1. Suppose that in a two-factor ANOVA (with factors A and B), a significant interaction is found. Suppose in addition that the F-test for factor A is significant. The latter result is potentially misleading because:

(a) We must also consider the F-test for factor B.

(b) The value of SSE might be too large.

(c) The levels of factor A may only be significantly different from each other when a particular level of B is used.

(d) We cannot be sure in this case whether or not the sums of squares add up to SST.

(e) Interactions are hard enough to explain without having to explain a significant factor A effect as well.

Suppose there are two levels of B. An interaction allows for the possibility that at level 1 of B there is no difference between levels of A, while at level 2 of B there are differences between levels of A. It would thus be misleading to say that there is an effect due to A because this is not true when level 1 of B is used. This whole principle was also discussed in detail in class.

2. Suppose one is using a χ^2 goodness-of-fit test to test the fit of a particular model that does not completely specify all the unknown parameters. The most *ideal* way to estimate the unknown parameters in this case is to:

(a) Use the sample mean and variance.

(b) Use the method of moments.

(c) Use the method of maximum likelihood.

(d) Use the method of minimum χ^2.

(e) Do so while watching the NFL playoffs.

See pg. 187 of the lecture notes for Chapter 3.

3. A genetics experiment on characteristics of tomato plants provided the following data on the number of offspring expressing four phenotypes.

Phenotype	Frequency
1 - Tall, cut-leaf	926
2 - Dwarf, cut-leaf	293
3 - Tall, potato-leaf	288
4 - Dwarf, cut-leaf	104
Total	1611

It was of interest to test the following hypothesis, where π_i = the proportion of tomato plants expressing phenotype i, $i = 1, 2, 3, 4$:

$$H_0 : \pi_1 = \frac{9}{16}, \quad \pi_2 = \frac{3}{16}, \quad \pi_3 = \frac{3}{16}, \quad \pi_4 = \frac{1}{16}.$$

The value of the χ^2 statistic for testing this hypothesis is 1.469. Which of the following statements is correct for a test with level $\alpha = 0.1$?

(a) The P-value is greater than 0.1 and so there is enough evidence to conclude that the phenotype proportions differ from those in H_0.

(b) The P-value is greater than 0.1 and so there is not enough evidence to conclude that the phenotype proportions differ from those in H_0.

(c) The P-value is less than 0.1 and so there is enough evidence to conclude that the phenotype proportions differ from those in H_0.

(d) The P-value is less than 0.1 and so there is not enough evidence to conclude that the phenotype proportions differ from those in H_0.

(e) Run for your lives! The killer tomatoes are attacking!

This corresponds to a Multinomial experiment with $k = 4$ categories and the χ^2 goodness-of-fit test with category probabilities fully specified. From pg. 179-181 of Chapter 3 notes: under H_0, the test statistic χ^2 follows a χ^2 distribution with $k - 1 = 3$ degrees of freedom, and we reject H_0 at level $\alpha = 0.1$ if $\chi^2 > \chi^2_{k-1,\alpha} = \chi^2_{3,0.1} = 6.251$. The observed value of χ^2 is: $\chi^2_{obs} = 1.469 < 6.251$. So we cannot reject H_0 at level α based on the observed data. Further, the P-value $= P(\chi^2_{k-1} > \chi^2_{obs}) = P(\chi^2_3 > 1.469) > 0.1$. So we fail to reject H_0 at level α. (Note: the actual P-value will be 0.689 but you did *not* need to compute that here!)

4. The 2008 General Social Survey produced the following count data for $n = 955$ families:

	Marital Happiness		
Income	Not	Pretty	Very
Above	123	105	7
Average	291	151	17
Below	172	83	6

The value of the χ^2 statistic for testing independence of income and marital happiness is 12.84. Which of the following is the correct conclusion for a test with $\alpha = 0.05$?

(a) The P-value is smaller than 0.05 and we therefore conclude that income and marital happiness are independent.

(b) The P-value is smaller than 0.05 and we therefore conclude that income and marital happiness are not independent.

(c) The P-value is larger than 0.05 and we therefore conclude that income and marital happiness are not independent.

(d) The P-value is larger than 0.05 and we therefore conclude that income and marital happiness are independent.

(e) Having a high income is the secret to happiness in marriage.

This corresponds to a contingency table based χ^2 test for independence of two characteristics (income and marital happiness) with $I = 3$ and $J = 3$ categories (see Chapter 3). Under H_0, the test statistic χ^2 follows a χ^2 distribution with degrees of freedom $= (I - 1)(J - 1) = 4$, and its observed value is $\chi^2_{obs} = 12.84$. Hence, the P-value for the test is $P(\chi^2_4 > \chi^2_{obs}) = P(\chi^2_4 > 12.84)$. Since $12.84 > \chi^2_{4,0.05} = 9.488$, the P-value must be smaller than 0.05. So we reject H_0 at level $\alpha = 0.05$, and conclude that income and marital status are not independent.

5. In Small Town, USA, the ages and smoking status of adult residents are distributed as:

		Age		
		18–30	31–45	Over 45
Smoking status	Nonsmoker	2000	3500	3000
	Smokes $<$ one pack a day	400	800	400
	Smokes $\geq$ one pack a day	200	450	150

An adult is randomly selected from Small Town, USA, and it turns out that this adult is *not* a nonsmoker. The probability that this person is between the ages of 31 and 45 is:

(a) $4750/10,900$.

(b) $1250/10,900$.

(c) $450/800$.

(d) $800/1600$.

(e) $1250/2400$.

Let A be the event that the person is between ages 31 and 45, and let B be the event that the person is not a nonsmoker. We then want to find $P(A|B)$ which is given by:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{(800+450)/10,900}{2400/10,900} = \frac{1250}{2400}.$$

Note here that the total number of people = 10,900 and we also used the following: $P(B) = P(\text{not a nonsmoker}) = 1 - P(\text{nonsmoker}) = 1 - (2000 + 3500 + 3000)/10900 = 2400/10900$.

6. An experimenter investigated the effects of two stimulant drugs, labeled A and B. She had a total of 20 rats, and randomly assigned 4 rats to each of the following five treatment groups: placebo, Drug A low, Drug A high, Drug B low, and Drug B high. Twenty minutes after injection of the drug, each rat's activity level was measured. This led to the following (partially filled) ANOVA table:

Source	df	Sum of squares	Mean square	F
Treatments		181.3		
Within treatments		159.0		
Total	19	340.3		

The value of the F-statistic for testing the hypothesis that the mean activity level is the same for all five treatments is:

(a) 0.534.

(b) 9.72.

(c) 3.192.

(d) 1.140 .

(e) 4.276.

The problem falls under one-way ANOVA with $k = 5$ treatments and a total sample size of $N = 20$. From the ANOVA table above, the sum of squares due to treatments (SSTr) = 181.3 and sum of squares due to error (SSE) = 159. Hence, using the formula from pg. 120-125 of Chapter 2A notes, the required F-statistic for testing equality of means is:

$$F = \frac{MSTr}{MSE} = \frac{SSTr/(k-1)}{SSE/(N-k)} = \frac{(181.3/4)}{(159/15)} = 4.276.$$

7. In Question **6**, the type of experimental design used is:

(a) A split-plot design.

(b) A randomized block design.

(c) A completely randomized design.

(d) A crossover design.

(e) None of the above.

See pg. 161 of Chapter 2B notes. As discussed in class, the one-way ANOVA we have learnt is the appropriate statistical analysis for data from a completely randomized design (CRD).

8. In a simple linear regression analysis, the residuals are:

(a) $y_i - (\hat{\beta}_0 + \hat{\beta}_1 x_i)$, for $i = 1, \ldots, n$.

(b) Used to check whether the variability of y tends to increase with x.

(c) Used to check whether or not the relationship between the response variable y and the independent variable (or covariate) x is linear.

(d) All of the above.

(e) None of the above.

See pg. 17-19 of Chapter 1A notes. Since this was straight out of the notes, you will **not** get any partial credit for picking out only one of these answers, i.e. options (a), (b) or (c). You get credit only if you mark the *most* correct and inclusive option which is (d).

9. In a simple linear regression analysis based on a sample of 100 observations, the 95% confidence interval (CI) for the regression parameter β_1 turned out to be $(0.27, 0.52)$. Then, which of the following is the best and most meaningful interpretation of this CI?

(a) About 95% of the time the true parameter β_1 lies within this interval.

(b) About 95% of the time the parameter estimate $\hat{\beta}_1$ lies within this interval.

(c) If one repeats the data collection process many times and constructs a 95% CI for β_1 each time, then about 95% of these intervals will contain the parameter estimate $\hat{\beta}_1$.

(d) If one repeats the data collection process many times and constructs a 95% CI for β_1 each time, then about 95% of these intervals will contain the true parameter β_1.

(e) About 95% of the time the true response variable lies within this interval.

We discussed the correct interpretation and subtleties of CIs many times in class. Option (d) is *always* the best and *only* correct interpretation of a CI, in general. Remember that the phrase 'confidence' is associated with a CI because the CI *itself* is a *random* interval (it will change if the sample changes)! On the other hand, the true parameter is always fixed! Hence, option (a) is certainly not correct. Options (b), (c) and (e) are also trivially incorrect.

10. I graded a quiz for 12 students in my class. Their scores (out of 10 points) are as follows:

$$7,\ 5,\ 5,\ 7,\ 7,\ 9,\ 8,\ 6,\ 9,\ 8,\ 6,\ 7.$$

Suppose I want to test the following hypotheses for the median (M) of the population of my students' scores: $H_0 : M = 6.5$ vs. $H_a : M > 6.5$, and suppose I use the Sign test to do so. Then, the Z-statistic and the approximate P-value for this Sign test are closest to:

(a) Z-statistic = -1.1547; P-value $\approx$ 0.125.

(b) Z-statistic = -1.1547; P-value $\approx$ 0.875.

(c) Z-statistic = 1.1547; P-value $\approx$ 0.875.

(d) Z-statistic = 1.1547; P-value $\approx$ 0.125.

(e) Z-statistic = 1.1547; P-value $\approx$ 0.25.

The number of observations greater than the null value 6.5 of the median is given by $Y = 8$. Then, using pg. 222-224 of Chapter 4 notes, the Z-statistic for the Sign test equals:

$$Z = \frac{Y - n/2}{\sqrt{n}/2} = \frac{8 - 12/2}{\sqrt{12}/2} = \frac{2}{\sqrt{12}/2} = 1.1547.$$

Since the alternative H_a is one-sided and 'right-sided', we reject for large values of Z. Under H_0, $Z \sim N(0, 1)$. Hence, the P-value for this test is $P(Z > Z_{obs}) = P(Z > 1.1547)$ where $Z \sim N(0, 1)$. Using the Normal table, this P-value roughly equals $1 - 0.8749 = 0.1251$.