Note: *Throughout this exam, unless specifically stated otherwise, all notations/symbols and abbreviations used have their usual standard meanings, as used/defined in the lecture notes.*

Read the following information to answer **Questions 1–4**:

Polynomial models of degree 1 to 6 were fitted to a set of $n = 100$ observations of (x, y) pairs. Fitted models and a scatterplot of the data are shown on **pg. 8** and information obtained from `R` is given below. Use this information and the plots to answer **Questions 1–4** below.

Polynomial degree	SSR	AIC	BIC
1	149.85	478.36	486.18
2	156.43	479.36	489.78
3	401.20	434.34	447.36
4	401.64	436.23	451.86
5	418.89	433.90	452.14
6	418.92	435.90	456.74

SST = 808.9614

1. The value of adjusted R^2 for the fourth degree polynomial model is closest to:

(a) 0.4753.

(b) 0.7572.

(c) 0.4826.

(d) 436.23.

(e) 0.3165.

The R^2 value in this case is $R^2_4 = SSR/SST = 401.64/808.9614 = 0.4965$. Using the formula for adjusted R^2 on pg. 64 of Chapter 1B notes with $n = 100$ and $k = 4$ in this case, we have:

$$R^2_{\text{adj},4} = \frac{(n-1)R^2_4 - k}{n-1-k} = \frac{99(0.4965) - 4}{99 - 4} = 0.4753.$$

2. Suppose you were presented with the plots of the third and fifth degree polynomials (as in **pg. 8**) and were asked to choose between the two, *strictly* on the basis of the plots. Then:

(a) The fifth degree polynomial is preferable because the two curves look almost the same, but the fifth degree model is more complex.

(b) The third degree polynomial is preferable because simpler models are always preferable.

(c) The third degree polynomial is preferable because the two curves look almost the same, but the third degree model is simpler.

(d) The fifth degree model is preferable because more complex models are always preferable.

(e) Not even William of Occam could decide between the two.

When two fitted models (and the fitted curves) are virtually the same but one is simpler, then the simpler model is always the better choice following the principle of Occam's Razor.

3. Based on the plots and all the other given information, the best model choice would be:

(a) Either the polynomial of degree 1 or degree 2.

(b) The polynomial of degree 3.

(c) The polynomial of degree 4.

(d) The polynomial of degree 5.

(e) The polynomial of degree 6.

AIC and BIC prefer the fifth and third degree models, respectively. However, since the plots of those two models are very similar, it therefore makes sense to choose the simpler model.

4. The estimate of the error standard deviation, σ, based on the third degree model is:

(a) $\sqrt{401.20/96}$.

(b) $\sqrt{808.9614/96}$.

(c) $\sqrt{(808.9614 - 401.20)/96}$.

(d) $401.20/96$.

(e) $(808.9614 - 401.20)/96$.

For a kth degree polynomial model, the estimate of σ is:

$$\hat{\sigma} \;=\; \sqrt{\frac{SSE}{n-k-1}} \qquad \text{(using the formula from pg. 53 of Chapter 1B notes).}$$

The answer then follows from the fact that we have $n = 100$, and that for the third degree polynomial model, $k = 3$ and $SSE = SST - SSR = 808.9614 - 401.20$.

5. Suppose that x is the length and Y is the weight of smallmouth bass (a type of fish) in a large Texas lake. A sample of smallmouth bass was caught at this lake and a simple linear regression model for Y versus x was fitted using the observed data. Using all the results we studied in class regarding inference for simple linear regression, the following two intervals were produced, one of which is a 95% confidence interval for the average weight (in pounds) of smallmouth bass that are 14 inches long, and the other is a 95% prediction interval for the weight (in pounds) of a particular smallmouth bass that is 14 inches long:

$$1.50 \pm 0.15 \quad \text{and} \quad 1.50 \pm 0.50.$$

Which of the following is correct?

(a) This information implies that the average weight of a 21 inch long smallmouth bass is about 2.25 lbs.

(b) The interval 1.50 ± 0.15 is the prediction interval and the other is the confidence interval.

(c) The interval 1.50 ± 0.15 is the confidence interval and the other is the prediction interval.

(d) Both (a) and (b) are correct.

(e) Both (a) and (c) are correct.

The prediction interval for Y given x is *always* wider than the confidence interval (with the same confidence level) for the *average* of Y at the same x, since the former accounts for more uncertainty, i.e. the additional variability in Y (due to the noise ϵ) given x. This is also seen from pg. 37 of Chapter 1A notes and was discussed at length in class. Hence, (c) is correct. Also, (a) is not correct since the given information isn't enough to estimate $E(Y)$ at $x = 21$.

6. Suppose the weights (Y, in pounds) of female Golden Retriever dogs between the ages of 3 and 12 months follow the simple linear regression model:

$$Y = 11.3 + 4.9x + \epsilon \quad \text{with} \;\; \epsilon \sim N(0, \sigma^2),$$

where x is their age (in months) and ϵ has a Normal distribution with mean 0 and standard deviation $\sigma = 5$ pounds for each x. Then, the probability that a 6 month old female golden retriever will weigh between 35 and 50 pounds is:

(a) 0.0456.

(b) 0.0957.

(c) 0.1585.

(d) 0.8415.

(e) 0.9544.

Using the linear model, we have that for a given age x, the weight $Y \sim N(11.3 + 4.9x, 5^2)$. We are asked to find $P(35 < Y < 50)$ *given* $x = 6$. Hence, when $x = 6$, we have:

$$\begin{aligned} P(35 < Y < 50 \mid x = 6) &= P\left(\frac{35 - 11.3 - 4.9 * 6}{5} < \; Z \; < \frac{50 - 11.3 - 4.9 * 6}{5}\right) \\ &= P(-1.14 < Z < 1.86) \qquad \text{where } \; Z \sim N(0,1), \\ &= 0.9686 - (1 - 0.8729) \qquad \text{(using the Normal table)}, \\ &= 0.8415. \end{aligned}$$

7. In the setting of Question **6**, suppose that a female Golden Retriever is defined to be 'large' if she is at or above the 80th percentile of the weights for dogs of her age. Then, what is the smallest weight that qualifies a 9 month old female Golden Retriever as 'large'?

(a) 44.9 pounds.

(b) 50.2 pounds.

(c) 59.6 pounds.

(d) 63.6 pounds.

(e) 70 pounds.

A dog of age x months is defined to be 'large' if her weight is at or above the 80th percentile of the distribution of the weight Y of all dogs of the same age. From the linear model, we have that for a given age x, the weight $Y \sim N(11.3+4.9x, 5^2)$. Hence, to find the minimum weight for a 9 month old dog to be deemed 'large', we need to find the 80th percentile of the $N(11.3+4.9x, 5^2)$ distribution when $x = 9$ months. Now, the 80th percentile of the standard Normal, i.e. $N(0,1)$, distribution is approximately 0.84 (from the Normal table). Hence, the corresponding 80th percentile of the $N(11.3+4.9x, 5^2)$ distribution with $x = 9$ is given by:

$$(5 * 0.84) + (11.3 + 4.9 * 9) \;=\; 59.6 \text{ pounds.}$$

In the above, we used the fact if z_p denotes the $100p^{th}$ percentile of the $N(0,1)$ distribution, then the $100p^{th}$ percentile of the $N(\mu, \sigma^2)$ distribution is given by: $\sigma z_p + \mu$ for any μ and σ^2. Thus, a 9 month old female dog must weigh at least 59.6 pounds for it to be called 'large'.

Read the following information to answer **Questions 8–11**:

The following information is from a report on the determination of silver content of galena crystals grown in a closed hydrothermal system over a range of temperatures. Let x denote the crystallization temperature in degrees centigrade, and Y the silver content in mol%. A simple linear regression was fitted for Y versus x. The observed data are as follows:

x	398	292	352	575	568	450	550	408	484	350	503	600
y	0.15	0.05	0.23	0.43	0.23	0.4	0.44	0.44	0.45	0.09	0.59	0.63

Some summary statistics from the regression analysis are as follows:

$$\sum_{i=1}^{12} x_i = 5530, \qquad \sum_{i=1}^{12} y_i = 4.13, \qquad \sum_{i=1}^{12}(x_i - \bar{x})(y_i - \bar{y}) = 155.4983,$$

$$\sum_{i=1}^{12}(x_i - \bar{x})^2 = 113642, \qquad \sum_{i=1}^{12}(y_i - \bar{y})^2 = 0.39709, \qquad \sum_{i=1}^{12}(y_i - \hat{y}_i)^2 = 0.18432.$$

Use this information to answer **Questions 8–11** below.

8. The least-squares estimates of the intercept and slope parameters, respectively, are:

(a) $\hat{\beta}_0 = 0.2164$ and $\hat{\beta}_1 = 0.224$.

(b) $\hat{\beta}_0 = 0.42$ and $\hat{\beta}_1 = 0.05$.

(c) $\hat{\beta}_0 = 0.2164$ and $\hat{\beta}_1 = -0.224$.

(d) $\hat{\beta}_0 = -0.2864$ and $\hat{\beta}_1 = 0.001368$.

(e) $\hat{\beta}_0 = -0.80$ and $\hat{\beta}_1 = 0.001368$.

Using the formulae from pg. 14 of Chapter 1A notes, the slope and intercept estimates are:

$$\begin{aligned}\hat{\beta}_1 &= \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2} = \frac{155.4983}{113642} = 0.001368,\\ \hat{\beta}_0 &= \bar{y} - \hat{\beta}_1 \bar{x} = \frac{4.13}{12} - \hat{\beta}_1 \frac{5530}{12} = -0.2864.\end{aligned}$$

9. To test $H_0 : \beta_1 = 0$ against $H_a : \beta_1 \neq 0$, the test statistic to be used is:

(a) $\hat{\beta}_1 / (\hat{\sigma}\sqrt{1/12 + 1/113642})$

(b) $(\hat{\beta}_0 + \hat{\beta}_1)/\hat{\sigma}$.

(c) $\hat{\beta}_1/\hat{\sigma}$.

(d) $\sqrt{12}\,\hat{\beta}_1/\hat{\sigma}$.

(e) $(337.108)\hat{\beta}_1/\hat{\sigma}$.

Using the formula from pg. 32 of Chapter 1A notes the test statistic T in this case will be:

$$T = \frac{\hat{\beta}_1 - 0}{\hat{\sigma}/\sqrt{\sum_{i=1}^n (x - \bar{x})^2}} = \frac{(337.108)\hat{\beta}_1}{\hat{\sigma}}.$$

10. The proportion of variance in silver content that is explained by temperature is:

(a) 0.668.

(b) 0.956.

(c) 0.810.

(d) 0.464.

(e) 0.536.

We know (from pg. 25, Chapter 1A notes) that this proportion is R^2, or SSR/SST. We are given here that $SSE = \sum_{i=1}^{12}(y_i - \hat{y}_i)^2 = 0.18432$ and $SST = \sum_{i=1}^{12}(y_i - \bar{y})^2 = 0.39709$. So

$$R^2 = SSR/SST = (SST - SSE)/SST = (0.39709 - 0.18432)/0.39709 = 0.536.$$

11. Given that the quantity: $1/12+(500-5530/12)^2/113642$ equals 0.0968, a 95% confidence interval for the *average* silver content of galena crystals grown in a closed hydrothermal system at 500 degrees centigrade is:

(a) $(\hat{\beta}_0 + 500\hat{\beta}_1) \pm 1.96\hat{\sigma}\sqrt{0.0968}$.

(b) $(\hat{\beta}_0 + 500\hat{\beta}_1) \pm 1.96\hat{\sigma}\sqrt{1 + 0.0968}$.

(c) $(\hat{\beta}_0 + 500\hat{\beta}_1) \pm 2.228\hat{\sigma}\sqrt{0.0968}$.

(d) $(\hat{\beta}_0 + 500\hat{\beta}_1) \pm 2.228\hat{\sigma}\sqrt{1 + 0.0968}$.

(e) Cannot be determined from the information given.

Just use the formula on pg. 35 of Chapter 1A notes for the confidence interval of $E(Y)$ at a given $x = x_0 = 500$. The answer follows directly by noting that $1/n+(x_0-\bar{x})^2/\sum_{i=1}^{n}(x_i-\bar{x})^2 = 1/12+(500-5530/12)^2/113642 = 0.0968$ (given), along with use of the t-table which gives the 97.5th percentile of the t_{n-2} distribution in this case to be: $t_{n-2;\alpha/2} = t_{10;0.025} = 2.228$.

12. A simple linear regression model is fitted to $n = 20$ observations of (x, y) pairs, and the following summary statistics were computed:

$$\sum_{i=1}^{n}(y_i - \bar{y})^2 = 202.71 \qquad \text{and} \qquad \sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2 = 157.25.$$

In this case, the estimated error variance $\hat{\sigma}^2$ is:

(a) 157.25/202.71.

(b) 157.25/18.

(c) $(202.71 - 157.25)/18$.

(d) $1 - 157.25/202.71$.

(e) Cannot be determined from the information given.

Using the summary statistics, $SSR = \sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2 = 157.25$ and $SST = \sum_{i=1}^{n}(y_i - \bar{y})^2 = 202.71$. Hence, $SSE = SST - SSR = 202.71 - 157.25$. The estimate of the error variance is given by: $\hat{\sigma}^2 = SSE/(n-2)$ which therefore equals $(202.71 - 157.25)/18$ since $n = 20$.

13. Two random variables X and Y have a population correlation coefficient, ρ, that equals 0. We may thus conclude that:

(a) X and Y are independent.

(b) X and Y are not independent.

(c) X and Y could be independent.

(d) $E(X) = E(Y) = 0$.

(e) Donkeys can fly.

See pg. 43 of the Chapter 1A notes. Remember correlation does *not* necessarily imply causation - two random variables may be uncorrelated (i.e. $\rho = 0$) and *still* may be dependent!

14. A researcher has data $(x_1, y_1), \ldots, (x_n, y_n)$ and wants to fit the following linear model:

$$y_i = \beta x_i + \epsilon_i, \quad i = 1, \ldots, n,$$

i.e. a linear regression model *without* an intercept parameter. Here, $\epsilon_1, \ldots, \epsilon_n$ are independent and each $\epsilon_i \sim N(0, \sigma^2)$ as usual. Define $\bar{y} = \sum_{i=1}^n y_i/n$ and $\bar{x} = \sum_{i=1}^n x_i/n$. Then, the correct expression for the least-squares estimate $\hat{\beta}$ of β for this model is given by:

(a) $\hat{\beta} = \bar{x}/\bar{y}$.

(b) $\hat{\beta} = \bar{y}/\bar{x}$.

(c) $\hat{\beta} = \sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y}) / \sum_{i=1}^n (x_i - \bar{x})^2$.

(d) $\hat{\beta} = \sum_{i=1}^n x_i y_i / \sum_{i=1}^n x_i^2$.

(e) Cannot be determined from the information given.

The appropriate error sum of squares for the model being considered is given by:

$$f(\beta) = \sum_{i=1}^n (y_i - \beta x_i)^2.$$

We need to find the value of β that minimizes $f(\beta)$ and this minimizer will be the required least-squares estimate $\hat{\beta}$ of β. Taking the derivative of $f(\beta)$ with respect to β, we have:

$$f'(\beta) = \frac{d}{d\beta} f(\beta) = 2\sum_{i=1}^n (y_i - \beta x_i)(-x_i).$$

Setting this derivative equal to 0 and solving for β gives the reqd. solution/minimizer $\hat{\beta}$ as:

$$-2\sum_{i=1}^n (y_i - \beta x_i)x_i = 0 \;\Rightarrow\; \sum_{i=1}^n (y_i - \beta x_i)x_i = 0 \;\Rightarrow\; \sum_{i=1}^n x_i y_i = \sum_{i=1}^n \beta x_i^2 \;\Rightarrow\; \hat{\beta} = \frac{\sum_{i=1}^n x_i y_i}{\sum_{i=1}^n x_i^2}.$$

15. Suppose that a response variable Y is such that $Y = \alpha e^{\beta x}\epsilon$, where ϵ is a random variable and α, β and x are constants. Then, the correct expressions for $\mathrm{Var}(Y)$ and $\mathrm{Var}(\log Y)$ are:

(a) $\mathrm{Var}(Y) = \mathrm{Var}(\epsilon)$ and $\mathrm{Var}(\log Y) = \mathrm{Var}(\log \epsilon)$.

(b) $\mathrm{Var}(Y) = \alpha e^{\beta x}\mathrm{Var}(\epsilon)$ and $\mathrm{Var}(\log Y) = \mathrm{Var}(\log \epsilon)$.

(c) $\mathrm{Var}(Y) = \alpha^2 e^{2\beta x}\mathrm{Var}(\epsilon)$ and $\mathrm{Var}(\log Y) = \log\alpha + \beta x + \mathrm{Var}(\log \epsilon)$.

(d) $\mathrm{Var}(Y) = \alpha^2 e^{2\beta x}\mathrm{Var}(\epsilon)$ and $\mathrm{Var}(\log Y) = \log\alpha + \log(\beta x) + \mathrm{Var}(\log \epsilon)$.

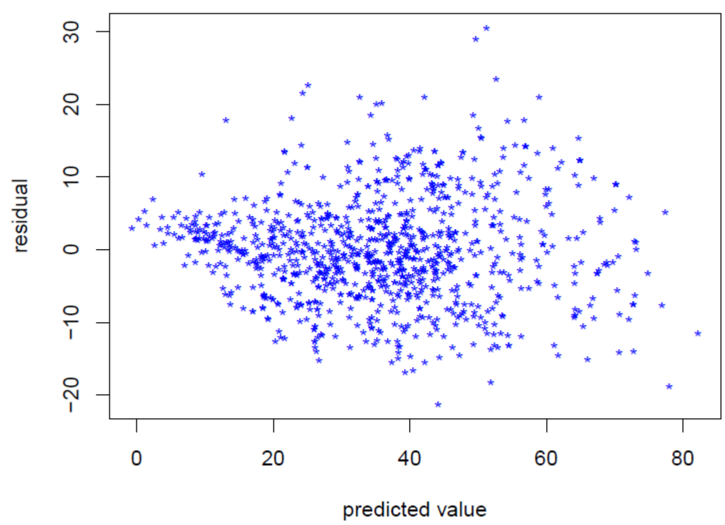
(e) $\mathrm{Var}(Y) = \alpha^2 e^{2\beta x}\mathrm{Var}(\epsilon)$ and $\mathrm{Var}(\log Y) = \mathrm{Var}(\log \epsilon)$.

Since α, β and x are constants (i.e. fixed and non-random), the variances are given by:

$$\begin{aligned}\mathrm{Var}(Y) &= \mathrm{Var}\left(\alpha e^{\beta x}\epsilon\right) = \left(\alpha e^{\beta x}\right)^2 \mathrm{Var}(\epsilon) = \alpha^2 e^{2\beta x}\mathrm{Var}(\epsilon) \quad \text{and} \\ \mathrm{Var}(\log Y) &= \mathrm{Var}(\log\alpha + \beta x + \log\epsilon) = \mathrm{Var}(\log\epsilon).\end{aligned}$$

Throughout, we used the basic properties: $\mathrm{Var}(cZ) = c^2\mathrm{Var}(Z)$ and $\mathrm{Var}(c+Z) = \mathrm{Var}(Z)$ for any random variable Z and any (non-random) constant c. We also used the basic properties: $\log(ab) = \log a + \log b$ and $\log a^c = c\log a$ for any $a > 0, b > 0$ and any real number c.

16. The residual plot below was obtained from a polynomial regression analysis.



From this plot we can see that:

(a) The residuals show no pattern.

(b) The residuals are clearly Normally distributed.

(c) The residual variance increases somewhat as the predicted value increases.

(d) Both (b) and (c) are true.

(e) The residuals were obviously computed incorrectly.

The answer should be quite obvious. Similar plots were shown several times in class in various contexts (e.g. pg. 19-22 and pg. 76-79 of Chapter 1A and 1B notes). We also discussed the diagnosis, possible causes and remedies for this type of phenomenon in details in class.

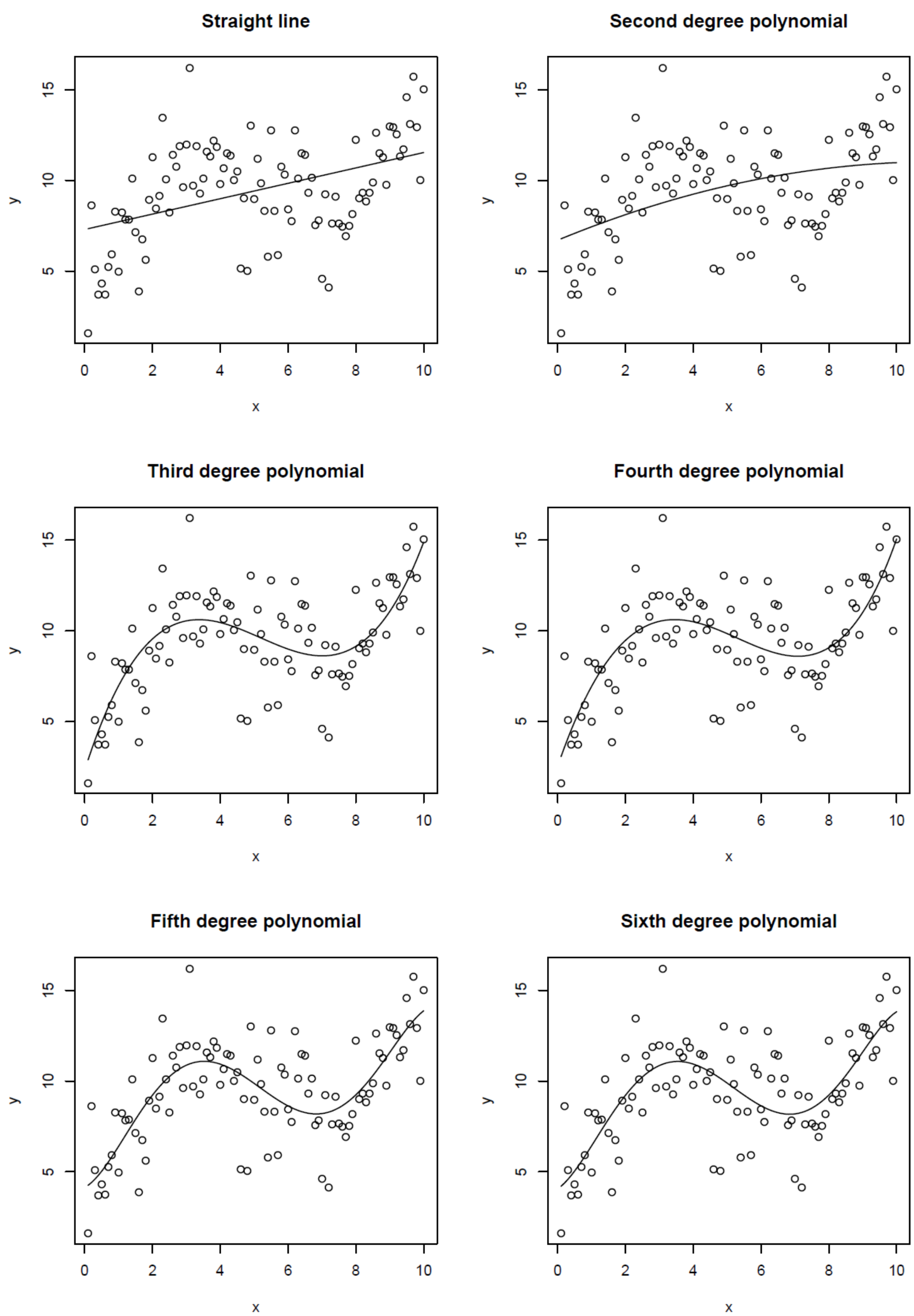


Table A.3 The Cumulative Distribution Function for the Standard Normal Distribution: Values of $\Phi(z)$ for Nonnegative z

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

Table A.4 Percentiles of the *T* Distribution

df	90%	95%	97.5%	99%	99.5%	99.9%
1	3.078	6.314	12.706	31.821	63.657	318.309
2	1.886	2.920	4.303	6.965	9.925	22.327
3	1.638	2.353	3.183	4.541	5.841	10.215
4	1.533	2.132	2.777	3.747	4.604	7.173
5	1.476	2.015	2.571	3.365	4.032	5.893
6	1.440	1.943	2.447	3.143	3.708	5.208
7	1.415	1.895	2.365	2.998	3.500	4.785
8	1.397	1.860	2.306	2.897	3.355	4.501
9	1.383	1.833	2.262	2.822	3.250	4.297
10	1.372	1.812	2.228	2.764	3.169	4.144
11	1.363	1.796	2.201	2.718	3.106	4.025
12	1.356	1.782	2.179	2.681	3.055	3.930
13	1.350	1.771	2.160	2.650	3.012	3.852
14	1.345	1.761	2.145	2.625	2.977	3.787
15	1.341	1.753	2.132	2.603	2.947	3.733
16	1.337	1.746	2.120	2.584	2.921	3.686
17	1.333	1.740	2.110	2.567	2.898	3.646
18	1.330	1.734	2.101	2.552	2.879	3.611
19	1.328	1.729	2.093	2.540	2.861	3.580
20	1.325	1.725	2.086	2.528	2.845	3.552
21	1.323	1.721	2.080	2.518	2.831	3.527
22	1.321	1.717	2.074	2.508	2.819	3.505
23	1.319	1.714	2.069	2.500	2.807	3.485
24	1.318	1.711	2.064	2.492	2.797	3.467
25	1.316	1.708	2.060	2.485	2.788	3.450
26	1.315	1.706	2.056	2.479	2.779	3.435
27	1.314	1.703	2.052	2.473	2.771	3.421
28	1.313	1.701	2.048	2.467	2.763	3.408
29	1.311	1.699	2.045	2.462	2.756	3.396
30	1.310	1.697	2.042	2.457	2.750	3.385
40	1.303	1.684	2.021	2.423	2.705	3.307
80	1.292	1.664	1.990	2.374	2.639	3.195
∞	1.282	1.645	1.960	2.326	2.576	3.090