

A random sample of 200 observations of the continuous random variable  $X$  was taken and the values are summarised in the following table.

Interval	$0 \leq x < 0.5$	$0.5 \leq x < 1$	$1 \leq x < 1.5$	$1.5 \leq x < 2$	$2 \leq x < 2.5$	$2.5 \leq x < 3$
Observed frequency	5	23	40	41	46	45

It is required to test the goodness of fit of the distribution with probability density function  $f$  given by

$$f(x) = \begin{cases} \frac{1}{9}x(4-x) & 0 \leq x \leq 3, \\ 0 & \text{otherwise.} \end{cases}$$

Most of the relevant expected frequencies, correct to 2 decimal places, are given in the following table.

Interval	$0 \leq x < 0.5$	$0.5 \leq x < 1$	$1 \leq x < 1.5$	$1.5 \leq x < 2$	$2 \leq x < 2.5$	$2.5 \leq x < 3$
Expected frequency	$p$	$q$	37.96	43.52	43.52	37.96

**(a)** Show that  $p = 10.19$  and find the value of  $q$ . [3]

**(b)** Carry out a goodness of fit test, at the 5% significance level, to test whether  $f$  is a satisfactory model for the data. [4]