The random variable X has the geometric distribution Geo(p).

(a) Show that the probability generating function of X is $\frac{pt}{1-qt}$, where $q = 1-p$. [3]

(b) Use the probability generating function of X to show that $\mathrm{Var}(X) = \frac{q}{p^2}$. [5]

Kenny throws an ordinary fair 6-sided dice repeatedly. The random variable X is the number of throws that Kenny takes in order to obtain a 6. The random variable Z denotes the sum of two independent values of X.

(c) Find the probability generating function of Z. [2]