# Differential Cohomology and Virasoro Central Extensions

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# Motivation

Virasoro groups is a $\mathbf{R}$ family of central extension of $\mathrm{Diff}^+(S^1)$, the group of orientation preserving smooth automorphism of $S^1$. The central extension is describe by the Bott-Thurston cocyle. The goal of this talk is to give a novel geometric description these central extensions, using differential cohomology. thus affirmativaly answering a conjecture of Freed-Hopkins.

# Bott-Thurston cocycles

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## Definition

The Virasoro group $\widetilde{\Gamma}_\lambda$, for $\lambda \in \mathbf{R}$, is a $U(1)$ central extension of $\mathrm{Diff}^+(S^1)$, described by the Bott-Thurston cocycle
$B_\lambda : \mathrm{Diff}^+(S^1) \times \mathrm{Diff}^+(S^1) \to U(1)$:

$$B_\lambda(\gamma_1, \gamma_2) = \exp\left(-\frac{i\lambda}{48\pi}\int_{S^1} \log(\gamma_1' \circ \gamma_2)\, \mathrm{d}(\log(\gamma_2))'\right) \tag{1}$$

for $\gamma_1, \gamma_2 \in \mathrm{Diff}^+(S^1)$, viewed as morphisms $S^1 \to S^1$.

# Central Extensions

Let's briefly review what is a central extension:

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**Definition**

Let $G$ be a group and $A$ be an abelian group, a central extension of $G$ by $A$ is a group $\tilde{G}$ with short exact sequence:

$$0 \to A \to \tilde{G} \to G \to 1 \tag{2}$$

such that subgroup $A \subset \tilde{G}$ is in the center, that is, it commutes with every element of $\tilde{G}$.

# Group cohomology

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**Proposition**

*Let $G$ be a discrete group, then the isomorphism class of central extensions of $G$ by $A$ is classified by group cohomology class $H^2(G; A)$.*