A particle $P$ of mass $m$ is attached to one end of a light elastic string of natural length $a$ and modulus of elasticity $\frac{4}{3}mg$ . The other end of the string is attached to a fixed point $O$ on a rough horizontal surface. The particle is at rest on the surface with the string at its natural length. The coefficient of friction between $P$ and the surface is $\frac{1}{3}$ . The particle is projected along the surface in the direction $OP$ with a speed of $\frac{1}{2}\sqrt{ga}$ .
Find the greatest extension of the string during the subsequent motion. [5]