The random variable X has probability generating function $\mathrm{G}_X(t)$ given by

$$\mathrm{G}_X(t) = \tfrac{1}{5} + pt + qt^2,$$

where p and q are constants.

(a) Given that $\mathrm{E}(X) = 1.1$, find the numerical value of $\mathrm{Var}(X)$. [4]

The random variable Y has probability generating function $\mathrm{G}_Y(t)$ given by

$$\mathrm{G}_Y(t) = \tfrac{2}{3}t\left(1 + \tfrac{1}{2}t^2\right).$$

The random variable Z is the sum of independent observations of X and Y.

(b) Find the probability generating function of Z. [2]

(c) Find $\mathrm{P}(Z > 2)$. [1]

(d) State the most probable value of Z. [1]