

A random sample of 50 values of the continuous random variable  $X$  was taken. These values are summarised in the following table.

Interval	$1 \leq x < 1.5$	$1.5 \leq x < 2$	$2 \leq x < 2.5$	$2.5 \leq x < 3$	$3 \leq x < 3.5$	$3.5 \leq x \leq 4$
Observed frequency	3	3	8	11	13	12

It is required to test the goodness of fit of the distribution with probability density function  $f$  given by

$$f(x) = \begin{cases} \frac{1}{24} \left( \frac{4}{x^2} + x^2 \right) & 1 \leq x \leq 4, \\ 0 & \text{otherwise.} \end{cases}$$

The expected frequencies, correct to 4 decimal places, are given in the following table.

Interval	$1 \leq x < 1.5$	$1.5 \leq x < 2$	$2 \leq x < 2.5$	$2.5 \leq x < 3$	$3 \leq x < 3.5$	$3.5 \leq x \leq 4$
Expected frequency	4.4271	$a$	6.1285	8.4549	$b$	14.9678

**(a)** Show that  $a = 4.6007$  and find the value of  $b$ . [3]

**(b)** Carry out a goodness of fit test, at the 10% significance level, to test whether  $f$  is a satisfactory model for the data. [6]