



A fixed smooth solid sphere has centre  $O$  and radius  $a$ . A particle of mass  $m$  is projected downwards with speed  $\sqrt{\frac{1}{6}ag}$  from the point  $A$  on the surface of the sphere, where  $OA$  makes an angle  $\alpha$  with the upward vertical through  $O$  (see diagram). The particle moves in part of a vertical circle on the surface of the sphere. It loses contact with the sphere at the point  $B$ , where  $OB$  makes an angle  $\beta$  with the upward vertical through  $O$ .

Given that  $\cos \alpha = \frac{2}{3}$ , find the value of  $\cos \beta$ .

[5]