

A particle P of mass m is free to move on the smooth inner surface of a fixed hollow sphere of radius a. The centre of the sphere is O. The points A and A' are on the inner surface of the sphere, on opposite sides of the vertical through O; the radius OA makes an angle α with the downward vertical and the radius OA' makes an angle β with the upward vertical. The point B is on the inner surface of the sphere, vertically below O. The point B' is on the inner surface of the sphere and such that OB' makes an angle 2β with the upward vertical through O (see diagram). It is given that $\cos \alpha = \frac{1}{16}$.

- (i) P is projected from A with speed u along the surface of the sphere downwards towards B. Subsequently it loses contact with the sphere at A'. Show that $u^2 = \frac{1}{8}ag \ 1 + 24\cos\beta$. [5]
- (ii) P is now projected from B with speed u along the surface of the sphere towards B'. Subsequently it loses contact with the sphere at B'. Find $\cos \beta$. [6]
- (iii) In part (i), the reaction of the sphere on *P* when it is initially projected at *A* is *R*. Find *R* in terms of *m* and *g*.