The random variable X has probability generating function $G_X(t)$ given by

$$G_X(t) = k(1 + 3t + 4t^2),$$

where k is a constant.

(a) Show that $E(X) = \frac{11}{8}$. [3]

The random variable Y has probability generating function $G_Y(t)$ given by

$$G_Y(t) = \tfrac{1}{3}t^2(1 + 2t).$$

The random variables X and Y are independent and $Z = X + Y$.

(b) Find the probability generating function of Z, expressing your answer as a polynomial in t. [2]

(c) Use your answer to part **(b)** to find the value of $\mathrm{Var}(Z)$. [3]

(d) Write down the most probable value of Z. [1]