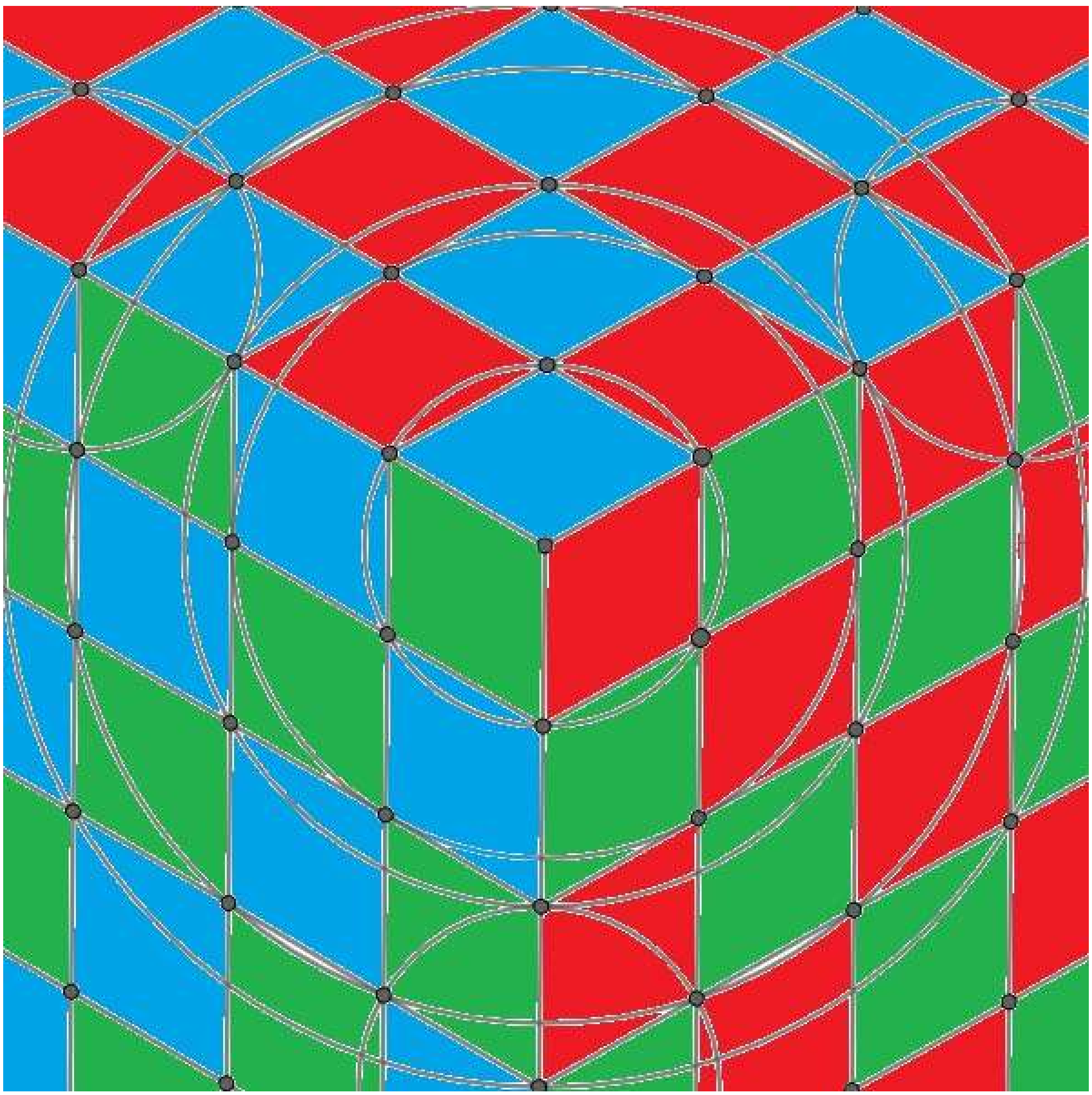
A New Model for Any-Dimensional Space

Let’s start with a therapeutic image. We have lots of biases, some ancient, some modern. While you look at it, please repeat with me:

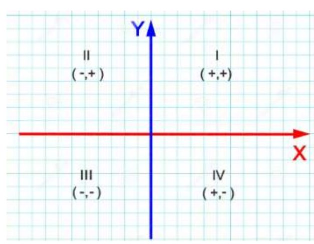
This is not a cube. It is not a corner. It’s The Optimist Plane. A plane with no negative coordinates. This is not a cube. It is not a corner. It is an optimist plane. A plane with no negative coordinates. This is not a cube….”.



We woke up to a world that seems to be a planar surface. We stand up as trees, orthogonal to this plane, projecting to the sky. We are bilateral beings; we have been for at least 500 million years. We have both an orthogonal and bilateral/bipolar bias. We ran with the notion that negative numbers are as real as the positive numbers, I disagree.

In modern times, these biases, led to quadrature and cubism biases. We build rectangles all around. We went with the idea that 1 unit of area is the area of a square of side 1, and that 1 unit of volume is a cube of side 1. I will argue these are just conventions, nothing fundamental about these. We can have area and volume defined differently and everything would be fine, trust me on that for now. Around 400 years ago we gravitate to use the Cartesian coordinate system for most things in math, one of the most versatile systems found to date. Polar coordinates in a plane are cool, but Spherical coordinates have awful symmetry. You need two angles; one is the azimuthal angle, measured against the orthogonal projection of a plane. Ha! Orthogonality creeps up again.

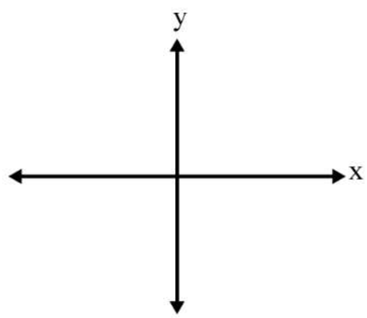
A further critic to the Cartesian coordinate system

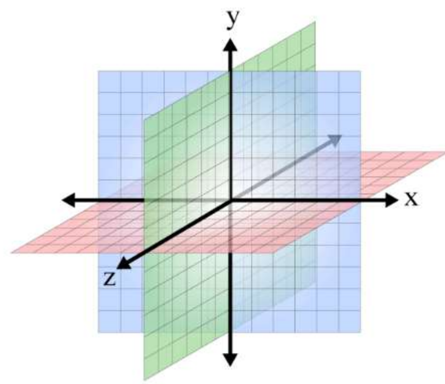


In the Cartesian plane we have four regions, or quadrants. But their relationships/interfaces are not that symmetrical. A given region has:

- Oneself: the given region.
- Two neighboring regions: regions joined by a ray, a linear interface, not just a point.
- One opposite region: intersecting the given region only at the origin. This opposite region is a neighbor of a neighbor.

We have 2 regions in 1D space ("one dimension"). On the "real" numbers line, you have the region of the positives and the region of the negatives. And we just went from 2 to 4 to get to a plane or 2D space ("two dimensions"). Smells suboptimal.



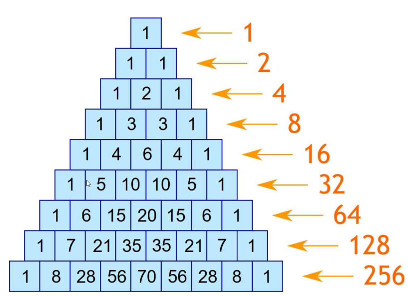


Now, take a look at 3D space ("three dimensions") above. It has 8 regions:

- Oneself: the given region.
- Three regions that are neighbors.
- Three regions that are neighbors of a neighbor.
- One opposite region: Neighbor of a neighbor of a neighbor.

Then 2, 4, 8, … Will we keep doubling regions as we go into more dimensions? This is exponential, with complexity increasing linearly.

This Cartesian regional pattern follows the Triangle of Pascal:



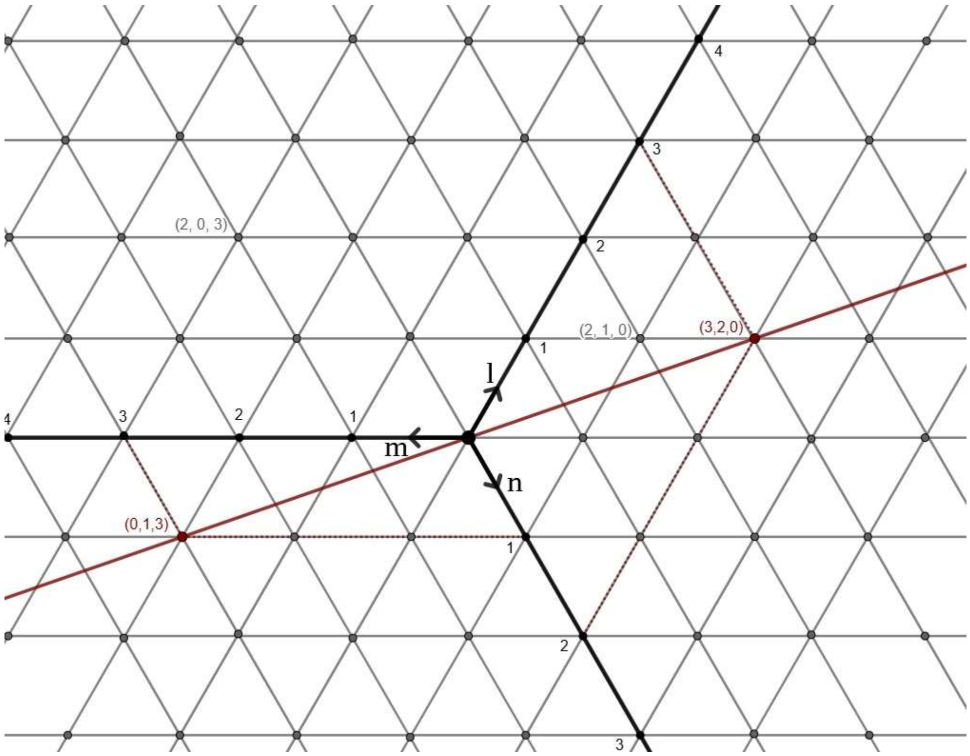
In 4D, the 16 regions would follow the neighboring chain 1, 4, 6, 4, 1. This pattern starts with the given region and ends with the opposite region: a neighbor of a neighbor of a neighbor of a neighbor of a neighbor.

The Cartesian system displays what we can call a component-combinatorial space-coverage gap. XY covers a fourth of the *plane-space*, since we use angles of 90°. If you were to use 120° you'd cover a third! Can you cover half? No. [Can you cover almost half: yes! Play with your parallelograms. Take the limit of the largest angle of a parallelogram approaching 180°. The diamond-parallelogram (two isosceles sharing a) is your bisector, orthogonal to the line that's missing coverage. Ok… so you can almost cover the plane with two magnitudes, just missing a line, the line tangential to this angle that approaches 180°. Oh man! If I could just cover that gap… Well you can! All you need is an ∞ symbol notion. (3, ∞) is the point at (3,0) in the closed coordinate systems (Cartesian or Trangular). The origin is the destination!]

¡We want something less complex! Can we have it all?

- A coordinate system without negative numbers. and I don't like complex numbers either.
- The number of regions increases one by one as I go from line to plane to 3D-space, 4D, and so on. And that every region is neighbor to all others.
- Multiplying 2 components gets me area, multiplying 3 components gets me volume, and so on.
- I can do with it anything I can do with the Cartesian system, and more!
- Computational efficiency equal or better. (fingers crossed – so far so good)

Let me share a sneak peek into the Optimist Plane, to start flexing prior beliefs. And a bit later I'll start from scratch: the point, <skip the segment/espheral 😉 >, the ray, the line, to get to the plane.



For coordinates (l, n, m), one of the coordinates is always zero on the standard form.

To normalize a point where all 3 coordinates are greater than zero ($massaction$ ל is a made-up term *mass action time energy ל? God knows we only have hypothesis*):

$$(l, n, m) \text{ where } l > 0, n > 0, m > 0, \text{calculate the } massaction \text{ ל} = \min(l, n, m) \\ \rightarrow (l - \text{ל}, n - \text{ל}, m - \text{ל})$$

I chose letters l, n, m as I want a mnemonic for the geometry.

First, a line will have just two vectors in opposite directions, two magnitudes, l and n. Get it? The line goes with *lin*e.

So, we'll call the models for now:

0. The O-model: No magnitudes or vectors, the *space* is just a point, the number zero. The Origin and The Destination. Everything else is in between. The single point shared by all regions. The equality of all components
1. **The *l*-model**: **One** quantity: *l*, a one vector space (a magnitude and a direction). The vector has an origin, and the magnitude carries the distance from it. If there is only one vector, one direction, we can ignore directions. So, for a vector or point, we only care about the magnitude: a number that is zero or a positive real:

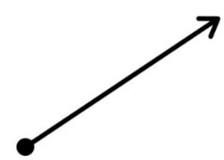
   $$\mathbb{R}_0^+ = \{x \in \mathbb{R} \mid x = 0 \ \vee x \in \mathbb{R}^+\}$$

   I like to call them True Numbers. "Zero or a positive real" is too loong to say. And I'd like a symbol, like a T for True Numbers. The F for fractional numbers, $\{0, \mathbb{Q}^+\}$ too loong. Naturals include zero already.

   $$\mathbb{T} \quad \mathbb{F}$$

   Fractionals are finite compute for 100% accuracy. Then you enter gradual levels of recursivity to complete True Numbers with those requiring infinity compute for 100% accuracy.

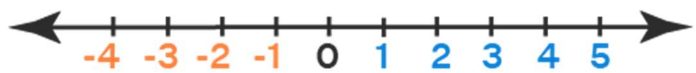
   The geometric locus is a ray. We call it *geometric place* in Spanish. We can call it the *ray*-*space* here.

   We will represent the location of a point as (*l*). For example (1) is the point at distance one of the origin.

   A lot goes on here. Number Theory has a huge property in this *ray*-space alone.

2. The *ln*-model, a line: is made of two vectors in opposite direction. This is the real number line you're familiar with. The *space* is a *line* in geometric sense: the *line-space*. We are looking at the *geometric place* of two vectors, we just display them sharing origin and as opposites without loss of generality. You may also call it "one dimension": 1D. However, notice we are already at the third element here: a point, a ray, and now a line. I argue the *dimensions* naming convention is misleading, it's another example of "off-by-one" error-bias, as we are carrying two vectors here.



I don't use negative numbers on this model, we don't on vectors in Physics. Its reality is limited. Instead, the point at 3 will be represented as a pair (3,0) and the point at -2 is (0, 2). Boom! We got rid of negatives on the bipolar number line!

Then, if we have two opposite vectors, you can relate *l* to the positive ray and *n* to the negative ray. Two opposite rays form a line. You have two regions: one for positives and one for negatives. Regions are mutually exclusive for a point other than at the origin: a point can be in only one region of space or at an interface or union.

The two regions intersection is just a point here: the origin.

Positive or negative (+,-) can be (l, n) or (x, y). But these two magnitudes are only zero or positive. And at least one of them is zero on the line, in the standard form (*massaction* aside). Either both or one of (l, n) is zero in a fundamental way of representing them in *line*-space. Want to see the *massaction* in action? Add another vector into *plane*-space.

On the point (l, n) on the *line-space*, if both $l > 0$ and $n > 0$, then we can find the point by finding the minimum of both, the *massaction* of (l, n), and subtract it from each component to find the standard form:

To normalize a point where all 3 coordinates are greater than zero:

$$(l, n) \text{ where } l > 0, n > 0, \text{calculate the } massaction\ \text{ל} = \min(l, n)$$
$$\Rightarrow (l - \text{ל}, n - \text{ל})$$

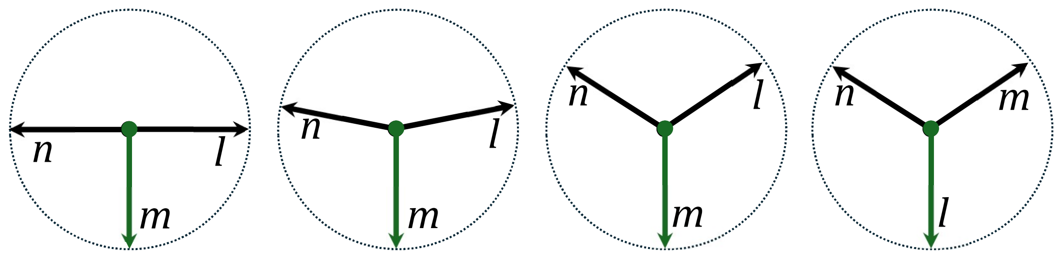
Notice that for a point, you always have at least one zero, the zero denotes which vector is absent, pointing to the region or ray where points like this are not: the only region where this component or *dimension* is absent. If all regions are neighboring all others, this will come nicely: identify a region by the absent coordinate.

Forky asks a question: “But then, what is minus? Negatives? And how to interpret -1?” Minus is an operation, subtraction, opposite to addition. It follows that -1 is not a true number. It’s a shortcut for a vector resulting from rotating ‘one vector on the primer region’ around ‘the origin’ into the opposite direction. In this sense, the negatives are the **result** of an *opposite* operation, a *reflection* over ‘the origin point’ geometrically speaking. And then, -1 is notation for the opposite of vector +1, and its direction contingent in whatever direction +1 was. Without loss of generality (WLOG), positives and negatives are just a *summary view* of a 2-truths vector, a *differential* view. Negative numbers cannot be interpreted without the notion of what positive means. They are just a mirror illusion.

In the real line there is no ambiguity in directions as there are just two and we choose a primer. So, we use negatives to reflect the symmetry of the two opposite vectors, defaulting to positive (we don't say 'plus one', just 'one'), and let the reflections on the other vector be the negative: -1 is the point | vector that is the reflection over zero | origin | destination of the point | vector at 1 | +1. This is what led us to the mirror illusion.

3. The *Plane-Space*: the *lnm*-model, or *m*-space for short

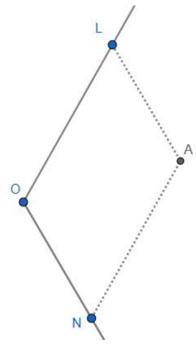
Now you have a line, marked with an origin and divided into two regions or rays. You want to know what happens when you add a third component or dimension.



We inserted a third component *m* and then got three uneven regions (if we just wanted half of a plane maybe…). We pull down the third until we divide into three symmetric regions, and components make angles of 120°, or 2π/3 radians.

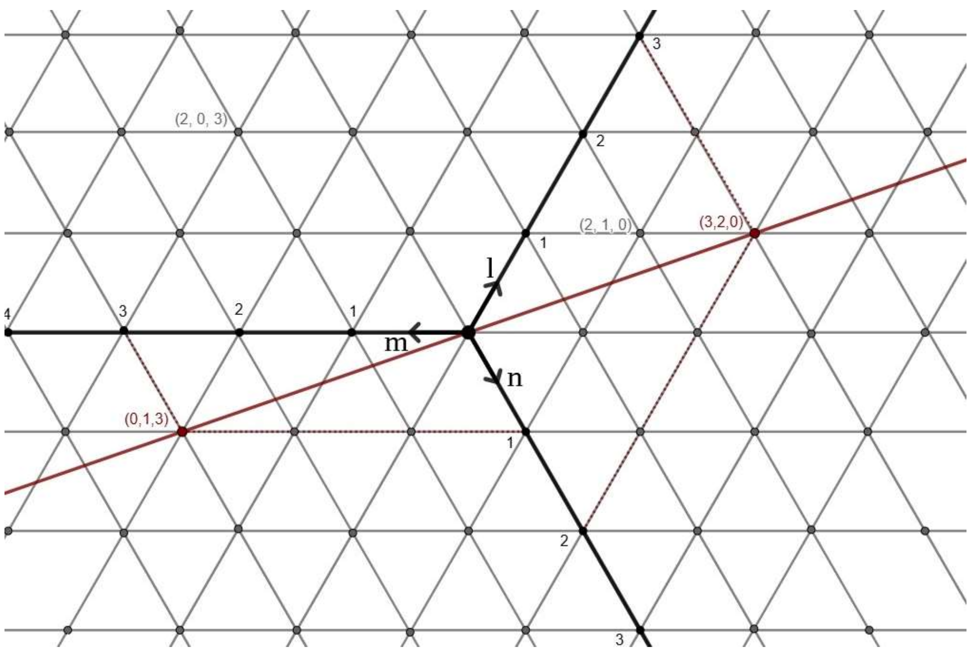
For the purposes of this discussion, I use the term trangulation to mean locating a point within a defined angle (or region) using coordinate-based or geometric constraints.

I'll stick to the parallelograms, so we'll draw parallels by the coordinates on the two rays that determine the region. Note I will use the word *rays* to refer to *l*, *n* and *m*, and not *axis*, as axis has a bipolar bias. Try it here:
https://www.geogebra.org/geometry/nf7yeca8



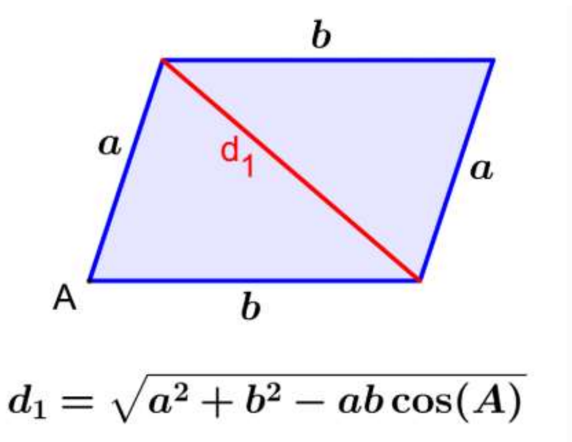
Check out how are the points (3, 2, 0), (0, 1, 3) and (2, 0, 3). Note how the zero hints the region on each.

- Region *ln* has *m* in zero.
- Region *nm* has *l* in zero.
- Region *lm* has *n* in zero.

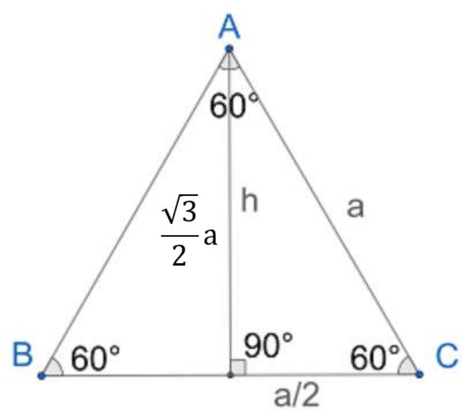


The distance of a point to the origin

Let's review the diagonal of a parallelogram. Simply applying *Law of Cosines*.



Also, let's review some of the properties of the equilateral triangle:



Also:

$sin\left(\frac{\pi}{3} = 60°\right) = \frac{\sqrt{3}}{2}$ $cos\left(\frac{\pi}{3} = 60°\right) = \frac{1}{2}$ $cos\left(\frac{2\pi}{3} = 120°\right) = -\frac{1}{2}$

The diagonal, the distance from the point to the origin:

$$d^2 = l^2 + n^2 - ln\,cos\left(\frac{\pi}{3}\right)$$

$$d^2 = l^2 + n^2 - ln$$

So, the distance from a point to the origin in the *plane-space* is:

$$d = \sqrt{l^2 + n^2 - ln}$$

The formula of distance has changed as compared to Cartesian, but distance is consistent with Cartesian and Geometry.

The Circle

A circle centered at the origin, the *geometric place* of equidistant points to the origin, with a radius *r*, is defined by the equation:

$$r^2 = l^2 + n^2 + m^2 - ln - lm - nm$$

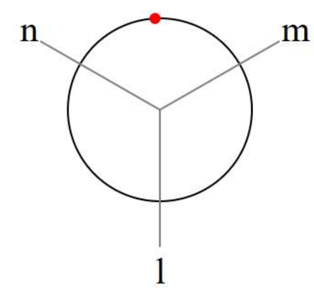
Where did that come from?

This can be inferred from analyzing the three regions of the circle:

$$r^2 = l^2 + n^2 - ln, \text{when } m = 0$$

$$r^2 = l^2 + m^2 - lm, \text{when } n = 0$$

$$r^2 = n^2 + m^2 - nm, \text{when } l = 0$$



And then try to imagine the equation that describes the three cases by making one of *l*, *n*, or *m* zero:

$$\boxed{r^2 = l^2 + n^2 + m^2 - ln - lm - nm}$$

Again, the equation of the circle in the *plane-space*:

$$\boxed{r^2 = l^2 + n^2 + m^2 - ln - lm - nm}$$

And if two are zero, say m and n, then $r^2 = l^2$ (note: I am omitting a reasoning with limits).

Hold on! But what about points that are not normalized? Will all of this hold?

Let's see:

…insert a simple proof by adding +k and doing simple algebra

The Plane Point, Massaction and Triquations

Ok, that's a punching title. Let's break it down: ….