

Lecture 2.3: Matrix product

Optimization and Computational Linear Algebra for Data Science

Matrix-vector product

Consider a linear map $L : \mathbb{R}^m \rightarrow \mathbb{R}^n$ and its associated matrix $\tilde{L} \in \mathbb{R}^{n \times m}$.

Question: Can we use the matrix \tilde{L} to compute the image $L(x)$ of a vector $x \in \mathbb{R}^m$?

Proposition

For all $x \in \mathbb{R}^m$ we have

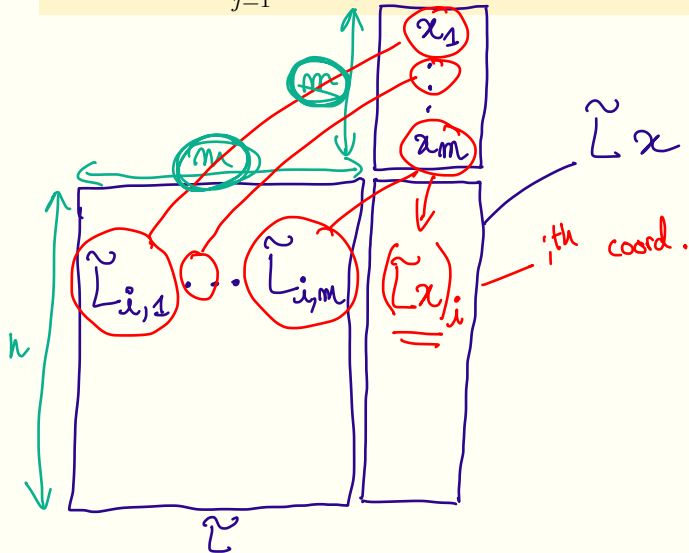
$$L(x) = \tilde{L}x$$

where the “matrix-vector” product $\tilde{L}x \in \mathbb{R}^n$ is defined by

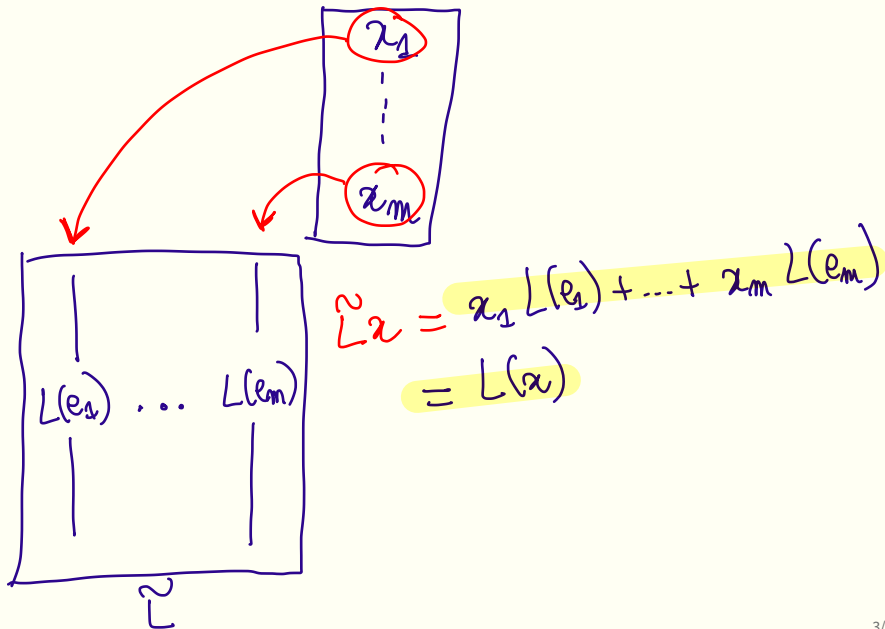
$$(\tilde{L}x)_i = \sum_{j=1}^m \tilde{L}_{i,j} x_j \quad \text{for all } i \in \{1, \dots, n\}.$$

Visualizing the formula

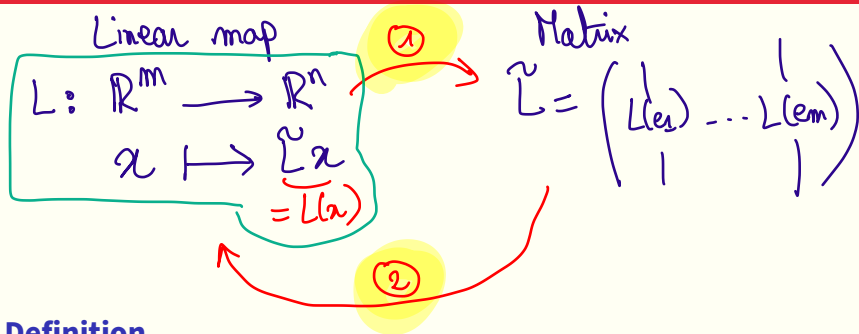
$$(\tilde{L}x)_i = \sum_{j=1}^m \tilde{L}_{i,j} x_j = \tilde{L}_{i,1} x_1 + \tilde{L}_{i,2} x_2 + \cdots + \tilde{L}_{i,m} x_m$$



Why do we have $L(x) = \tilde{L}x$?



Linear map associated to a matrix



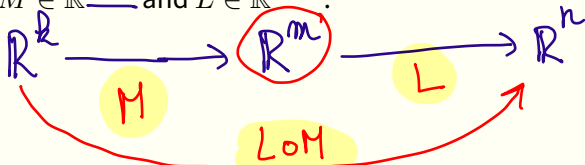
Definition

The linear map associated to a matrix $\tilde{L} \in \mathbb{R}^{n \times m}$ is the map

$$\boxed{\begin{array}{ccc} L: & \mathbb{R}^m & \rightarrow \mathbb{R}^n \\ & x & \mapsto \tilde{L}x. \end{array}}$$

Matrix product

Let $M \in \mathbb{R}^{m \times k}$ and $L \in \mathbb{R}^{n \times m}$.



Definition - Proposition

- ❖ The matrix product LM is the $n \times k$ matrix of the linear map $L \circ M$.
- ❖ Its coefficients are given by the formula:

$$(LM)_{i,j} = \sum_{\ell=1}^m L_{i,\ell} M_{\ell,j} \quad \text{for all } 1 \leq i \leq n, \quad 1 \leq j \leq k.$$

Visualizing the formula

$$(LM)_{i,j} = \sum_{\ell=1}^m L_{i,\ell} M_{\ell,j} = L_{i,1} M_{1,j} + \cdots + L_{i,m} M_{m,j}$$

