Supplementary Materials – OmniNet: Omnidirectional Jumping Neural Network with Height-awareness for Quadrupedal Robots

Abstract

This document mainly demonstrates the detailed computation process of computing the desired joint angle during aerial phase using the analytical Inverse Kinematic (IK) and the meaning of the involved variables [1].

I. Analytical Inverse Kinematic

A foot height h_{feet} is predefined and set to be the same for each leg. Based on the theory in [1] and assuming the joint velocity to be zero, we can compute the angle of calf, thigh, and hip respectively. In Table I, several geometrical properties of the robot are listed, which will be used to compute the calf angle.

TABLE I: Geometical Properties

Variables	**Symbols**	
hip origin	p^o_{hip}	$\mathbb{R}^3$
foot position	h_{feet}	$\mathbb{R}^3$
hip length	l_{hip}	$\mathbb{R}$
thigh length	l_{thigh}	$\mathbb{R}$
calf length	l_{calf}	$\mathbb{R}$

$$l_{ft} = \sqrt{l^2_{fh} - l^2_{hip}} \tag{1}$$

$$l_{fh} = \sqrt{h^2_{feet} - (p^o_{hip})^2} \tag{2}$$

$$\theta_{calf} = \arccos(\frac{l^2_{thigh} + l^2_{calf} - l^2_{ft}}{2 l_{thigh} l_{calf}} - \pi) \tag{3}$$

The relative angles of thigh and hip joint of the same leg are computed using some intermediate variables $\boldsymbol{u'}$,$\boldsymbol{v'}$, $\boldsymbol{u_t}$, $\boldsymbol{v_t}$, $\boldsymbol{u_h}$, $\boldsymbol{v_h}$, $\boldsymbol{\omega}$, $\boldsymbol{l_1}$, $\boldsymbol{l_2}$ and $\boldsymbol{r}$ which are gained through the robot's geometrical properties and predefined feet end-effector height h_{feet}, as follows:

TABLE II: Intermediate Variables

Variables	**Symbols**	
Relative position from foot to calf	p_{fc}	$\mathbb{R}^3$
Relative position from foot to thigh	p_{ft}	$\mathbb{R}^3$
A chosen point in the axis of hip joint	w_h	$\mathbb{R}^3$
Calf origin	p^o_{calf}	$\mathbb{R}^3$
Thigh origin	p^o_{thigh}	$\mathbb{R}^3$
Unit axis vector of thigh joint	w_t	$\mathbb{R}^3$
Unit axis vector of hip joint	w_t	$\mathbb{R}^3$
Relative position from foot to calf	p^o_{fc}	$\mathbb{R}^3$
Relative position from calf to thigh	p^o_{ct}	$\mathbb{R}^3$
Rotation matrix around the axis by its angle	$R_{axis}(\theta)$	$\mathbb{R}^{3\times 3}$

$$p_{fc} = R_{\omega_{calf}}(\theta_{calf}) p^o_{fc} \tag{4}$$

$$v' = p^a_{hip} - p^o_{hip} \tag{5}$$

$$u' = p^o_{calf} + p_{fc} - p^o_{hip} \tag{6}$$

$$u_t = u' - \omega_t u'^T \omega_t \tag{7}$$

$$v_t = v' - \omega_t v'^T \omega_t \tag{8}$$

$$\delta = \sqrt{h_{feet}^2 - (p_{hip}^a)^2} \tag{9}$$

$$\delta' = \sqrt{\delta^2 - (\omega_t^T(p_{calf}^o + p_{fc} - p_{hip}^o - p_{hip}^a))^2} \tag{10}$$

$$\theta_1 = \arctan\frac{\omega_t^T u_t \times v_t}{u_t^T v_t} \tag{11}$$

$$\theta_2 = \arccos\frac{u_t^T u_t + v_t^T v_t - \delta'^2}{2||u_t||||v_t||} \tag{12}$$

$$\theta_{thigh} = \theta_1 - sign(\theta_1)\theta_2 \tag{13}$$

The deduction process of the hip angle is similar to that of thigh angle:

$$p_{fc} = R_{\omega_{thigh}}(\theta_{thigh})p_{ct}^o + p_{fc} \tag{14}$$

$$v' = h_{feet} - p_{hip}^o \tag{15}$$

$$u' = p_{thigh}^o + p_{ft} - p_{hip}^o \tag{16}$$

$$u_h = u' - \omega_h u'^T \omega_h \tag{17}$$

$$v_h = v' - \omega_h v'^T \omega_h \tag{18}$$

$$\theta_{hip} = \arctan(\frac{\omega_h^T u_h \times v_h}{u_h^T v_h}) \tag{19}$$

References

[1] R. M. Murray, Z. Li, and S. S. Sastry, *A mathematical introduction to robotic manipulation*. CRC press, 2017.