%Appendix 1

Calculation of power distribution.

%%%%% talking about how to calculate the power in each section base on FIT knowledge. In %Appendix\_power\_distribution

This section aims for offering a set of numerical functions to calculating the power distribution along beam propagation in a photonic waveguide. The simplest idea is to compute the power in waveguide and base by integrating power flow density (or Poynting vector $\vec{S}$) at their cross-sections respectively. Meanwhile CST MWS has constructed a cuboid like Fig.\quad\ref{Afig:app\_power\_distribution01}, which contains all engaged objects such as waveguide and TLF, as a total calculation space which is discretized through Finite Integral method (FIT) and all variables(see \cite{script\_FeldSim} or section Finite Integration Method) from FIT are also available from CST MWS. In following it will be introduced how to calculate the power distribution of a Fiber-to-Chip model (see chapter modelling).

\begin{figure}[ht]

\centering

\includegraphics[width=0.5 \textwidth]{bilder/basic\_waveguide\_efield}

\caption{Calculation Cuboid in CST}

\label{Afig:app\_power\_distribution01}

\end{figure}

The first step is to choose a working plane. Suppose a working plane through point z$\_{0}$ at z axis in the total calculation space Fig.\quad\ref{Afig:app\_power\_distribution01} is given, the point index n$\_{0}$ can be determined by its coordinate z$\_{0}$. The plane Fig. \quad\ref{Afig:app\_power\_distribution02} is divided into small elemental pieces in FIT. The base cross-section is given by four points coordinates (x$\_{1}$, y$\_{1}$),(x$\_{2}$, y$\_{2}$),(x$\_{3}$, y$\_{3}$) and (x$\_{4}$, y$\_{4}$). Their points indexes (n$\_{1}$, n$\_{2}$, n$\_{3}$, n$\_{4}$) can also be derived.

\begin{figure}[ht]

\centering

\includegraphics[width=0.5 \textwidth]{bilder/basic\_waveguide\_efield}

\caption{Cross-section at z$\_{0}$}

\label{Afig:app\_power\_distribution02}

\end{figure}

In the next step it is necessary to prepare variables such as elemental

As is in \ref{script\_FeldSim } the complete elemental plane matrix D$\_{A}$ is given by (\ref{Aeq:da\_matrix}):

\begin{equation}

D\_{A}=Diag\left\{Ax(1),\cdots,Ax(Np),Ay(1),\cdots,Ay(Np), Az(1),\cdots,Az(Np)\right\}

\label{Aeq:da\_matrix}

\end{equation}

In the working cross-section only z-components of elemental plane matrix D$\_{A}$ are needed:

\begin{equation}

D\_{Az}=D\_{A}(2\*Np+1:3\*Np, 2\*Np+1:3\*Np)

\label{Aeq:daz\_matrix}

\end{equation}

Construct a auxiliary matrix $A\_{base}$, which composed of only $1$ and $0$ like Fig\quad\ref{ Afig:app\_Auxiliary\_matrix}, to indicate all points indexes which are included in base cross-section.

\begin{equation}

A\_{base}=Diag\left\{0,\cdots 0,P1,0,\cdots 0,P2,0,\cdots 0\right\}

\label{Aeq:A\_matrix}

\end{equation}

\begin{figure}[ht]

\centering

\includegraphics[width=0.5\textwidth]{bilder/basic\_waveguide\_efield}

\caption{structure of the auxiliary matrix $A$}

\label{Afig:app\_Auxiliary\_matrix}

\end{figure}

Where P$\_{x}$ are submateix:

\begin{equation}

P\_{x}=Diag\left\{1,\cdot,1\right\}\_{(n\_{2}-n\_{1})\*(n\_{2}-n\_{1})}

\end{equation}

And m is given by:

\begin{equation}

M=(n\_{3}-n\_{1})/n\_{x}=(n\_{4}-n\_{2})/n\_{x}

\end{equation}

Then

\begin{equation}

D\_{Abase}=A\_{guide}\*D\_{A}

\end{equation}

Pick z-components of the Poynting vector $S$:

\begin{equation}

S\_{z}=S(2\*Np+1:3\*Np)

\end{equation}

At last the power in base at the plane (for z=z$\_{0}$) can be counted up by:

\begin{equation}

P\_{base}(z)=sum(D\_{Aguide}\*S\_{z})

\end{equation}

By analogous procedures power in guide cross-section$P\_{guide}$ and in total cross-section $P\_{total}$ are derived. For observing the power distribution the results are processed by normalization:

\begin{align}

\eta\_{guide}&=\frac{P\_{guide}}{P\_{total}}\\

\eta\_{base}&=\frac{P\_{base}}{P\_{total}}\\

\eta\_{air}&=1-\eta\_{guide}-\eta\_{base}

\end{align}