CSP

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1 CSP Operators

Definition 1.1.1: Nondeterministic Composition

A Theory of Communicating Sequential Processes P568 / P9
If P and Q are processes, the combination $P \sqcap Q$ is a process that behaves exactly like P or like Q, but the choice between them is wholly nondeterministic: It is made autonomously by the process (or by its implementer) and cannot be influenced or even observed by the environment. Thus $P \sqcap Q$ can do (or refuse to do) everything that P or Q can do (or refuse to do)

$$P \xrightarrow{s} R \vee Q \xrightarrow{s} R \implies (P \sqcap Q) \xrightarrow{s} R$$

The process determined by this law is simply

$$P \sqcap Q = P \cup Q$$

Definition 1.1.2: Parallel Composition

A Theory of Communicating Sequential Processes P572 / P13
The combination $(P||Q)$ is intended to behave like both P and Q, progressing in parallel. Thus an event can occur only when both P and Q are able to participate in it simultaneously. The same is therefore true of sequences of events:

$$P \xrightarrow{s} P' \text{ and } Q \xrightarrow{s} Q' \implies (P||Q) \xrightarrow{s} (P'||Q')$$

The process determined by this law is defined as

$$(P||Q) = \{(s, X \cup Y)|(s, X) \in P \text{ and } (s, Y) \in Q\}$$

Definition 1.1.3: Conditional Composition

A Theory of Communicating Sequential Processes P573 / P14
The process $P \square Q$ behaves either like P or like Q, but the choice can be influenced by the environment on the very first step.
If the environment offers an event a that is possible for P but not Q, then P is selected.
If the environment offers an event that is possible for both processes, the selection between them is nondeterminate and the environment doesn't get a second chance to influence it. Thus

$$P \xrightarrow{\langle a \rangle s} R \vee Q \xrightarrow{\langle a \rangle s} R \implies (P \square Q) \xrightarrow{\langle a \rangle s} R$$

Before occurance of the first event, P and Q may progress independently:

$$P \xrightarrow{\tau} P' \text{ and } Q \xrightarrow{\tau} Q' \implies (P \,\Box\, Q) \xrightarrow{\tau} (P' \,\Box\, Q')$$

The process determined by these laws is defined

$$\begin{aligned}(P \,\Box\, Q) =& \{(\tau, X) \mid (\tau, X) \in P \text{ and } (\tau, X) \in Q\} \\ & \cup \{(s, X) \mid s \neq \tau \text{ and } ((s, X) \in P \vee (s, X) \in Q)\}\end{aligned}$$

Definition 1.1.4: Concealment

Let b be an event that is regarded as an internal operation of a process P. Define $P \backslash b$ s the process that behaves like P except that every occurrence of b is removed from its traces. It therefore satisfies the law

$$P \xrightarrow{s} Q \implies (P \backslash b) \xrightarrow{s \backslash b} (Q \backslash b)$$

where $s \backslash b$ is formed from s by removing all occurrances of b
The required definition is

$$\begin{aligned}P \backslash b =& \{(s \backslash b, X) \mid (s, X \cup \{b\}) \in P\} \\ & \cup \{((s \backslash b)t, X) \mid \forall n.(sb^n, \emptyset) \in P \text{ and } (t, X) \in \text{CHAOS}\}\end{aligned}$$

where sb^n is s followed by n occurrences of b

1.2 List of processes

- **Nondeterministic Composition**

$$P \xrightarrow{s} R \vee Q \xrightarrow{s} R \implies (P \sqcap Q) \xrightarrow{s} R$$

The process determined by this law is simply

$$P \sqcap Q = P \cup Q$$

The proof tree for this is:

$$P \sqcap Q \xrightarrow{\tau} P \qquad P \sqcap Q \xrightarrow{\tau} Q$$

The equivalent ACP expression is:

$$\mathscr{I}(P \sqcap Q) = \tau.\mathscr{I}(P) + \tau.\mathscr{I}(Q)$$

- **Parallel Composition**

$$P \xrightarrow{s} P' \text{ and } Q \xrightarrow{s} Q' \implies (P||Q) \xrightarrow{s} (P'||Q')$$

The process determined by this law is defined as

$$(P||Q) = \{(s, X \cup Y)|(s, X) \in P \text{ and } (s, Y) \in Q\}$$

The proof tree for this is:

$$\frac{P \xrightarrow{\sigma} P' \quad (\sigma \notin S)}{P||Q \xrightarrow{\sigma} P'||Q} \qquad \frac{P \xrightarrow{\sigma} P' \quad (s \in S)}{P||Q \xrightarrow{\sigma} P'||Q'} \qquad \frac{Q \xrightarrow{\sigma} Q' \quad (\sigma \notin A)}{P||Q \xrightarrow{\sigma} P||Q'}$$

The equivalent ACP expression is:

$$\mathscr{I}(P||Q) =$$

- **Conditional Composition**

$$P \xrightarrow{\tau} P' \text{ and } Q \xrightarrow{\tau} Q' \implies (P \,\Box\, Q) \xrightarrow{\tau} (P' \,\Box\, Q')$$

The process determined by these laws is defined

$$\begin{aligned}(P \,\Box\, Q) =& \{(\tau, X) \mid (\tau, X) \in P \text{ and } (\tau, X) \in Q\} \\ & \cup \{(s, X) \mid s \neq \tau \text{ and } ((s, X) \in P \vee (s, X) \in Q)\}\end{aligned}$$

As defined in An improved Failures model for communicating processes, Roscoe:

$$\mathcal{D}(P \,\Box\, Q)_e = \mathcal{D}(P)_e \cup \mathcal{D}(Q)_e$$

$$\begin{aligned}\mathcal{F}(P \,\Box\, Q)_e =& \{(\tau, X) \mid (\tau, X) \in \mathcal{F}(P)_e \cap \mathcal{F}(P)_e\} \\ & \cup \{(s, X) \mid s \neq \tau \text{ and } ((s, X) \in \mathcal{F}(P)_\rho \cup \mathcal{F}(P)_e)\} \\ & \cup \{(s, X) \mid s \in \mathcal{D}(P \,\Box\, Q)_e\}\end{aligned}$$

and For divergent sets and failure sets respectively

The proof tree for this is:

$$\frac{P \xrightarrow{a} P'}{P \,\Box\, Q \xrightarrow{a} P'} \qquad \frac{P \xrightarrow{\tau} P'}{P \,\Box\, Q \xrightarrow{\tau} P' \,\Box\, Q} \qquad \frac{Q \xrightarrow{a} Q'}{P \,\Box\, Q \xrightarrow{a} Q'} \qquad \frac{Q \xrightarrow{\tau} Q'}{P \,\Box\, Q \xrightarrow{\tau} P \,\Box\, Q'}$$

The equivalent ACP Expression is:

$$\mathscr{I}(P \,\Box\, Q)$$

- **Concealment**

$$P \xrightarrow{s} Q \implies (P \backslash b) \xrightarrow{s\backslash b} (Q \backslash b)$$

where $s\backslash b$ is formed from s by removing all occurrences of b

The required definition is

$$\begin{aligned} P\backslash b = &\{(s\backslash b, X) \mid (s, X \cup \{b\}) \in P\} \\ &\cup \{((s\backslash b)t, X) \mid \forall n.(sb^n, \emptyset) \in P \text{ and } (t, X) \in \text{CHAOS}\} \end{aligned}$$

where sb^n is s followed by n occurrences of b

The proof tree is:

$$\frac{P \xrightarrow{\sigma} P' \quad (\sigma \notin S)}{P\backslash A \xrightarrow{\sigma} P'\backslash A} \qquad \frac{P \xrightarrow{s} P' \quad (s \in S)}{P\backslash A \xrightarrow{\tau} P'\backslash A}$$

The equivalent ACP Expression is:

$$\mathscr{I}(P\backslash A) = \tau_A(\mathscr{I}(P))$$