Lecture 8.2: Taylor’s formula

Optimization and Computational Linear Algebra for Data Science

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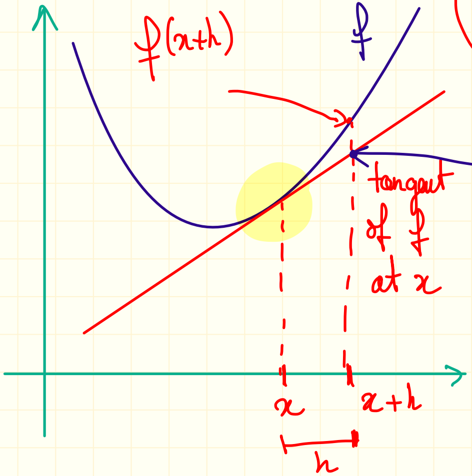
Functions of one variable

$$\begin{array}{cccc} f: & \mathbb{R} & \to & \mathbb{R} \\ & x & \mapsto & f(x) \end{array}$$

$$\boxed{f(x+h) \simeq f(x) + f'(x) \cdot h.}$$

for "h small"

Taylor's formula of order 1.



$f(x) + f'(x)\, h.$

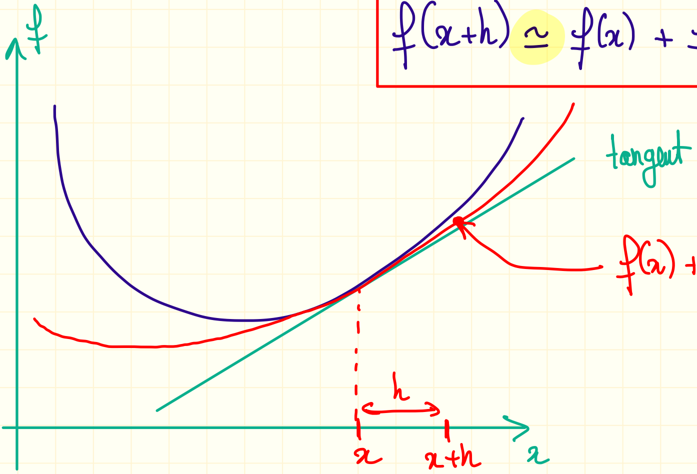
Functions of one variable

$$
\begin{array}{cccc}
f: & \mathbb{R} & \to & \mathbb{R} \\
 & x & \mapsto & f(x)
\end{array}
$$

Taylor's formula of order 2:

$$\boxed{f(x+h) \simeq f(x) + f'(x)\cdot h + \frac{1}{2} f''(x)\, h^2}$$

"for small h"



$$f(x) + f'(x)h + \frac{1}{2} f''(x) h^2$$

Functions of n variables

$$\begin{array}{rccl} f: & \mathbb{R}^n & \to & \mathbb{R} \\ & x & \mapsto & f(x) = f(x_1, \dots, x_n) \end{array}$$

- Taylors' formula of order 1: for all $x \in \mathbb{R}^n$

$$f(x+h) \simeq f(x) + \langle \nabla f(x), h \rangle$$

for $h \in \mathbb{R}^n$ "small"

- Taylor's formula of order 2: for all $x \in \mathbb{R}^n$

$$f(x+h) \simeq f(x) + \langle \nabla f(x), h \rangle + \frac{1}{2} \underbrace{h^T H_f(x) h}$$

for $h \in \mathbb{R}^n$ "small"