The random variable X has probability generating function $G_X(t)$ given by

$$G_X(t) = ct(1+t)^5,$$

where c is a constant.

(a) Find the value of c. [1]

(b) Find the value of $E(X)$. [2]

The random variable Y is the sum of two independent values of X.

(c) Write down the probability generating function of Y and hence find $\text{Var}(Y)$. [4]

(d) Find $P(Y = 5)$. [2]