The random variable X has the binomial distribution B(n, p).

(a) Write down an expression for $\mathrm{P}(X = r)$ and hence show that the probability generating function of X is $(q + pt)^n$, where $q = 1 - p$. [3]

(b) Use the probability generating function of X to prove that $\mathrm{E}(X) = np$ and $\mathrm{Var}(X) = np(1 - p)$. [5]