



A particle of mass  $m$  is attached to one end of a light inextensible string of length  $a$ . The other end of the string is attached to a fixed point  $O$ . The point  $A$  is such that  $OA = a$  and  $OA$  makes an angle  $\alpha$  with the upward vertical through  $O$ . The particle is held at  $A$  and then projected downwards with speed  $\sqrt{ag}$  so that it begins to move in a vertical circle with centre  $O$ . There is a small smooth peg at the point  $B$  which is at the same horizontal level as  $O$  and at a distance  $\frac{1}{3}a$  from  $O$  on the opposite side of  $O$  to  $A$  (see diagram).

- (i) Show that, when the string first makes contact with the peg, the speed of the particle is  $\sqrt{ag(1 + 2 \cos \alpha)}$ . [2]

The particle now begins to move in a vertical circle with centre  $B$ . When the particle is at the point  $C$  where angle  $CBO = 150^\circ$ , the tension in the string is the same as it was when the particle was at the point  $A$ .

- (ii) Find the value of  $\cos \alpha$ . [10]