The random variable X has probability generating function $G_X(t)$ given by

$$G_X(t) = k(1+3t+4t^2),$$

where k is a constant.

(a) Show that
$$E(X) = \frac{11}{8}$$
.

The random variable Y has probability generating function $G_Y(t)$ given by

$$G_Y(t) = \frac{1}{3}t^2(1+2t).$$

The random variables X and Y are independent and Z = X + Y.

- (b) Find the probability generating function of Z, expressing your answer as a polynomial in t. [2]
- (c) Use your answer to part (b) to find the value of Var(Z). [3]
- (d) Write down the most probable value of Z.