7 A smooth sphere with centre O and of radius a is fixed to a horizontal plane. A particle P of mass m is projected horizontally from the highest point of the sphere with speed u, so that it begins to move along the surface of the sphere. The particle P loses contact with the sphere at the point Q on the sphere, where OQ makes an angle θ with the upward vertical through O.

(a) Show that $\cos\theta = \dfrac{u^2 + 2ag}{3ag}$. [4]

It is given that $\cos\theta = \frac{5}{6}$.

(b) Find, in terms of a and g, an expression for the vertical component of the velocity of P just before it hits the horizontal plane to which the sphere is fixed. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Find an expression for the time taken by P to fall from Q to the plane. Give your answer in the form $k\sqrt{\frac{a}{g}}$, stating the value of k correct to 3 significant figures. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....