



A uniform solid circular cone, of vertical height  $4r$  and radius  $2r$ , is attached to a uniform solid cylinder, of height  $3r$  and radius  $kr$ , where  $k$  is a constant less than 2. The base of the cone is joined to one of the circular faces of the cylinder so that the axes of symmetry of the two solids coincide (see diagram). The cone and the cylinder are made of the same material.

- (a) Show that the distance of the centre of mass of the combined solid from the vertex of the cone is  $\frac{(99k^2 + 96)r}{18k^2 + 32}$ . [4]

The point  $C$  is on the circumference of the base of the cone. When the combined solid is freely suspended from  $C$  and hanging in equilibrium, the diameter through  $C$  makes an angle  $\alpha$  with the downward vertical, where  $\tan \alpha = \frac{1}{8}$ .

- (b) Given that the centre of mass of the combined solid is within the cylinder, find the value of  $k$ . [4]