Nine balls labelled 1, 2, 3, 4, 5, 6, 7, 8, 9 are placed in a bag. Kai selects three balls at random from the bag, without replacement. The random variable X is the number of balls selected by Kai that are labelled with a multiple of 3.

(a) Find the probability generating function $G_X(t)$ of X. [3]

The balls are replaced in the bag.

Jacob now selects two balls at random from the bag, without replacement. The random variable Y is the number of balls selected by Jacob that are labelled with an even number.

(b) Find the probability generating function $G_Y(t)$ of Y. [2]

The random variable Z is the sum of the number of balls that are labelled with a multiple of 3 selected by Kai and the number of balls that are labelled with an even number selected by Jacob.

(c) Find the probability generating function of Z, expressing your answer as a polynomial. [3]

(d) Use the probability generating function of Z to find $E(Z)$. [2]