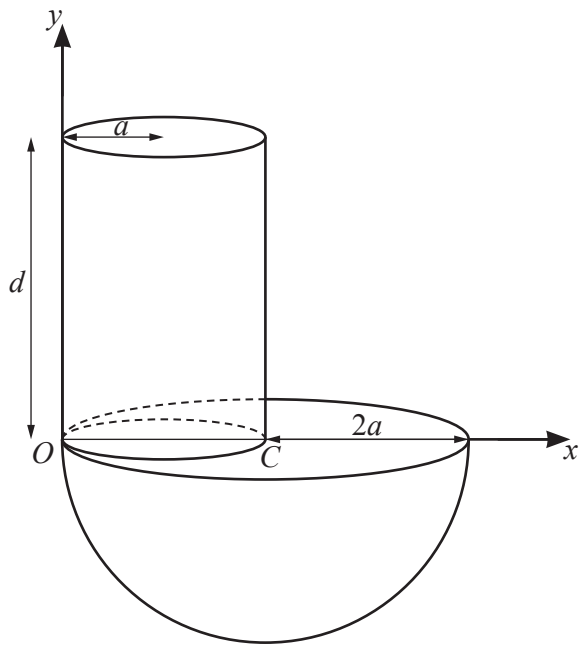
4



An object is formed from a solid hemisphere, of radius $2a$, and a solid cylinder, of radius a and height d. The hemisphere and the cylinder are made of the same material. The cylinder is attached to the plane face of the hemisphere. The line OC forms a diameter of the base of the cylinder, where C is the centre of the plane face of the hemisphere and O is common to both circumferences (see diagram). Relative to axes through O, parallel and perpendicular to OC as shown, the centre of mass of the object is $(\overline{x},\overline{y})$.

(a) Show that $\overline{x} = \dfrac{32a^2 + 3ad}{16a + 3d}$ and find an expression, in terms of a and d, for $\overline{y}$. [5]

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The object is placed on a rough plane which is inclined to the horizontal at an angle θ where $\sin\theta = \frac{1}{6}$. The object is in equilibrium with CO horizontal, where CO lies in a vertical plane through a line of greatest slope.

(b) Find d in terms of a. [3]