A particle P of mass m is moving in a horizontal circle with angular speed ω on the smooth inner surface of a hemispherical shell of radius r. The angle between the vertical and the normal reaction of the surface on P is θ.

(a) Show that $\cos\theta = \dfrac{g}{\omega^2 r}$. [3]

The plane of the circular motion is at a height x above the lowest point of the shell. When the angular speed is doubled, the plane of the motion is at a height $4x$ above the lowest point of the shell.

(b) Find x in terms of r. [4]