An Encoding of the CSP Stopping operator to ACP$_\tau$

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1.1 Prerequisites

Definition 1.1.1: Subsets of A

We define some subsets of A which we will use in our encodings

- $A_0 \subseteq A$ is the set of actions that actually get used in processes
- $H_0 = A - A_0$ is the set of working space operators, or any other action that doesn't get used
- $H_1 = A_0 \uplus \{\texttt{first}, \texttt{next}, \texttt{left}, \texttt{right}\}$ is the set of actions, plus some working operators

In general, $A_0 \subseteq H_1 \subseteq A$.
Note that the silent step is not defined in A, and we will define A_τ to be $A \cup \{\tau\}$

Definition 1.1.2: F1

Define a function $f_1 : A \to A$ where

$$f_1(a_{\texttt{first}}) = a_{\texttt{first}}$$
$$f_1(a_{\texttt{next}}) = a$$

Definition 1.1.3: Communications

Define Communications where

$$a|\texttt{first} = a_{\texttt{first}}$$
$$a|\texttt{next} = a_{\texttt{next}}$$
$$a|\texttt{left} = a_{\texttt{left}}$$

We define triggering as the same as external choice

Definition 1.1.4: Triggering

The operator $\Gamma(P)$ turns a trace of a process P, $a.b.c. \ldots$ into the trace

$$a_{\texttt{first}}.b.c. \ldots$$

and is defined as

$$\Gamma(P) = \rho_{f_1}[\partial_{H_1}(p||\texttt{first}(\texttt{next}^\infty))]$$

We also define an operator which will tag every action in a process with the identifier of "`left`".

Definition 1.1.5: Left-Tagging

The operator $\Phi(P)$ turns a trace of a process P, $a.b.c$ into the trace

$$a_{\texttt{left}}.b_{\texttt{left}}.c_{\texttt{left}}$$

and is defined as

$$\Phi(P) = \partial_{H_1}\left(P || (\texttt{left})^{\infty}\right)$$

1.2 Translating the CSP Stopping Operator

The stopping operator $\triangle$ is defined with the following rules:

$$\frac{P \xrightarrow{\alpha} P'}{P \triangle Q \xrightarrow{\alpha} P' \triangle Q} \qquad \frac{Q \xrightarrow{\tau} Q'}{P \triangle Q \xrightarrow{\tau} P \triangle Q'} \qquad \frac{Q \xrightarrow{a} Q'}{P \triangle Q \xrightarrow{a} Q'}$$

In other words, we can take an external choice from P without interrupting the operator, in addition to internal choices from Q. However, the moment an external choice is made from Q, the process will then never return to P.

For our encoding, we take $H_1 = \{\texttt{first}, \texttt{next}, \texttt{left}, \texttt{origin}, \texttt{split}\}$ as defined in 1.1.1. We then modify Definition 1.1.3 to include additional communications for the operators `origin` and `split`

Definition 1.2.1: Communications Modified

In addition to the communications defined in 1.1.3:

$$a|\texttt{first} = a_{\texttt{first}} \quad a|\texttt{next} = a_{\texttt{next}} \quad a|\texttt{left} = a_{\texttt{left}}$$

We define communications for the operators `origin` as follows:

$$a_{\texttt{left}}|\texttt{origin} = a_{\texttt{post}} \quad a_{\texttt{first}}|\texttt{split} = a_{\texttt{post}}$$

Definition 1.2.2: Functions Modified

In addition to the function f_1 defined in 1.1.2,

$$f_1(a_{\texttt{first}}) = a_{\texttt{first}} \quad f_1(a_{\texttt{next}}) = a$$

We define a compatibility function to prevent issues with transitivity. Let $f_2 : A \to A$ where

$$f_2(a_{\texttt{post}}) = a$$

We can now define an encoding of the CSP Stopping operator $\triangle$ in ACP_τ. We start off with a new process, which we will call σ. This is defined as the process

$$\sigma = (\texttt{origin})^\infty.(\texttt{split})^\infty$$

Or, visualised as a process graph:

$$\begin{tikzcd} \circ \arrow[r, "\texttt{origin}"] & \bullet \arrow[loop above, "\texttt{origin}"] \arrow[r, "\texttt{split}"] & \bullet \arrow[loop above, "\texttt{split}"] \end{tikzcd}$$

From this, an encoding can be written in the following way:

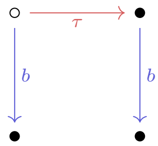
$$\mathscr{T}(P \triangle Q) = \partial_{H_0}\left(\rho_{f_2}\left[(\Phi(\mathscr{T}(P))||\sigma)||\Gamma(\mathscr{T}(Q))\right]\right)$$

Similarly to external choice, this is also Rooted Branching Bisimilar.

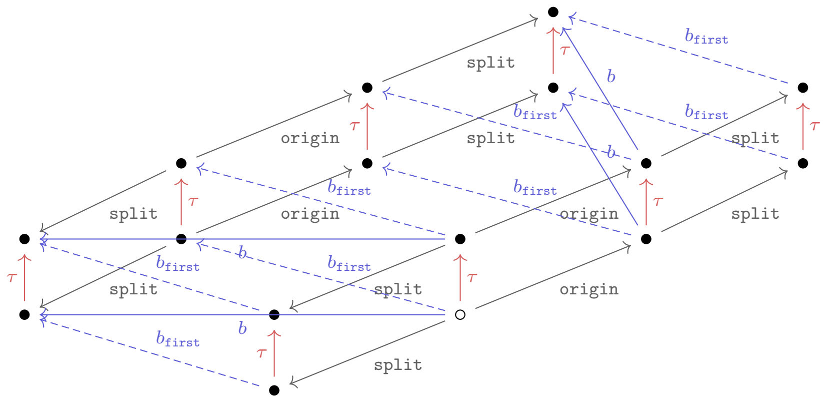
A counterexample to the encoding being Strongly Bisimilar is with the trivial example

$$P = \tau,\ Q = b$$

This should yield the following process graph:



However, it yields the following



Which reduces down to: