%Appendix 1

Calculation of power distribution.

%%%%% talking about how to calculate the power in each section base on FIT knowledge. In %Appendix\_power\_distribution

This section aims for offering a set of numerical functions to calculating the power distribution along beam propagation in a photonic waveguide. The simplest idea is to compute the power in waveguide and base by integrating power flow density (or Poynting vector $\vec{S}$) at their cross-sections respectively. Meanwhile CST MWS has constructed a cuboid like Fig.\quad\ref{Afig:app\_power\_distribution01}, which contains all engaged objects such as waveguide and TLF, as a total calculation space which is discretized through Finite Integral method (FIT) and all variables(see \cite{script\_FeldSim} or section Finite Integration Method) from FIT are also available from CST MWS. In following it will be introduced how to calculate the power distribution of a Fiber-to-Chip model (see chapter modelling).

\begin{figure}[ht]

\centering

\includegraphics[width=0.5 \textwidth]{bilder/basic\_waveguide\_efield}

\caption{Calculation Cuboid in CST}

\label{Afig:app\_power\_distribution01}

\end{figure}

The first step is to choose a working plane. Suppose a working plane through point z$\_{0}$ at z axis in the total calculation space Fig.\quad\ref{Afig:app\_power\_distribution01} is given, the point index n$\_{0}$ can be determined by its coordinate z$\_{0}$. The plane Fig. \quad\ref{Afig:app\_power\_distribution02} is divided into small elemental pieces in FIT. The base cross-section is given by four points coordinates (x$\_{1}$, y$\_{1}$),(x$\_{2}$, y$\_{2}$),(x$\_{3}$, y$\_{3}$) and (x$\_{4}$, y$\_{4}$). Their points indexes (n$\_{1}$, n$\_{2}$, n$\_{3}$, n$\_{4}$) can also be derived.

\begin{figure}[ht]

\centering

\includegraphics[width=0.5 \textwidth]{bilder/basic\_waveguide\_efield}

\caption{Cross-section at z$\_{0}$}

\label{Afig:app\_power\_distribution02}

\end{figure}

In the next step it is necessary to prepare variables such as elemental

As is in \ref{script\_FeldSim } the complete elemental plane matrix D$\_{A}$ is given by (\ref{Aeq:da\_matrix}):

\begin{equation}

D\_{A}=Diag\left\{Ax(1),\cdots,Ax(Np),Ay(1),\cdots,Ay(Np), Az(1),\cdots,Az(Np)\right\}

\label{Aeq:da\_matrix}

\end{equation}

In the working cross-section only z-components of elemental plane matrix D$\_{A}$ are needed:

\begin{equation}

D\_{Az}=D\_{A}(2\*Np+1:3\*Np, 2\*Np+1:3\*Np)

\label{Aeq:daz\_matrix}

\end{equation}

Construct a auxiliary matrix $A\_{base}$, which composed of only $1$ and $0$ like Fig\quad\ref{ Afig:app\_Auxiliary\_matrix}, to indicate all points indexes which are included in base cross-section.

\begin{equation}

A\_{base}=Diag\left\{0,\cdots 0,P1,0,\cdots 0,P2,0,\cdots 0\right\}

\label{Aeq:A\_matrix}

\end{equation}

\begin{figure}[ht]

\centering

\includegraphics[width=0.5\textwidth]{bilder/basic\_waveguide\_efield}

\caption{structure of the auxiliary matrix $A$}

\label{Afig:app\_Auxiliary\_matrix}

\end{figure}

Where P$\_{x}$ are submateix:

\begin{equation}

P\_{x}=Diag\left\{1,\cdot,1\right\}\_{(n\_{2}-n\_{1})\*(n\_{2}-n\_{1})}

\end{equation}

And m is given by:

\begin{equation}

M=(n\_{3}-n\_{1})/n\_{x}=(n\_{4}-n\_{2})/n\_{x}

\end{equation}

Then

\begin{equation}

D\_{Abase}=A\_{guide}\*D\_{A}

\end{equation}

Pick z-components of the Poynting vector $S$:

\begin{equation}

S\_{z}=S(2\*Np+1:3\*Np)

\end{equation}

At last the power in base at the plane (for z=z$\_{0}$) can be counted up by:

\begin{equation}

P\_{base}(z)=sum(D\_{Aguide}\*S\_{z})

\end{equation}

By analogous procedures power in guide cross-section$P\_{guide}$ and in total cross-section $P\_{total}$ are derived. For observing the power distribution the results are processed by normalization:

\begin{align}

\eta\_{guide}&=\frac{P\_{guide}}{P\_{total}}\\

\eta\_{base}&=\frac{P\_{base}}{P\_{total}}\\

\eta\_{air}&=1-\eta\_{guide}-\eta\_{base}

\end{align}

%Appendix\_spot\_size

In this section we will introduce the process of calculating beam spot size from data in CST MWS. As definition of spot size, the target is the distance from beam center to the point where the power density is $1/e^\_{2}$ of the peak value. Fig. \ref{Afig:beam\_cuboid} is the calculation cuboid in the CST MWS. The gaussian beam propagates along the z-axis in the simulation of this work. So the spot diameter is the function of z coordinate $d=f(z)$. In order to calculate the spot diameter we cut a working plane at each z-coordinate. Like the step in Appendix \ref{app:powwer\_distribution}, we assume a working plane through the point $z\_{0}$. In this work the simulation of beam propagation is symmetric on both x-axis and y-axis. Thus only quarter of the beam cross-section is the working plane like Fig. \ref{Afig:beam\_crosssection } in CST MWS. Supposing the point $n\_{0}$ is the beam center of this plane the peak value of the power flow density is $|S(n\_{0})|$. The next step is to find the point $n\_{1}$ where $|S(n\_{1})|=1/e^\_{2}|S(n\_{0})|$. $n\_{1}$ is also among the range $[n\_{0}, n\_{0}+n\_{x}n\_{y}]$. Point $n\_{1}$ leads to its coordinate ($x\_{1},y\_{1}$). Then the beam spot radium of this plane is given by \ref{eq:spot\_radium}. The spot size is twice of this radium. We can draw the spot size curve by jointing values of the spot size in each corss-section.

\begin{figure}[!ht]

\centering

\includegraphics[width=0.5\textwidth]{bilder/beam\_cuboid}

\caption{Beam propagation in simulation cuboid.}

\label{Afig:beam\_cuboid}

\end{figure}

\begin{figure}[!ht]

\centering

\includegraphics[width=0.5\textwidth]{bilder/beam\_crosssection}

\caption{Beam cross-section at through point n\_{0}.}

\label{Afig:beam\_crosssection}

\end{figure}

\begin{equation}

R=sqrt(x\_{1}^{2}+y\_{1}^{2})

\label{eq:spot\_radium}

\end{equation}