Keira has two unbiased coins. She tosses both coins. The number of heads obtained by Keira is denoted by X.

(a) Find the probability generating function $G_X(t)$ of X. [1]

Hassan has three coins, two of which are biased so that the probability of obtaining a head when the coin is tossed is $\frac{1}{3}$. The corresponding probability for the third coin is $\frac{1}{4}$. The number of heads obtained by Hassan when he tosses these three coins is denoted by Y.

(b) Find the probability generating function $G_Y(t)$ of Y. [3]

The random variable Z is the total number of heads obtained by Keira and Hassan.

(c) Find the probability generating function of Z, expressing your answer as a polynomial. [3]

(d) Use the probability generating function of Z to find $E(Z)$. [2]

(e) Use the probability generating function of Z to find the most probable value of Z. [1]