Investigation of the Zenith-Angle Dependence of Muon Flux in Hong Kong, Geneva, and Leiden

Chan Ho Sang, Chan Yan Yan, Cheng Pui Yan, Fung Kin Yip, Hung Wing Hong, Lau Woon Sing
Department of Physics, the Chinese University of Hong Kong, Shatin, Hong Kong.
The Hong Kong Academy for Gifted Education
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This paper aims to determine the zenith angular dependence on muon flux I on different city. Muon total count was being measured using MIT cosmic watch mounted to a theodolite. Muon flux at 0 zenith angle I_0 is computed. The experiment was being conducted in Hong Kong, Geneva and Leiden. Experimental results shows that the angular depedence on the muon flux agreed with the widely accepted results $I = I_0 cos^n\theta$, and the value of n is closed to the widely accepeted results 1.85 ± 0.10 for measurement in Geneva and Leiden but not Hong Kong. The measurement of n with $n = 1.409 \pm 0.309$ is dissatisfactory in Hong Kong with an error to measured value percentage 21.901. The value of measured n shows a possible positive correlation with latitude in the northen hemisphere as shown from the increment of n from $n = 1.573 \pm 0.167$ in Geneva to $n = 2.226 \pm 0.213$ in Leiden, with error to measured value percentage of 10.598 and 9.578 respectively. The measurement of I_0 in unit of $10^{-3} cm^{-2} s^{-1} sr^{-1}$ gives the values of 2.244 ± 0.169 in Hong Kong, 2.626 ± 0.111 in Geneva and 2.454 ± 0.099 in Leiden, which shows no direct relations with increasing latitude, with a significant deviation reflected by the discrepancy to error ratio of 26.317 in Hong Kong, 24.724 in Geneva, 25.442 in Leiden, when compared with the widely accepted results of 8.56 ± 0.24, which is not expected. We conclude that the measurement of I_0 is dissatisfactory, experimental error still exist and remained to be eliminated.

I. INTRODUCTION AND THEORY

A. Zenith Angular dependnce on Muon Count Rate

Muons are elementary particle that believed to be the second generation of the standard model, with charge same as an electron but mass 207 times that of an electron. Cosmic muons are secondary particle generated by high energy cosmic ray, mainly protons, hitting the atmosphere with carbon or beryllium. The product mainly consist a positive pion, which decays into a muon. Muon travels at nearly speed of light, hence even with short life-time, muons could reach the earth surface due to time-dilation in earth frame.

Muons interact very weakly with matter, However, energy lost due to bombardment with atmospheric nuclei may also reduce muon energy, which in turns reduce the time dilation effect, such that muons decay without reaching the earth surface. In particular, the energy lost depends on the path length D that the muon travelled to reach the position of detection. Prashant Shukla[1] shows that the ratio of integrated muon flux I inclined at an zenith angle θ to that of zero inclination is proportional to the energy lost of muon in inclined direction to that of non-inclined direction. The energy lost ratio, is also proportional to the ratio of path length of inclinded muon and non-inclinded muon. He therefore concluded that

$$I(\theta) = I_0 D(\theta)^{-(n-1)} \tag{1}$$

Where n is a value to be determined. He also shows that the path length of muon travelled at an angle θ is given by

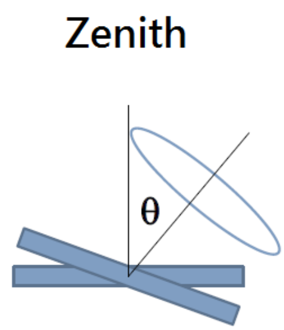

FIG. 1. Diagram showing how the zenith angle was defined, with reference to Sumanta Pal Et.al[8]

$$D(\theta) = \sqrt{\frac{R^2}{d^2} cos^2\theta + 2\frac{R}{d} + 1} - \frac{R}{d} cos\theta \tag{2}$$

Where R is the Earth radius while d is the distance from Earth surface to atmosphere. In case for flat earth, where $R <<< d$, it reduced to

$$D(\theta) = \frac{1}{cos\theta} \tag{3}$$

Which gives the relations

$$I(\theta) = I_0 cos^{n-1}\theta \tag{4}$$

The relations agrees with most of the previous results by different literature such as Mehmet Bektasoglu et.al[2], Inna Shteinbuk et.al[3] and Greider et.al[4]. The parameter I is called the directional intensity[4] or the muon flux[1]. It is defined as the number of muons, incident upon an element of area, per unit time, within an element of solid angle, while I_0 is the muon flux at 0 zenith angle.[4] It carries a unit of $cm^{-2}s^{-1}sr^{-1}$ where sr is the unit of solid angle. In particular, the value of n is not a universal constant. Sumanta Pal et.al[4] shows that the value of n depends on energy of muon, latitude and altitude. The schematic diagram on how θ was defined was included in Fig.1. This expression, however, assume a flat earth and thus the expression is a good approximation only in limited rang e of angle, in particular, Mehmet Bektasoglu et.al[2] shows that this expression is valid for $\theta <= 75^o$.

Nevertheless, this expression assume the following.

1.Muon are produced at the same radial position at the atmosphere from the center of earth

2.Muon travel in straight path D after being produced with no deviation from the original direction

The muon count rate C received using a flat detector of area A, suspended by a solid angle Ω, is then approximately the muon flux times area times solid angle. Moreover, the exponent $n-1$ can be arbitary replaced by n without loss of generality.

$$C(\theta) \approx I(\theta)A\Omega = I_0 A\Omega cos^n\theta = C_0 cos^n\theta \tag{5}$$

In this paper, we aimed to reproduce the results generated by several previous literature by using the MIT cosmic watch, we aimed to reproduce the angular dependence of muon by finding I_0 from C_0 and the value of n. We would like to compared it with the widely accepted value. Experiment will be conduct in Hong Kong, Geneva and Leiden, we aimed to find I_0 and n in those places and to find out any difference.

II. METHODOLOGY

A. Angular Dependence On Muon Count Rate

1. Experimental Apparatus

The experimental setup is shown in Fig.2 and the schematic diagram is shown in Fig.3. Two MIT cosmic watch detectors, with detection area $A = 25cm^2$ has been implemented to ensure a coincidence of muon count are to be considered as a true count, noise from the detectors can therefore be reduced. The detectors were placed back-to-back with the first detector at upper position while the second detector that determine coincidence count placing at the lower position. The combined detectors is shown in Fig.3. The lower detector is being connected to a computer for data collection. They were stuck together by plastic tape to ensure no slipping

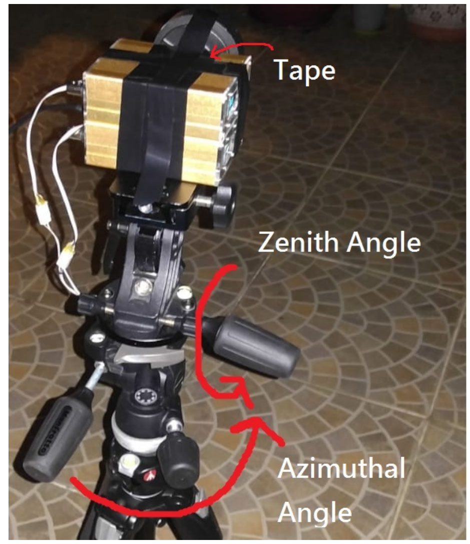

FIG. 2. Experimental Setup, showing how Zenith and Azimuthal angle is varied and the plastic tape

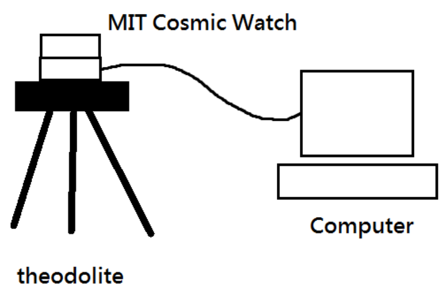

FIG. 3. Schematic Diagram of the experimental setup, showing how and which Cosmic Watch is connected to the computer

off motion. The detecting plate inside the detector were seperated as a distance of $2l = 3.5 \pm 0.1cm$. The combined detectors are being held to the theodolite, which in turn can varies zenith angle and azimuthal angle.

2. Measurement Method

Each Muon counting were taken in approximately 10 minutes. Count rate could be calculated by total count divided by the recording time interval. Erros in total number of count, time, and errors propagation in count rate can be found in the appendix.

By rotating the theodolite, muon count measurement was taken for each zenith angle in the interval of 15^o from 0^o to 75^o inclusive. The value of zenith angle θ can be read from the angle marker implemented in the theodolite. However, as Greider et.al[4] pointed that, due to Earth Magnetic field effect and the positive charge possess by the primary Pion, there could be

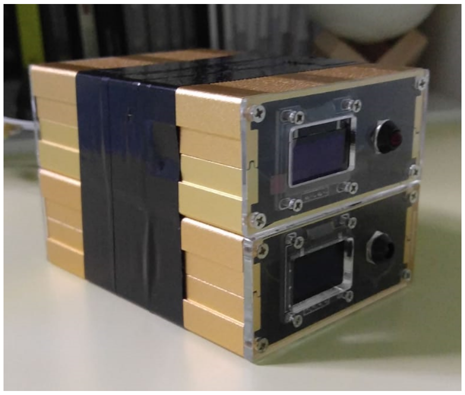

FIG. 4. Showing how two MIT cosmic watch combined by plastic tapes

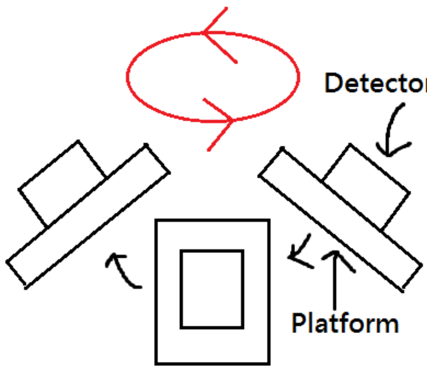

FIG. 5. Schematic diagram showing how to measure count rate in azimuthal direction for each zenith angle. The platform of theodolite and the combined detectors are drawn. The count rate is to be measured in 90^o interval in azimuthal direction and average count rate could then be taken

east-west asymmetry in muon count rate. Although he pointed that this effect is less significant at ground level, to reduce asymmetry accounting for the systematic error, for a particular zenith angle, the muon count measurement will taken for each azimuthal angle, with the interval of 90^o from 0^o to 270^o inclusive, count rate for each azimuthal angle can be obtained. Average of these 4 datum will be taken. Averaged muon count rate at a particular zenith angle C_{avg} can be obtained and used for curve fitting. The Uncertainty in zenith angle, the average muon count rate, can be found in the appendix.

After getting muon count rate for each zenith angle, a graph of muon count rate as a function of cosine to power n of zenith angle will be plotted and error bar will be include. By using OriginPro software, one could determine the value of C_0 and n with standard error calculated by the software. The value of I_0 will be calculated by dividing C_0 to the area of detector and

	C_0 Count Per Seond	I_0 $10^{-3}cm^{-2}s^{-1}sr^{-1}$	n	R^2
Hong Kong	0.165±0.012	2.244±0.169	1.409±0.309	0.915
Geneva	0.193±0.007	2.626±0.111	1.573±0.167	0.982
Leiden	0.181±0.006	2.454±0.099	2.226±0.213	0.988

TABLE I. Summarizing important experimental quantity calculated on different places

	Discrepancy	Percentage Error	Discrepancy to error ratio	Error to measured value percentage
Hong Kong	0.441	23.818	4.406	21.901
Geneva	0.277	14.990	2.773	10.598
Leiden	0.375	20.321	3.759	9.578

TABLE II. Summarizing important error parameter of n compared with accepted results $n = 1.85 \pm 0.10$ by Gredier et.al[4] calculated on different places

solid angle suspended. The area is given by the MIT cosmic watch website as $A = 25cm^2$ which is taken to be exact. Together with the sepeartion between detecting of two detector, the solid angle suspended is calculated using the formula given by Richard J. Mathar et.al[7].

$$\Omega = 4sin^{-1}(\frac{25}{25+4l^2}) \tag{6}$$

Where l is in $1.75 \pm 0.05cm$. The value is calculated as $2.943 \pm 0.068sr$. The error calculation of all the quantity can be found in the appendix.

The procedure will be taken in Hong Kong, Geneva and Leiden to compare and to discover any difference. Although no previous literature has shown that there is a strong or weak correlation between day-night variation, which in turns is the self rotational motion of Earth with respect to the sun, to the muon count rate, we proposed to measure muon in the same period of time, starting from night 9:00 pm, to prevent if any, effect due to day-night variation to muon count rate.

	Discrepancy in unit 10^{-3}	Percentage Error	Discrepancy to error ratio	Error to measured value percentage
Hong Kong	6.316	73.787	26.317	7.530
Geneva	5.934	69.320	24.724	4.220
Leiden	6.106	71.332	25.442	4.035

TABLE III. Summarizing important error parameter of I_0 compared with accepted results $I_0 = 8.56 \pm 0.24$ by Shukla, P. et.al[1] calculated on different places

III. EXPERIEMTNAL RESULTS AND DATA ANALYSIS

A. Angular Dependence On Muon Count Rate

The measurement procedure had been done in Hong Kong, Geneva and Leiden, the calculation results has been implemented in table.1 with suitable uncertainty. The best fit curve of the experimental results in these location has been included. We see that the measured data do have a good fitting with the theoretical curve for measurement in Geneva and in Leiden, which can be seen from the R^2 for the non-linear fit results which both of them are close to 1. The measured data in Hong Kong, however, do not show good fitting with theoretical curve, which can be seen from the lower $R^2 = 0.915$ and the exceptional outlier data presented in the curve.

Comparing with widely accepted results from Greider et.al[4] which is $n = 1.85 \pm 0.10$, we see that the value of n for Leiden which is $n = 2.226 \pm 0.213$, and the value of n for Geneva which is $n = 1.573 \pm 0.167$ are very close it. The slightly deviation can be explained by the non universal constant feature of the value of n, which depends also on latitude. The value of n for Hong Kong $n = 1.409 \pm 0.309$ is the most deviated from the accepted value. The discrepancy, percentage error, and discrepancy to error ratio and error to measued value percentage of n calculated on different places has been included in table 2. We could see that the quantity indicating how large error the measured value is, the error to measured value percentage, is the largest for the measurement in Hong Kong, which is 21.901 percent. While this quantity is smaller for measurement in Geneva and Leiden, which are 10.598 and 9.578 percent respectively. We do conclude that the measurement of n in Hong Kong is dissatisfactory and experimental error exist and are to be eliminated. The value of n shows a increment when the location move from Geneva to Leiden, which has a increasing latitude. We do conclude that there may be a positive correlation of the value n in Eq(5) with the latitude on Earth at northern hemisphere.

The measured I_0 in unit of $10^{-3}cm^{-2}s^{-1}sr^{-1}$ are 0.165 in Hong Kong, 0.193 in Geneva and 0.181 in Leiden. The value of I_0 first increase then decrease with latitude in the nothern hemisphere, we conclude that it shows no correlation with latitude in the nothern hemisphere. Compared with the accepted value $I_0 = 8.56 \pm 0.24$ in the same unit by Shukla, P. et.al[1]. The discrepancy, percentage error, discrepancy to error ratio, error to measured value ratio has been included in table.3. Although the error to measured value of I_0 in Hong Kong, Geneva and Leiden are 7.530, 4.220 and 4.035 percent respectively, the discrepancy to error ratio are 26.317, 24.724 and 25.442 Respectively. It indicated that the measurement of I_0 deviates significantlyfrom the widely accepted value. Althought the value of I_0

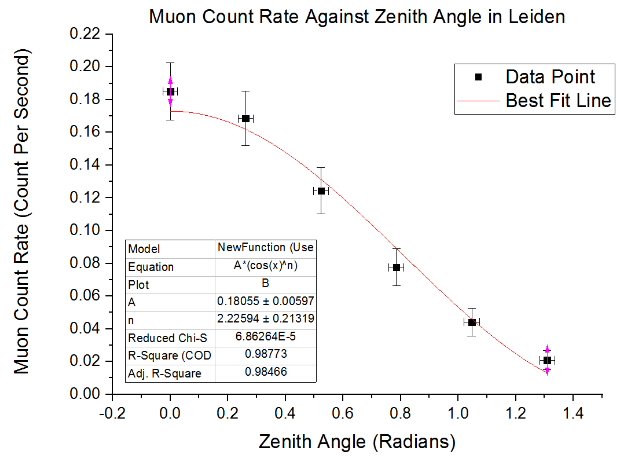

FIG. 6. Muon count rate against zenith angle in Leiden, with suitable statisitcal results included

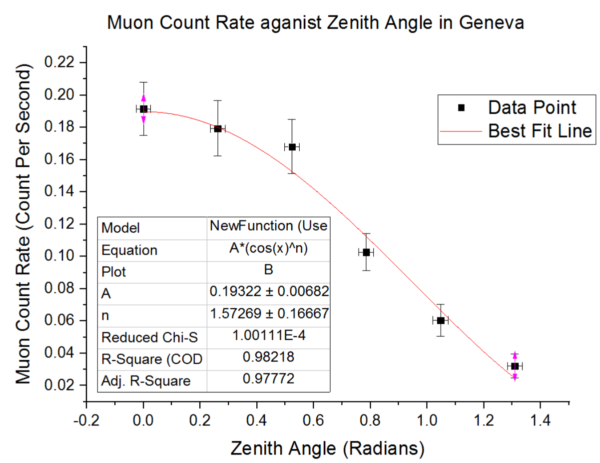

FIG. 7. Muon count rate against zenith angle in Geneva, with suitable statisitcal results included

do depends on different factors[4], we do not expect this huge amount of deviation. It indicated that the calculation of I_0 is dissatisfactory with significant error. More experimental errors exist and are to be eliminated and they are to be discussed.

IV. ERRORS ANALYSIS AND IMPROVEMENT

The experimental results on determining I_0 shows dissatisfactory results with large deviation from the widely accepeted results. Experimental error exist and are remained to be eliminated. This session summarize some of the significant experimental errors that leads to the dissatisfactory results.

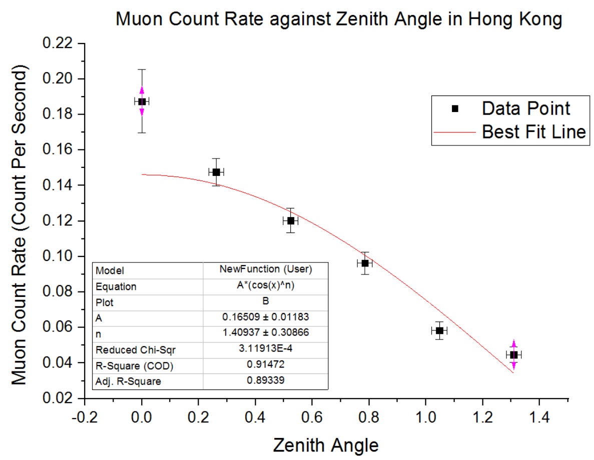

FIG. 8. Muon count rate against zenith angle in Hong Kong, with suitable statisitcal results included

A. Angular Dependence On Muon Count Rate

The major source of error in determining Angular Dependence on Muon Count Rate included the following.

Limited number of count

According to poisson distribution, the relative error of the muon total count is given by $\frac{1}{\sqrt{N}}$, hence to minimize the measurement error, large number of total count is required. However, due to limited time provided, we could only measure total count in the interval of 10 minutes, and hence we could only get at most get around hundred of count. The limited number of count greatly inrease the measurement error.

Moreover, the limited time of data recording may also contribute error to the count rate since the calculated count rate may not eventually reach the steady value in this limited amout of time. To improve this situation, we proposed to take data with longer period of time and to get more count.

Finite solid angle suspended by combined detectors

Two detectors were placed back to back and coincidence count is to be noticed as a true signal, however, the distance between the two counter is finite and hence the suspended solid angle is large, it would then included muon count for a finite range of solid angle even the detector is set to measure muon count at specific zenith angle. In other words, the count is over-estimated.

To improve this situdation, we proposed to increase the distance between the two counter by inserting durable material and to use a better theodolite that could sustain a larger mass on top of it.

Non straight path length travelled by muon

In the derivation of equation (4), we assume that the muon travel staright path length $D(\theta)$ to reach the detector, in practice, the muon will travel zig-zag path due to collision with nuclei. Hence the actual path length is longer than that of the theoretical model, hence the muon count rate is less than that of theoretical value.

To improve this situation, we proposed to improve the model deriving equation (4) to include scattering motion of muon.

Fake coincidence signal

Although coincidence signal is considered as true count to reduce noises. It is possible that both detectors record a fake count from noise simutaenously. The probability of this situation depends on the quality of detecting plate. Therefore to improve this situation, we propose to use a detector with detecting plate with better quality.

Limited precision on angle measurement

As stated in the appendix, the standard error in angle measurement is about 2.5^o, which is large. It is the limited quality on the angle measurement tools of the theodolite that give rise to the large standard error.

To improve this situation, we proposed to use a theodolite with a more precise angle measurement device.

Rare but possible muon travelled at high energies

The formula (4) is a good approximation for zenith angle $\theta < 75^o$ and for most of the range of energy of muon. However, high energy muon can also be produced and reach the Earth Surface. In that case, the angular dependence of the muon flux should be modified.

To improve this situation, we proposed to use a modified angular dependence equation for curve fitting and data analysis.

Muon not generated at the radial position on atmosphere from center of Earth

In the derivation in (4), we assumed that muon are generated at the same radial position at atmosphere from the center of Earth. However in practice, it is not the case. The Pion decay may not all happen at the same radial position. It changes the path length that muon needed to travel, which in turn changes the muon count rate.

To improve this situation, we proposed that we can improve the model that derive equation (4).

Varying atmospheric condition

In performing the experiment, we are not able to held the atmospheric condition to remain constant overtime. However, previous studies by A Maghrabi et.al[5][6] shows that there is a correlation between muon count rate and various atmospheric parameter, such as temperature, pressure and humidity. The change in atmospheric condition may contribute error to our experiment. However, we concluded that this is an intrinsic property of the atmosphere which are not able to be altered and eliminated.

Non-flat Earth

The equation (4) depends on flat earth approximation such that the equation fails when angle is large. In realistic case, the Earth is obviously not flat. To included this factor, we propose to use a more realistic angular dependence equation for curve fitting and data analysis to reduce experimental error.

V. CONCLUSION

From the investigating angular dependence on muon flux in Leiden, Hong Kong and Geneva experiment, we conclude that the angular dependence follows the widely accepted relations $I = I_0 cos^n\theta$. The value of measured n in Hong Kong is dissatisfactory with error the measured value percentage 21.901. The value of measured n shows a possible positive correlation with latitude in the northen hemisphere as shown from the increment of n from $n = 1.573 \pm 0.167$ in Geneva to $n = 2.226 \pm 0.213$ in Leiden, with error to measured value percentage of 10.598 and 9.578 respectively.

The measurement of I_0 in unit of $10^{-3}cm^{-2}s^{-1}sr^{-1}$ gives the value of $= 2.244 \pm 0.169$ in Hong Kong, 2.626 ± 0.111 in Geneva and 2.454 ± 0.099 in Leiden. The value of I_0 shows no direct relation with latitude in the northen hemisphere. Nevertheless, the measurement is dissatisfactory with the discrepancy to error ratio of 26.317, 24.724 and 25.442 respectively. We conclude that the experiment is dissatisfactory for measurement of n in Hong Kong and I_0 for Hong Kong, Geneva and Leiden. Significant experimental error exist and are to be eliminated.

A. Angular Dependence on Muon Count Rate

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VII. APPENDIX:ERROR ANALYSIS ON EXPERIMENTAL PARAMETER

The standard error in total count N is given by the poisson distribution as

$$\delta N = \sqrt{N} \tag{7}$$

The standard error in single time measurement is $0.001/2 = 0.0005$, but since the time elapsed is calculated by ending time minus starting time, the standard error in time elapsed t is then

$$\delta t = 0.001s \tag{8}$$

The standard error in angle of measurement is given by the least signficant digit of angle divided by half

$$\delta\theta = \frac{3^o}{2} = 1.5^o \tag{9}$$

The standard error of count rate C is given by error of propagation formula

$$\frac{\delta C}{C} = \sqrt{(\frac{\delta N}{N})^2 + (\frac{\delta t}{t})^2} \tag{10}$$

The average count rate is calculated by averaging count rate measured for 4 azimuthal angle. The standard error of averaged count rate at each zenith angle C_{avg} is given by error of propagation formula.

$$\delta C_{avg} = \sqrt{\sum_{i=1}^{4} \frac{C_i}{4}} \tag{11}$$

Where C_i, $i = 1, 2, 3, 4$ are the 4 count rate measured for 4 azimuthal angle.

The standard error of l for which $2l = 3.5 \pm 0.1cm$ is given by

$$\delta l = \frac{0.1}{2} = 0.05 \tag{12}$$

The standard error of solid angle is given by error of propagation formula.

$$\delta\Omega = \frac{800l}{\sqrt{1 - \frac{625}{(25+4l^2)^2}}(25 + 4l^2)^2}\delta l \tag{13}$$

The standard error of muon flux at 0 zenith angle is given by error of propagation formula.

$$\frac{\delta I_0}{I_0} = \sqrt{(\frac{\delta C_0}{C_0})^2 + (\frac{\delta\Omega}{\Omega})^2} \tag{14}$$

The error in C_0 and n are calculated by the curve fitting software.

Since the area of detection A is given from the MIT cosmic watch website, we would consider it as exact without error.