

A particle  $P$ , of mass  $m$ , is able to move in a vertical circle on the smooth inner surface of a sphere with centre  $O$  and radius  $a$ . Points  $A$  and  $B$  are on the inner surface of the sphere and  $AOB$  is a horizontal diameter. Initially,  $P$  is projected vertically downwards with speed  $\sqrt{\left(\frac{21}{2}ag\right)}$  from  $A$  and begins to move in a vertical circle. At the lowest point of its path, vertically below  $O$ , the particle  $P$  collides with a stationary particle  $Q$ , of mass  $4m$ , and rebounds. The speed acquired by  $Q$ , as a result of the collision, is just sufficient for it to reach the point  $B$ .

- (i) Find the speed of  $P$  and the speed of  $Q$  immediately after their collision. [7]

In its subsequent motion,  $P$  loses contact with the inner surface of the sphere at the point  $D$ , where the angle between  $OD$  and the upward vertical through  $O$  is  $\theta$ .

- (ii) Find  $\cos \theta$ . [5]