A bag contains 4 red balls and 6 blue balls. Rassa selects two balls at random, without replacement, from the bag. The number of red balls selected by Rassa is denoted by X.

(a) Find the probability generating function, $G_X(t)$, of X. [2]

Rassa also tosses two coins. One coin is biased so that the probability of a head is $\frac{2}{3}$. The other coin is biased so that the probability of a head is p. The probability generating function of Y, the number of heads obtained by Rassa, is $G_Y(t)$. The coefficient of t in $G_Y(t)$ is $\frac{7}{12}$.

(b) Find $G_Y(t)$. [3]

The random variable Z is the sum of the number of red balls selected and the number of heads obtained by Rassa.

(c) Find the probability generating function of Z, expressing your answer as a polynomial. [3]

(d) Use the probability generating function of Z to find $E(Z)$. [2]