

A bag contains 4 red balls and 6 blue balls. Rassa selects two balls at random, without replacement, from the bag. The number of red balls selected by Rassa is denoted by  $X$ .

- (a) Find the probability generating function,  $G_X(t)$ , of  $X$ . [2]

Rassa also tosses two coins. One coin is biased so that the probability of a head is  $\frac{2}{3}$ . The other coin is biased so that the probability of a head is  $p$ . The probability generating function of  $Y$ , the number of heads obtained by Rassa, is  $G_Y(t)$ . The coefficient of  $t$  in  $G_Y(t)$  is  $\frac{7}{12}$ .

- (b) Find  $G_Y(t)$ . [3]

The random variable  $Z$  is the sum of the number of red balls selected and the number of heads obtained by Rassa.

- (c) Find the probability generating function of  $Z$ , expressing your answer as a polynomial. [3]
- (d) Use the probability generating function of  $Z$  to find  $E(Z)$ . [2]