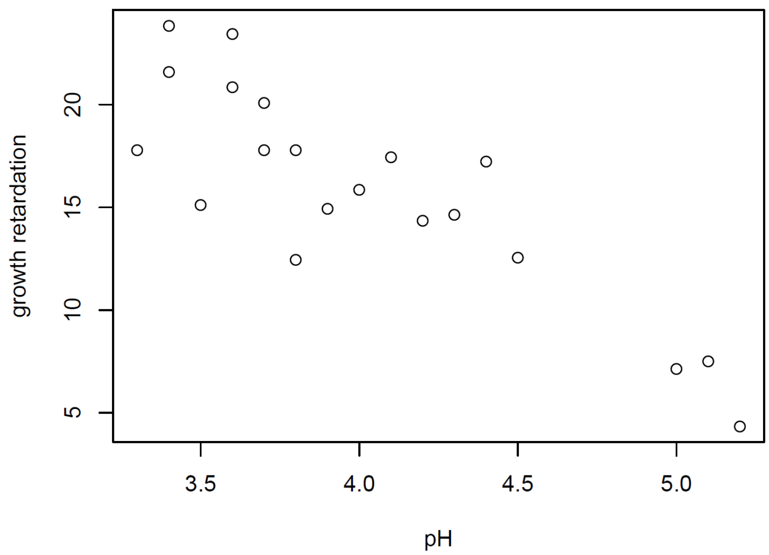
**Note**: *Throughout this exam, unless specifically stated otherwise, all notations/symbols and abbreviations used have their usual standard meanings, as used/defined in the lecture notes.*

Read the following information to answer **Questions 1–4**:

Forest scientists were interested in determining the impact of soil acidity on the retardation of tree growth. Data from $n = 20$ tree stands were collected. The measured variables were $x$ = soil pH and $y$ = index of growth retardation, and a simple linear regression model for $y$ versus $x$ was used. A scatterplot of the data and some summary statistics are given below.




$$\sum_{i=1}^{20} x_i = 80.5, \quad \sum_{i=1}^{20} y_i = 316.84, \quad \sum_{i=1}^{20}(x_i - \bar{x})(y_i - \bar{y}) = -49.022,$$

$$\sum_{i=1}^{20}(x_i - \bar{x})^2 = 6.2375, \quad \sum_{i=1}^{20}(y_i - \bar{y})^2 = 518.6051, \quad \sum_{i=1}^{20}(y_i - \hat{y}_i)^2 = 133.3295.$$

Use the information above to answer the **next four questions (Questions 1–4).**

**1.** The least-squares estimates of the intercept and slope parameters, respectively, are:

(a) $\hat{\beta}_0 = -7.86$ and $\hat{\beta}_1 = 47.48$.

(b) $\hat{\beta}_0 = 47.48$ and $\hat{\beta}_1 = -7.86$.

(c) $\hat{\beta}_0 = 15.84$ and $\hat{\beta}_1 = -7.86$.

(d) $\hat{\beta}_0 = -15.79$ and $\hat{\beta}_1 = 7.86$.

(e) $\hat{\beta}_0 = 213.16$ and $\hat{\beta}_1 = -49.022$.

**2.** A 99% confidence interval for $\beta_1$ is given by:

(a) $\hat{\beta}_1 \pm 2.879(2.72)$.

(b) $\hat{\beta}_1 \pm 2.552(2.72)$.

(c) $\hat{\beta}_1 \pm 2.879(1.09)$.

(d) $\hat{\beta}_1 \pm 2.552(1.09)$.

(e) $\hat{\beta}_1 \pm 2.101(2.72)$.

**3.** The proportion of variance in growth retardation that is explained by soil acidity is:

(a) 0.257.

(b) 0.743.

(c) 0.012.

(d) 0.611.

(e) 518.6.

**4.** An estimate of $\sigma$, the error standard deviation, is given by:

(a) 518.6051/19.

(b) 133.3295/18.

(c) $\sqrt{133.3295/18}$.

(d) $\sqrt{518.6051/19}$.

(e) Cannot be determined from the information given.

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**5.** Having fitted a simple linear regression model to a set of $n$ observations of $(x_i, y_i)$ pairs, a good way to check if there is evidence of a nonlinear relationship between $y$ and $x$ is to:

(a) Determine the sum of squared of the residuals.

(b) Examine a plot of the residuals versus $x_i$.

(c) Examine a histogram of the residuals.

(d) Do both (b) and (c).

(e) Call Dr. Chakrabortty late at night for his opinion.

**6.** The lifetime $Y$ (in thousands of holes drilled) for drill bits is related to the speed ($x$) of the drill through a simple linear regression model as follows:

$$Y = 6.0 - 0.017x + \epsilon \quad \text{with} \;\; 60 \leq x \leq 100, \quad \text{where}$$

$\epsilon$ has a Normal distribution with mean 0 and variance $\sigma^2 = 0.40$. Which of the following is the *highest* drill speed $x$ such that at least 50% of drill bits used at that speed have lifetimes greater than 4.5? (**Hint:** for Normal distributions, the 50th percentile also equals the mean.)

(a) 60.

(b) 75.9.

(c) 88.2.

(d) 112.5.

(e) 136.1.

**7.** In the setting of Question **6**, what proportion of drill bits used at a speed $x = 75$ have lifetimes larger than 4?

(a) 0.0351.

(b) 0.1251.

(c) 0.5000.

(d) 0.8749.

(e) 0.9649.

**8.** Which of the following statements about simple linear regression is **not** correct?

(a) A 90% prediction interval for the response $Y$ at a given $x = x_0$ is wider than a 90% confidence interval for the mean of $Y$ at the same $x = x_0$.

(b) The coefficient of determination, $R^2$, always lies between 0 and 1.

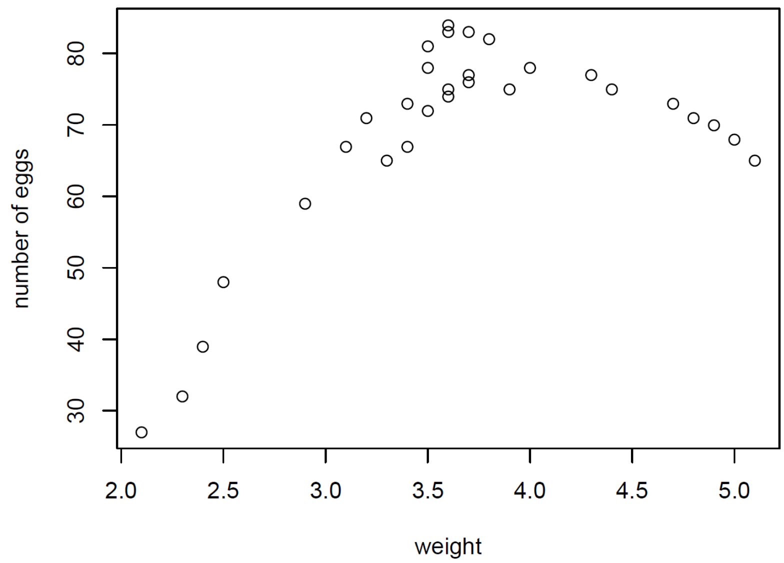
(c) A 95% confidence interval for the mean of $Y$ at a given value $x = x_0$ far from $\bar{x}$ will be narrower than a 95% confidence interval for the mean of $Y$ at $x = \bar{x}$.

(d) The average increase in $Y$ for a 1-unit increase in $x$ is $\beta_1$ (i.e. the slope parameter).

(e) The parameter $\beta_0$ equals the $y$–intercept of the true regression line.

Read the following information to answer **Questions 9–12**:

In a study of the reproductive success of grasshoppers, an entomologist collected a sample of $n = 30$ female grasshoppers. He recorded $y =$ the number of mature eggs produced and $x =$ the body weight (in grams) for each grasshopper. A scatterplot of the data is given below.




Polynomial models of degree 1 to 5 were fitted to these data. Information obtained from `R` is given below. Use this information to answer the **next four questions (Questions 9–12)**.

| Polynomial degree | $R^2$ | BIC | Estimated model |
|---|---|---|---|
| 1 | 0.367 | 240.9 | $27.80 + 11.24x$ |
| 2 | 0.942 | 172.5 | $-155.71 + 116.01x - 14.30x^2$ |
| 3 | 0.942 | 175.8 | $-171.48 + 130.32x - 18.43x^2 + 0.38x^3$ |
| 4 | 0.945 | 177.7 | $112.68 - 210.76x + 130.48x^2 - 27.74x^3 + 1.94x^4$ |
| 5 | 0.945 | 181.1 | $77.18 - 156.83x + 98.47x^2 - 18.46x^3 + 0.63x^4 + 0.073x^5$ |

$$\text{SST} = 6066.17, \quad \bar{y} = 68.83, \quad \bar{x} = 3.65.$$

**9.** Based on the information given, the best choice for the polynomial degree is:

(a) Degree 1, because it has the highest value of BIC.

(b) Degree 2, because it has the lowest value of BIC.

(c) Degree 3, because of Occam's razor.

(d) Either degree 4 or degree 5, because they have the largest $R^2$ values.

(e) Cannot be determined from the information given.

**10.** Using the third degree model, an estimate of the *average* number of mature eggs produced by female grasshoppers weighing 3 grams is:

(a) 61.52.

(b) 62.76.

(c) 60.15.

(d) 63.87.

(e) 68.83.

**11.** The entomologist wants to predict the number of mature eggs that will be produced by a particular grasshopper (affectionately known as Jumpy) that weighs $x_0 = 4.5$ grams. Let $\hat{\mu}(x)$ denote the $k$th degree polynomial that was finally chosen by the entomologist (according to some criteria of his liking). The following information was determined using `R`:

$$\hat{\mu}(4.5) = 76.78, \quad \hat{\sigma} = 3.605, \quad \text{estimated standard error of } \hat{\mu}(4.5)\text{: } SE_{\hat{\mu}(4.5)} = 0.988.$$

Then, we can be 95% sure that the number of eggs Jumpy produces will lie in the interval:

(a) $76.78 \pm t_{29-k;0.025}(0.988)$.

(b) $76.78 \pm t_{29-k;0.025}\sqrt{(3.605)^2 + (0.988)^2}$.

(c) $76.78 \pm 1.96(3.605)$.

(d) $76.78 \pm 3.605$.

(e) $3.605 \pm 1.96(76.78)$.

**12.** The estimate of the error variance, $\sigma^2$, based on the fourth degree model is given by:

(a) 0.945.

(b) 6066.17(0.945)/25.

(c) 6066.17/30.

(d) 6066.17(1 − 0.945)/25.

(e) Cannot be calculated from the information given.

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**13.** A researcher has data $(x_1, y_1), \ldots, (x_n, y_n)$ and wants to fit the following linear model:

$$y_i = \beta_0 + \beta_1(1/x_i) + \epsilon_i, \quad i = 1, \ldots, n, \quad \text{where} \quad x_i \neq 0 \quad \text{and}$$

$\epsilon_i \sim N(0, \sigma^2)$ and $\epsilon_1, \ldots, \epsilon_n$ are independent. (You saw this model for the 'wind speed' data in class and in the homeworks.) Define $\bar{y} = \sum_{i=1}^n y_i/n$, $\bar{x} = \sum_{i=1}^n x_i/n$ and $\bar{x}_{inv} = \sum_{i=1}^n (1/x_i)/n$. Then, the correct expression for the least-squares estimate $\hat{\beta}_1$ of $\beta_1$ in this model is given by:

(a) $\hat{\beta}_1 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}$.

(b) $\hat{\beta}_1 = \frac{\sum_{i=1}^n \{(1/x_i) - (1/\bar{x})\}(y_i - \bar{y})}{\sum_{i=1}^n \{(1/x_i) - (1/\bar{x})\}^2}$.

(c) $\hat{\beta}_1 = \frac{\sum_{i=1}^n \{(1/x_i) - \bar{x}_{inv}\}(y_i - \bar{y})}{\sum_{i=1}^n \{(1/x_i) - \bar{x}_{inv}\}^2}$.

(d) $\hat{\beta}_1 = \frac{\sum_{i=1}^n \{(1/x_i) - \bar{x}_{inv}\}\{(1/y_i) - (1/\bar{y})\}}{\sum_{i=1}^n \{(1/x_i) - \bar{x}_{inv}\}^2}$.

(e) $\hat{\beta}_1 = \frac{\sum_{i=1}^n \{(1/x_i) - (1/\bar{x})\}\{(1/y_i) - (1/\bar{y})\}}{\sum_{i=1}^n \{(1/x_i) - (1/\bar{x})\}^2}$.

**14.** In polynomial regression, the least squares estimates of the regression coefficients are the solution of a system of equations, the so-called 'normal equations'. These equations:

(a) Are non-linear and have an explicit solution.

(b) Are linear and do not have an explicit solution.

(c) Are non-linear and do not have an explicit solution.

(d) Are linear and have an explicit solution.

(e) Have befuddled budding statisticians for a good 150 years.

**15.** Suppose that a response variable $Y > 0$ is such that $\log Y = \alpha + \beta x + \varepsilon$, where $\varepsilon$ is a random variable, and $\alpha$, $\beta$, $x$ are constants (i.e. non-random), and the logarithm is taken to the base $e$. Then, the correct expressions for $E(Y)$ and $\mathrm{Var}(Y)$ are:

(a) $E(Y) = \alpha + \beta x + E(\varepsilon)$ and $\mathrm{Var}(Y) = \mathrm{Var}(\varepsilon)$.

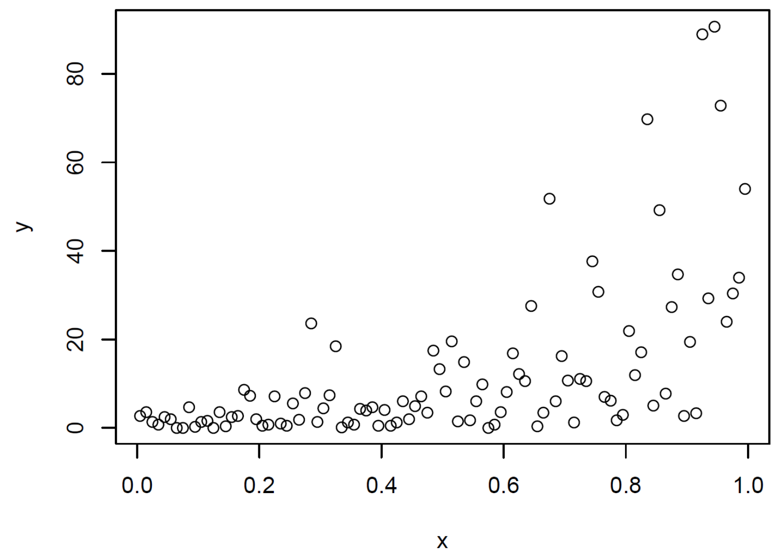
(b) $E(Y) = e^{\alpha} e^{\beta x} e^{E(\varepsilon)}$ and $\mathrm{Var}(Y) = e^{\mathrm{Var}(\varepsilon)}$.

(c) $E(Y) = e^{\alpha} e^{\beta x} E(e^{\varepsilon})$ and $\mathrm{Var}(Y) = e^{2\alpha} e^{2\beta x} \mathrm{Var}(e^{\varepsilon})$.

(d) $E(Y) = e^{\alpha} e^{\beta x} e^{E(\varepsilon)}$ and $\mathrm{Var}(Y) = e^{2\alpha} e^{2\beta x} e^{\mathrm{Var}(\varepsilon)}$.

(e) $E(Y) = e^{\alpha} + e^{\beta x} + E(e^{\varepsilon})$ and $\mathrm{Var}(Y) = \mathrm{Var}(e^{\varepsilon})$.

**16.** Consider the following scatterplot of a regression data $(x_1, y_1), \ldots, (x_{100}, y_{100})$.




Which of the following is the *best* answer? (Remember: there is **no** partial credit!)

(a) It appears that the variance of the response variable $Y$ is increasing with $x$.

(b) It appears that the variance of the response variable $Y$ is decreasing with $x$.

(c) The variance of the response variable $Y$ is approximately constant over all $x$.

(d) Using a log transformation of the response $Y$ is a good idea in this case.

(e) Both (a) and (d) are correct.

**Table A.3** The Cumulative Distribution Function for the Standard Normal Distribution: Values of $\Phi(z)$ for Nonnegative $z$

| $z$ | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |

## Table A.4 Percentiles of the *T* Distribution

| *df* | 90% | 95% | 97.5% | 99% | 99.5% | 99.9% |
|---|---|---|---|---|---|---|
| 1 | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 | 318.309 |
| 2 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 22.327 |
| 3 | 1.638 | 2.353 | 3.183 | 4.541 | 5.841 | 10.215 |
| 4 | 1.533 | 2.132 | 2.777 | 3.747 | 4.604 | 7.173 |
| 5 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 5.893 |
| 6 | 1.440 | 1.943 | 2.447 | 3.143 | 3.708 | 5.208 |
| 7 | 1.415 | 1.895 | 2.365 | 2.998 | 3.500 | 4.785 |
| 8 | 1.397 | 1.860 | 2.306 | 2.897 | 3.355 | 4.501 |
| 9 | 1.383 | 1.833 | 2.262 | 2.822 | 3.250 | 4.297 |
| 10 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 4.144 |
| 11 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 4.025 |
| 12 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 3.930 |
| 13 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | 3.852 |
| 14 | 1.345 | 1.761 | 2.145 | 2.625 | 2.977 | 3.787 |
| 15 | 1.341 | 1.753 | 2.132 | 2.603 | 2.947 | 3.733 |
| 16 | 1.337 | 1.746 | 2.120 | 2.584 | 2.921 | 3.686 |
| 17 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 3.646 |
| 18 | 1.330 | 1.734 | 2.101 | 2.552 | 2.879 | 3.611 |
| 19 | 1.328 | 1.729 | 2.093 | 2.540 | 2.861 | 3.580 |
| 20 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | 3.552 |
| 21 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 3.527 |
| 22 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 | 3.505 |
| 23 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | 3.485 |
| 24 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | 3.467 |
| 25 | 1.316 | 1.708 | 2.060 | 2.485 | 2.788 | 3.450 |
| 26 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 | 3.435 |
| 27 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | 3.421 |
| 28 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 | 3.408 |
| 29 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 | 3.396 |
| 30 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 | 3.385 |
| 40 | 1.303 | 1.684 | 2.021 | 2.423 | 2.705 | 3.307 |
| 80 | 1.292 | 1.664 | 1.990 | 2.374 | 2.639 | 3.195 |
| $\infty$ | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 3.090 |