

Rotating White Dwarfs As Dark Matter Detectors

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We show that rotating white dwarfs admixed with dark matter have interesting properties that may be used to reveal the presence of dark matter. Even though such objects follow universal relations among the I (moment of inertia), Love (tidal Love number), and Q (quadrupole moment) that are robust with respect to different normal matter equations of state, these relations are sensitive to the dark matter fraction. Since each white dwarf may have a different dark matter fraction, the $I - \text{Love} - Q$ relations for dark matter admixed white dwarfs span bands above those without dark matter admixture. Furthermore, the limiting mass of dark matter admixed rotating white dwarfs can be increased beyond those without any dark matter for some rotational rules. Ultra-massive white dwarfs with a total mass of at least $2.6 M_\odot$ could be formed. The accretion-induced collapse of such an object may lead to a $2.6 M_\odot$ compact object, such as the one discovered in the gravitational-wave event GW190814.

I. INTRODUCTION

A. Dark Matter-Admixed Astrophysical Objects

Dark Matter (DM) constitutes more than 95% of the mass in a typical galaxy [1] and 23% of the mass-energy budget of the Universe [2]. It is natural to hypothesize that stellar objects comprising NM and DM may form. Indeed, DM could be captured by NM in a region where there is a high concentration of DM particles [3–5]. It is possible that stars contain a DM component. The effect of DM on stellar evolution could be significant depending on the stellar evolution models [4, 6–11]. Therefore, anomalous stars could be used as probes for astrophysical DM. Besides, studies have been made on the effect of admixing DM on structures and dynamics of compact and exotic objects. They include white dwarfs (WD) [12–14], neutron stars [Leung et al. 2020 in prep, 12, 15–25], and quark stars [26–30]. Recently, studies have been made on Bosonic DM-admixed compact objects [Lee et al. 2020 submitted, 31–34] and Bosonic dark stars [35–39]. DM-admixed astrophysical objects have now opened up a new window to search for astrophysical DM.

B. The I -Love- Q Relations

Yagi and Yunes [40] discovered universal relations for neutron stars that are independent of the EOS among the (all rotationally-induced) moment of inertia I , mass

quadrupole moment Q , and tidal Love number λ_T . These so-called $I - \text{Love} - Q$ relations are believed to emerge from the strong dependence of I , Q , and λ_T on the structure of the outer envelope of the neutron star EOS. A later study revealed that most modern neutron star EOSs are stiff enough to be modeled as an incompressible EOS, which leads to the universality [41]. However, it was pointed out that the similarity of different neutron star EOSs at the outer-most part of a compact object cannot account for such universality, because they can differ from each other by as much as $\sim 17\%$ [42]. The reason behind the $I - \text{Love} - Q$ relations could be an approximate symmetry - isodensity self-similarity, which emerges as one considers stellar objects that are compact [43]. Ever since the discovery of the $I - \text{Love} - Q$ relations for neutron stars, similar theoretical studies have been made and confirmed the existence of the universal relations for polytropic stars [44, 45], WDs [46–49], quark and strange stars [50–53], and neutron stars with other kinds of realistic EOSs and extreme conditions [54–60].

The $I - \text{Love} - Q$ information of a compact star is imprinted in its gravitational-wave signature [61–64]. Even though the sensitivity of current gravitational-wave detectors does not allow direct measurement of the $I - \text{Love} - Q$ numbers, methods for obtaining and testing the universal relations have been proposed [40, 65]. In addition, the $I - \text{Love} - Q$ relations have also been applied to constrain gravitational parity violation [66], extra dimensions [67], gravitational theories [68], neutron star equation of states [52, 69–71] and moment of inertia [72]. Gravitational-wave measurements of the $I - \text{Love} - Q$ relations could provide an excellent laboratory for testing and understanding fundamental physics.

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C. White Dwarfs Rotations

Stellar rotation is a natural phenomenon. The observation of the westward movements of sunspots suggests rotational motions of the Sun. Doppler shifts of the stellar spectral lines indicate that most stars exhibit a certain degree of rotations [73], which induce additional mixing of stellar materials to explain the isotope distribution near the stellar surface that deviates from those with the non-rotating model [74, 75]. Due to the large reduction in the size of a molecular cloud during its collapse phase, a small initial angular momentum could result in a rapidly rotating proto-star [76, 77]. However, most of the angular momentum of a proto-star would be carried away through wind-magnetic field interactions, leading to a slowly rotating new-born star [78–80]. In addition, diffusion of material from the core to the surface of a WD lowers its final spin [81, 82]. Therefore, the primordial angular momentum of a low-mass star that eventually ends up as a WD contributes to only a small portion of the final total angular momentum. Indeed, dimensional analysis suggests that extra angular momentum transported to WDs is necessary to explain the observed WD rotation rates [83, 84]. Possible scenarios include accretion from a companion star [85, 86] and mergers between two or more WDs [87–90]. Rotating WDs have been proposed to be progenitors of neutron stars formed by core contractions [91–95], as well as super-luminous thermonuclear supernovae, due to the fact that rotating WDs could support more mass than their traditional Chandrasekhar limit [96–100]. Recently, studies have been made on finite-temperature corrections to modelling of rotating WDs [101, 102], as well as effects of the strong magnetic field on the equilibrium structures of WDs [103–106], for which the WD rotation takes a critical role.

D. Motivations

A number of new telescopes such as the LISA and LSST are expected to begin construction or operation in the upcoming decade. It is estimated that over 150 million WDs could be detected in the final phase of the 10-year LSST survey [107], and that over 25,000 WD binaries could be observed through the LISA telescope [108]. Furthermore, it was shown that over 10,000 super-luminous Type Ia supernovae per year may be captured by the LSST [109], and even more is expected for the total population of Type Ia supernovae. Previous studies have shown that the effect of DM admixture on WDs and Type Ia supernovae could be significant [13, 110, 111]. Hence, the overwhelmingly large amount of data for WDs and Type Ia supernovae makes them promising indirect DM detecting channels.

Rotating WDs are shown to be ubiquitous, but the effect of DM admixture on rotating WDs has never been considered. Moreover, there are studies of the

$I - \text{Love} - Q$ universality for exotic objects such as dark stars [112] and neutron stars admixed with quark matter [113], but none have been made on DM-admixed (rotating) WDs. Here, we extend previous studies of DM-admixed WDs to rotating WDs. Our goal is to provide predictions to facilitate searches for DM through astrophysical observations. The plan of the paper is as follows: Section II describes our method of constructing DM-admixed rotating WDs and obtaining the $I - \text{Love} - Q$ numbers. Section III is a summary and discussion of the results. Section IV concludes our study.

II. METHODOLOGY

A. Equation of Hydrostatic Equilibrium

We compute a series of DM-admixed rotating WDs (DMRWs) by solving the Newtonian hydrostatic equations, including the centripetal force:

$$\vec{\nabla} P_i = -\rho_i \vec{\nabla} \Phi + \rho_i \omega(s)_i^2 s \delta_{i2} \hat{s}. \quad (1)$$

Here, the subscript $i = 1(2)$ denotes the DM (NM) quantities, and ρ , P , and ω are the density, pressure, and angular speed for the fluid element. s is the perpendicular distance from the rotation axis, and \hat{s} is the unit vector orthogonal to and pointing away from that axis. The angular speed is assumed to be a function of s only. δ_{i2} is the Kronecker-Delta function, indicating that only the NM is rotating. Φ is the gravitational potential governed by the two-fluid Poisson equation:

$$\nabla^2 \Phi = 4\pi G(\rho_1 + \rho_2). \quad (2)$$

The use of the Newtonian framework is justified since the rotation speed and compactness of WDs are small. Following Eriguchi and Mueller [114], Hachisu [115] and Aksenov and Blinnikov [116], we can integrate the equation of equilibrium:

$$\int \frac{dP_i}{\rho_i} = -\Phi + \delta_{i2} \int \omega(s)_i^2 s ds + C_i, \quad (3)$$

where C_i is an integration constant. In particular, following Hachisu [115], we define:

$$\int \frac{dP_i}{\rho_i} = H_i, \quad (4)$$

$$\int \omega(s)_i^2 s ds = -h_i^2 \psi_i, \quad (5)$$

where H is the enthalpy, ψ is the rotational potential, and h^2 is a constant to be determined. So, the equation of equilibrium can be written in the integral form as:

$$H_i + \Phi + \delta_{i2} h_i^2 \psi_i = C_i. \quad (6)$$

The equations of equilibrium for the DM and NM will be simultaneously solved by iteration to obtain the equilibrium configurations.

B. Rotation Rules

We have considered rotation profiles for the NM from Hachisu [115] and Yoshida [102] including (1) the rigid rotation:

$$\omega(s)_2^2 = \Omega_2^2, \quad (7)$$

and (2) Keplerian-flow-like rotation profile:

$$\omega(s)_2^2 = k_2^2/(d^{3/2} + s^{3/2}). \quad (8)$$

The parameter d governs the degree of differential rotation. We choose $d = 0.1$ for this work. We integrate the angular velocity to obtain the effective potential of the rotation for the rigid rotation:

$$\psi_2 = s^2/2 \quad (9)$$

The effective potential for the Kepler rule is:

$$\psi_2 = -\frac{1}{9} \left[-\frac{6\sqrt{s}}{s^{\frac{3}{2}} + d^{\frac{3}{2}}} + \frac{1}{d} \ln \left(\frac{(\sqrt{d} + \sqrt{s})^2}{d + s - \sqrt{sd}} \right) - \frac{2\sqrt{3}}{d} \tan^{-1} \left(\frac{1 - 2\sqrt{s/d}}{\sqrt{3}} \right) \right]. \quad (10)$$

C. The Self-Consistent Method

We follow Hachisu [115] to adopt an axial symmetric spherical grid. We adopt dimensionless units so that we set the gravitational constant G , maximum NM density $\rho_{\text{Max}2}$ and NM equatorial radius $r_{\text{eq}2}$ to be one. The computational domain is described by the radial coordinate r and $\mu = \cos \theta$, where θ is the polar angle. We divide μ and r into KDIV and NDIV equal portions, respectively. Therefore,

$$\begin{cases} r_j = r_0 \frac{j-1}{\text{NDIV}-1}, & (1 \leq j \leq \text{NDIV}), \\ \mu_k = \frac{k-1}{\text{KDIV}-1}, & (1 \leq k \leq \text{KDIV}). \end{cases} \quad (11)$$

Here, r_0 is the size of the computational domain. In this work, when there is no DM component, we choose KDIV = 257, $r_0 = \frac{16}{15}$, and NDIV = 257. When there is a DM admixture, r_0 and NDIV are enlarged to accommodate the DM fluid. We compute the gravitational potential in spherical coordinate using the multipole expansion method:

$$\Phi(\mu, r) = -4\pi G \int_0^\infty dr' \int_0^1 d\mu' \times \sum_{n=0}^{\text{LMAX}} f_{2n}(r', r) P_{2n}(\mu) P_{2n}(\mu') \rho(\mu', r'), \quad (12)$$

where $P_{2n}(\mu)$ is the Legendre polynomial, and LMAX is the maximum number of moments. We choose LMAX = 16, and

$$f_{2n}(r', r) = \begin{cases} \frac{r'^{2n+2}}{r^{2n+1}}, & r' < r, \\ \frac{r^{2n}}{r'^{2n-1}}, & r' > r. \end{cases} \quad (13)$$

To compute the equilibrium structure of a pure NM rotating WD, we need to specify the boundary condition for which $\rho_2 = 0$. The equatorial boundary is set at $r_{a2} = 1, \theta = \frac{\pi}{2}$, while the axis boundary is set at $0 \leq r_{b2} \leq 1, \theta = 0$. The axis-ratio of the NM is defined as $\kappa_2 = r_{b2}/r_{a2}$. We need one more parameter to completely specify the system. We choose it to be the radial position of the DM equatorial boundary r_{a1} . Now we describe the procedure for obtaining the equilibrium structures:

1. Specify $\rho_{\text{Max}2}$, r_{b2} and r_{a1} ;
2. Solve the two-fluid equations (6) iteratively.

In general, the DM mass changes during iterations. Therefore, we use the bisection method to vary r_{b1} to obtain the targeted DM mass. In solving Eq. (6), we choose to terminate the iterations once h_i , C_i and the maximum of H_i for both fluids converge, i.e., their relative changes are less than 10^{-10} .

D. Extracting the I-Love-Q Information

Since we have computed the equilibrium structures for DMRWDs in a self-consistent way, we can determine I , λ_T and Q by post-processing the density profiles¹. The moment of inertia of a self-gravitating fluid is given as:

$$I = \int (\rho_1 + \rho_2) s^2 d\tau, \quad (14)$$

while the mass quadrupole moment is:

$$Q = \int (\rho_1 + \rho_2) r^2 P_2(\cos \theta) d\tau, \quad (15)$$

where $d\tau$ is the volume element and $P_2(x)$ is the $l = 2$ Legendre polynomial. Following Boshkayev *et al.* [46], we scale I , and Q by:

$$\begin{aligned} I &\rightarrow \left(\frac{c^2}{G} \right)^2 \frac{I}{M^3}, \\ Q &\rightarrow \left(\frac{Mc^2 Q}{J^2} \right). \end{aligned} \quad (16)$$

Here, M is the total mass, and J is the total angular momentum of the DMRWD. On the other hand, λ_T is related to the tidal deformability k_2 as [46]:

$$\begin{aligned} \lambda_T &= \frac{2R^5}{3G} k_2, \\ k_2 &= \frac{3 - \eta_2(R)}{2(2 + \eta_2(R))}. \end{aligned} \quad (17)$$

¹ We expect that the relativistic effect of DMRWDs are small, provided that the WD is not rapidly rotating and $\rho_{\text{Max}2}$ is below the nuclear matter density.

Here, R is the stellar radius, which we take to be the maximum between the NM and DM radii. η_2 is obtained by solving Radau's equation for the unperturbed spherically symmetric configuration [117]:

$$\frac{d}{dr}(r\eta_2) = 6(1 - D(r)(\eta_2 + 1)) - \eta_2(\eta_2 - 2). \quad (18)$$

Here, $D(r) = \frac{4\pi r^3 \rho(r)}{3m(r)}$ is the average density of the enclosed mass at a radial distance r . The boundary condition is $\eta_2(0) = 0$. We then scale λ_T as [46]:

$$\lambda_T \rightarrow \frac{c^{10} \lambda_T}{G^4 M^5}. \quad (19)$$

Previous studies on the $I - \text{Love} - Q$ relations assumed slowly rotating WDs, in which the equilibrium structures are computed based on the Hartle formalism. In this work, we would mimic small perturbations from the hydrostatic equilibrium by post-processing density profiles that are computed at $\kappa_2 = \frac{239}{240}$.

E. Equations of State

We adopt the ideal degenerate Fermi gas equation of state (EOS) for the DM. Following the previous studies on DMWD [111], we choose the DM particle mass to be 0.1 GeV. To explore the $I - \text{Love} - Q$ relations, we adopt several EOSs for the NM, including the ideal degenerate Fermi gas [118, 119] and Harrison-Wheeler (HW) EOSs [120]. There are several parameterized deleptonization formulae to describe the electron capture at high density. In particular, we choose the formulae given in Arutyunyan *et al.* [121] (ASC), in Liebendorfer [122] (G15, N13), Cabezón *et al.* [123] (S15), and Dessart *et al.* [124], Abdikamalov *et al.* [125] (VUL).

III. RESULTS AND DISCUSSION

A. The Deviation of the $I - \text{Love} - Q$ Universality

We have computed the $I - \text{Love} - Q$ relations for DMRWDs using several EOSs mentioned in section II E. We construct a few sequences of DMRWD models by varying the maximum NM density ρ_{Max2} from 10^6 g cm^{-3} to $10^{10.5} \text{ g cm}^{-3}$. Here, we consider DMRWD with fixed DM fractions $\epsilon = M_1/(M_1 + M_2)$, and we computed DMRWD models with ϵ ranging from 0 to 0.3 in each series.

When there is no DM admixture, the $I - \text{Love} - Q$ relations for WDs computed by using different EOSs lie approximately on the same line for all rotation rules. In particular, the maximum deviations from the best-fit line are within $\sim 1\%$ for the rigid-rotation rules, and $\sim 0.2 - 0.4\%$ for the Kepler-rotation rule. Our results also agree with previous studies on the $I - \text{Love} - Q$

relations of slowly rotating WDs, particularly for the universality lines and the corresponding maximum deviations. For rigidly rotating DMRWDs, we find that admixing a small fraction (e.g. $\sim 5\%$) of DM makes no change to the $I - \text{Love} - Q$ universality. Most of the deviations due to the DM admixture are within the uncertainties of the pure NM model. However, when the dark matter fraction is larger than (e.g. $\sim 10\%$) the $I - \text{Love} - Q$ relations for different EOS are all shifted upwards to higher Q values relative to those without DM admixtures as shown in Figure 1 (a) and Figure 2 (a). The relative deviation between the universal lines for DMRWDs and those for pure NM models can be larger than $\sim 1\%$ in the log scale, and the deviation is larger with higher DM fraction. The results for DMRWDs rotating in the Kepler rule are also similar, as shown in Figures 3 (a) and 4 (a).

In reality, each WD may acquire a different amount of DM. Therefore, WDs would scatter within a band of $I - \text{Love} - Q$ relations spanned by different ϵ . We show these bands together with the uncertainties of the pure NM models in Figures 1 (b) and 2 (b) for rigidly rotating, and in Figures 4 (b) and 3 (b) for Kepler rotating DMRWDs, respectively. They are computed by taking differences between the best-fit lines for DM-admixed models with those of the pure NM models. In particular, since the uncertainties (shown as the blue band) of the pure NM model are small, any significant deviation that lies above the pure NM version of the $I - \text{Love} - Q$ relations is possibly a signature of a sub-solar mass scale of DM admixture.

The changes of the universality of the $I - \text{Love} - Q$ relations when DM is admixed could be understood through Equation 16. Since $J \sim \bar{\omega} M R^2$, and $Q \sim M R^2$, the scaled Q will be proportional to $1/R^2$. When DM is admixed, the strong gravitational force from the DM would contract the NM to make R smaller, the scaled Q would be increased, which causes the upward shifting of the universal lines. We show the total angular momentum of a slowly rotating DMRWD in Figure 5 for a better illustration, where the reduction of J due to an increasing DM fraction ϵ could be seen.

Taylor *et al.* [49] demonstrated that the $I - \text{Love} - Q$ relations for white dwarfs depend on the degree of differential rotation (the parameter d). However, we find that the upward shifting of the DM-admixed $I - \text{Love} - Q$ relations from that of the pure NM model is independent of whether the WD is rotating rigidly or differentially. We have computed two entirely different rotational profiles² for which the rigid rotation resembles the $d \rightarrow \infty$ limit, and we observe the same

² Although not shown in this article, we have also computed the $I - \text{Love} - Q$ relations for DMRWDs rotating in the j-const rule

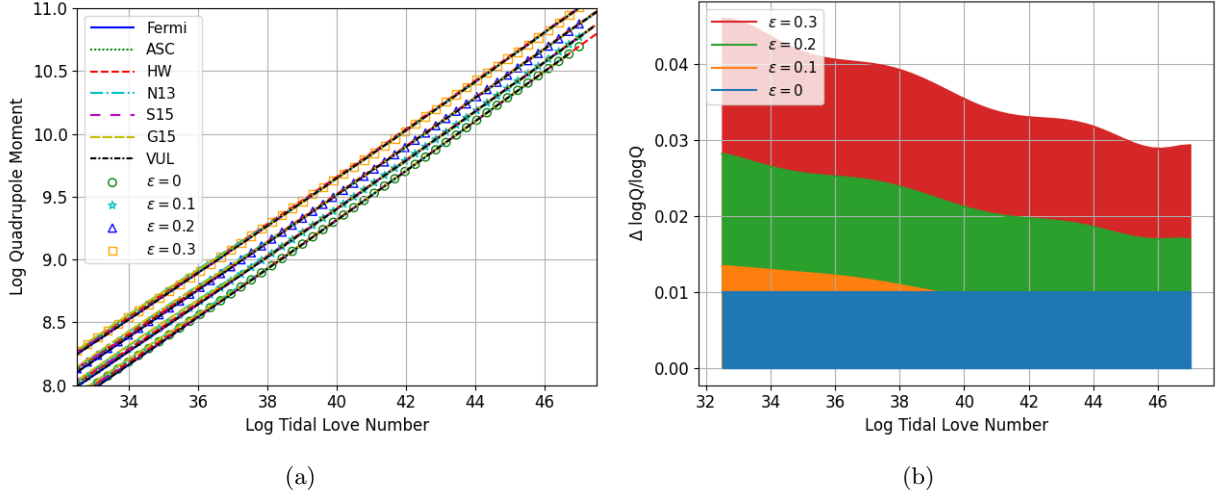


FIG. 1: (a) $\log_{10}Q$ vs. $\log_{10}\lambda_T$ relations for slowly, rigidly rotating DMRWDs having different amounts of ϵ . Models without DM are indicated as $\epsilon = 0$. The scatter plot is best-fit lines generated for each amount of ϵ , and it is obtained using a high-order polynomial fitting. (b) The relative deviation $\Delta \log_{10}Q / \log_{10}Q = (\log_{10}Q_{\text{DM}} - \log_{10}Q_{\text{NM}}) / (\log_{10}Q_{\text{NM}})$ for different amount of ϵ . $\Delta \log_{10}Q$ is computed by taking differences between the best-fit lines of DM-admixed models with the pure NM model, except for the $\epsilon = 0$ line, which is taken as the MAXIMUM uncertainties of 1% of the $Q - \text{Love}$ relations for the pure NM model. The coloured bands of parameter spaces are spanned by universal relations of the DM-admixed models and could indicate how much DM is being admixed into the WD. For instance, WDs lying in the green strip are having DM fraction of $0.1 < \epsilon < 0.2$.

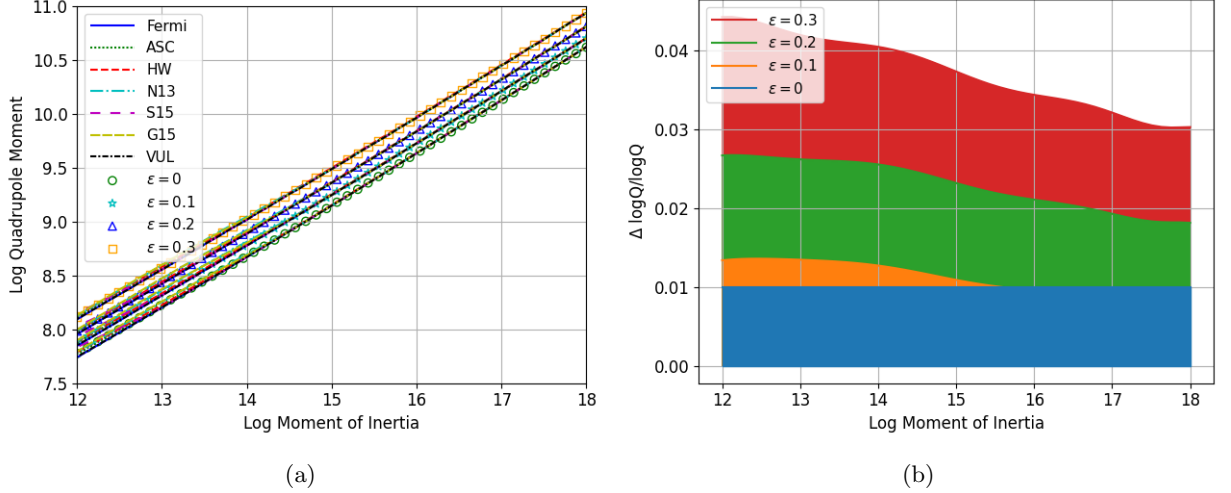


FIG. 2: Same as Figure 1, but for the $\log_{10}Q$ vs $\log_{10}I$ relation.

upward shifting of the $I - \text{Love} - Q$ relations due to DM admixtures. Therefore, we expect similar behavior of the $I - \text{Love} - Q$ relations for different values of d . We note that the universality of the $I - \text{Love} - Q$ relations is violated for hot WDs [47]. However, our DMRWD models produce deviations from the NM version of the universal line that are rather different from those of

hot WDs. Nonetheless, given the high Fermi energy of degenerate electrons, cold EOS for WDs will remain a good approximation.

Gravitational waves would be emitted in a binary WD merger event [126–129]. Although the signal is too weak to be detected in the current gravitational-wave detectors, it was estimated that a binary WD merger event could be resolvable for the next generation of detectors [130, 131]. In fact, a method for estimating WD masses by extracting the tidal information

[115]. The upward shifting of the $I - \text{Love} - Q$ relations due to DM admixtures are also observed.

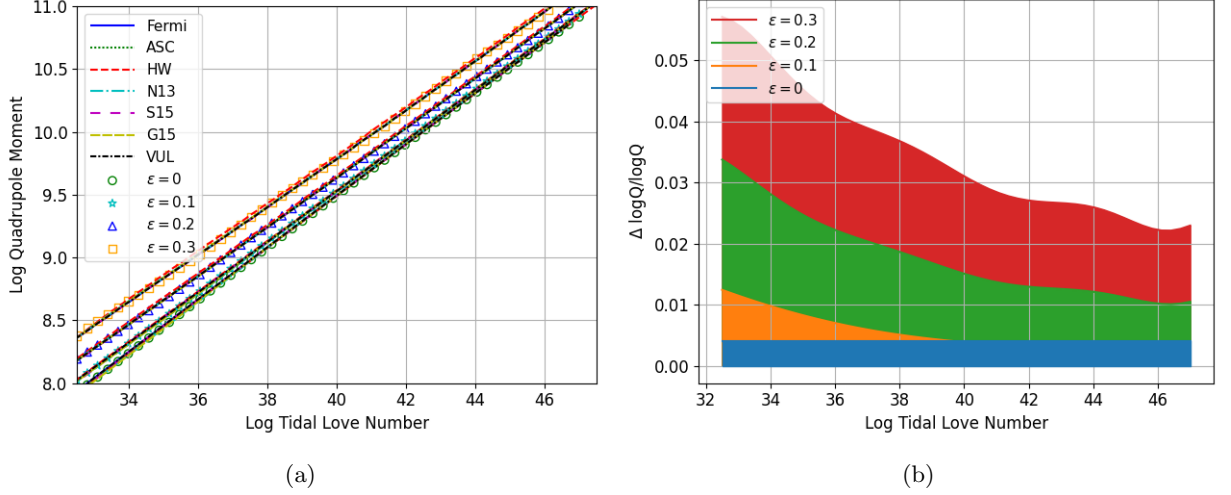


FIG. 3: Same as Figure 1, but for DMRWD rotating in the Kepler rule. Here, the blue shaded region represents MAXIMUM uncertainties of 0.4%.

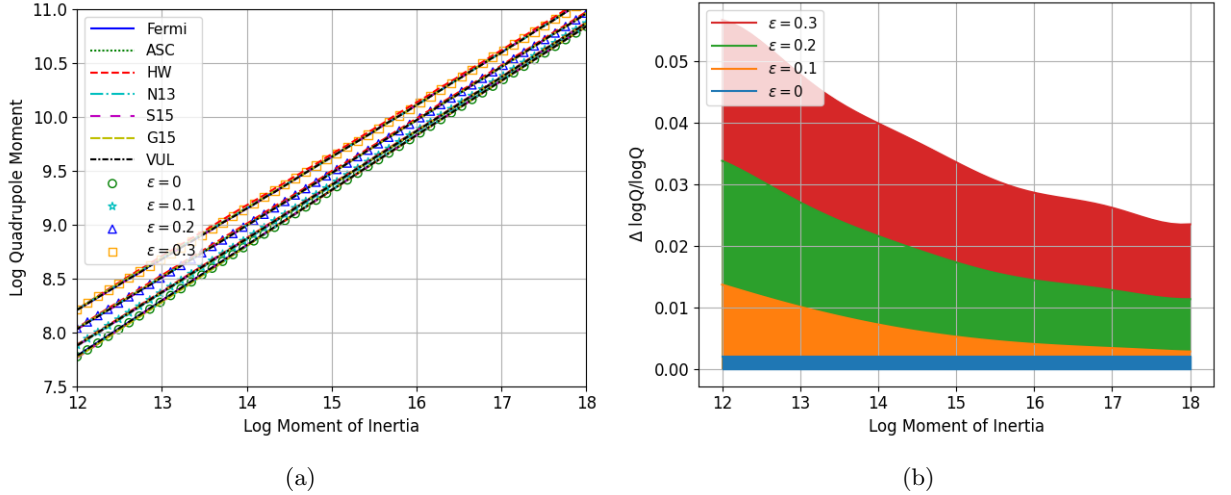


FIG. 4: Same as Figure 3, but for the $\log_{10}Q$ vs $\log_{10}I$ relation, with MAXIMUM uncertainties of 0.2%.

from the gravitational-wave signatures has been proposed [132]. Hence, we anticipate that measurements and testing of the $I - \text{Love} - Q$ relations for white dwarfs via gravitational-wave measurement, could help reveal the existence of DM inside rotating WDs, providing in-direct searches for astrophysical dark matter.

B. Ultra-Massive White Dwarf and a Formation Path for a $2.6 M_{\odot}$ Neutron Star

In contrast to the rigid rotation, a differentially rotating WD can have a much smaller κ_2 , but the star is still below the mass stripping limit [133–136]. These flattened configurations often resemble the shape of that of a ‘hamburger’ or ‘doughnut’ [137, 138]. In addition, a differentially rotating white dwarf can also

TABLE I: Stellar parameters of a $2.6 M_{\odot}$ DMRWD rotating in the Kepler rule. The density contours of both the DM and NM of this DMRWD are shown in Figure 8.

Total Mass (M_{\odot})	2.61
DM Mass (M_{\odot})	0.20
NM Mass (M_{\odot})	2.41
r_{eq1} (10^8 cm)	0.736
r_{eq2} (10^8 cm)	0.874
$\text{Log}_{10}\rho_{\text{Max1}}$ (gcm^{-3})	9.204
$\text{Log}_{10}\rho_{\text{Max2}}$ (gcm^{-3})	10.5
f_2 (Hz)	1.41
σ	0.136
κ_1	0.90
κ_2	1/3

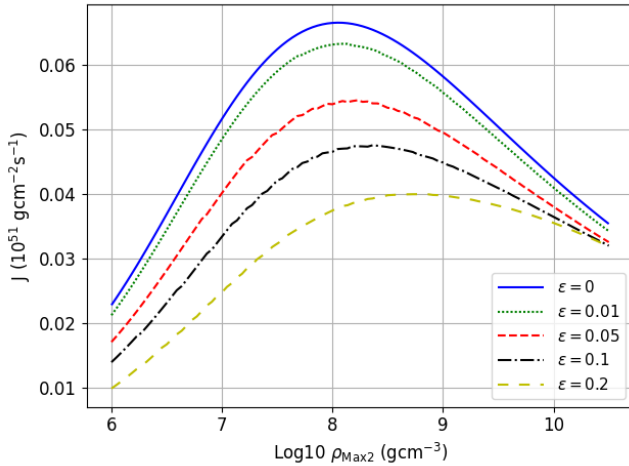


FIG. 5: Total angular momentum J versus the Log_{10} NM maximum density $\rho_{\text{Max}2}$ for slowly rotating DMRWD for different DM fractions ϵ . This is for DMRWDs rotating in the Kepler rule. The plot for rigidly rotating DMRWDs is similar to this plot and hence omitted.

support a massive component that is way beyond the traditional Chandrasekhar limit [88, 102, 139–142] up to $4 M_{\odot}$ [133]. However, such a massive configuration could not last long, as the secular instability sets in for $\sigma > 0.14$ [86, 143–146]. Here, $\sigma \equiv T/|W|$, the ratio of the total kinetic energy T to the magnitude of the total gravitational energy $|W|$, measures how stable the rotating configurations would be against secular and dynamical instabilities. The former is believed to be driven by gravitational-wave emission as the dissipation mechanism [147–151]. We find that DMRWDs rotating in the Kepler rule could reach $\sigma > 0.14$ if κ_2 is small enough. However, we consider the configurations of different DMRWDs rotating in the Kepler rule only for $\sigma < 0.14$.

An apparent effect of admixing DM to compact objects would be the alteration of its limiting mass. Leung *et al.* [13] showed that admixing DM with particle mass of 1 GeV would significantly reduce the Chandrasekhar mass, whereas admixing Fermionic DM with particle mass of around 0.1 GeV would increase the Chandrasekhar mass if one includes more than certain fractions of DM [Leung et al. 2020 submitted, 111]. Here, we study how the limiting mass of differentially rotating WDs would change in the presence of DM. In particular, we compute the total and NM masses against the maximum NM density from 10^9 g cm^{-3} to $10^{14} \text{ g cm}^{-3}$ for DMRWDs rotating in the Kepler rule, with a fixed DM mass ranging from $0.001 M_{\odot}$ to $0.2 M_{\odot}$. We show their results in Figure 6 (a) and (b) respectively. The ideal degenerate Fermi gas EOS is assumed. The apparent maximum total and NM masses at finite $\rho_{\text{Max}2}$

are due to the upper limit of $\sigma = 0.14$ we have set. To better understand the turning point, we can consider a fixed $\rho_{\text{Max}2}$. When $\rho_{\text{Max}2}$ is large, the central part of the NM becomes more compact. Although this would increase $|W|$, a larger fraction of NM rotates faster so that T also increases. If the NM mass increases indefinitely, T will eventually increase faster than $|W|$, surpassing the limit $\sigma = 0.14$. The upper limiting mass of NM in the high $\rho_{\text{Max}2}$ region that do not surpass $\sigma = 0.14$ would be reduced, creating the turning point.

We also find that the mass-radius relations for the DMRWD behave differently in the large density limit. Admixing DM into a Kepler-rotating WD does not necessarily reduce the NM mass when compared to the pure NM model (with the same $\rho_{\text{Max}2}$ and $\sigma = 0.14$). For instance, model with $0.2 M_{\odot}$ of DM could support more NM than the pure NM model for $\rho_{\text{Max}2} > 10^{11} \text{ g cm}^{-3}$. The maximum NM and total masses occur at a finite $\rho_{\text{Max}2}$. We observe that their values first decrease to a minimum, then increase as more DM is admixed. In addition, the corresponding value of $\rho_{\text{Max}2}$ becomes smaller. The increase of the maximum mass, and the shifting of the turning point could be qualitatively understood as the following: when the DM admixture is small, the NM content is reduced to accommodate the DM. The maximum total mass is therefore lowered. The reduction in the NM content reduces its contribution to T . On the other hand, the DM contributes to $|W|$. The turning point for $\rho_{\text{Max}2}$ is thus shifted to higher density, where the WD finds itself more compact and thus rotating significantly faster until it surpasses the $\sigma = 0.14$ limit.

However, the situation is different when more DM is admixed. We show in Figure 9 the equatorial rotational frequency f_2 against $\rho_{\text{Max}2}$. The strong gravitational force from the massive DM contracts the NM to make it more compact and thus faster rotating. The increase in the centrifugal force supports more NM mass, thus increasing the maximum NM and total mass. Although the NM is faster rotating, the contributions to $|W|$ from the NM and DM overcome that in $|T|$. Hence, the limit $\sigma = 0.14$ has not been surpassed even though there are more masses. However, to compensate the increase in the compactness due to an increase in the NM and DM mass, the turning point for $\rho_{\text{Max}2}$ should move to the less dense region, or else the WD would be even more compact and thus faster rotating, that the requirement $\sigma \leq 0.14$ would be violated.

The maximum possible mass of a pure NM, stably rotating WD could be $\sim 2 M_{\odot}$ [93, 102, 115]. It was pointed out that ultra-massive WD models with mass up to $3 M_{\odot}$ could be accounted for by rotating WDs having strong magnetic fields [105, 152–155]. However, the limiting value of σ has not been considered. Moreover, it was later pointed out that the ultra-massive magnetic

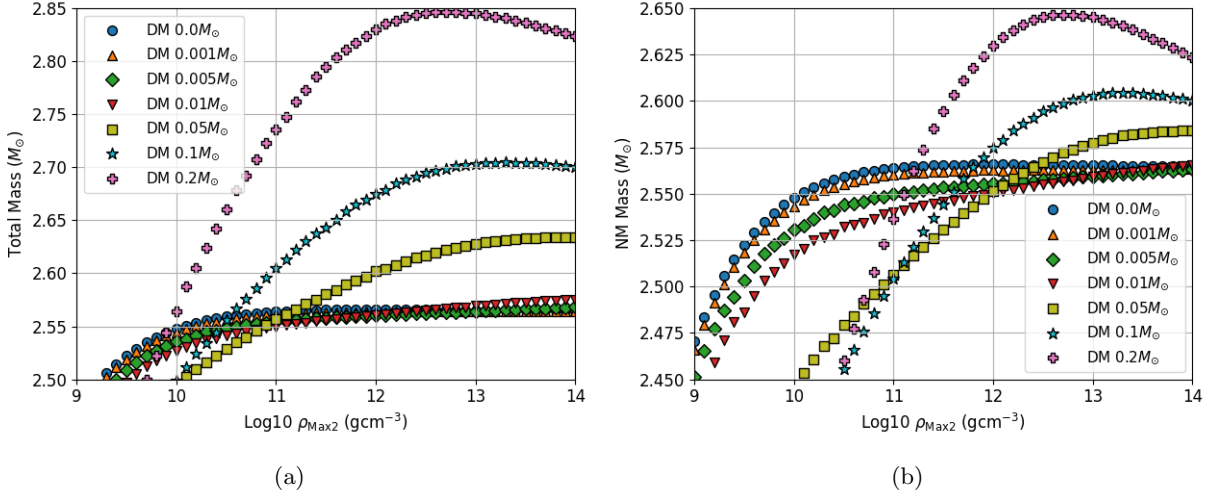


FIG. 6: (a) Total mass of DMRWDs rotating in the Kepler rule and at the secular limit $\sigma = 0.14$ vs. $\rho_{\text{Max}2}$ in the log scale. The label listed after each legend indicate the amount of DM mass (in M_{\odot}) being admixed. (b) Same as (a), but for the NM mass.

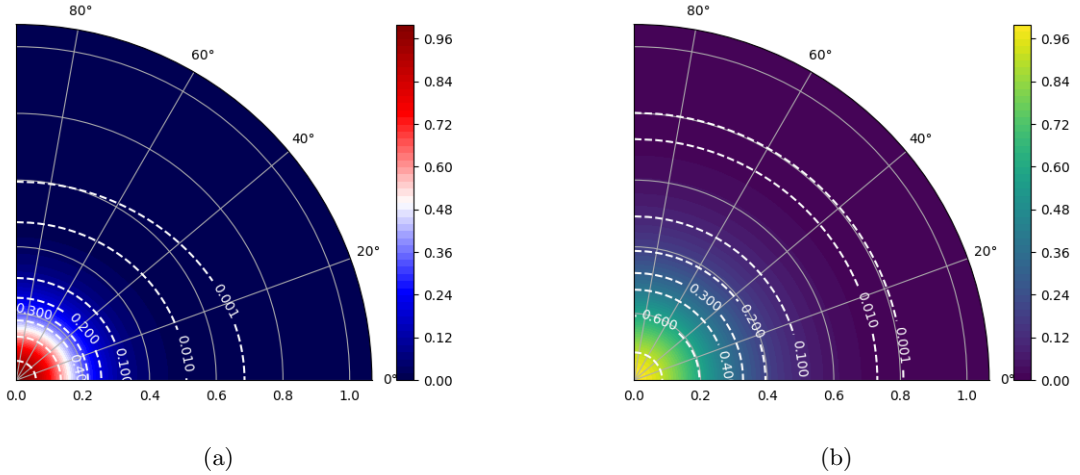


FIG. 7: (a) NM density contour for a rigidly rotating WD admixed with $0.1 M_{\odot}$ of DM. Here, the NM mass is $1.39 M_{\odot}$, $\rho_{\text{Max}2} = 10^{10.5} \text{ g cm}^{-3}$, and $\kappa_2 = 0.7$. The density is normalized to $\rho_{\text{Max}2}$. The radial distance is in unit of $r_{\text{eq}2} = 1.23 \times 10^8 \text{ cm}$. The angular frequency is $f_2 = 1.50 \text{ Hz}$ at the equatorial surface. (b) Same as (a), but for the DM density normalized to $\rho_{\text{Max}1} = 10^{9.042} \text{ g cm}^{-3}$. Here $r_{\text{eq}1} = 0.79 \times 10^8 \text{ cm}$, while $\kappa_1 = 0.99$.

WD models are far from equilibrium because the Lorentz force has been neglected [156]. In addition, the stability of such WDs is limited by pycnonuclear reactions and electron captures [106]. More recent studies give a maximum mass of $\sim 2 M_{\odot}$ [104, 157]. Here, and in Yoshida [102], the maximum mass of a Kepler rotating, pure NM WD is shown to be around $2.58 M_{\odot}$.

We show that DMRWDs rotating in the Kepler rule can support a total mass larger than that without DM admixture while being free from secular instability. For example, a DMRWD with a total mass of $2.7 M_{\odot}$ can be obtained with $0.2 M_{\odot}$ of DM admixtures. Here, we pro-

pose DMRWD as an alternative model to construct ultra-massive WDs. For example, we show in Figure 8 the NM and DM density contours of a DMRWD model that is rotating in the Kepler rule (see Table I for its stellar parameters). It has a total mass of $2.61 M_{\odot}$ and $\rho_{\text{Max}2} \approx 10^{10.5} \text{ g cm}^{-3}$, with around $0.2 M_{\odot}$ of DM admixture (which is less than 8% of the total mass). Admixing more DM would create stable DMRWDs with an even larger mass. We note that the shifting of the turning points may indicate other kinds of instabilities³. However, some of the

³ We thank the anonymous referee for pointing this out.

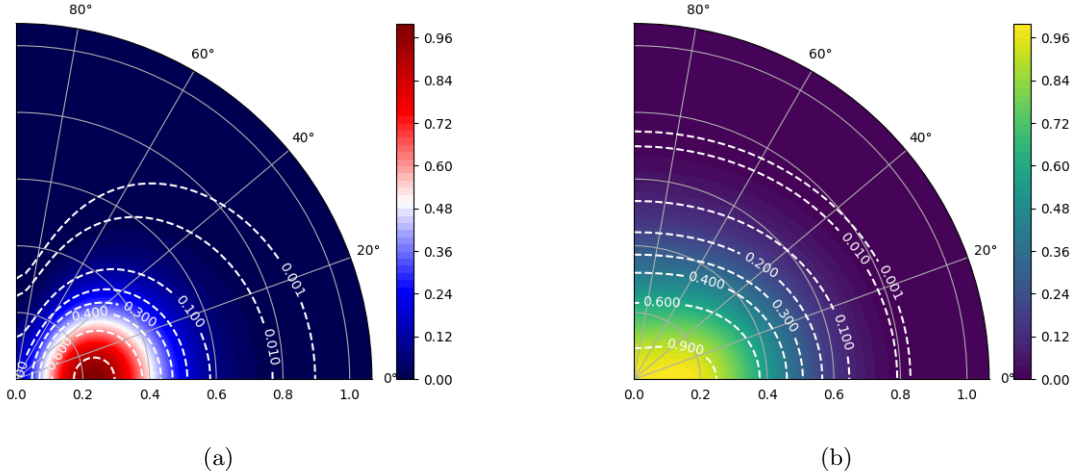


FIG. 8: Same as Figure 7, but for DMRWDs rotating in the Kepler rule, with $0.2 M_{\odot}$ of DM admixed. See Table I for details of the parameters.

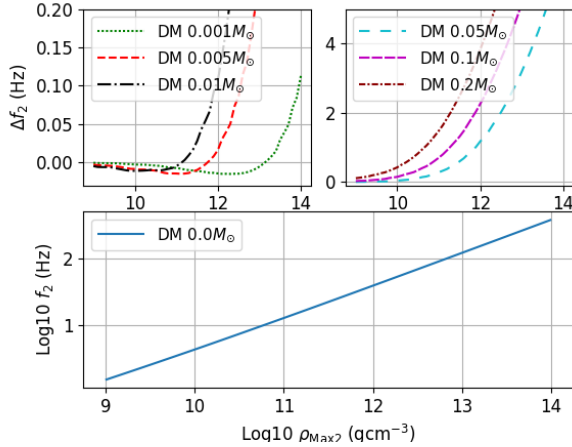


FIG. 9: Same as Figure 6, but for the equatorial angular frequency f_2 against $\rho_{\text{Max}2}$. In the lower panel, the $\log_{10} f_2$ was shown against $\rho_{\text{Max}2}$ for the pure NM model. In the upper panel, the differences between f_2 of the DM-admixed models and that of the pure NM model, Δf_2 was shown. As more DM is being admixed, Δf_2 increased and hence the NM is rotating faster.

models that we have constructed are far from the turning points, and so they should be free from these potential instabilities. The investigation of such instabilities will be an interesting future work. A final remark is that the increase of total mass at a finite $\rho_{2\text{Max}}$ with DM admixed is rotational rule dependent. For instance, we could not find the same phenomenon for j-const rotating DMRWD. The angular velocity profile could be parameterised as follow:

$$\omega(s)_2^2 \propto \frac{1}{(d^a + s^a)^b} \quad (20)$$

A systematic study on the criteria of a and b such that total mass could be increased when DM is admixed, will also be an interesting future work.

The recent detection of gravitational waves from a merger event (GW190814) between a $23 M_{\odot}$ black hole and a $2.6 M_{\odot}$ compact object [158] is puzzling since the mass of the latter falls within the black hole-neutron star mass gap [159]. To resolve this issue, models using modified gravity theories [160], quark matter EOS [161, 162], primordial black holes [163], other neutron star EOSs [164–169], rotation of the neutron star [170, 171], and combinations of these have been proposed [172]. There are also proposals based on admixture of DM in the neutron star recently [Lee et al. in prep, Leung et al. in prep, 173].

Besides the iron-core collapse of massive stars, the accretion-induced collapse (AIC) of WDs has been proposed to be another channel for forming neutron stars. AIC occurs when the mass of a WD containing an Oxygen-Neon core increases towards the Chandrasekhar limit through stable accretion from a companion object [174–176], though a binary WD merger seems to be another possible scenario [177]. The collapse is triggered by electron capture in the degenerate matter [178]. On the other hand, pycnonuclear burning is also possible in such an extremely dense core. Hence the ultimate fate of an Oxygen-Neon WD would depend on the competition between nuclear runaway and electron capture [179]. However, it was later found that the central temperature of Oxygen-Neon WDs is insufficient for explosive oxygen-neon burning [180]. Even if deflagration can take place, it fails to unbind the WD, and this directly leads to a collapse for a wide range of parameters [181–183].

Although its exact value is not clear [184], the typical

runaway density should be $\sim 10^{10} \text{ g cm}^{-3}$ [185, 186]. A critical density of a few times of $10^{10} \text{ g cm}^{-3}$ has also been adopted in multi-dimensional simulations of rotating collapse [124, 125]. We see that DMRWD can be a suitable progenitor that none of the previous rotating WD models could produce. A $2.6 M_{\odot}$ DMRWD with $\rho_{\text{Max2}} = 10^{10.5} \text{ g cm}^{-3}$ (which is just about the critical density) can be obtained, and it can be a progenitor for AIC. The collapse of such a WD can form a DM-admixed neutron star that is rapidly rotating while having a total mass of $\sim 2.6 M_{\odot}$. This gives rise to another possible explanation of the origin of the $2.6 M_{\odot}$ compact object. The AIC of a rapidly rotating WD [187, 188] and a rotating WD admixed with a point DM [184, 189] have been numerically investigated. It will be an interesting future research to analyze the possibilities of forming a $2.6 M_{\odot}$ DM-admixed rotating neutron star through multi-dimensional simulations.

C. Non-Trivial Multipole Moments for DM

A rigidly rotating DMRWD would have κ_2 greater than 0.65. Also, most of the NM is concentrated near the center. The DM remains highly spherical (see Figure 7 (b)). However, for differentially-rotating DMRWDs the NM would have a deformed structure with most of its mass shifted off from the rotational-axis (see Figure 8 (a)). The DM reacts to this and forms a deformed ellipsoidal structure (see Figure 8 (b)). Such a deformation would induce non-trivial higher-order moments for the DM component. The rapid changes of such high order moments of the DM would imprint a gravitational-wave signature when the DMRWDs approach their end-stage of evolution. Some of the possible scenarios would be AIC [94, 176, 190] and thermonuclear supernovae [191–194]. The gravitational-wave signatures from the DM component of a collapsing DMRWD were studied by Leung *et al.* [189] and Zha *et al.* [184]⁴. It will be interesting to investigate the corresponding gravitational-wave signatures of a deformed and extended component of DM in a collapsing and rotating white dwarf.

D. Limitations

We have assumed that the DM is non-rotating for two reasons. First, we are considering non-interacting DM particles. Such particles have random individual motions such as rotation with respect to the stellar center, but on average they would not have a collective motion. Second, even though accretion can change the collective motion of the DM component, it was pointed

out by Iorio [195] that a neutron star in the galaxy could accrete DM at a rate of 10^7 kg s^{-1} , which is so small that there could be no considerable amount of DM build-up at the outer envelope in the time scale of the age of the Universe. We expect that the change of the angular momentum which scales as $\sim \rho v r$ should be negligible so that the non-rotation approximation is also valid throughout the evolution from the zero-age main-sequence to the formation of DMRWD. Yet the rotational motion of DM should be considered if one studies DMRWD with a self-interacting DM EOS.

We have estimated the onset of secular instability in terms of the stability parameter $\sigma = 0.14$ for a two-fluid star, in which the DM is non-rotating. A more careful analysis would require performing perturbative analysis (numerically) on the criteria that govern the onset of secular instability in terms of σ . We leave such a study for the future. Furthermore, we have naively generalized Radau’s equation (Equation IID) to a two-fluid star. The legitimacy of such a generalization should also be verified. We will show in Appendix A that we obtain Equation IID by taking the Newtonian limit from a relativistic version of a similar differential equation that had been established for the two-fluid case.

We have discussed the maximum mass of DMRWDs in the Newtonian framework. Since the maximum rotational speed of DMRWDs is much below the speed of light, the relativistic effect of the fluid motion can be neglected. However, the relativistic gravitational effect cannot be neglected when $\rho_{\text{Max2}} \rightarrow 10^{14} \text{ g cm}^{-3}$. General relativistic corrections should be calculated for the maximum mass and $I - \text{Love} - Q$ relations. Such a future study can also help reveal whether the $I - \text{Love} - Q$ universality holds for a DMRWD under strong gravity.

We hypothesize that the $2.6 M_{\odot}$ compact object discovered in the gravitational-wave event GW190814 could be a $2.6 M_{\odot}$ DM-admixed rotating neutron star formed by the AIC of a DMRWD. However, further analysis will be needed to confirm two key assumptions used: 1. A $2.6 M_{\odot}$ DM-admixed rotating neutron star exists for some realistic neutron star EOSs, and 2. the AIC of such a DMRWD leads to a stable neutron star supported by rotation. We leave such a study for the future.

IV. CONCLUSION

The effects of admixing DM to rotating WDs are investigated. We find that the DMRWDs produce universal $I - \text{Love} - Q$ relations that deviate from those of the pure NM models, with larger deviation for a larger DM fraction. Since each WD may have a different DM fraction, the WD $I - \text{Love} - Q$ relations span bands above the universal lines for the pure NM models. This can be

⁴ They assumed a uniform rotation for the NM plus a small and spherical DM component.

used as an in-direct method to identify DMRWDs. Furthermore, admixing DM to a Kepler-rotating WD could increase its maximum possible mass to a value of larger than $2.6 M_\odot$ while still free from secular instabilities. Thus, we propose DMRWD as an alternative model for ultra-massive WDs. We also hypothesize that the $2.6 M_\odot$ compact object discovered in the gravitational-wave event GW190814 could be a DM-admixed rotating neutron star formed by the AIC of a DMRWD rotating in the Kepler rule. We also observe that the non-rotating DM reacts to the deformed NM so that it reshapes into an ellipsoidal structure, and possesses a non-trivial multipole moment, which can be detected through the gravitational waves emitted when the DMRWD is rotating and exploding as a supernova or undergoing AIC.

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Appendix A: Generalizing the Tidal Equation to the Two-Fluid Case

Radau's equation which gives the tidal love number can be obtained in an alternative way by considering the Poisson equation [196]:

$$H''(r) + \frac{2}{r}H'(r) - \left[\frac{6}{r^2} - 4\pi G\rho(r) \frac{d\rho(r)}{dP(r)} \right] H(r) = 0. \quad (\text{A1})$$

Here, $H(r)$ represents the Eulerian change of the Newtonian gravitational potential [44]. A change of the variable $Y(r) = r \frac{H'(r)}{H(r)}$ transforms the second-order equation to a first-order equation [44]:

$$Y'(r) + \frac{Y(r)}{r} + \frac{Y(r)^2}{r} = \frac{6}{r} + \frac{4\pi r^3}{m(r)} \frac{d\rho(r)}{dr}, \quad (\text{A2})$$

where we have made use of the chain rule $\frac{d\rho(r)}{dP(r)} = \frac{d\rho(r)}{dr} \frac{dr}{dP(r)}$ and the hydrostatic equation $\frac{dp(r)}{dr} = -\frac{Gm(r)\rho(r)}{r^2}$. We can define $y(r) = Y(r) - \frac{4\pi r^3}{m(r)}$ to obtain [44]:

$$y'(r) + \left[\frac{1}{r} + \frac{8\pi r^2 \rho(r)}{m(r)} \right] y(r) + \frac{y(r)^2}{r} = \frac{6}{r} - \frac{16\pi r^2 \rho(r)}{m(r)}. \quad (\text{A3})$$

Rearranging terms, and denoting $D(r) = \frac{4\pi r^3 \rho(r)}{3m(r)}$, we have:

$$ry'(r) + y(r)^2 + y + 6D(r)(y(r) + 2) - 6 = 0, \quad (\text{A4})$$

This is to be solved with initial condition $y(0) = -1$, and the tidal deformability is given as $k_2 = \frac{2-y(R)}{2[3+y(R)]}$, where R is the stellar radius. By inspection, we make a substitution $y = \eta - 1$ to transform the equation as:

$$r\eta(r)' + \eta(r)(\eta(r) - 1) + 6D(r)(\eta(r) + 1) - 6 = 0, \quad (\text{A5})$$

for which we recover Radau's equation with $\eta = \eta_2$. We notice that there are two-fluid generalizations to calculate the tidal love number of hybrid stars under the general relativistic framework [Leung et al. 2020, in prep, 197]. Therefore, the key to justifying our naive generalization of Equation IID to the two-fluid situation is to take the Newtonian limit of the relativistic version of Equation A1 that has been established for the two-fluid system. The relativistic version of the equation is [196]:

$$\begin{aligned} rY'(r) + Y(r)^2 + Y(r)e^{\lambda(r)}[1 + 4\pi r^2(p(r) - \rho(r))] + r^2Q(r) &= 0, \\ Q(r) &= 4\pi e^{\lambda(r)} \left[5\rho(r) + 9p(r) + \frac{\rho(r) + p(r)}{dp/d\rho} \right] - 6 \frac{e^{\lambda(r)}}{r^2} - (\nu'(r))^2, \end{aligned} \quad (\text{A6})$$

where we have adopted geometric units $c = G = 1$. Here, $\lambda(r)$ and $\nu(r)$ are related to the metric elements. The term $\frac{\rho(r)+p(r)}{dp/d\rho}$ should be treated carefully in the two-fluid case. Fortunately, it can be decomposed into the contributions from NM and DM if they do not interact with each other [Leung et al. 2020, in prep, 197]:

$$\frac{\rho(r) + p(r)}{dp/d\rho} = \frac{\rho_1(r) + p_1(r)}{dp_1/d\rho_1} + \frac{\rho_2(r) + p_2(r)}{dp_2/d\rho_2}. \quad (\text{A7})$$

In the Newtonian limit $e^{\lambda(r)} \approx 1$, $\nu'(r) \approx 0$ and $p \ll \rho$, we have:

$$Q(r) \approx 4\pi \left[\rho_1(r) \frac{d\rho_1(r)}{dp_1} + \rho_2(r) \frac{d\rho_2(r)}{dp_2} \right] - \frac{6}{r^2}. \quad (\text{A8})$$

We make use of the hydrostatic equation $\frac{dp_i(r)}{dr} = -\frac{Gm(r)\rho_i(r)}{r^2}$ to obtain:

$$Q(r) \approx -4\pi \frac{r^2}{m(r)} \left[\frac{d\rho_1(r)}{dr} + \frac{d\rho_2(r)}{dr} \right] - \frac{6}{r^2}. \quad (\text{A9})$$

Since the densities of DM and NM can be added as scalars, we substitute this expression into Equation A6 and use $4\pi r^2(p(r) - \rho(r)) \approx 0$ to get:

$$rY'(r) + Y(r)^2 + Y(r) - \left(4\pi \frac{r^4}{m(r)} \frac{d\rho(r)}{dr} + 6 \right) = 0, \quad (\text{A10})$$

which is just Equation A1 with $\rho = \rho_1 + \rho_2$ and $m(r)$ the total enclosed mass.

Appendix B: The Formation of DMRWD

We consider a scenario similar to that presented in Leung *et al.* [13], where the star is born with an inherent admixture of DM, contributing an extra gravitational force to the zero-age main-sequence star. We assume a spherically symmetric cloud of NM and DM having constant densities ρ_1 and ρ_2 respectively. Their individual radii could be computed by $R = (3M/4\pi\rho)^{1/3}$. In particular, we consider the situation with the DM radius R_1 being larger than that of the NM, R_2 . The total energy E (gravitational + kinetic) is:

$$E = -\left(\frac{3}{5} \frac{GM_1^2}{R_1} + \frac{3}{5} \frac{GM_2^2}{R_2} + \frac{3}{2} \frac{GM_1M_2}{R_1} - \frac{3}{10} \frac{GM_1^2R_1^2}{R_1^3} \right) + \frac{3}{2} NkT + \frac{1}{2} M_1 v_1^2. \quad (\text{B1})$$

Here, v_1 is the DM thermal velocity, $N = M_2/m_H$ is the total number of NM nuclei, and m_H is the molecular mass of hydrogen. Furthermore, we assume $M_1 \sim 0.1 M_\odot$, $M_2 \sim 10.0 M_\odot$. For a typical collapsing molecular cloud, we have $T \sim 150$ K and $\rho_2 \sim 10^8 m_H \text{ cm}^{-3}$, and hence $R_2 = 3.05 \times 10^{16} \text{ cm}$ is smaller than the Jeans radius. The maximum velocity of DM $v_{1\text{max}}$ for it to be bounded by the combined gravitational force is obtained by solving $E(R_2) = 0$. We find $v_{1\text{max}} \sim 1.27 \times 10^6 \text{ cm s}^{-1}$. For a given $v_1 < v_{1\text{max}}$, we would fix R_2 and vary R_1 to look for solution where $E < 0$. However, the most probable DM speed (assuming a Maxwell distribution) is $v_{p1} \sim 10^7 \text{ cm s}^{-1}$. To take this into account, the bounded DM fraction is given by f :

$$f = \frac{\int_0^{u_1} u^2 \exp(-u^2) du}{\int_0^\infty u^2 \exp(-u^2) du}. \quad (\text{B2})$$

Here, $u = v/v_{p1}$, and $u_1 = v_1/v_{p1}$. We take a particular $v_1 = 1.23 \times 10^6 \text{ cm s}^{-1}$, and give two sets of solutions in terms of (R_1, ρ_1) to show that the requirement of $E < 0$ could be satisfied: $(1.71 \times 10^{18} \text{ cm}, 3860 \text{ GeV/cm}^3)$ and $(6.10 \times 10^{16} \text{ cm}, 8.48 \times 10^7 \text{ GeV/cm}^3)$. The required DM density in the first set of solutions is based on the state-of-the-art simulations, which showed that the DM density at the galactic bulge could be $\sim 3600 \text{ GeV cm}^{-3}$ [198]. The required DM density in the other set of solution is much larger. However, such a value is possible near the galactic center, and values with a similar order of magnitude have been adopted in studying the effect of DM annihilation on main-sequence stars [199, 200]. In conclusion, our estimations that take into account the DM velocity dispersion show that it is possible to trap a DM of $0.1 M_\odot$ during the star-forming phase, provided that the molecular cloud is in the vicinity of the galactic center. Note that the DM and NM have different ambient densities, implying that they have different free-fall times, and in principle, the DM would not follow the trajectory of the NM.

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