

A machine is used to produce metal rods. When the machine is working efficiently, the lengths, x cm, of the rods have a normal distribution with mean 150 cm and standard deviation 1.2 cm. The machine is checked regularly by taking random samples of 200 rods. The latest results are shown in the following table.

Interval	$146 \leq x < 147$	$147 \leq x < 148$	$148 \leq x < 149$	$149 \leq x < 150$
Observed frequency	1	2	23	52

$150 \leq x < 151$	$151 \leq x < 152$	$152 \leq x < 153$	$153 \leq x < 154$
69	36	15	2

As a first check, the sample is used to calculate an estimate for the mean.

(i) Show that an estimate for the mean from this sample is close to 150 cm. [2]

As a second check, the results are tested for goodness of fit of the normal distribution with mean 150 cm and standard deviation 1.2 cm. The relevant expected frequencies, found using the normal distribution function given in the List of Formulae (MF10), are shown in the following table.

Interval	$x < 147$	$147 \leq x < 148$	$148 \leq x < 149$	$149 \leq x < 150$
Observed frequency	1	2	23	52
Expected frequency	1.24	8.32	30.94	59.50

$150 \leq x < 151$	$151 \leq x < 152$	$152 \leq x < 153$	$153 \leq x$
69	36	15	2
59.50	30.94	8.32	1.24

(ii) Show how the expected frequency for $151 \leq x < 152$ is obtained. [3]

(iii) Test, at the 5% significance level, the goodness of fit of the normal distribution to the results. [7]