

Toby has a bag which contains 6 red marbles and 3 green marbles. He randomly chooses 3 marbles from the bag, without replacement. The random variable  $X$  is the number of red marbles that Toby obtains.

**(a)** Find the probability generating function of  $X$ . [3]

Ling also has a bag which contains 6 red marbles and 3 green marbles. He randomly chooses 2 marbles from his bag, without replacement. The random variable  $Y$  is the number of red marbles that Ling obtains. It is given that the probability generating function of  $Y$  is  $\frac{1}{8} + \frac{3}{4}t + \frac{1}{8}t^2$ .

The random variable  $Z$  is the total number of red marbles that Toby and Ling obtain.

**(b)** Find the probability generating function of  $Z$ , expressing your answer as a polynomial in  $t$ . [3]

**(c)** Use the probability generating function of  $Z$  to find  $\text{Var}(Z)$ . [4]