Keira has two unbiased coins. She tosses both coins. The number of heads obtained by Keira is denoted by *X*.

(a) Find the probability generating function  $G_X(t)$  of X. [1]

Hassan has three coins, two of which are biased so that the probability of obtaining a head when the coin is tossed is  $\frac{1}{3}$ . The corresponding probability for the third coin is  $\frac{1}{4}$ . The number of heads obtained by Hassan when he tosses these three coins is denoted by Y.

(b) Find the probability generating function  $G_Y(t)$  of Y. [3]

The random variable Z is the total number of heads obtained by Keira and Hassan.

- (c) Find the probability generating function of Z, expressing your answer as a polynomial. [3]
- (d) Use the probability generating function of Z to find E(Z).
- (e) Use the probability generating function of Z to find the most probable value of Z. [1]