

# Modelling Concurrent Systems Notes

Made by Leon :)

## 1 Process Algebras

### Definition 1.1.1: ACP, CCS, CSP

The syntax of ACP, CCS, and CSP is defined as:

Operation	ACP	CCS	CSP
Termination	$\epsilon$	0	STOP
Deadlock	$\delta$		
Action	$a$		
Sequential Composition	$P.Q$		
Action Prefixing		$a.P$	$a \rightarrow P$
Alternative Choice	$P + Q$	$P + Q$	
External Choice			$P \sqcap Q$
Internal Choice			$P \sqcap Q$
Parallel Composition	$P \parallel Q$	$P \mid Q$	$P \parallel_A Q$
Restriction	$\partial_H(P)$	$P \setminus a$	
Abstraction	$\tau_I(P)$		$P/a$
Relabelling		$P[f]$	$P[f]$

### Differences between ACP, CCS, and CSP

- **Action:** CCS and CSP require Action Prefixing, while ACP can perform sequential composition on any process. This also requires CCS and CSP processes to feature the inaction 0/STOP, while ACP is not restricted to this.
- **Choice:** ACP and CCS have an operator which can perform both External and Internal Choice. CSP Differentiates internal choices from external ones, and internal actions within  $\sqcap$  do not count as a choice in CSP.
- **Parallel Composition:**
  - ACP actions follow a communication function to decide what to synchronise, i.e.  $\gamma(a, b)$
  - CCS actions can only synchronise with its conjugate counterpart, i.e.  $a$  and  $\bar{a}$
  - CSP actions can only synchronise over the same action, i.e.  $a$  and  $a$
- **Restriction, Abstraction, Relabelling:**
  - Relabelling just doesn't exist in base ACP, lol
  - CCS combines communication and abstraction into one step - every synchronisation results in a  $\tau$ .
  - CSP combines Parallel Composition and Restriction into one step, as CSP Parallel Composition doesn't feature left-over Left Merges.

### Definition 1.1.2: The GSOS Format

General Structured Operational Semantics (GSOS) operations are compositional.

#### Rules of GSOS

- Its source has the form  $f(x_1, \dots, x_{ar(f)})$  with  $f \in \Sigma$  and  $x_i \in V$
- The left hand sides of its premises are variables  $x_i$  with  $1 \leq i \leq ar(f)$
- The right hand sides of its positive premises are variables that are all distinct, and that do not occur in its source
- Its target only contains variables that also occur in its source or premises

#### GSOS Semantics of ACP

$$\begin{array}{c}
 (a.P) \xrightarrow{a} P \quad P + Q \xrightarrow{a} P \quad P + Q \xrightarrow{a} Q \\
 \\
 \frac{P \xrightarrow{a} P'}{P \parallel Q \xrightarrow{a} P' \parallel Q} \quad \frac{P \xrightarrow{a} P' \quad Q \xrightarrow{b} Q' \quad a|b=c}{P \parallel Q \xrightarrow{a} P' \parallel Q'} \\
 \\
 \frac{Q \xrightarrow{a} Q'}{P \parallel Q \xrightarrow{a} P \parallel Q'} \quad \frac{P \xrightarrow{a} P' \quad (\alpha \notin A)}{\partial_H(P) \xrightarrow{a} \partial_H(P')} \quad \frac{\langle S_X \mid S \rangle \xrightarrow{a} P'}{\langle X \mid S \rangle \xrightarrow{a} P'}
 \end{array}$$

#### GSOS Semantics of CSP

$$\begin{array}{c}
 (a \rightarrow P) \xrightarrow{a} P \quad P \sqcap Q \xrightarrow{\tau} P \quad P \sqcap Q \xrightarrow{\tau} Q \\
 \\
 \frac{P \xrightarrow{a} P'}{P \sqcap Q \xrightarrow{a} P'} \quad \frac{P \xrightarrow{\tau} P'}{P \sqcap Q \xrightarrow{\tau} P' \sqcap Q} \quad \frac{Q \xrightarrow{a} Q'}{P \sqcap Q \xrightarrow{a} Q'} \\
 \\
 \frac{Q \xrightarrow{\tau} Q'}{P \sqcap Q \xrightarrow{\tau} P \sqcap Q'} \quad \frac{P \xrightarrow{a} P'}{f(P) \xrightarrow{f(\alpha)} f(P')} \quad \frac{P \xrightarrow{a} P' \quad (\alpha \notin A)}{P \parallel_A Q \xrightarrow{a} P' \parallel_A Q} \\
 \\
 \frac{P \xrightarrow{a} P' \quad Q \xrightarrow{a} Q' \quad (a \in A)}{P \parallel_A Q \xrightarrow{a} P' \parallel_A Q'} \quad \frac{Q \xrightarrow{a} Q' \quad (\alpha \notin A)}{P \parallel_A Q \xrightarrow{a} P \parallel_A Q'} \\
 \\
 \mu p. P \xrightarrow{\tau} P[\mu p. P/p]
 \end{array}$$

### Definition 1.1.3: CSP Expansion Theorem

Let  $P := \sum_{i \in I} \alpha_i P_i$  and  $Q := \sum_{j \in J} \beta_j Q_j$ . Then

$$P \mid Q = \sum_{i \in I} \alpha_i (P_i \mid Q) + \sum_{j \in J} \beta_j (P \mid Q_j) + \sum_{\substack{i \in I, j \in J \\ \alpha_i = \bar{\beta}_j}} \tau. (P_i \mid Q_j)$$

Any guarded CCS expression can be written into a bisimulation equivalent CCS expression of the form  $\sum_{i \in I} \alpha_i P_i$ . This is called **head normal form**

## 2 Semantics and shit like that

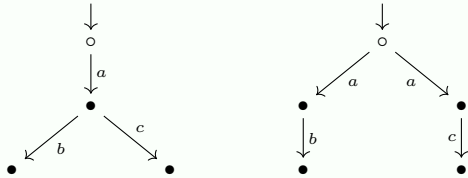
### Definition 2.0.1: Trace Semantics

- **Completed Trace:** A start to finish trace of a process.
- **Partial Trace:** From the start of a process to any point, including the end. Clearly,  $CT(P) \subseteq PT(Q)$
- **Strong vs Weak:** Weak PT and CT means that two processes are equivalent with all instances of  $\tau$  omitted.
- **Infinite Trace Semantics:** Differs from different types of divergence. Stronger than CT and PT

### Definition 2.0.2: Bisimulation Semantics

- True Bisimulation
- Weak Bisimilarity
- Branching Bisimilarity
- Rooted Branching Bisimilarity

### Definition 2.0.3: The GOAT of Process algebra



### Definition 2.0.4: Failure Semantics

### Definition 2.0.5: Consistent Colouring

### Definition 2.0.6: Safety

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