String theory ??

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1 Last time

Last time we had that $T_{zz} = \sum_n^{-n-2} L_n$, with $z = e^w$, then $T_{ww} = \sum_n (L_n - \mu_0 \delta_{n,0} e^{-nw}$. z is the cylinder quantization and w is the standard Minkowski plane. From this we calculated that $\mu_0 = c/24$. We also have OPE

$$T(z')T(z) = \frac{c/2}{(z'-z)^4} + \frac{2T(z)}{(z'-z)^2} + \frac{\partial_z T}{z'-z} + ...$$

, where ... is something regular, thus doesn't contribute to line integrals. Note that the vacuum is sl_2 invariant.

2 Extending to the Riemann Sphere

2.1 Extends to the origin

Extending to orogin z = 0: $lim_{z \mapsto 0} \mathcal{O}(z, \bar{z})|0>$ is finite. In particular, for $T(z) = \sum_n z^{-n-2} L_n$, $lim_{z \mapsto 0} T(z)|0>$ finite is equivalent to $L_n|0>=0$ for $n \geq -1$. Note that this automatically implied that the vaccum is sl_2 invariant. This was automatic for free scalar, with $a^\dagger n = a_{-n}, n \geq 0$, and $a_n|0>=0, n \geq 0$, and our normal ordering convention

$$L_n = 1/2 \sum_{m=-\infty}^{\infty} : a_{n-m} a_m :$$

. From this, we see that $lim_{z\mapsto 0}T(z) = L_{-2}|0>= 1/2a_{-1}^2|0>$, more generally, $lim_{z\mapsto 0}\mathcal{O}(z,\bar{Z})|0>:= |mathcalO>$, this is the operator-state correspondence.

normal ordering symbol I don't know

Now we can say anything about operators to thing we can say about the states:

If $\mathcal{O}$ is primary, we have $[L_n, \mathcal{O}z, \bar{z}] = (n+1)z^n h\mathcal{O}(z,\bar{z}) + z^{n+1}\partial_z\mathcal{O}(z,\bar{Z})$, now we take the limit as $z\mapsto 0$: for $n\geq 1$, $limz\mapsto 0[L_n, \mathcal{O}(z,\bar{z})|0>= L_n|\mathcal{O}>= 0$ the last part is by the calculation above about the commutator. $\lim_{z\mapsto 0}[L_0, \mathcal{O}(z,\bar{z})]|0>= L_0|\mathcal{O}>= h|\mathcal{O}>$, and $\lim_{z\mapsto 0}[L_{-1}, \mathcal{O}(z,\bar{z})]|0>= |\partial_z\mathcal{O}>$. States that satisfies this are called the primary states.

Similiarly, we have descendent states, where L_{-1} doesn't vanish as in the primary states, one example of this would be $\partial_z\mathcal{O}$, which by Jacobi identity doesn't vanish. They all look like this, this is why they are called descendent states.

2.2 extending $\mathbf{z} = \infty$

Let $y = -1/z$, in changing of coordinates, we see that $T_{yy} = \frac{dz}{dy}^2 T_{zz}(z) + c/12\{z,y\} = z^4 T_{zz}(z)$. $\{\}$ is the Scharwzian derivative .

I don't know how to spell it

That tells me how the stress-energy tensor transform:

So given a correlation function$< T(z)O_1(z_1,\bar{z}_1)...\mathcal{O}_n(z_n,\bar{z}_n) >\ \ z^{-4}$ as $z\mapsto\infty$, the z^{-4} is there to ensure that it cancels the z^4 from the stress energy tensor.

Thus we define $<\mathcal{O}| :=< 0|lim_{z\mapsto\infty}z^{2L_0}\bar{z}^{2\tilde{L}_0}$, $< 0|L_n = 0forn\leq 1$.

Note that for a primary operator $\mathcal{O}(y,\bar{y}) = \frac{dz}{dy}^h(d\bar{z})(d\bar{y})\mathcal{O}(z,\bar{z})$.

Sow we learn that

1. states are organized into representations of Virasoro. We have the primary states $|mathcalO>$ with $L_0|\mathcal{O}>= h|\mathcal{O}>$, $L_n|\mathcal{O}>= 0, n\geq 1$. And it has descendents $\ldots L_{-3}^{n_3}L_{-2}^{n_2}L_{-1}^{n_1}|\mathcal{O}>$, then $sym_{k=1}^{\infty}kn_k = N$, N is called the level, and L_0 has this a eigenstate with eigenvalue $= h+N$.
2. Unitarity: fro any state (primary or descendent), we have $<\psi|\psi>\geq 0$, if it is =0, then it is called a null state, corresponding operator vanishes

by the equation of motion. Since $L_0^\dagger = L_0 \Rightarrow h$ is real. $|L{-}1|^2 =< |\mathcal{O} > |L_1 L_{-1}||\mathcal{O} >>= 2h < |\mathcal{O} > ||\mathcal{O} >>$, thus $h \geq 0$, $h = 0$ iff $l_{-1}0 = |\partial_z \mathcal{O} >= 0$, which is equivalent to $\mathcal{O} = \bar{\ddagger}$. One example of this is $\bar{T}_{\bar{z}\bar{z}}$, which has $[L_{-1}, \bar{T}_{\bar{z}\bar{z}}(\bar{z})] = 0$. So we see an example of null state: if $\mathcal{O}$ is a primary operator with $h = 0$, then its descendent $\partial_z \mathcal{O}$ has the corresponding state a null state.

What about null state at level two: there are more conditions (it will involve both c and h): Let's start with a primary state, study the matrix of inner products of descendants at level two:

I don't know how to write matrices

$A = (< \mathcal{O}|L_2 L_{-2}|\mathcal{O} >< \mathcal{O}|L_2 L_{-1}^2|\mathcal{O} >$ .... The calculation goes that

I also don't know how to quickly to bra-ket symbols, as well as operators of a Hamiltonian

$[L_2, L_{-2}] = 4L_0 + c/2$, and $[L_1^2, L_{-1}^2] = 4L_0(2L_0 + 1)$.

We will get more and more conditions at each level. The goal is write out a condition for the zero being the determinant of the n-th descendent of a primary.

Why do we care about the determinant being positive? Because if this inner product is positive definite, then the determinant better be positive.

Todo list