

The variables  $x$  and  $y$  satisfy the differential equation

$$\frac{dy}{dx} = ky^3 e^{-x},$$

where  $k$  is a constant. It is given that  $y = 1$  when  $x = 0$ , and that  $y = \sqrt{e}$  when  $x = 1$ . Solve the differential equation, obtaining an expression for  $y$  in terms of  $x$ . [7]