The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} \frac{1}{6}\left(x^{-\frac{1}{3}} - x^{-\frac{2}{3}}\right) & 1 \leqslant x \leqslant 27, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the cumulative distribution function of X. [3]

The random variable Y is defined by $Y = X^{\frac{1}{3}}$.

(b) Find the probability density function of Y. [3]

(c) Find the exact value of the median of Y. [2]