

A uniform square lamina ABCD has sides of length 10 cm. The point E is on BC with EC = 7.5 cm, and the point F is on DC with CF = x cm. The triangle EFC is removed from ABCD (see diagram). The centre of mass of the resulting shape ABEFD is a distance \overline{x} cm from CB and a distance \overline{y} cm from CD.

(a) Show that
$$\bar{x} = \frac{400 - x^2}{80 - 3x}$$
 and find a corresponding expression for \bar{y} . [4]

The shape *ABEFD* is in equilibrium in a vertical plane with the edge *DF* resting on a smooth horizontal surface.

(b) Find the greatest possible value of x, giving your answer in the form $a + b\sqrt{2}$, where a and b are constants to be determined. [3]