

Jason has three biased coins. For each coin the probability of obtaining a head when it is thrown is $\frac{2}{3}$. Jason throws all three coins. The number of heads obtained is denoted by X .

(a) Find the probability generating function $G_X(t)$ of X . [3]

Jason also has two unbiased coins. He throws all five coins. The number of heads obtained from the two unbiased coins is denoted by Y . It is given that $G_Y(t) = \frac{1}{4} + \frac{1}{2}t + \frac{1}{4}t^2$. The random variable Z is the total number of heads obtained when Jason throws all five coins.

(b) Find the probability generating function of Z , expressing your answer as a polynomial. [3]

(c) Find $E(Z)$. [2]