

Modelling Concurrent Systems Notes

Leon Lee

September 24, 2024

Contents

1	Introduction to Concurrent Systems	3
1.1	Lecture 1 - Specification and Implementation	3
1.2	Deadlocks	5
1.3	Relations	6

1 Introduction to Concurrent Systems

1.1 Lecture 1 - Specification and Implementation

The main topics that are covered in this course:

- Formalising specifications as well as implementations of concurrent systems
- Studying the criteria for deciding whether an implementation meets a specification
- Techniques for proving whether an implementation meets a specification

Both specifications and implementations can be represented by means of **models of concurrency** such as **Labelled Transition Systems (LTSs)** or **Process Graphs**.

Definition 1.1.1: Process Graphs and LTSs

A **process graph** is a triple (S, I, \rightarrow) , defined by the following:

- S is a set of **states**
- $I \in S$ is an **initial state**
- \rightarrow is a set of triples (s, a, t) with $s, t \in S$, and a an **action** drawn from a set **Act**

A **Labelled Transition System (LTS)** is a *process graph* without the initial state (but sometimes LTS is used as a synonym for process graph i.e. with the initial state)

Sometimes we will use process graphs with a fourth component $\checkmark \subseteq S$ indicating the **final** states of the process: those in the system can terminate successfully

Specifications and implementations can not only be represented by LTSs or other models of concurrency, but also by **process algebraic expressions**, where complex processes are built up from constants for atomic actions using operators for *sequential*, *alternative*, and *parallel composition*.

The most popular process algebraic languages from literature are:

- **CCS**: the Calculus of Communicating Systems
- **CSP**: Communicating Sequential Processes
- **ACP**: the Algebra of Communicating Processing

We will be using ACP, focusing on the *partially synchronous parallel composition* operator

Definition 1.1.2: ACP

The syntax of **ACP**, the **Algebra of Communicating Processes** features the operations

1. ϵ : **Successful termination** (only in the optional extension ACP_ϵ)
2. δ : **Deadlock**
3. a : **Action constant** for each action a
4. $P \cdot Q$: **Sequential Composition**
5. $P + Q$: **Summation, Choice, or Alternative Composition**
6. $P \parallel Q$: **Parallel Composition**
7. $\partial_H(P)$: **Restriction**, or **Encapsulation** for each set of visible actions H
8. $\tau_I(P)$: **Abstraction** for each set of visible actions I (only in the optional extension ACT_τ)

Note: There is also left and right parallel operators, P

The atomic actions of ACP consist of all a, b, c etc from a given set A of visible actions, and one special action τ , that is meant to be internal and invisible to the outside world.

For each application, a partial **communication function** $\gamma : A \times A \rightarrow A$ is chosen that tells for each two visible actions a and b whether they synchronise (namely if γ is defined), and if so, what is result of their synchronisation: $\gamma(a, b)$. The communication function is required to be commutative and associative. The invisible action cannot take part in synchronisations.

Definition 1.1.3: ACP in terms of Process Graphs

Below is the **ACP** operations in terms of process graphs extended with a predicate \checkmark that signals successful termination

- ϵ is the graph with one state and no transition. This one state is the initial state, and is marked with \checkmark
- δ is the graph with one state and no transitions. This one state is the initial state. It is not marked as terminating.
- a is a graph with two states (and initial and a final one) and one transition between them, labelled a . The final state is marked with \checkmark
- $G \cdot H$ is the process that first performs G , and upon successful termination of G proceed with H .
- $G + H$ is obtained by taking the union of copies of G and H with disjoint sets of states, and adding a fresh state **root** which will be the initial state of $G + H$. For each transition $I_G \xrightarrow{a} s$ in G , where I_G denotes the initial state of G , there will be an extra transition **root** $\xrightarrow{a} s$, and likewise, for each transition $I_H \xrightarrow{a} s$ in H , where I_H denotes the initial state of h , there will be an extra transition **root** $\xrightarrow{a} s$. **root** is labelled with \checkmark if either I_G or I_H is.
- $G||H$ is obtained by taking the Cartesian product of the states of G and H ; that is, the states of $G||H$ are pairs (s, t) with s a state from G and t a state from H . The initial state of $G||H$ is the pair of initial states of G and H . A state (s, t) is labelled \checkmark iff both s and t are labelled \checkmark . The transitions are
 - $(s, t) \xrightarrow{a} (s', t)$ whenever $s \xrightarrow{a} s'$ is a transition in G
 - $(s, t) \xrightarrow{a} (s, t')$ whenever $t \xrightarrow{a} t'$ is a transition in H
 - $(s, t) \xrightarrow{c} (s', t')$ whenever $s \xrightarrow{a} s'$ is a transition in G , and $t \xrightarrow{b} t'$ is a transition in H , and $\gamma(a, b) = c$

Intuitively, $G||H$ allows all possible interleavings of actions from G and actions from H . In addition, it enables actions to synchronise with their communication partners.

- $\partial_H(G)$ is just G , but with all actions in G omitted. It is used to remove the remnants of unsuccessful communication, so that the synchronisation that is enabled by parallel composition, is enforced.
- $\tau_I(G)$ is just G , but with all actions in I renamed into τ .

These semantics are of the **denotational** kind. Here "denotational" entails that each constant denotes a process graph (up to **isomorphism**) and each ACP operator denotes an operation on process graphs (creating a new graph out of one or two argument graphs)

1.2 Deadlocks

Deadlock means that a process isn't "terminated" but it cannot go further
ACP- ϵ is an alternative model where non-finishing states can also terminate

1.3 Relations

Definition 1.3.1: Equivalence Relation

A binary relation \sim on a set D is an **equivalence relation** if it is

- **Reflexive:** $p \sim p$ for all processes $p \in D$
- **Symmetric:** If $p \sim q$ then $q \sim p$
- **Transitive:** If $p \sim q$ and $q \sim r$ then $p \sim r$

Definition 1.3.2: Equivalence Class

If \sim is an equivalence relation on D , and p is in D , then $[p]_{\sim}$ denotes the set of all processes in D that are equivalent to p . Such a set is called an **equivalent class**

Equivalence relations that split a set into more sets are called **finer** or **more discriminating**, and equivalence relations that split a set into less sets are called **coarser** or **more identifying**. The finest relation is e - every element has its own class, and the coarsest relation is the universal equivalence - everything is equivalent to itself

Definition 1.3.3: Equivalence Relation

A binary relation on a set D is a **preorder** (\sqsubseteq) if it is

- **Reflexive:** $p \sqsubseteq p$ for all processes $p \in D$
- **Transitive:** If $p \sqsubseteq q$ and $q \sqsubseteq r$ then $p \sqsubseteq r$