The variables y and θ satisfy the differential equation

$$(1+y)(1+\cos 2\theta)\frac{\mathrm{d}y}{\mathrm{d}\theta} = \mathrm{e}^{3y}.$$

It is given that $y = 0$ when $\theta = \frac{1}{4}\pi$.

Solve the differential equation and find the exact value of $\tan\theta$ when $y = 1$. [9]