2		
Problem	10	A
MIDOROIL	40.	1.

a) Let $A = U \Sigma V^T$ be the SUD of A. We write r = naule A and n = naule A.

Claim: $Ker(A) = Span(15_{r+1}, ..., 15_m)$. Indeed, for $n = i \in 2_{r+1}, ..., my$, $\Sigma V = \Sigma = 0$ because V is orthogonal and only the r first diagonal elements of Σ are non-zero.

This proves that for all i) 1+1, v; E KeulA). Since KeulA) is a subspace, we get to Span(vr+1,...,vm) C KeulA).

Now, by the rank-nullity theorem:

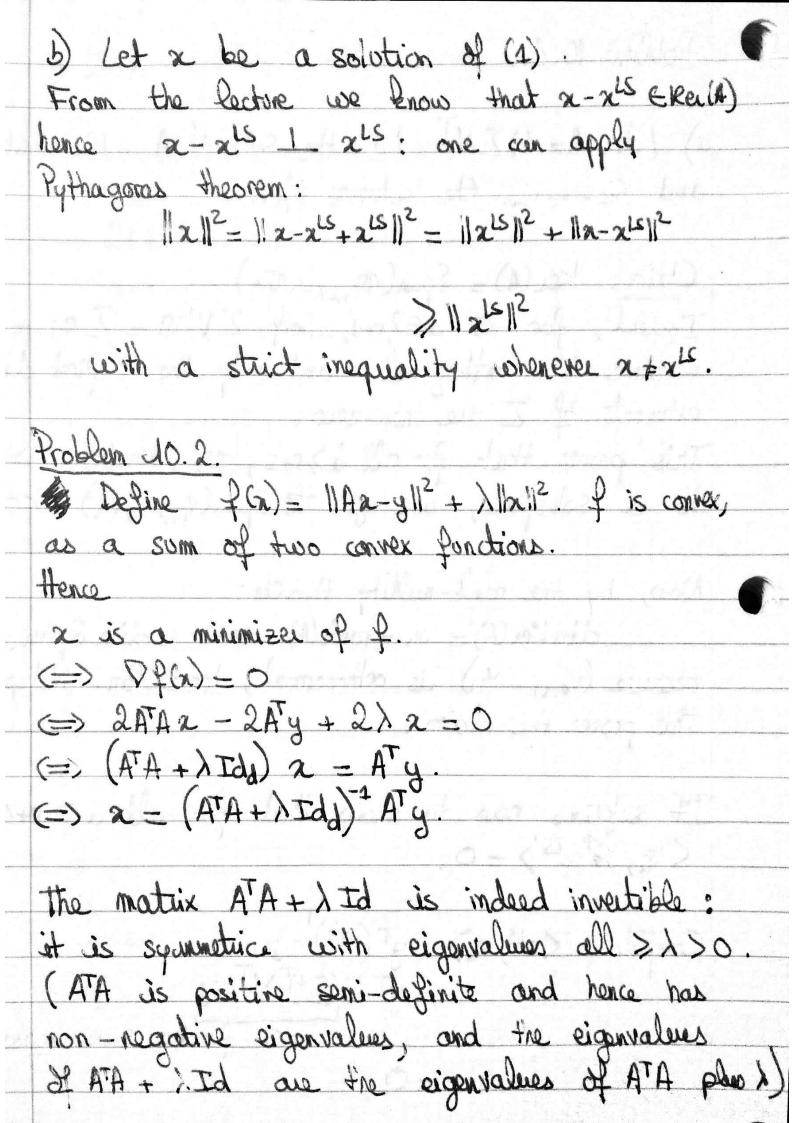
dim Ker (4) = m - rank (4) = m - r = dim Span (vm - vm) because (vm, -vm) is orthonormal, hence lin. independent. This proves the claim.

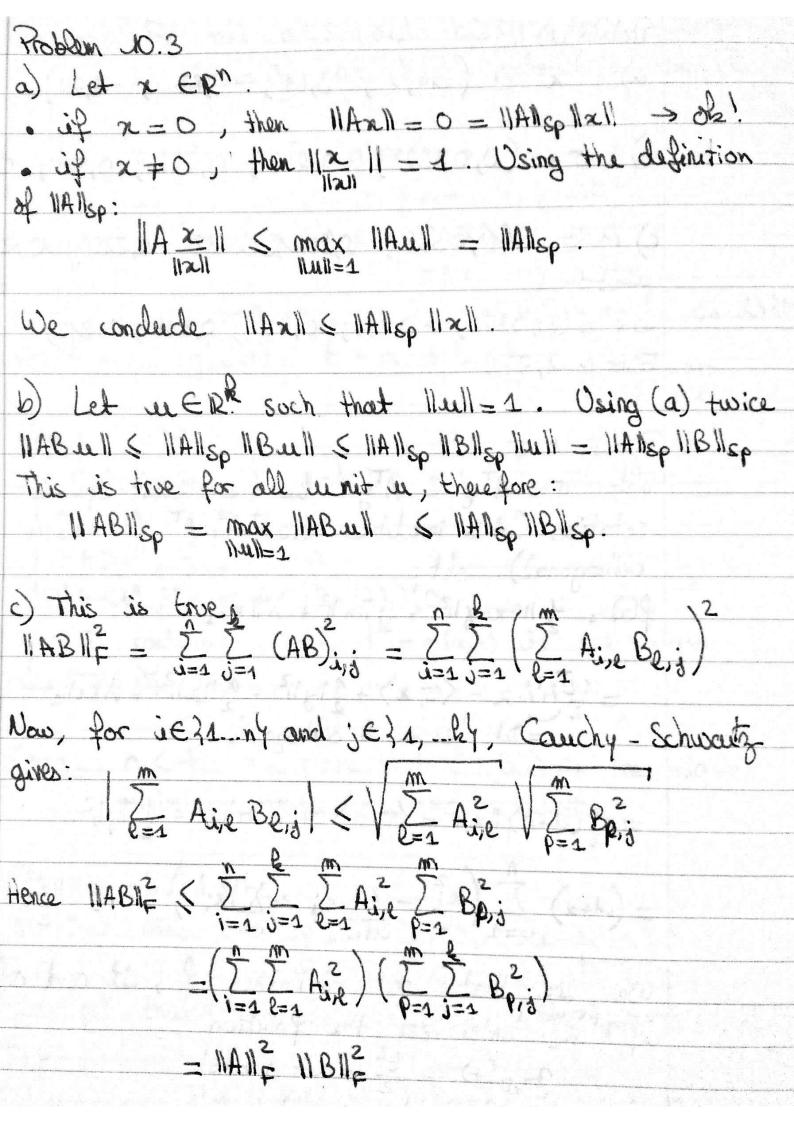
It suffices now to show that for all i > 1+1, (oi, \$\frac{1}{2}(\sigma) = 0.

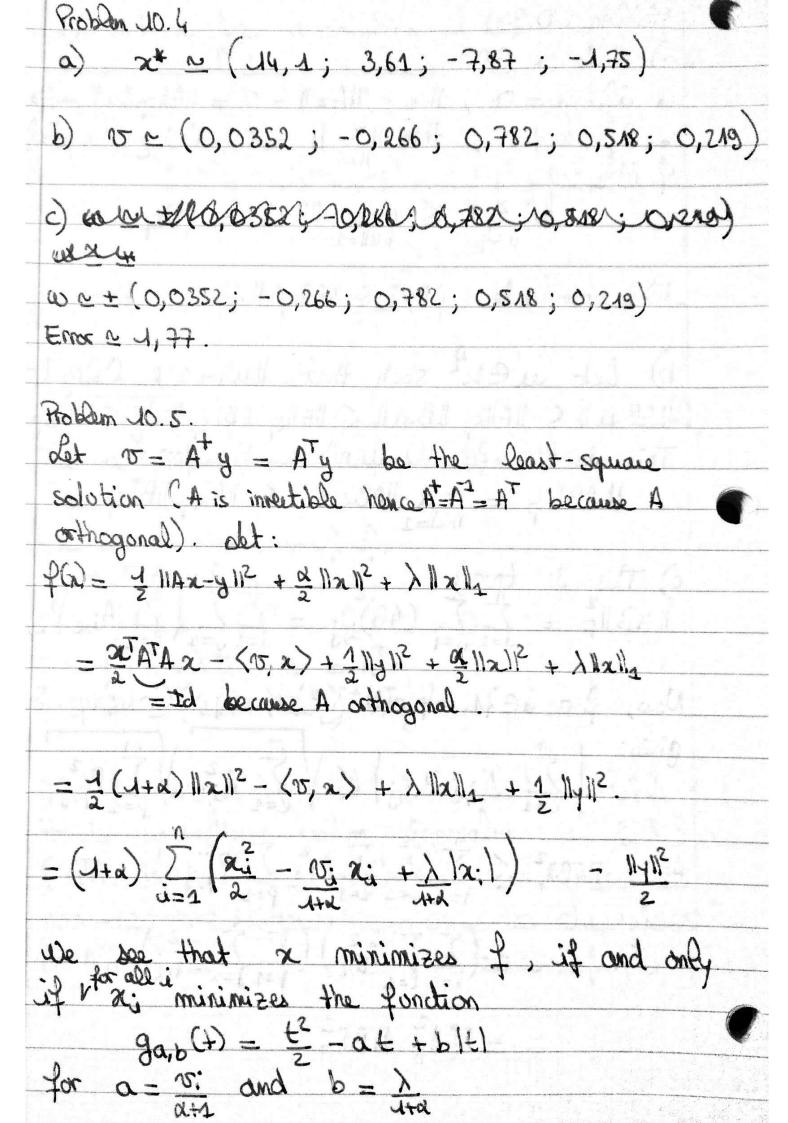
Compote (215, vi) = yT (A+) vi = yT U(I+) VT vi

= 0 for the some reason as above

=0.







Lenuma: For all a ER and b>0 the function
$g_{a,b}(t) = \frac{t^2}{2} - at + b t $ admits a unique
minimizer on R, given by:
$t^* = m(a,b) \stackrel{\text{def}}{=} \begin{cases} a-b & \text{if } a \ge b \\ 0 & \text{if } -b \le a \le b \end{cases}$ $(a+b) \stackrel{\text{def}}{=} a \le -b.$
that a is continuous on it and differentiale on 18/100
For $t \neq 0$, $g_{a,b}(t) = t - a + b \text{ sign}(t)$ $+1 \text{ if } t \neq 0$.
Case 1: $a>b$. In that case $t^* = a-b>0$ and we see that $ g'a,b(t^*)=0 $ $ f'a,b(t^*)=0 $
$\forall t > t^*$, $g_{a,b}(t) > 0$ $\forall t < t^*$, $s,t \neq 0 \neq g_{a,b}(t) \neq 0$. $g_{a,b}$ is continous, hence $t^* = \eta(a,b)$ is the unique minimizer of $g_{a,b}$.
Case 2: a <-b: We prove that t=n(a,b) is the unique minimizer enough the same arguments.
Case3: -bsasb.
In that case, (garb(+) > 0 for all +>0
In that case, $\begin{cases} g'a_{1b}(t) > 0 & \text{for all } t > 0 \\ g'a_{1b}(t) < 0 & \text{for all } t < 0 \end{cases}$ We get that $t^* = \eta(a_{1b})$ is the unique minimizer of
Ja,b. This makes the Romma

Ising the lemma we get that the enique minimizer of f is given by: