

A 6-sided dice,  $A$ , with faces numbered 1, 2, 3, 4, 5, 6 is biased so that the probability of throwing a 6 is  $\frac{1}{4}$ . The random variable  $X$  is the number of 6s obtained when dice  $A$  is thrown twice.

**(a)** Find the probability generating function of  $X$ . [2]

A second dice,  $B$ , with faces numbered 1, 2, 3, 4, 5, 6 is unbiased. The random variable  $Y$  is the number of 6s obtained when dice  $B$  is thrown twice.

The random variable  $Z$  is the total number of 6s obtained when both dice are thrown twice.

**(b)** Find the probability generating function of  $Z$ , expressing your answer as a polynomial. [3]

**(c)** Find  $\text{Var}(Z)$ . [3]

**(d)** Use the probability generating function of  $Z$  to find the most probable value of  $Z$ . [1]