2		
Problem	10	A
MIDOROIL	40.	1.

a) Let $A = U \Sigma V^T$ be the SUD of A. We write r = naule A and n = naule A.

Claim: $Ker(A) = Span(15_{r+1}, ..., 15_m)$. Indeed, for $n = i \in 2_{r+1}, ..., my$, $\Sigma V = \Sigma = 0$ because V is orthogonal and only the r first diagonal elements of Σ are non-zero.

This proves that for all i) 1+1, v; E KeulA). Since KeulA) is a subspace, we get to Span(vr+1,...,vm) C KeulA).

Now, by the rank-nullity theorem:

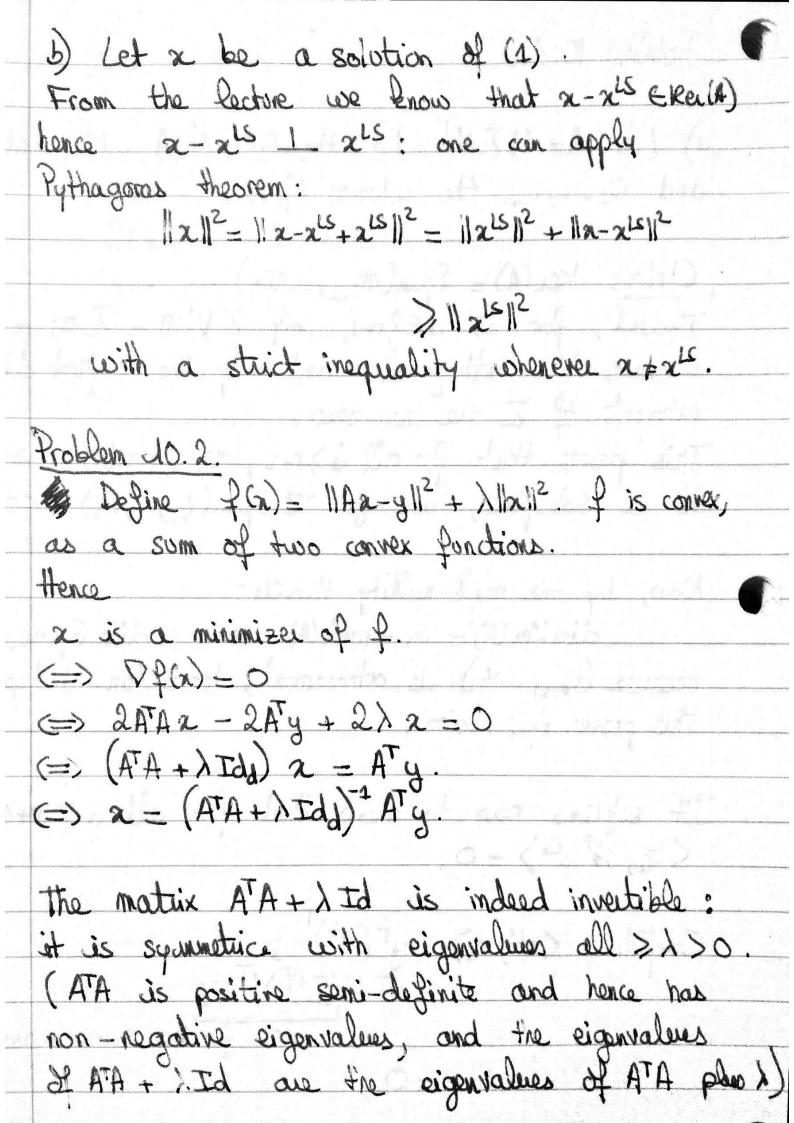
dim Ker (4) = m - rank (4) = m - r = dim Span (vm - vm) because (vm, -vm) is orthonormal, hence lin. independent. This proves the claim.

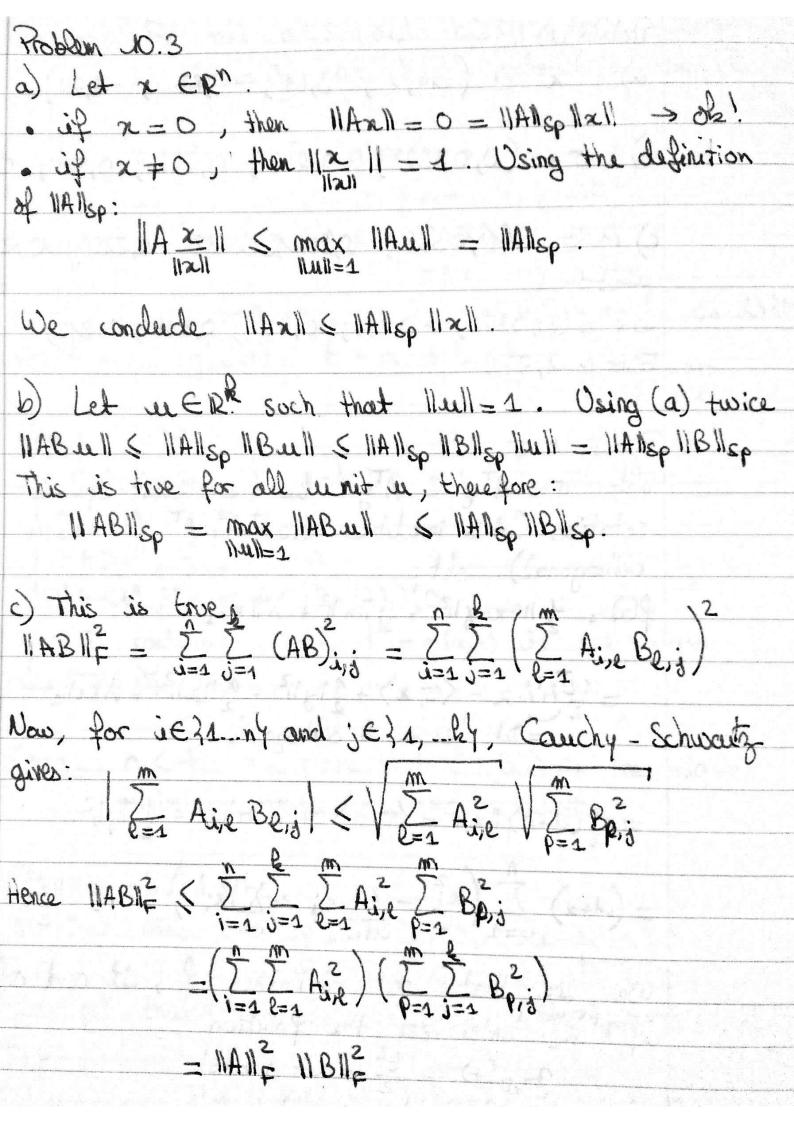
It suffices now to show that for all i > 1+1, (oi, \$\frac{1}{2}(\sigma) = 0.

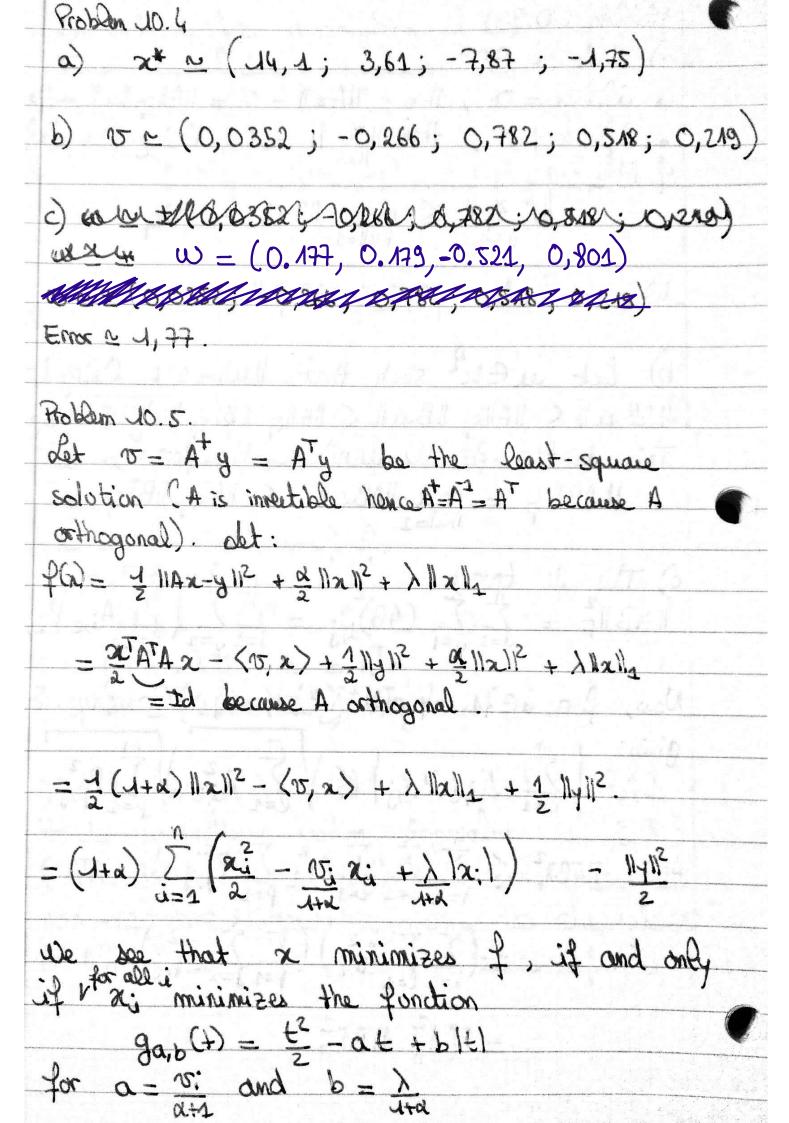
Compote (215, vi) = yT (A+) vi = yT U(I+) VT vi

= 0 for the some reason as above

=0.







Lemma: for all $a \in \mathbb{R}$ and $b \geqslant 0$ the function $g_{a,b}(t) = \frac{t^2}{2} - at + b t $ admits a unique
minimizer on R, given by:
$t^* = m(a,b) \stackrel{\text{def}}{=} \begin{cases} a-b & \text{if } a > b \\ 0 & \text{if } -b \leq a \leq b \end{cases}$ $(a+b) \stackrel{\text{def}}{=} a \leq -b.$
$t=m(a,b)=0$ if $-b \leq a \leq b$
(a+b if a \le -b.
that a is continuous on it and distributed and KIADY
For $t \neq 0$, $g_{a,b}(t) = t - a + b \text{ sign}(t)$ The sign $t \neq 0$ if $t \neq 0$.
(ave 1: a)b.
In that case t= a-b>0 and we see that
$ g_{a,b}(t^*) = 0$ $ \forall t > t^*, g_{a,b}(t) > 0$ $\forall t < (t^*, s, t \neq 0 \neq g_{a,b}(t) \times 0$
H + < + st + + 0 ≠ gab(+) X0.
ga, b is continous, hence $t^* = n(a, b)$ is the unique minimizer of ga, b.
Casel: a <-b: We more that t=n(a,b) is the unique
Casel: a <-b: We prove that t=n(a,b) is the unique minimizer enough the same arguments.
Case 3: -b < a < b. In that case : $\begin{cases} g'a_{1}b(t) > 0 & \text{for all } t > 0 \\ g'a_{1}b(t) < 0 & \text{for all } t < 0 \end{cases}$ We get that $t^{+} = m(a_{1}b)$ is the unique minimizer of
In that case, { ga, 6(+) > 0 for all 6>0
g'a, b(t) <0 for all t(0
We get that 6t = m(a, b) is the unique minimizer of
a,b
This moves the Romma

Ising the lemma we get that the enique minimizer of f is given by: