A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O and P is held with the string taut and horizontal. The particle P is projected vertically downwards with speed $\sqrt{(2ag)}$ so that it begins to move along a circular path. The string becomes slack when OP makes an angle θ with the upward vertical through O.

(i) Show that
$$\cos \theta = \frac{2}{3}$$
. [5]

(ii) Find the greatest height, above the horizontal through O, reached by P in its subsequent motion. [4]