

Large N limit into the Grassmannians

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1 Introduction

Consider N real scalar bosons with potential in dimension 2. The action is

$$\mathcal{L}(\phi) := \int_{\mathbb{R}^2} d^2x (|d\phi|^2 + g/4! (|\phi|^2 - 1)^2),$$

where $\phi : \mathbb{R}^2 \rightarrow \mathbb{R}^N$ is our field.

In previous talks, we know that in dimension 3, we have spontaneous symmetry breaking and we get $N - 1$ massless goldstone bosons in the low energy limit.

But in dimension 2, there is no symmetry breaking (WHY??), we will have massive particles, and low energy is the σ -model into S^{N-1} .

The Lagrangian becomes

$$\mathcal{L}(\phi) = \int_{|\phi|=1} d^2x |d\phi|^2$$

, this is classically a CFT (because the hodge star operator $*$ in two dimension at the middle level depends conformally on the metric). However, there is an anomaly at the quantum level and this is not a (quantum) CFT.

2 Large N limit

We have the generating function for the correlation functions

$$Z(J) = \int D\phi \exp[\mathcal{L} + J\phi]$$

, this is called a generating function because taking derivatives with respect to $J(x)$, then set $J = 0$ gives me the correlation functions.

To impose that ϕ lives on S^{N-1} we add a field σ , so we also have a term

$$\int_{\sigma \in C^\infty(\mathbb{R})} \exp[i \int_{\mathbb{R}^2} \sigma (|\phi|^2 - 1)]$$

. Now we have the term

$$1/2 \phi (\Delta/\gamma + \sigma) \phi$$

, which after integrate over $D\phi$, we see that

$$Z(J) = \int D\sigma \exp[-N/2 \log(\text{Tr}(\Delta/\gamma + i\sigma)) + 1/2 \int_{\mathbb{R}^2} \sigma + J(\delta + i\sigma)^{-1} J]$$

. Now we are going to take the large N limit and do a saddle-point approximation.