A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle P is held with the string taut and the string makes an angle θ with the downward vertical through O. The particle P is projected at right angles to the string with speed $\frac{1}{3}\sqrt{10ag}$ and begins to move downwards along a circular path. When the string is vertical, it strikes a small smooth peg at the point A which is vertically below O. The circular path and the point A are in the same vertical plane. After the string strikes the peg, the particle P begins to move in a vertical circle with centre A. When the string makes an angle θ with the upward vertical through A the string becomes slack (see diagram). The distance of A below O is $\frac{5}{9}a$.

(a) Find the value of $\cos\theta$. [6]

(b) Find the ratio of the tensions in the string immediately before and immediately after it strikes the peg. [4]