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$$\begin{aligned}(\neg p \wedge (p \to q)) \to \neg q &\equiv \neg(\neg p \wedge (p \to q)) \vee \neg q \\ &\equiv \neg(\neg p \wedge (\neg p \vee q)) \vee \neg q \\ &\equiv \neg(\neg p) \vee \neg q \\ &\equiv p \vee \neg q\end{aligned}$$

不是重言式

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$$\begin{aligned}(\neg q \wedge (p \to q)) \to \neg p &\equiv \neg(\neg q \wedge (\neg p \vee q)) \vee \neg p \\ &\equiv (q \vee \neg(\neg p \vee q)) \vee \neg p \\ &\equiv (q \vee (p \wedge \neg q)) \vee \neg p \\ &\equiv ((q \vee p) \wedge (q \vee \neg q)) \vee \neg p \\ &\equiv ((p \vee q) \wedge T)) \vee \neg p \\ &\equiv q \vee p \vee \neg p \\ &\equiv T\end{aligned}$$

是重言式

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i.Alice is a mathematics major. Therefore, Alice is either a mathematics major or a computer science major.

p:Alice is a mathematics major.

q:Alice is a computer science major.

rule of inference:$p \Rightarrow (p \vee q)$

ii. Jerry is a mathematics major and a computer science major. Therefore, Jerry is a mathematics major.

p: Jerry is a mathematics major;

q: Jerry is a computer science major.

rule of inference: $(p \vee q) \Rightarrow p$

iii. If it is rainy, then the pool will be closed. It is rainy. Therefore, the pool is closed.

p: It is rainy;

q: The pool is closed.

rule of inference: $(p \to q) \wedge p \Rightarrow q$

iv. If it snows today, the university will close. The university is not closed today. Therefore, it did not snow today.

p:It snows today;

q:The university is closed today.

rule of inference:$(p \to q) \wedge \neg q \Rightarrow \neg q$

v.If I go swimming, then I will stay in the sun too long. If I stay in the sun too long, then I will sunburn. Therefore, If I go swimming, then I will sunburn.

p: I go swimming;

q: I stay in the sun too long;

r: I sunburn.

rule of inference:$(p \to q) \wedge (q \to r) \Rightarrow (q \to r)$

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i.$A \cup (B \cup C) = (A \cup B) \cup C$ 显然，这是成立的，证明略

ii.$A \cap (B \cap C) = (A \cap B) \cap C$ 显然，这是成立的，证明略

iii.

$$\begin{aligned} x \in A \cup (B \cap C) &\Leftrightarrow x \in A \text{ or } x \in (B \cap C) \\ &\Leftrightarrow (x \in A \text{ or } x \in B) \text{ and } (x \in A \text{ or } x \in C) \\ &\Leftrightarrow x \in (A \cup B \text{ and } A \cup C) \\ &\Leftrightarrow x \in (A \cup B) \cap (A \cup C) \end{aligned}$$

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整数域内，是否是满射？

i. $f(m,n) = m + n$ 是

ii. $f(m,n) = m^2 + n^2$ 不是

iii. $f(m,n) = m$ 是

iv. $f(m,n) = |n|$ 不是

v. $f(m,n) = m - n$ 是

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$\sum_{k=1}^{5}(k+1) = 20$

$\sum_{j=0}^{4}(-2)^j = 11$

$\sum_{i=1}^{10} 3 = 30$

$\sum_{=0}^{8}(2^{j+1} - 2^j) = 511$