

George throws two coins,  $A$  and  $B$ , at the same time. Coin  $A$  is biased so that the probability of obtaining a head is  $a$ . Coin  $B$  is biased so that the probability of obtaining a head is  $b$ , where  $b < a$ . The probability generating function of  $X$ , the number of heads obtained by George, is  $G_X(t)$ . The coefficients of  $t$  and  $t^2$  in  $G_X(t)$  are  $\frac{5}{12}$  and  $\frac{1}{12}$  respectively.

(a) Find the value of  $a$ . [2]

The random variable  $Y$  is the sum of two independent observations of  $X$ .

(b) Find the probability generating function of  $Y$ , giving your answer as a polynomial in  $t$ . [3]

(c) Find  $\text{Var}(Y)$ . [3]