FNLP Exam Notes

Made by Leon :)

1 smooth and stuff

Definition 1.0.1: Maximum Likelihood Estimates (MLE)

$$P_{RF}(x) = \frac{C(x)}{N}$$

$C(x)$ is the count of x in the dataset, and N is the total number of items in the dataset

- **Problem 1 (Sparse data problem)**: If the count of an item is 0, then the probability will also be 0 - you want the model to be able to calculate sentences with new words in them. **Solution**: Smoothing
- **Problem 2**: Cannot reliably find probability of sentences (the chance of “skibidi sigma gyatt rizz” being already in a corpus is very low). **Solution**: use n-gram models

Definition 1.0.2: n-gram models

Turn a sentence $P(S = w_1 \dots w_n)$ into joint probabilities $P(w_1, \dots, w_n)$. We have $P(X, Y) = P(Y|X)P(X)$. So

$$\begin{aligned} P(a, b, c) &= P(c|a, b)P(a, b) \\ &= P(c|a, b)P(b|a)P(a) \end{aligned}$$

n-gram model just estimates probability to n probabilities

- **Trigram**: $P(w_i|w_1, w_2, \dots, w_{i-1}) \approx P(w_i|w_{i-2}, w_{i-1})$
- **Bigram**: $P(w_i|w_1, w_2, \dots, w_{i-1}) \approx P(w_i|w_{i-1})$
- **Unigram**: $P(w_i|w_1, w_2, \dots, w_{i-1}) \approx P(w_i)$

To be able to detect edges of sentences, add <s> and <\s> on sentence edges to be factored into the n-gram model

skibidi rizz $\implies$ <s> skibidi rizz <\s>

therefore a bigram like P(<\s>|rizz) will detect the end of a sentence
Usually, **negative log probs** will be used instead of regular decimals, as the probabilities will get small fast and floating precision issues will happen.

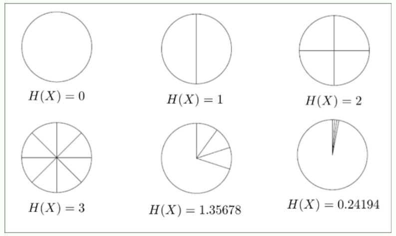
- Probabilities from 0 to 1, but negative log probs go from 0 to ∞
- Log probs are added instead of multiplied like regular probabilities

Definition 1.0.3: Entropy

Entropy of a random variable X:

$$H(X) = \sum_x -P(x) \log_2 P(x)$$

also the expected value of $-\log_2 P(X)$
Higher entropy means less predictable



For $w_1 \dots w_n$ with large n, per-word cross-entropy is well approximated by

$$H_M(w_1 \dots w_n) = -\frac{1}{n} \log_2 P_M(w_1 \dots w_n)$$

Lower cross-entropy $\implies$ model is better at predicting next word
Perplexity: $2^{\text{cross-entropy}}$

Definition 1.0.4: Add-one and Lidstone smoothing

Add one smoothing

$$P_{+1}(w_i|w_{i-2}, w_{i-1}) = \frac{C(w_{i-2}, w_{i-1}, w_i) + 1}{C(w_{i-2}, w_{i-1}) + v}$$

where v is the vocabulary size

Add-α smoothing

$$P_{+\alpha}(w_i|w_{i-1}) = \frac{C(w_{i-1}, w + i) + \alpha}{C(w_{i-1}) + \alpha v}$$

Choosing an α: Use a three-way data split: **training set** (80-90%), **held-out/development set** (5-10%), and **test set** (5-10%)

- Train model (estimate probabilities) on training set with different values of α
- Choose the α that minimizes cross-entropy on development set
- Report final results on test set

More generally, use development set for evaluating different models, debugging, and optimizing choises. This avoids overfitting to the training set and even to the test set

Definition 1.0.5: Good-Turing Smoothing

$$c* = (c + 1)\frac{N_{c+1}}{N + c} \qquad P*_c = \frac{c*}{N} = (c + 1)\frac{\frac{N_{c+1}}{N_c}}{N}$$

- N_c is the number of occurances with count c
- $P*_c$ is the probability of an item with count c
- $c*$ is the good-turing smoothed version of count
- N is total count

random items

- Probability the next observation is new

$$P(unseen) = \frac{N_1}{N}$$

- Probability the next observation is a specific new object

$$P_{GT} = \frac{1}{N_0}\frac{N_1}{N} \implies c* = \frac{N_1}{N_0}$$

Problems with Good-Turing

- Assumes we know the vocabulary size
- Doesn’t allow ”holes” in the counts (if $N_i > 0, N_{i-1} > 0$)
- Applies discounts even to high-frequency items
- Assigns equal probability to all unseen events, same with add-α, e.g. “w rizz” vs “w indowpane” shouldn’t be equal

Definition 1.0.6: Interpolation and backoff

Interpolation: Combines higher and lower order N-gram models, since they have different strengths and weaknesses

- high-order N-grams are sensitive to more context, but have sparse counts
- low-order N-grams have limited context but robust counts

If P_N is N-gram estimate

$$P_{\text{INT}}(w_3|w_1, w_2) = \lambda_1 P_1(w_3) + \lambda_2 P_2(w_3|w_2) + \lambda_3 P_3(w_3|w_1, w_2)$$

Katz-backoff: Trust the highest order language model that contains the N-gram. Requires an adjusted prediction model:
$P*(w_i|w_{i-N+1}, \dots, w_{i-1})$ and backoff weights: $\alpha(w_1, \dots, w_{N-1})$
Kneser-Ney: Takes diversities of histories into account

- count of distinct histories for a word

$$N_{1+}(\circ w_i) = |\{w_{i-1} : c(w_{i-1}, w_i) > 0\}|$$

- In KN smoothing, replace raw counts with count of histories:

$$P_{\text{ML}}(w_i) = \frac{C(w_i)}{\sum_w C(w)} \implies P_{\text{KN}}(w_i) = \frac{N_{1+(\circ w_i)}}{\sum_w N_{1+}(\circ w)}$$

Method use cases:

- Uniform probabilities - add-α, Good-Turing
- Probabilities from lower-order n-grams - interpolation, backoff
- Probability of appearing in new contexts - Kneser-Ney

2 Text Classification

Categorizing the *content* of the text. e.g.

- Spam detection (binary classification: spam/not spam)
- Sentiment analysis (binary / multiway)
  - movie, restaurant, product reviews (pos/neg, or 1-5 stars)
  - political argument (pro/con or pro/con/neutral)
- Topic classification (multiway: sport/finance/travel/etc)

Or, categorizing the *author* of the text (authorship attribution)

- Native language identification (e.g. to tailor language tutoring)
- Diagnosis of disease (psychiatric or cognitive impairments)
- Identification of gender/dialect/educational background (e.g. forensics [legal matters], advertising/marketing)

n-gram models are not as useful for classification - often we can just consider a **bag of words** and not worry about the order that the words come in

Definition 2.0.1: Naive Bayes

Given document d and set of categories C we want to assign d to the most probable category $\hat{c}$

$$\hat{c} = \arg\max_{c \in C} P(c|d) = \arg\max_{c \in C} P(d|c)P(c)$$

Represent d as the set of features (words) it contains: $f_1, f_2, \ldots, f_n$

$$P(d|c) = P(f_1, f_2, \ldots, f_n|c)$$

Then make **naive Bayes assumption** that features are conditionally independent given the class

$$P(f_1, f_2, \ldots, f_n|c) \approx P(f_1|c)P(f_2|c)\ldots P(f_n|c)$$

i.e. the probability of a word happening depends **only** on the class, not on words occuring before/after (n-gram), or even what other words occurred at all. Basically we only care about the **count** of each feature in a document

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Naive Bayes classifier: Given a document with features $f_1, f_2, \ldots, f_n$ and set of categories C, choose the class $\hat{c}$ where

$$\hat{c} = \arg\max_{c \in C} P(c)\prod_{i=1}^{n} P(f_i|c)$$

- $P(c)$ is the **prior probability** of class c before observing any data. normally estimated with MLE:

$$\hat{P}(c) = \frac{N_c}{N}$$

  - N_c is the number of training documents in class c
  - N is the total number of training documents.

  Therefore, $\hat{P}(c)$ is the proportion of training documents in class c
- $P(f_i|c)$ is the probability of seeing feature f_i in class c. Normall estimated with simple smoothing:

$$\hat{P}(f_i|c) = \frac{\text{count}(f_i, c) + \alpha}{\sum_{f \in F}(\text{count}(f, c) + \alpha)}$$

  - $\text{count}(f_i, c)$: the number of times f_i occurs in class c
  - F: the set of possible features
  - α: the smoothing parameter, optimized on held-out data

Same with n-gram models, usually uses **negative log probabilities** - adjusted equation:

$$\hat{c} = \arg\min_{c \in C} +(-\log P(x) + \sum_{i=1}^{n} -\log P(f_i|c))$$

This amounts to classification using a linear function (in log space) of the input features. Therefore Naive bayes is called a **linear classifier**

Issues with choosing features

- Sentiment analysis might need domain-specific non-sentiment words e.g. "quiet" or "memory" for computer reviews
- Stopwords might be useful features for other tasks, e.g. People with schizophrenia use more 2nd-person pronouns, and people with depression use more 1st-person
- Probably better to use too many irrelevant feaetures than not enough relevant ones

Problems with annotation: Usually hard to come by already annotated text - ergo you need someone to label text. On the other hand there is usually a lot of unannotated texts.
Solution: Use semi-supervised learning

1. Train NB on labeled data alone
2. Predict labels on unlabelled data
3. Re-estimate NB, but now using also self-labelled data

Self Training

- **Advantages**: Simplicity and applicable to any classifier
- **Disadvantages**: Does not account for uncertainty of a classifier, and no theoretical motivation
- To make it work needs discarding low-confidence predictions, and curriculum (start with examples similar to labeled data)

Expectation Maximisation for Semi-supervised Learning

- Train NB on labelled data alone
- Make soft prediction on unlabelled data ("E-step")
- Recompute NB parameters using the soft counts

Self-training for NB is known as "hard EM"
Advantages of Naive Bayes

- Very easy to implement
- Very fast to train and classify new docs (good for huge datasets)
- Doesn't require as much training data as some other methods (good for small datasets)
- Usually works reasonably well
- Should be the baseline method for any classification task

Evolving past naive Bayes:

- Assuming that all features are conditionally independent can have some issues, and often we have enough training data for a better model.
- Adding multiple feature types (e.g. words and morphemes) often leads to even stronger correlations between features
- Accuracy of classifier can sometimes still be ok, but it will be highly overconfident in its decision, e.g. NB sees 5 features that all point to class 1, treats them as 5 independent sources of evidence - like asking 5 friends for an opinion when some got theirs from each other

Definition 2.0.2: Maximum Entropy / Logistic Regression

Most commonly **multinomial logistic regression**. **multinomial** if more than two possible classes, otherwise just **logistic regression**
Like Naive Bayes, assign a document x to class $\hat{c}$ where

$$\hat{c} = \arg\max_{c \in C} P(c|x)$$

unlike Naive Bayes, model $P(c|x)$ directly instead of using Baye's rule

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Discrimination

- Trained to discriminate correct vs wrong values of c given input x
- Need not be probabilistic
- Examples: artificial neural networks, decision trees, nearest neighbour methods, support vector machines
- Here we only consider one method: MaxEnt models which are probabilistic

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Feature Functions: Like Naive Bayes, MaxEnt models use **features** we think will be useful for classification.
However, features are treated different in the two models

- NB: Features are **directly observed** (e.g. words in doc): no difference between features and data
- MaxEnt: We will use $\vec{x}$ to represent the observed data. Features are **functions** that depend on both observations $\vec{x}$ and class c

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Classification with MaxEnt
Choose the class that has highest probability according to

$$P(c|\vec{x}) = \frac{1}{Z}\exp\left(\sum_i w_i f_i(\vec{x}, c)\right)$$

- Normalization constant $Z = \sum_{c'} \exp(\sum_i w_i f_i(\vec{x}, c))$
- Inside brackets is just a dot product $\vec{w} \cdot \vec{f}$
- $P(c|\vec{x})$ is a **monotonic function** of this dot product
- So, we will end up choosing the class for which $\vec{w} \cdot \vec{f}$ is highest

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Training the model Given annotated data, choose weights that make the labels most probable under the model That is, given items $x^{(1)} \ldots x^{(N)}$ with labels $c^{(1)} \ldots c^{(N)}$, choose

$$\hat{w} = \arg\max_{\vec{w}} \sum_j \log P(c^{(j)}|x^{(j)})$$

This is called **conditional maximum likelihood estimation** (CMLE)
Like MLE, CMLE will overfit, so use **regularization** to avoid that

Relation to Naive Bayes - Naive Bayes is also a linear classifier, and can be expressed in the same form. Should the features actually be independent they would converge to the same solution as the amount of training data increases
Downside to MaxEnt models

- Supervised MLE in generative models is easy - compute counts and normalize
- Supervised CMLE in MaxEnt is not so easy

- requires multiple iterations over the data to gradually improve weights (using gradient ascent)
- Each iteration computese $P(c^{(j)}|x^{(j)})$ for all j, and each possible $c^{(j)}$
- This can be time-consuming, especially if there are a large number of classes and/or thousands of features to extract from each training example

Robustness: MaxEnt and Naive Bayes

- Imagine that in training there is one very frequent predictive feature, e.g. in training sentiment data contained emoticons but not at test time
- The model can quickly learn to rely on this feature
  - model is confident on examples with emoticons
  - the gradient on these examples gets close to zero
  - the model does not learn other features
- In MAxEnt, a feature weight will depend on the precense of other predictive features
- Naive Bayes will rely on all features - the weight of a feature is not affected by how predictive other features are
- This makes NB more robust than (basic) MaxEnt when test data is (distributionally) different from training data

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