Large N limit intot the Grassmannians

Charlie Reid

2/26/2020

Contents

1 Introduction

Consider N real scalar bosons with potential in dimension 2. The action is

$$\mathcal{L}(\phi) := \int_{\mathbb{R}^2} d^2x \ (|d\phi|^2 + g/4!(|\phi|^2 - 1)^2,$$

where $\phi : \mathbb{R}^2 \to \mathbb{R}^N$ is our field.

In previous talks, we know that in dimension¿3, we have spontaneous symmetry breaking and we get $N-1$ massless goldston bosons in the low energy limit.

But in dimension 2, there is no symmetry breaking (WHY??), we will have massive particles, and low energy is the σ-model into S^{N-1}.

The Lagrangian becomes

$$\mathcal{L}(\phi) = \int_{|\phi|=1} d^2x|d\phi|^2$$

, this is classically a CFT (because the hodge star operator * in two dimension at the middle level depends conformally on the metric). However, there is a anomaly at the quantum level and this is not a (quantum) CFT.

2 Large N limit

We have the generating function for the correlation functions

$$Z(J) = \int D\phi exp[\mathcal{L} + J\phi]$$

, this is called a generating function because taking derivatives with respect to $J(x)$, then set $J = 0$ gives me the coorrelation functions.

To impose that ϕ lives on S^{N-1} we add a field σ, so we also have a term

$$\int_{\sigma \in C^\infty(\mathbb{R})} exp[i \int_{\mathbb{R}^2} \sigma(|phi|^2 - 1)]$$

. Now we have the term

$$1/2\phi(\Delta/\gamma + \sigma)\phi$$

, which after integrate over $D\phi$, we see that

$$Z(J) = \int D\sigma exp[-N/2 \ log \ (Tr(\Delta/\gamma + i\sigma)) + 1/2 \int_{\mathbb{R}^2} \sigma + J(\delta + i\sigma)^{-1} J$$

. Now we are going to take the large N limit and do a saddle-point approximation.