The variables x and θ satisfy the differential equation

$$x\cos^2\theta\frac{\mathrm{d}x}{\mathrm{d}\theta} = 2\tan\theta + 1,$$

for $0 \leqslant \theta < \frac{1}{2}\pi$ and $x > 0$. It is given that $x = 1$ when $\theta = \frac{1}{4}\pi$.

(i) Show that $\dfrac{\mathrm{d}}{\mathrm{d}\theta}(\tan^2\theta) = \dfrac{2\tan\theta}{\cos^2\theta}$. [1]

(ii) Solve the differential equation and calculate the value of x when $\theta = \frac{1}{3}\pi$, giving your answer correct to 3 significant figures. [7]