**MSc Mathematical Modelling & Machine Learning**
**Dissertation project topics - 2022/23**

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**01. Bifurcation analysis using neural networks**
Supervisor: Dr Andreas Amann

In traditional numerical methods of bifurcation analysis stable and unstable solutions of a given dynamical system are followed under variation of parameters. The idea of this project is to instead use a neural network for this task. The network is to be trained through a cost function which involves the equations of the dynamical system directly.

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**02. Multifunctionality in recurrent neural networks based on LSTMs**
Supervisor: Dr Andreas Amann

The concept of multifunctionality describes the situation where a trained system is able to fulfil a number of different tasks, without being retrained. From a dynamical perspective this corresponds to multistability between different attractors in the potentially high-dimensional internal phase space of the system. The aim of this project is to recover similar effects using LSTM type networks and study efficient ways to train a system for multiple attractors.

[1] Multifunctionality in a reservoir computer A Flynn, VA Tsachouridis, A Amann Chaos: 31, 013125 (2021). https://doi.org/10.1063/5.0019974

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**03. Understanding machine learning using methods from quantum field theory**
Supervisor: Dr Andreas Amann

While machine learning is widely used, its theoretical foundations are generally not well understood. Very recently however some progress was made in using methods originally invented in the context of quantum field theory to characterise the random variable appearing in deep and broad neural networks [1-3]. The ratio of depth versus breath is used as an expansion parameter similar to a coupling constant in quantum field theory. The aim of this project is to review the emerging literature in this field and to validate some of the predictions of this approach practically.

This project is only suitable for a student with a strong background in mathematics or theoretical physics.

[1] The Principles of Deep Learning Theory; Daniel A. Roberts, Sho Yaida, Boris Hanin; 2021; https://arxiv.org/abs/2106.10165
[2] Neural networks and quantum field theory; James Halverson, Anindita Maiti and Keegan Stoner; Mach. Learn.: Sci. Technol. 2 (2021) 035002. https://iopscience.iop.org/article/10.1088/2632-2153/abeca3/pdf
[3] Statistical field theory for neural networks; Moritz Helias, David Dahmen; 2019; https://arxiv.org/abs/1901.10416

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**04. Fragmented tipping in an ocean box model**
Supervisor: Dr Andrew Keane

Sophisticated Earth System Models have been used many times to simulate the shutdown of the Atlantic Meridional Ocean Circulation, and the associated Gulf Stream, due to anthropogenic forcing [1]. This forcing usually takes the form of an increase in $CO_2$ in the atmosphere or an increase in freshwater flux into the northern Atlantic Ocean. Upon close inspection of these simulations, smaller sudden decreases in the ocean circulation strength are observed before the total collapse of circulation occurs. This is possibly related to fragmented tipping [2].

In this project the student will investigate whether these observations in sophisticated models can be replicated in a simple ocean box model, such as the model in Ref. [3]. The box model will be designed to capture the way that sophisticated Earth System Models discretise the North Atlantic Ocean into small grid cells.

This project would suit a student with strengths in nonlinear dynamics and numerical methods.

[1] Lohmann, J., & Ditlevsen, P. D. (2021). Risk of tipping the overturning circulation due to increasing rates of ice melt. Proceedings of the National Academy of Sciences, 118(9).
[2] Bastiaansen, R., Dijkstra, H. A., & von der Heydt, A. S. (2022). Fragmented tipping in a spatially heterogeneous world. Environmental Research Letters, 17(4), 045006.
[3] Titz, S., Kuhlbrodt, T., Rahmstorf, S., & Feudel, U. (2002). On freshwater-dependent bifurcations in box models of the interhemispheric thermohaline circulation. Tellus A: Dynamic Meteorology and Oceanography, 54(1), 89-98.

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**05. Detecting and predicting volcanic eruptions**
Supervisor: Dr Andrew Keane

Volcano eruptions are undoubtedly a threat to societies across the globe. They can often be unexpected and cause great harm, resulting in loss of lives and livelihoods; for example, the 2019 eruption of White Island, New Zealand. The knock-on effects can cross international borders; for instance, the eruption of Eyjafjallajökull, Iceland, in 2010 brought most of Europe to a standstill.

The evolving dynamics of a volcano and its critical transitions between different physical states is not unlike the dynamics and transitions studied heavily in the Earth's climate system. Much recent research has focussed on critical transitions in climate (often called "climate tipping events"); for example, the transition between an 'on' and 'off' state of the Gulf Stream [1]. The fundamental bifurcation that underlies this transition is the same as that of a volcano that transitions from a non-erupting to an erupting state [2].

In this project the student will analyse the bifurcations of mathematical models for volcano dynamics. The student may also investigate evidence of critical transitions in observational data from monitored volcano sites. This project will suit a student with strengths in dynamical systems theory and numerical methods, as well as an interest in mathematical modelling.

[1] Lohmann, J., & Ditlevsen, P. D. (2021). Risk of tipping the overturning circulation due to increasing rates of ice melt. Proceedings of the National Academy of Sciences, 118(9).
[2] Slezin, Y. B. (2003). The mechanism of volcanic eruptions (a steady state approach). Journal of Volcanology and Geothermal Research, 122(1-2), 7-50.

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**06. Pseudo spectral methods in C#**
Supervisor: Dr Kieran Mulchrone

Pseudospectral methods are an efficient and accurate approach to solving nonlinear ODEs and PDEs. Finite difference methods approximate derivatives by local polynomial interpolations, whereas pseudospectral method use global interpolation to gain spectacular results. In this project, you will develop and test a set of classes implementing pseudospectral by closely following the examples and explanations of L.N. Trefethen in his book in Spectral Methods in Matlab. This project will suit a student interested in developing their programming and numerical skills.

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**07. Agent-based modelling of electricity markets constrained by data from the Irish market**
Supervisors: Dr Kieran Mulchrone and Joseph Collins

Electricity markets have come under scrutiny in recent times due to rising energy costs. The electricity market is a marginal market and works quite differently to typical markets in terms of determining the price. They comprise three types of player or agent:

1. producers willing to provide a fixed quantity of energy at a fixed price (a sell bid),
2. consumers who are willing to purchase a fixed quantity of energy at a specific price (a buy bid)
3. speculators who neither produce nor consume energy but effectively trade energy to make a profit (a buy or sell bid).

Agent based models are fundamentally simple and easy to implement. The aim of this project is to implement an agent-based model based on real data whereby agents can adjust their strategies based on simple rules or learning algorithms.

[1] Bower, J., Dean, D.W. 2000. Model-based Comparisons of Pool and Bilateral Markets for Electricity. The Energy Journal.

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**08. Stochastic population models with applications to cancer**
Supervisor: Dr Kieran Mulchrone

Deterministic dynamical models of population evolution are quite common and well known e.g., Malthusian growth or decay, Logistic models with a carrying capacity or models incorporating the Allee effect. Underlying these models are probabilities that birth and death processes occur which lead to deterministic models as population numbers are assumed to be large. However, when population numbers are small, stochastic effects are important. This effect can be modelled by adding stochastic terms or by using brute force probability simulations. This approach is well developed for chemical reactions. For example, a birth process could be modelled as a molecule producing another molecule. In this project you will write code to simulate population development which mimics the logistic situation and incorporates the Allee effect. Furthermore, this approach may be used to develop a simple model of cancer development whereby:

1. normal cells grow to a carrying capacity.
2. normal cells may mutate to a precancerous state.

3. precancerous cells may then be eliminated by the immune system.
4. if precancerous cells are not eliminated, they become established as a cancer cell.
5. cancer cell proliferate.

In this project you will develop a model of cell growth and simulate cancer development. Additionally you will establish process rates so that there is agreement between the model and cancer incidence data.

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**09. Modelling friendship networks in dairy cattle herds**
Supervisors: Dr Kieran Mulchrone, Kathleen O'Sullivan, Dr Ben Taylor, Dr Hassan Alkhayoun

Anecdotal evidence from farmers supports the premise that cows are able to form special bonds with other cows and prefer to spend a lot of time in their social groups. Until recently, not many behavioural studies investigated cow friendships. Research by McLennan et al. [1] on the social bonds of dairy cattle identified favoured relationships between them using an association index adapted from Ginsberg and Young [2] and work by Durrell et al [3]. These researchers investigated the importance of special cow relationships by a short-term social isolation test, separating cows from their preferred partner, "best friend", and from their non-partner. This research concluded that social bonds were affected by separation and that cows' social bonds were important to their welfare. They further suggested farmers attending to cows' social groups had many benefits such as improved milk yields, reduced stress and improved welfare. However, certain frequent farming practices, such as vet visits and cattle movement, necessitate separating cows. Research has established that after two weeks separation existing friendships were no longer displayed. Typically, large-scale dairy farms may regroup a herd four to 12 times a year. This regrouping of cows creates high levels of stress for them as they try to integrate into new groups. Research suggests that inhibiting cow friendship negatively impacts on cow welfare.

More recent work by Foris et al. [4] used social network analysis to examine how, after regrouping, familiarity influenced social networks in dairy cows. Other studies such as de Freslon et al. [5] focused on cows' grooming behaviours, referred to as allogrooming, to indicate which animals had established a relationship by employing stochastic actor-oriented modelling (SAOM) to map changing relationships within cows' social groups. They concluded that "social network dynamics can be used to better understand the complexity and non-linearity of cow relationships". Consequently, they proposed that improved management practices should be designed to be more in line with the natural social behaviour of cows.

Many studies describe the benefits of long-term social connection in cows. Animal behavioural scientists have found that cows interact in socially complex ways, develop friendships over time and even hold grudges against other cows. The objective of this project is to investigate the social network of cows, identify cow friendships and describe cow interactions. This work will be done in conjunction with Teagasc in Moorepark and will analyse GPS location data measured on a single cow herd collected over a 24-hour period for 5 consecutive days. In this project you will interpret the gps data in terms of a temporal network. Standard network measures are to be calculated and possibility non-standard friendship measures. In this way, you will describe how friendships evolve over time. Furthermore, by applying simple processes on the network, a model of how friendships evolve can be developed.

[1] McLennan, K. M. and McCormick, W., (2012). Farmyard Friends. The Biologist, 59(4); 18-22
[2] Ginsberg, J. R. and Young, T. P. (1992). Measuring association between individuals or groups in behavioural studies. Anim . Behav ., 44, 377-379.

[3] Durrell, J.L. et al. (2004). Do pigs form preferential associations? Applied Animal Behaviour Science, 89(1–2), 41-52.
[4] Foris, B. et al. (2021). Familiarity influences social networks in dairy cows after regrouping. J. Dairy Sci., 104(3), 3485-3494.
[5] de Freslon, I. et al. (2020). Understanding Allogrooming Through a Dynamic Social Network Approach: An Example in a Group of Dairy Cows. Front. Vet. Sci., 4, 7:535.

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**10. Darkness to light: Detecting objects of interest from historic Irish Marine Magnetometer data**

Supervisors: Dr Mohit Tunwal, Dr Aaron Lim and Dr Kieran Mulchrone

Ireland aims to produce 7GW of energy offshore by 2030. Consequently, the south coast of Ireland and the north Celtic Sea are subject to intense offshore renewable energy development plans. Natural seabed features and considerable anthropogenic features lie on the seabed such as shipwrecks, cable routes, pipelines, and most dangerously, unexploded ordnance (UXO) left over from WW1 and WW2 [1]. Although it has been several decades since these wars, the remaining UXO still pose a threat in 2023. The acquisition of geophysical data to map the distribution of UXO is costly, time intensive and requires considerable expertise.

INFOMAR have acquired approx. 20 years' worth of magnetometry data (used to map ferrous objects on the seabed) which are yet to be examined [2]. This is due to: a) the acquisition parameters are unsuitable for the energy industry and; b) considerable expertise is required to process the data [3]. This project aims to develop a Python code block that processes magnetic data for earth's magnetic field and water depth, and automatically classify magnetic signal types using machine learning. Finally, a software interface will be developed for processing any new data collected by marine surveys. The work will suit a student with an interest towards pursuing a career in machine learning and possibly the offshore energy industry.

[1] Mitchell, T., & Smith, K. (2014). Investigations of Marine Geohazards for Coastal Infrastructure Projects. Sea Technology.
[2] Bodus-Olkowska, I., & Uriasz, J. (2020). Ferromagnetic Underwater Target Detection Based on a Magnetic Anomaly Map. Geomatics and Environmental Engineering, 14(1), 34-45.
[3] Holt P. (2014). Marine Magnetometer Processing, 3H Consulting Ltd., Plymouth.

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**11. Martian landscape evolution: finding analogue through numerical modelling and machine learning**

Supervisors: Dr Mohit Tunwal, Dr Hassan Alkhayuon and Dr Kieran Mulchrone

A large number of images of Mars have been received on Earth from NASA's Perseverance rover. High-definition images sent by the rover shows a remarkable landscape consisting of ridges of sedimentary rocks. Ridges are formed in nature due to preferential erosion of rocks with contrasting erodibility. Martian ridges point to a vivid aqueous past environment and hold clues to a much longer geological history than currently understood [1].

In this project, the student will numerically model erosion of a landscape consisting of rocks with contrasting erodibility. The numerical model will be applied to erode 3D block models, with a known geological history, generated using FLUMY™ which is a numerical simulator. Erosion of 3D block models will generate ridge structures with known geological parameters. A machine

learning model will have to be developed to predict geological parameters from ridge data. The complete mathematical approach of predicting geological parameters from ridge data will be tested on example ridges from Earth which correspond to landscape and geological features of Mars. Finally, the model will be tested on Martian data to predict its geological past. This project will suit a candidate with an interest in numerical modelling and machine learning. No prior experience with geosciences or extra-terrestrial life is required!

[1] Cardenas, Benjamin T., Michael P. Lamb, and John P. Grotzinger. "Martian landscapes of fluvial ridges carved from ancient sedimentary basin fill." Nature Geoscience (2022): 1-7.

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**12. Exploring complex dynamical behaviour using Lyapunov exponents**
Supervisors: Prof Sebastian Wieczorek and Dr Andreas Amann

Nonlinear oscillators in science and engineering (such as electronic circuits, lasers, neurons or even a simple spring or mechanical pendulum) can exhibit a wealth of nonlinear behaviour when subject to external disturbances. The system response to external forcing will typically depend on the control parameters and will include periodic and quasiperiodic oscillations, chaos as well as interesting bifurcation transitions between these different types of behaviour. The aim of this computationally oriented project is to understand and implement numerically the concept of Lyapunov Exponents to explore dynamical complexity of selected nonlinear oscillators and unveil the graphical beauty of chaos.

The project will be organised about the following topics:

1. Dynamical Systems, initial value problems for ODEs, linearisation, phase space, theory of Lyapunov Exponents, examples of compact invariant sets in the phase space and bifurcations.
2. Numerical methods for direct time integration of initial value problems for ODEs.
3. One-parameter bifurcation diagrams.
4. Numerical methods for QR matrix decomposition including Gram-Shmidt orthonormalisation.
5. Numerical methods for computing Lyapunov.
6. Creating one- and two-parameter Lyapunov diagrams; see (http://imaginary.org/fr/node/96) and (https://element90.wordpress.com/2012/03/05/lyapunov-fractals/) for examples of visually stunning Lyapunov diagrams.

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