General Topology Notes

Made by Leon :) *Note: Any reference numbers are to the lecture notes*

1 Topological Spaces and Examples

Definition 1.1: Topological Space

A **topological space** is a pair $(X, \mathcal{T})$, where X is a nonempty set, and $\mathcal{T}$ is a collection of subsets of X which satisfies:

a) $\emptyset \in \mathcal{T}$ and $X \in \mathcal{T}$

b) if $U_\lambda \in \mathcal{T}$ for each $\lambda \in \Lambda$ (where Λ is some indexing set), then $\bigcup_{\lambda \in \Lambda} U_\lambda \in \mathcal{T}$

c) if $U_1, U_2 \in \mathcal{T}$, then $U_1 \cap U_2 \in \mathcal{T}$

The collection $\mathcal{T}$ is called the **topology** of the topological space, and the members of $\mathcal{T}$ are called the **open sets** of the topology

Example 1.7: Euclidean Spaces

Let $\mathbb{R}^n$ enote the n-dimensional Euclidean vector space with elements $x = (x_1, x_2, \dots, x_n)$ and $x_i \in \mathbb{R}$, and let

$$|x| = \sqrt{\sum_{i=1}^{n} x_i^2} \geq 0$$

be the length of x. ($\mathbb{R}^1 = \mathbb{R}$ is the real line). A subset U of $\mathbb{R}^n$ is **open (for the usual topology)** iff for each $a \in U$ there exists an $r > 0$ such that

$$|x - a| < r \implies x \in U.$$

The collection of open sets thus defined is called the **usual topology** on $\mathbb{R}^n$. Note that open balls $B(y, \rho) = \{x \in \mathbb{R}^n : |x - y| < \rho\}$ are open sets under this definition.

Example 1.8: Metric Spaces

A **metric space** (X, d) is a nonempty set X together with a function $d : X \times X \to \mathbb{R}$ with the following properties:

a) $d(x, y) \geq 0$ and $d(x, y) = 0 \iff x = y$

b) $d(x, y) = d(y, x)$

c) $d(x, y) \leq d(x, z) + d(z, y)$ (Triangle Inequality)

The function d is called the **metric**.

Let (X, d) be a metric space, x be a point in X, and $r > 0$. The **open ball** with center x and radius r is defined by

$$B(x, r) = \{y, , \in X : d(x, y) < r\}.$$

A subset U of X is **open (in the metric topology given by d)** iff for each $a \in U$ there is an $r > 0$ such that $B(a, r) \subseteq U$. Just like euclidean spaces, open balls are open in this sense.

Example 1.0.1: Other Topologies and Metrics

If $(X, \mathcal{T})$ is a topological space, and if X admits a metric whose metric topology is precisely $\mathcal{T}$, then we say that $(X, \mathcal{T})$ is **metrisable**

- Euclidean spaces with their usual topologies are metrisable.

1.9) The **Discrete Topology** is the topology of all subsets of a set X. We can define the **discrete metric** of X to be

$$d(x, y) = \begin{cases} 0 & \text{if } x = y \\ 1 & \text{otherwise} \end{cases}.$$

1.10) The **Trivial** or **Indiscrete Topology** is the topology $\mathcal{T} := \{\emptyset, X\}$ for a set X. This is a non-metrisable topology when X has more than one member.

1.14) Let $X = \{a, b, c\}$, where a, b, c are distinct. Then

$$\mathcal{T} = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$$

is a topology on X

1.15) Give $\mathbb{R}$ the topolgoy whose open subsets $U \subseteq \mathbb{R}$ are precisesly the subsets with finite complement $\mathbb{R} \backslash U$, or $U = \emptyset$. Then $\mathbb{R}$ with this topology is not metrisable. This is an example of a **Zariski Topology**

Proposition 1.11: Topology Equality

Let d, d' be metrics on the same set X, and let $\mathcal{T}$, $\mathcal{T}'$ be the corresponding metric topologies. If for real numbers A, $B > 0$ we have

$$d(x, y) \leq Ad'(x, y),\ d'(x, y) \leq Bd(x, y) \text{ for all } x,\ y \in X,$$

then $\mathcal{T} = \mathcal{T}'$.

Example 1.12: Example of Topology Equality

- The **Euclidean metric** on $\mathbb{R}^n$ is defined as:

$$d\big((x_1, x_2, \dots, x_n),\ (y_1, y_2, \dots, y_n)\big) = \sqrt{\sum_{i=1}^{n} (x_i - y_i)^2}$$

- The **Box metric** on $\mathbb{R}^n$ is defined as:

$$d(x, y) \leq \sqrt{n} d'(x, y),\ d'(x, y) \leq d(x, y)$$

By 1, these have the same topology.

Definition 1.16: Subspace Topology

Let $(X, \mathcal{T})$ be a topological space, and let $A \subseteq X$ be any subset. Then teh **subspace topology** on A consists of all sets of the form $U \cap A$ where $U \in \mathcal{T}$.

Definition 1.17: Closed Set

Let $(X, \mathcal{T})$ be a topological space. A subset $A \subseteq X$ is **closed** iff its complement $X \backslash A := \{x \in X \mid x \notin A\}$ is open in X. Note that a set being *closed* does not mean it isn't *open*. Sets that are both *closed* and *open* are called **clopen**.

Theorem 1.19: Properties of open and closed sets

Let $(X, \mathcal{T})$ be a topological space.

1. $\emptyset$ and X are closed.
2. The union of **finitely many** closed sets is an closed set.
3. The intersection of **any collection** of closed sets is a closed set.
4. The union of **any collection** of open sets is an open set.
5. The intersection of **finitely many** open sets is an open set

Definition 1.20: Properties of Topological Spaces

1. The **closure** of a set $A \subseteq X$ is
$$\overline{A} := \bigcap_{C \subseteq X \text{ closed};\ A \subseteq C} C.$$
2. The **interior** of a set $A \subseteq X$ is
$$\operatorname{int} A = A^\circ := \bigcap_{C \subseteq X \text{ open};\ A \subseteq C} C.$$
3. The **boundary** (or **frontier**) of a subset $A \subseteq X$ is
$$\partial A := \overline{A} \backslash A^\circ.$$
4. A subset A of X is **dense** in X iff $\overline{A} = X$.

$\overline{A}$ is closed, and contains A and is the smallest set with this property. So A is closed iff $\overline{A} = A$.
A° is open, and is contained in A, and is the largest set with this proprety. So A is open iff $A^\circ = A$.

Proposition 1.22: Relating Topological Properties

The closure of the complement is the complement of the interior:

$$\overline{X \backslash A} = X \backslash (A^\circ).$$

The interior of the complement is the complement of the closure:

$$(X \backslash A)^\circ = X \backslash \overline{A}.$$

Definition 1.23: Limit Points

Let $(X, \mathcal{T})$ be a topological space, and let $A \subseteq X$ be a subset. A **limit point** of A is a point $x \in X$ s.t. for every open subset $U \subseteq X$ with $x \in U$ there exists an element $a \in A \cup U$ with $a \neq x$.
Let A' be the set of limit points of A.
Note that this has nothing to do with limits of sequences.

Lemma 1.24: Limit Points and Open Balls

An element $x \in X$ in a metric space (X, d) is a limit point of a subset $A \subseteq X$ iff for every $\epsilon > 0$ there exists $a \in A$ with $0 < d(x, a) < \epsilon$, or iff there exists a sequence $a_1, a_2, a_3, \cdots$ of elements $a_i \in A$, with $a_i \neq x$ for all i, such that $d(x_i, a_i) \to 0$ as $i \to \infty$. This interpretation does not extend to general topological spaces.

Example 1.0.2: Examples of limit points

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Proposition 1.26: Union of Limit points

Let $(X, \mathcal{T})$ be a topological space, and suppose $A \subseteq X$. Then

$$\overline{A} = A \cup A'$$

Corollary 1.27

A subset $A \subseteq X$ is closed iff it contains all its limit points.

Theorem 1.30: Open and Closed sets in $\mathbb{R}$

Consider $\mathbb{R}$ with the usual topology.

1. A nonempty set U is open iff it can be written as a countable union of disjoint nonempty open intervals I_j:
$$U = \bigcup_{j=1}^{\infty} I_j.$$
2. A set F is closed iff it can be written as a countable intersection
$$F = \bigcap_{j=1}^{\infty} F_j$$
where each F_j is a finite union of closed intervals.

Definition 1.32: Hausdorff Spaces

A topological space $(X, \mathcal{T})$ is **Hausdorff** if for each $x, y \in X$ with $x \neq y$ there exist **disjoint** open sets U and V such that $x \in U$ and $y \in V$.

Any metrisable space is Hausdorff, The trivial topology n a set with more than one element is not Hausdorff.

Definition 1.33: Convergence of a Topological space

A sequence (x_n) of members of a topological space X converges to $x \in X$ if for every open set U containing x, there exists an N such that $n \geq N \implies x_n \in U$

Proposition 1.34: Convergence of Hausdorff Spaces

Suppose $(X, \mathcal{T})$ is Hausdorff. Then a sequence (x_n) can converge to at most one limit.

Definition 1.36: Cauchy and Completeness

Let (X, d) be a metric space.

1. A **Cauchy sequence** is a sequence (x_n) with each $x_n \in X$ with the property that for each $\epsilon > 0$, there exists an N such that $m, n \in N \implies d(x_m, x_n) < \epsilon$
2. (X, d) is **complete** if every Cauchy sequence converges.

Definition 1.37: Topology Basis

A **basis for a topology** on a set X is a collection $\mathcal{B}$ of subsets $B \subseteq X$ such that:

1. $X = \bigcup_{B \in \mathcal{B}} B$
2. The intersection of sets $B_1, B_2 \in \mathcal{B}$ in a set $B_1 \cap B_2 \in \mathcal{B}$

The **topology $\mathcal{T}$ generated by a basis $\mathcal{B}$** has open sets the arbitrary unions of basis elements $B_\lambda \in \mathcal{B}$:

$$U = \bigcup_{\lambda \in \Lambda} B_\lambda$$

(Don't forget to check that this really is a topology)

Example 1.38: Finite Intersections of open balls

For any metric space $(X, \mathcal{T})$ the finite intersections of open balls

$$B(x, r) = \{y \in X \mid d(x, y) < r\} \subseteq X \quad (r > 0, x \in X)$$

constitute a basis for the metric topology on X

$$\mathcal{B} = \{B(x_1, r_1) \cap B(x_2, r_2) \cap \cdots \cap B(x_k, r_k) \mid x_1, x_2, \ldots, x_k \in X, r_1, r_2, \ldots, r_k > 0\}$$

2 Continuous functions and Homeomorphisms

Definition 2.1: Continuity

Let $(X, \mathcal{T})$, $(Y, \mathcal{U})$ be topological spaces. A function $f : X \to Y$ is **continuous** iff

$$U \in \mathcal{U} \text{ implies } f^{-1}(U) \in \mathcal{T}.$$

That is, **inverse** images of open sets are open. Continuous functions are often called **maps** or **mappings** of topological spaces.

Proposition 2.6: Topological and Analytic Continuity

Let (X, d) and (Y, ρ) be metric spaces with their induced topologies $\mathcal{T}$ and $\mathcal{U}$ respectively. A function $f : X \to Y$ is continuous (topologically) iff it is continuous analytically: for every $a \in X$ and every $\epsilon > 0$ there exists $\delta > 0$ such that

$$d(x, a) < \delta \implies \rho(f(x), f(a)) < \epsilon$$

Definition 2.7: Homeomorphism

Let $(X, \mathcal{T})$ and $(Y, \mathcal{U})$ be topological spaces. A **homeomorphism** is a bijection $f : X \to Y$ such that both f and f^{-1} are continuous. Two topological spaces are **homeomorphic** if there is a homeomorphism between them.

Proposition 2.8: Open Homeomorphisms

Let $f : (X, \mathcal{T}) \to (Y, \mathcal{U})$ be a homeomorphism. Then U is open in Y iff $f^{-1}(U)$ is open in X.

Example 2.10: Examples of homeomorphisms

1. Let $(X, \mathcal{T})$ be an arbitrary topological space. Then the identity map
$$\iota : X \to X; \quad x \mapsto x$$
is continuous, and indeed a homeomorphism.
2. Suppose $(X, \mathcal{T})$, $(Y, \mathcal{U})$, and $(Z, \mathcal{W})$ are topological spaces, and that $f : X \to Y$ and $g : Y \to Z$ are continuous functions. Then their composition
$$g \circ f : X \to Z; \quad x \mapsto g(f(x))$$
is continuous.
3. For any topological spaces X, Y, and any element $y_0 \in Y$ the constant function
$$f_0 : X \to Y; \quad x \mapsto y_0$$
is continuous.

Proposition 2.14: Continuity and Closed sets

- Suppose $(X, \mathcal{T})$ and $(Y, \mathcal{U})$ are topological spaces and that $f : X \to Y$. Then f is continuous iff for every closed subset $F \subseteq Y$ its inverse image $f^{-1}(F)$ is closed in X.
- f is continuous iff the image of the closure of every subset $A \subseteq X$ is contained in the closure of the image, i.e., $\forall A \subseteq X$,
$$f(\overline{A}) \subseteq \overline{f(A)}$$

Proposition 2.18: The Punctured Sphere

Consider the n-dimensional sphere

$$\mathbb{S}^n = \{x \in \mathbb{R}^{n+1} : |x| = 1\}$$

with the metric topology inherited from $\mathbb{R}^{n+1}$. Let $x_0 \in \mathbb{S}^n$. Then $\mathbb{S}^n \backslash \{x_0\}$ is homeomorphic to $\mathbb{R}^n$.

3 Subspaces Revisited

Proposition 3.4: Hausdorff and Subspaces

Suppose $(X, \mathcal{T})$ is a Hausdorff topological space and suppose A is a subspace. Then A is Hausdorff.

Proposition 3.5: Continuity and Subspaces

Suppose $(X, \mathcal{T})$ and $(Y, \mathcal{U})$ are topological spaces and suppose A is a subspace of X. Let $f : X \to Y$ be continuous. Then $f|_A : A \to Y$ is continuous.

Corollary 3.6: Homeomorphisms and Exclusions

Suppose $(X, \mathcal{T})$ and $(Y, \mathcal{U})$ are homeomorphic via f. Then $X \backslash \{x_0\}$ is homeomorphic to $Y \backslash \{f(x_0)\}$

Definition 3.65: Disjoint Unions

Let $(X, \mathcal{T})$ and $(Y, \mathcal{U})$ be topological spaces. Their **disjoint union** $X + Y$ is the set $(X \times \{0\}) \cup (Y \times \{1\})$ with the topology consisting of all sets of the form

$$(T \times \{0\}) \cup (U \times \{1\}) \text{ such that } T \in \mathcal{T}, U \in \mathcal{U}$$

Definition 3.8: Product Topology

Let $(X, \mathcal{T})$, $(Y, \mathcal{U})$ be topological spaces. The **product topology** on their product $X \times Y$ consists of all sets of the form

$$T = \bigcup_{\alpha \in A} (U_\alpha \times V_\alpha)$$

where $\mathcal{A}$ is an arbitrary indexing set, and $U_\alpha \in \mathcal{U}$ and $V_\alpha \in \mathcal{V}$.

Lemma 3.9: Openness in Product Topologies

Let $(X, \mathcal{T})$ $(Y, \mathcal{U})$ be topological spaces. Then $T \subseteq X \times Y$ is open in the product topology if and only if for all $t \in T$ there exists $U \in \mathcal{U}$ and $V \in \mathcal{V}$ such that $t \in U \times V$ and $U \times V \subseteq T$.

Lemma 3.10: Product Topology is a topology

The product topology is indeed a topology. (lol)

Definition 3.11.5: Projection Maps

Let $(X, \mathcal{T})$ and $(Y, \mathcal{U})$ be topological spaces, and consider their product $X \times Y$ with the product topology. There are two natural maps Π_X and Π_Y, the projections of $X \times Y$ onto X and Y respectively, given by

$$\Pi_X : X \times Y \to X, \quad (x, y) \mapsto x$$

and

$$\Pi_Y : X \times Y \to Y, \quad (x, y) \mapsto y.$$

Theorem 3.12: Continuity of Projection Maps

Let $(X, \mathcal{T})$ and $(Y, \mathcal{U})$ be topological spaces and $\mathcal{T}$ the product topology on $X \times Y$. Then the projection maps Π_X and Π_Y are continuous. Moreover, $\mathcal{T}$ is the smallest topology on $X \times Y$ such that the projection maps are continuous.

Proposition 3.13: Continuity of compositions

Let X, Y, Z be topological spaces. Endow $X \times Y$ with the product topology. A function $f : Z \to X \times Y$ is continuous iff the functions $\Pi_X \circ f : Z \to X$ and $\Pi_Y \circ f : Z \to Y$ are both continuous.

Definition 3.14: Weak Topology

Suppose that X is a set. $(X_\lambda, \mathcal{T}_\lambda)$ is a family of topological spaces, and that $f_\lambda : X \to X_\lambda$ are functions. The **weak topology generated by** $\{f_\lambda\}$ is the smallest topology on X making all the f_λ continuous.

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