The random variable X is such that $P(X = r) = kr^2$ for r = 1, 2, 3, 4, where k is a constant.

(a) Find the value of
$$k$$
.

(b) Find the probability generating function $G_X(t)$ of X. [2]

The random variable Y has probability generating function $G_Y(t) = \frac{1}{4} + \frac{1}{2}t + \frac{1}{4}t^2$.

The random variable Z is the sum of X and Y.

- (c) Assuming that X and Y are independent, find the probability generating function $G_Z(t)$ of Z as a polynomial in t. [3]
- (d) Given that $E(Z) = \frac{13}{3}$, use $G_Z(t)$ to find Var(Z). [3]