Answers to Homework 1

Leonhard Horstmeyer (Leo Marlo#1048)

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Question 1

Let $S = \mathbb{Z}_7$ and let $+$ and $\star$ be addition and multiplication respectively on $\mathbb{Z}_7$, then

a) $4 + 4 = 1$

b) $3 \star 5 = 1$

c) The multiplicative inverse of 3 is 5 because $3 \star 5$ yields the multiplicative unit element 1. The additive inverse should be 4, because $3 + 4$ yields the additive unit element 0.

Question 2

Yes we can cosider $(\mathbb{Z}_7, +)$ to be a group. It is closed, because the modular thingy makes the range of target values lie between 0 and 6. There is a unit element, namely 0, for each element $a \in \mathbb{Z}_7$ there exists an element b such that $a + b = 0$, namely $b = 7 - a$.

Question 3

So $-13 \bmod 5 = (-13 + 15) \bmod 5 = 2 \bmod 5 = 2$.

Question 4

According to Wolfram alpha 2 is a positive root, the other two roots are complex numbers with non zero imaginary part so that positivity does not have a meaning unless one refers to just the real part, in which case they are both negative.