A particle P of mass 0.4 kg is projected horizontally along a smooth horizontal plane from a point O. After projection the velocity of P is $v\,\mathrm{m\,s^{-1}}$ and its displacement from O is x m. A force of magnitude $8x$ N directed away from O acts on P and a force of magnitude $(2\mathrm{e}^{-x} + 4)$ N opposes the motion of P. One end of a light elastic string of natural length 0.5 m is attached to O and the other end of the string is attached to P.

(i) Show that $v\dfrac{\mathrm{d}v}{\mathrm{d}x} = 20x - 10 - 5\mathrm{e}^{-x}$ before the elastic string becomes taut. [2]

(ii) Given that the initial velocity of P is $6\,\mathrm{m\,s^{-1}}$, find v when the string first becomes taut. [3]

When the string is taut, the acceleration of P is proportional to e^{-x}.

(iii) Find the modulus of elasticity of the string. [2]