

A 6-sided dice, A , with faces numbered 1, 2, 3, 4, 5, 6 is biased so that the probability of throwing a 6 is $\frac{1}{4}$. The random variable X is the number of 6s obtained when dice A is thrown twice.

(a) Find the probability generating function of X . [2]

A second dice, B , with faces numbered 1, 2, 3, 4, 5, 6 is unbiased. The random variable Y is the number of 6s obtained when dice B is thrown twice.

The random variable Z is the total number of 6s obtained when both dice are thrown twice.

(b) Find the probability generating function of Z , expressing your answer as a polynomial. [3]

(c) Find $\text{Var}(Z)$. [3]

(d) Use the probability generating function of Z to find the most probable value of Z . [1]