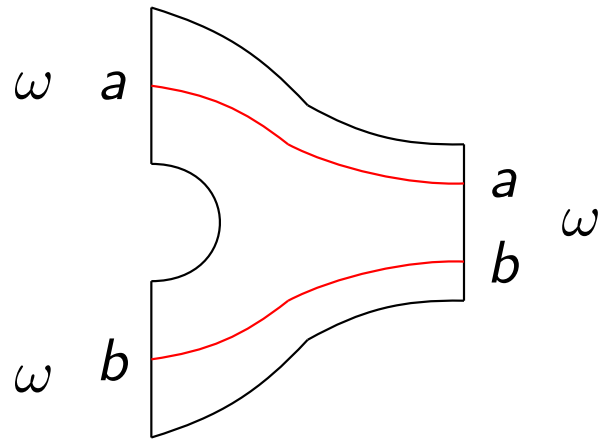
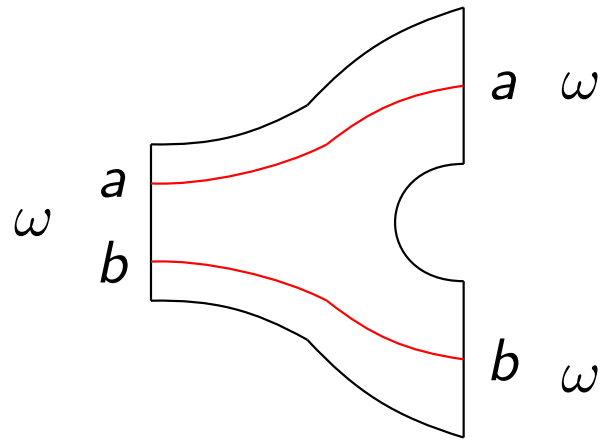
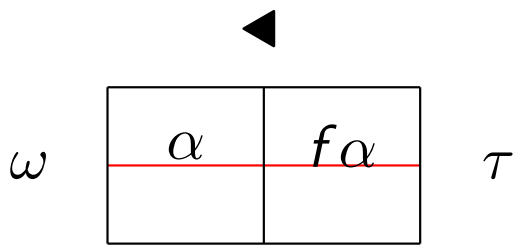
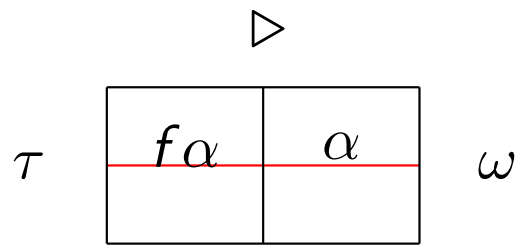




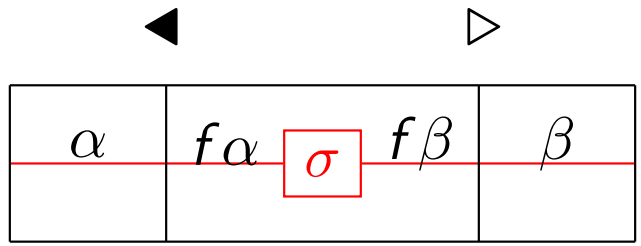




Explanations

Definition (Window, cowindow)

A *window* is a morphism in a layered prop of the following form:



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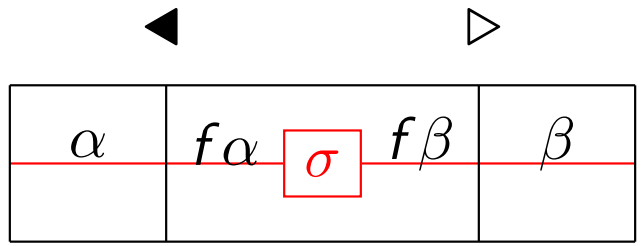
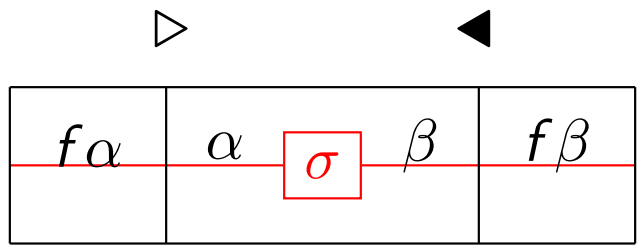


Dually, a *cowindow* is a morphism in a layered prop of the following form:



Example: glucose phosphorylation

after Jean Krivine

We wish to explain the chemical reaction rule:

$$\mathit{glucose} + \mathit{ATP} \longrightarrow \mathit{glucose\text{-}6\text{-}phosphate} + \mathit{ADP} + \mathit{hydrogen\ ion}$$

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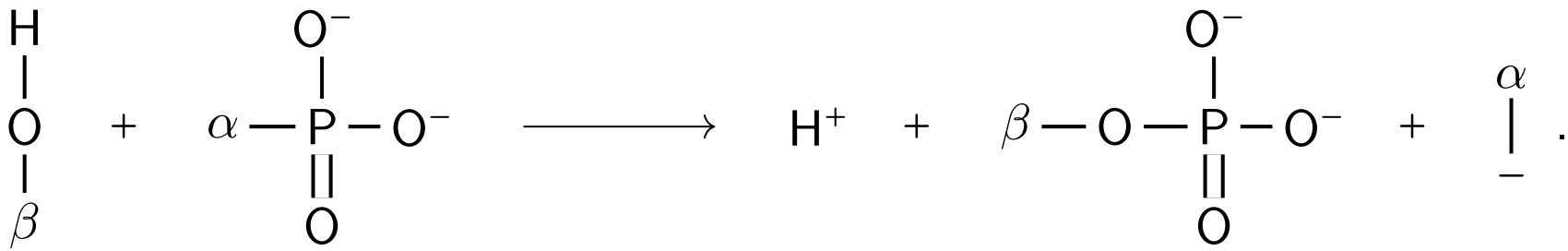

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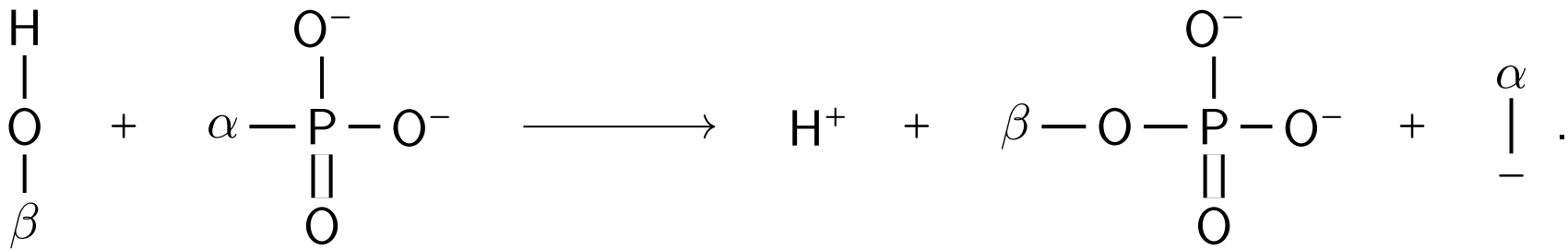

We draw this as an internal box:

.

in the layered prop.

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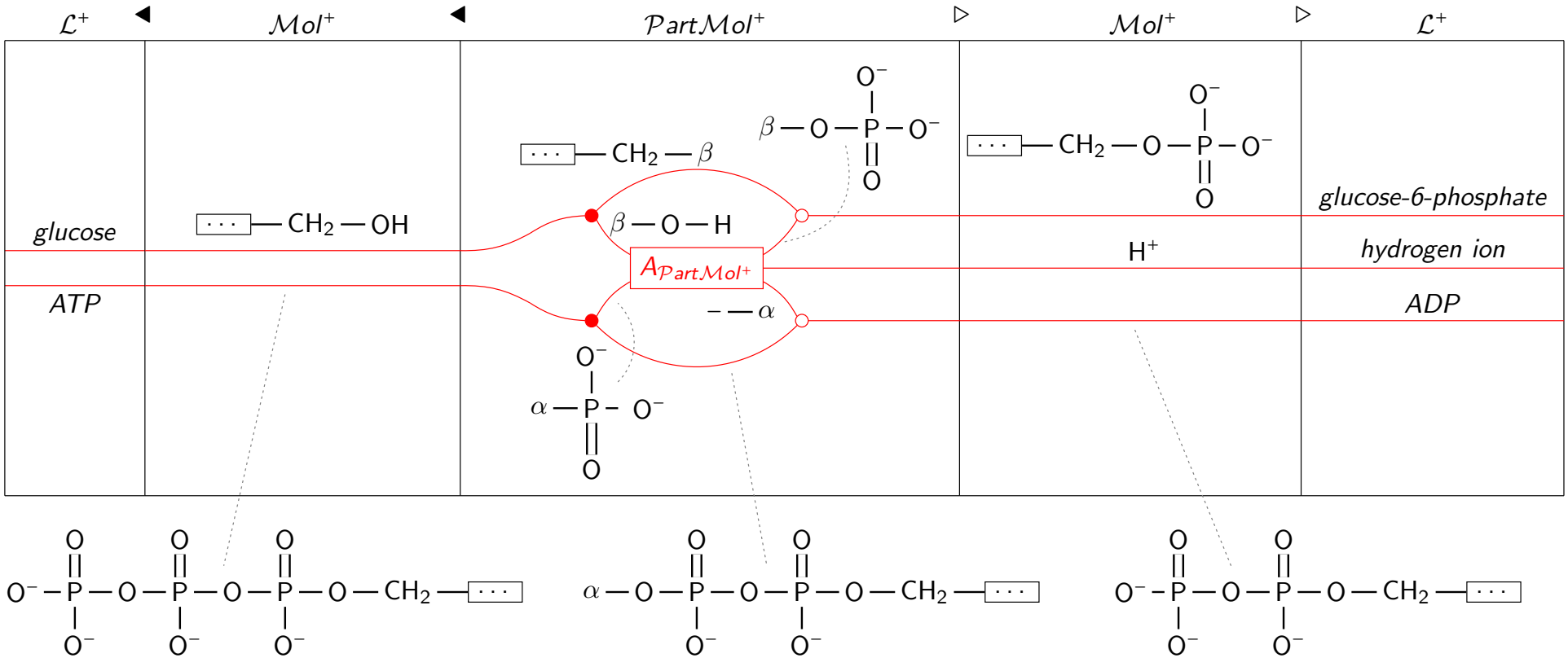

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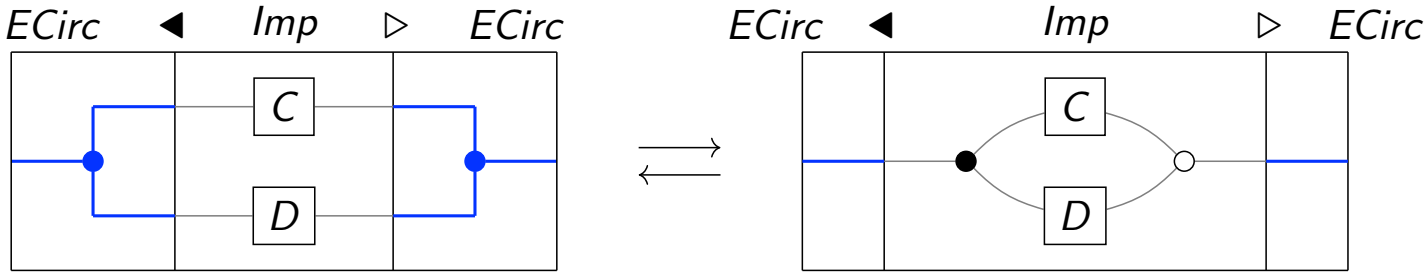

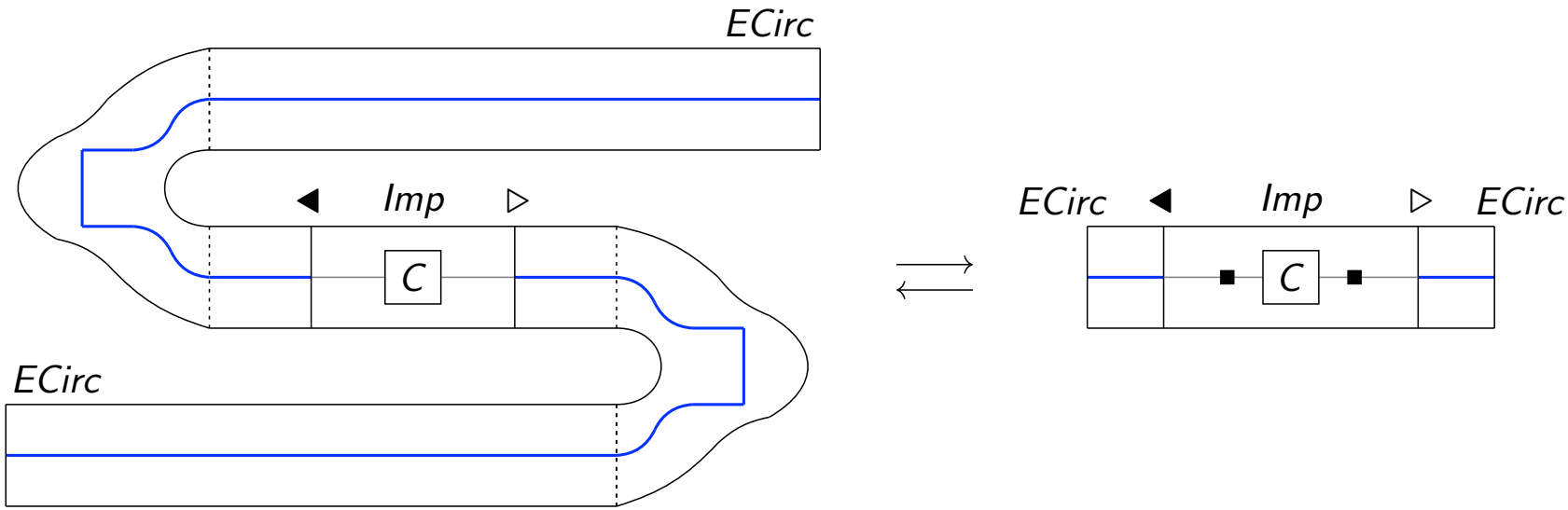

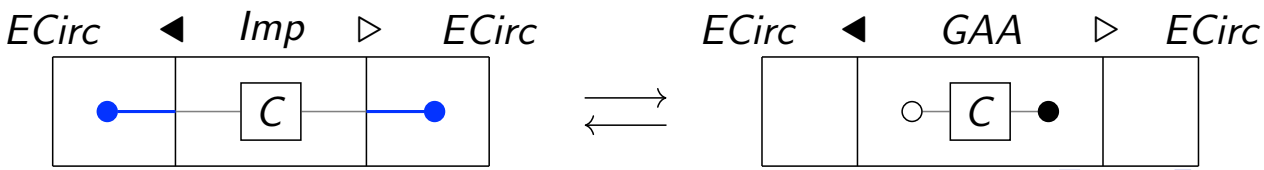

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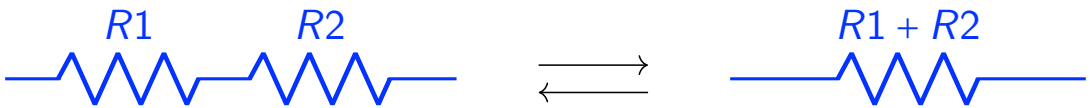
We wish to explain the rule for the sequential composition of resistors:



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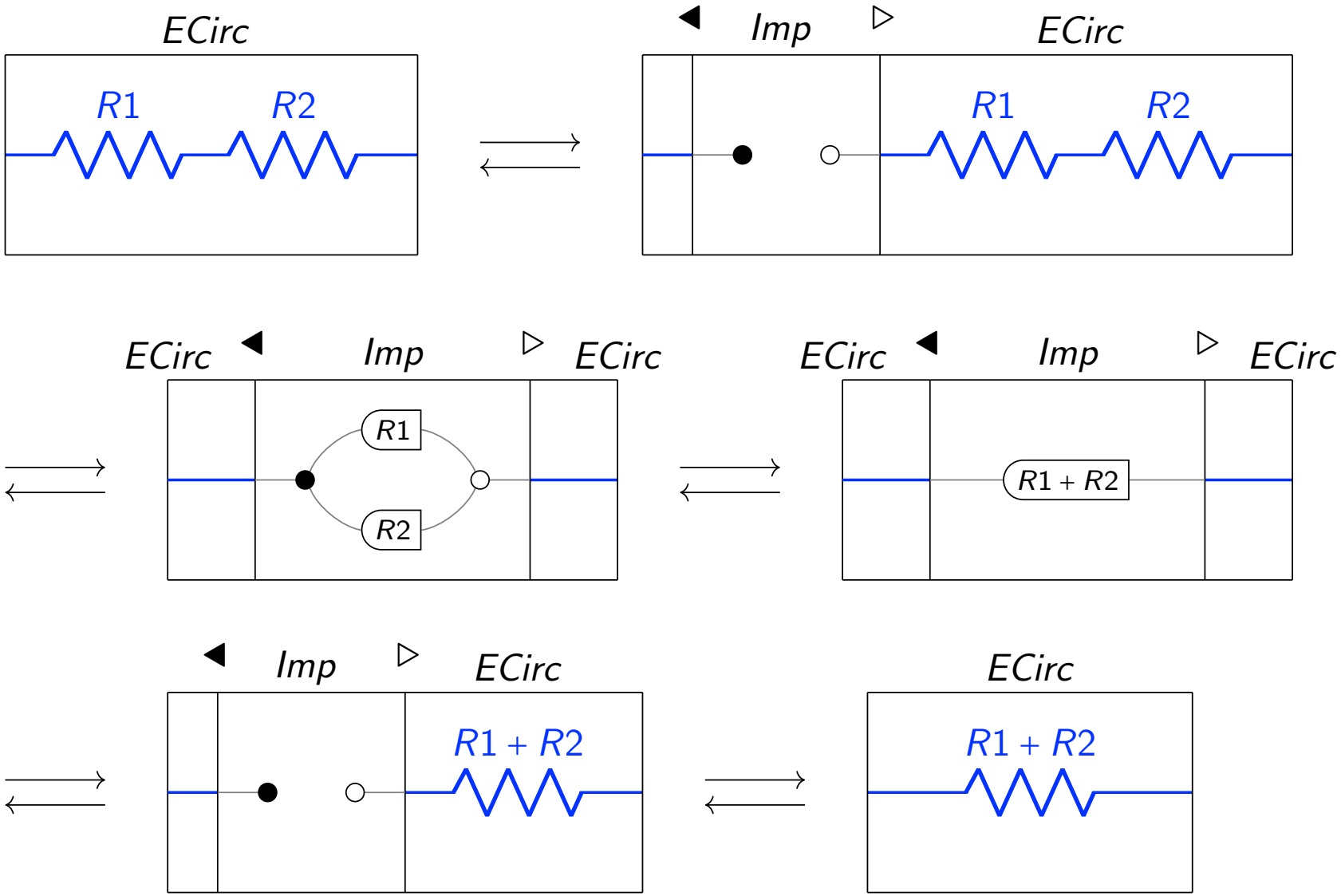

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- Guillaume Boisseau and Pawel Sobociński. *String Diagrammatic Electrical Circuit Theory.* arXiv:2106.07763v1.
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- Jean Krivine. *Systems biology.* ACM SIGLOG News 4.3, 2017.

Thank you for your attention!