

The diagram shows the continuous random variable X with probability density function f given by

$$f(x) = \begin{cases} \frac{1}{128} (4ax - bx^3) & 0 \le x \le 4, \\ c & 4 \le x \le 6, \\ 0 & \text{otherwise,} \end{cases}$$

where a, b and c are constants.

The upper quartile of X is equal to 4.

(a) Show that
$$c = \frac{1}{8}$$
 and find the values of a and b. [4]

(b) Find the exact value of the median of
$$X$$
. [3]

(c) Find
$$E(\sqrt{X})$$
, giving your answer correct to 2 decimal places. [3]