The random variable X is such that $\mathrm{P}(X = r) = kr^2$ for $r = 1,2,3,4,$ where k is a constant.

(a) Find the value of k. [1]

(b) Find the probability generating function $\mathrm{G}_X(t)$ of X. [2]

The random variable Y has probability generating function $\mathrm{G}_Y(t) = \frac{1}{4} + \frac{1}{2}t + \frac{1}{4}t^2$.

The random variable Z is the sum of X and Y.

(c) Assuming that X and Y are independent, find the probability generating function $\mathrm{G}_Z(t)$ of Z as a polynomial in t. [3]

(d) Given that $\mathrm{E}(Z) = \frac{13}{3}$, use $\mathrm{G}_Z(t)$ to find $\mathrm{Var}(Z)$. [3]