Digital Image Processing

Homework Assignment #2

Due: 9:10am, 11/4, 2022

程式作業使用環境

1、python3.9
2、openCV (cv2)
3、matplotlib

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Notes

1. Suppose that a digital image is subjected to histogram equalization. Show that a second pass of histogram equalization (on the histogram-equalized image) will produce exactly the same result as the first pass.

Pf:

first, define the process of histogram equalization below:

set f is all pixel of target image

C is the PDF of the gray scale value for Image

P is the CDF of C

than formula is: $P(f)$

final, set $u \in P(f)$, P' is CDF for $P(f)$, try to do 2 equalization

$P'(P(f)) = P'(u) = u = P(f) \Rightarrow$ they have same result

because it has uniform histogram

Notes

2. An image is filtered four times using a Gaussian kernel of size 3×3 with a standard deviation of 1.0. Because of the associative property of convolution, we know that equivalent results can be obtained using a single Gaussian kernel formed by convolving the individual kernels.

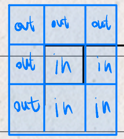
average filter

標準差

(a)* What is the size of the single Gaussian kernel?

(b) What is its standard deviation?

(a)



∵ every convolution add 1 row 1 col

∴ 4 times get $(1+4\times2)^2$ kernel

answer: $9*9$, fillter size is 9

(b) ∵ because of small Gaussian kernels can be replace by one Gaussian kernel

⇒ formula: $\sqrt{r_1^2+r_2^2+r_3^2+\cdots} = r'$, r_i is standard deviation

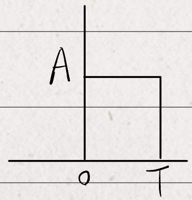
∴ $\sqrt{1^2+1^2+1^2+1^2}=\sqrt{4}=2$ ⇒ **new standard deviation = 2**

3. Repeat Example 4.1 in the textbook (page 211) but using the following function:

$f(t) = A$ for $0 \le t < T$ and $f(t) = 0$ for all other values of t. Explain the reason for any differences between your results and the results in the example.

Ans:

as question, $f(t) = \begin{cases} A, & 0 \le t < T \\ 0, & \text{others} \end{cases}$ can draw



$$F(u) = \int_{-\infty}^{\infty} f(t)\, e^{-j2\pi ut}\, dt = \int_0^T A\, e^{-j2\pi ut}\, dt$$

$$= \frac{-A}{2j\pi u} \times e^{-j2\pi ut} \Big|_0^T$$

$$= \frac{A}{2\pi j u}\left(e^{0j} - e^{-2\pi uTj}\right)$$

$$= \frac{A}{2\pi j u}\left(1 - e^{-2\pi uTj}\right)$$

(explain)

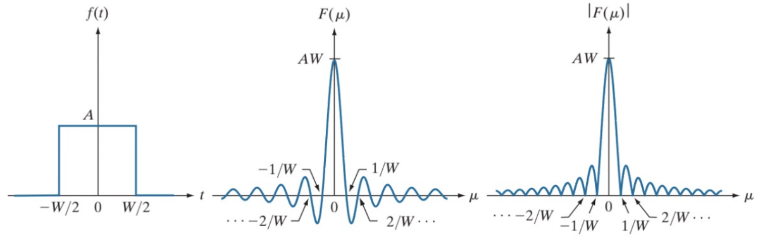
the reason why my result is different between example one is $f(t)$ has only $0 \sim T$ bigger than zero.

Therefore, $f(t)$ is asymmetric function. The result of $f(t)$ can not be transfer by only one trigonometric method. So, It should be transfer to 2 different period function.

EXAMPLE 4.1: Obtaining the Fourier transform of a simple continuous function.

The Fourier transform of the function in Fig. 4.4(a) follows from Eq. (4-20):

$$\begin{aligned} F(\mu) &= \int_{-\infty}^{\infty} f(t) e^{-j2\pi\mu t} dt = \int_{-W/2}^{W/2} A e^{-j2\pi\mu t} dt \\ &= \frac{-A}{j2\pi\mu}\left[e^{-j2\pi\mu t}\right]_{-W/2}^{W/2} = \frac{-A}{j2\pi\mu}\left[e^{-j\pi\mu W} - e^{j\pi\mu W}\right] \\ &= \frac{A}{j2\pi\mu}\left[e^{j\pi\mu W} - e^{-j\pi\mu W}\right] \\ &= AW \frac{\sin(\pi\mu W)}{(\pi\mu W)} \end{aligned}$$



a b c

FIGURE 4.4 (a) A box function, (b) its Fourier transform, and (c) its spectrum. All functions extend to infinity in both directions. Note the inverse relationship between the width, W, of the function and the zeros of the transform.

where we used the trigonometric identity $\sin\theta = (e^{j\theta} - e^{-j\theta})/2j$. In this case, the complex terms of the Fourier transform combined nicely into a real sine function. The result in the last step of the preceding expression is known as the *sinc* function, which has the general form

$$\text{sinc}(m) = \frac{\sin(\pi m)}{(\pi m)} \tag{4-23}$$

where $\text{sinc}(0) = 1$ and $\text{sinc}(m) = 0$ for all other *integer* values of m. Figure 4.4(b) shows a plot of $F(\mu)$.

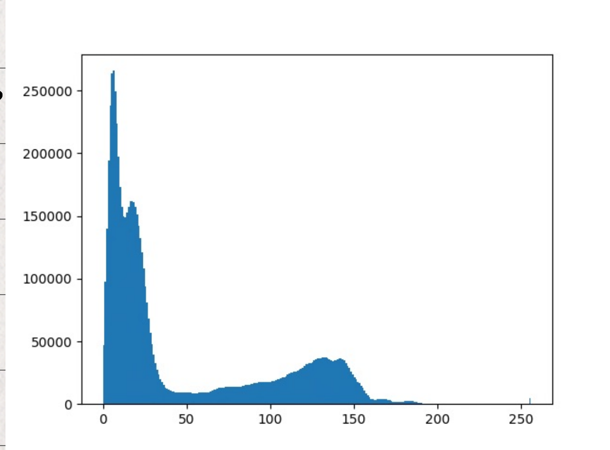
In general, the Fourier transform contains complex terms, and it is customary for display purposes to work with the magnitude of the transform (a real quantity), which is called the *Fourier spectrum* or the *frequency spectrum*:

$$|F(\mu)| = AW\left|\frac{\sin(\pi\mu W)}{(\pi\mu W)}\right|$$

Figure 4.4(c) shows a plot of $|F(\mu)|$ as a function of frequency. The key properties to note are (1) that the locations of the zeros of both $F(\mu)$ and $|F(\mu)|$ are inversely proportional to the width, W, of the "box" function; (2) that the height of the lobes decreases as a function of distance from the origin; and (3) that the function extends to infinity for both positive and negative values of μ. As you will see later, these properties are quite helpful in interpreting the spectra of two dimensional Fourier transforms of images.

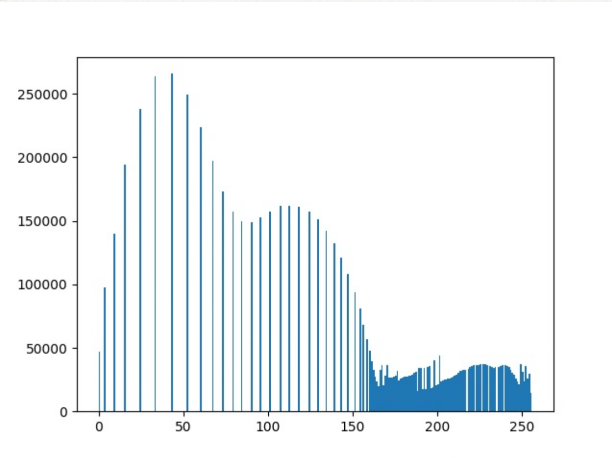
4. Write a program for histogram equalization, and test it with your own selfie took in a relatively dark environment so that we can clearly see the effect of histogram equalization in image enhancement. Please show the histograms of your selfie before and after histogram equalization and explain your results. (Note: You only have to work on the gray-scale image.)

before →



查看原始圖可以發現，色階基本集中在 0~50，100~150 2塊，分别是背景的黑和我臉的反光区

after →



經過 histogram equalization 後 可以明確發現暗部的位元減少，被拉往亮部填充，使畫面中本來因太暗而看不到的細節都可以被看見了

note: histogram 被拉為 uniform histogram，相同色階會被移到相同色階 只是不同色階間距改變