CS 186 Fall 2020 Midterm 1 (Online)

Do not share this exam until solutions are released.

Contents:

- The midterm has *6 questions*, each with multiple parts, and worth a total of *100 points*.

Taking the exam:

- You have *110 minutes* to complete the midterm.
- You may print this exam to work on it.
- For each question, submit only your *final answer* on examtool.
- For numerical answers, do not input any suffixes (i.e. if your answer is 5 I/Os, only input 5 and not 5 I/Os or 5 IOs) and do not use LaTeX.
- Some questions require you to show work and not doing so will result in **no credit**. You can do this by inputting an explanation in the text field. The text field supports LaTeX by using $$insert expression here$$ but don't worry too much about formatting.
- Make sure to save your answers in examtool at the end of the exam, although the website should autosave your answers as well.

Aids:

- You may use one page (double sided) of handwritten notes as well as a calculator.
- **You must work individually on this exam.**

Grading Notes:

- All I/Os must be written as integers. There is no such thing as 1.02 I/Os – that is actually 2 I/Os.
- 1 KB = 1024 bytes. We will be using powers of 2, not powers of 10
- Unsimplified answers, like those left in log format, will receive a point penalty.

1 Pre-Exam Question (0 points)

When was the last time CS186 lectures were taught in person? Give us the season and year (ex. Fall 2020).

Solution: Spring 2018

2 More Baseball! (20 points)

In this problem we will use a simplified version of the database we used in project 1:

```
CREATE TABLE Player (
    pid INTEGER PRIMARY KEY,
    age INTEGER NOT NULL,
    name VARCHAR[30] NOT NULL,
    salary INTEGER NOT NULL,
    rating INTEGER NOT NULL
);
CREATE TABLE Team (
    tid INTEGER PRIMARY KEY,
    name VARCHAR[30] NOT NULL,
);
CREATE TABLE MemberOf (
    pid INTEGER PRIMARY KEY,
    tid INTEGER NOT NULL,
    FOREIGN KEY (pid) REFERENCES Player
    FOREIGN KEY (tid) REFERENCES Team
);
```

You can assume none of the tables contains `NULL` values.

1. (4 points) Given the following contents of Player:

pid	name	age	salary	rating
1	"P1"	30	50000	3
2	"P2"	24	10000	3
3	"P3"	21	15000	2

   Write out the result for the following query in a comma-delimited form, where the first row represents the column names. For instance, if the query returned the following result:

pid	name	age	salary	rating
1	"P1"	30	50000	3

   Express it as:

   pid, name, age, salary, rating
   1, "P1", 30, 50000, 3

```
SELECT P1.rating AS rating, AVG(P1.age) AS avgAge
FROM Player P1
WHERE P1.age > 21
GROUP BY P1.rating
HAVING 1 < (SELECT COUNT(*)
            FROM Player P2
            WHERE P1.rating = P2.rating AND P2.age >= 21)
```

Your answer:

Solution:

rating	avgAge
3	27

Using the same schema as given previously, follow these instructions for the following parts.

Does each of the following query pairs always return the same results regardless of the contents of the involved tables? If so, write "Yes" below. Otherwise, write "No," and make up the contents of `Player` such that, when run with the two queries, they will return different results. You do not need to include any explanation for either answer.

For "No" answers, write the table contents in the comma-delimited form as in the first question. Again, if your answer is the following for the `Player` table:

Player

pid	name	age	salary	rating
1	"P1"	30	50000	3

Express it as:

Player:
pid, name, age, salary, rating
1, "P1", 30, 50000, 3

Note that some queries involve multiple tables so be sure to provide table names as in the above. Also, make sure your tuples satisfy all key constraints!

2. (4 points) Q1:

```
SELECT P1.name
FROM Player P1
WHERE NOT EXISTS (SELECT *
                  FROM Player P2
                  WHERE P2.age < 21 AND P1.rating <= P2.rating)
```

Q2:

```
SELECT P1.name
FROM Player P1
WHERE P1.rating > ANY (SELECT P2.rating
                       FROM Player P2
                       WHERE P2.age < 21)
```

Your answer:

Solution: No, these queries are not equivalent. Example data:

pid	name	age	salary	rating
1	"Alex"	27	1000	3
2	"Mike"	20	500	2
3	"Julie"	18	600	4
4	"Alice"	25	2000	5

Q1 will return only Alice but Q2 will return Alice, Julie, and Alex.

3. (4 points) Q1: $\sigma_{\texttt{salary<10k}}(\sigma_{\texttt{age}<21}(\texttt{Player}) \bowtie_{\texttt{Player.pid=MemberOf.pid}} (\sigma_{\texttt{tid>40}}(\texttt{MemberOf})))$
Q2: $\sigma_{\texttt{salary<10k}}(\texttt{Player} \bowtie_{\texttt{Player.pid=MemberOf.pid}} (\sigma_{\texttt{tid>40}}(\texttt{MemberOf})))$

Your answer:

Solution: No, these queries are not equivalent. Example data:
Person

pid	name	age	salary	rating
1	"Alex"	22	9000	3

MemberOf

pid	tid
1	45

Team

tid	name
45	"Oski"

Q1 will return nothing while Q2 will return the first tuple.

4. (4 points) Q1: $\gamma_{\texttt{Team.tid, max(salary)}}((\texttt{Player} \bowtie_{\texttt{Player.pid=MemberOf.pid}} \texttt{MemberOf}) \bowtie_{\texttt{Memberof.tid=Team.tid}} \texttt{Team})$
Q2: $\gamma_{\texttt{MemberOf.tid, max(salary)}}(\texttt{Player} \bowtie_{\texttt{Player.pid=MemberOf.pid}} \texttt{MemberOf})$

Your answer:

Solution: Yes.

5. (4 points) Which of the following returns the same results as the following relational algebra expression? Select all queries that return the same results. No explanations needed. There may be 0, 1, or more correct answers.

$\rho_{\texttt{rating->r, avg(salary)->sal}}(\gamma_{\texttt{rating, avg(salary)}}(\texttt{Player}))$

◯ Query A

```
SELECT P2.rating as r, AVG(P2.salary) as sal
FROM Player AS P2,
     (SELECT DISTINCT P1.salary AS salary
      FROM Player AS P1
      WHERE P1.rating = P2.rating) AS t
WHERE P2.rating = t.rating
```

√ **Query B**

```
SELECT DISTINCT P2.rating AS r,
       (SELECT AVG(t.salary)
        FROM (SELECT P1.salary AS salary
              FROM Player AS P1
              WHERE P1.rating = P2.rating) AS t) AS sal
FROM Player AS P2
```

◯ Query C

```
SELECT P2.rating AS r, AVG(P2.sal1) AS sal
FROM (SELECT P1.rating, P1.age, AVG(P1.salary) AS sal1
      FROM Player AS P1
      GROUP BY P1.rating, P1.age) AS P2
GROUP BY P2.rating
```

◯ Query D

```
SELECT P2.rating AS r,
       (SELECT AVG(t.salary)
        FROM (SELECT P1.salary AS salary
              FROM Player AS P1
              WHERE P1.rating = P2.rating) AS t) AS sal
FROM Player AS P2
```

3 Where's the Zoom Record(ing)? (25 points)

Zoom cloud recording has became too expensive for the university, so the CS 186 TAs have been hired to design a database to store lecture recording information. The TAs need your help in considering different designs!

For the rest of the questions consider the following schema:

```
CREATE TABLE zoomers (
    lecture_id INTEGER PRIMARY KEY,
    week INTEGER NOT NULL,
    topic VARCHAR[30] NOT NULL,
    num_thanks INTEGER,
    poppin_chat BOOLEAN
);
```

For questions 1-5, assume the following:

- `zoomers` is stored in a Heap File Page Directory implementation. There are 25 data pages and each header page has 6 entries.
- Data pages follow the slotted page layout as presented in lecture.
- On each data page, 8 bytes are reserved for the slot count and pointer to free space. Each slot is 8 bytes.
- Records are stored as variable length records
- The record header contains a bitmap to track all fields that can be `NULL`. The bitmap is as small as possible, rounded up to the nearest byte.
- Integers and pointers are 4 bytes. Booleans are 1 byte.
- Each page is 1 KB (1024 Bytes).
- There are no indices on any field.

1. (2 points) What is the maximum size of a `zoomers` record in bytes?

> **Solution:** The maximum record size is 48 bytes. 1 byte for null bitmap, 4 bytes for pointer to end of `topic`, 4 bytes each for `lecture_id`, `week`, `num_thanks`, 30 bytes for `topic`, and 1 byte for `poppin_chat`.

2. (2 points) What is the maximum number of `zoomers` records that can fit on a data page?

Solution: The minimum record size is $1 + 4 + 4 + 4 + 4 + 1 = 18$ bytes, when `topic` is an empty string. This comes from 1 byte for the bitmap, 4 bytes for each of the integers, 4 bytes for the pointer in the record header to the end of `topic, and 1 byte for the boolean. Subtracting 8 bytes from the page size of 1024 bytes for the slot count and pointer to free space, leaves 1016 bytes for slots and records. The maximum number of records that can fit on the slotted page is` $\lfloor \frac{1016}{18+8} \rfloor = 39$ `records.`

For questions 3-5, assume the queries are independent of each other; i.e. query 4 is run on a copy of the file that has never had query 3 run on it. The buffer is large enough to hold all data and header pages, and starts empty **for each question**.

3. (3 points) What is the **worst** case cost in I/Os for the following query?

```
DELETE FROM zoomers WHERE poppin_chat = FALSE;
```

Solution: 60 I/Os. In the worst case, there is a record with `popping_chat = FALSE`, on every data page. This involves reading all header and data pages, and writing all header and data pages. This is $2 * (5 + 25)$

4. (3 points) What is the **best** case cost in I/Os for the following query?

```
SELECT num_thanks FROM zoomers WHERE lecture_id = 186;
```

Solution: 2 I/Os. The best case is when the first directory entry on the first header page contains the matching record. Since `lecture_id` is a `PRIMARY KEY`, no other data pages need to be read.

5. (4 points) What is the **best** case cost in I/Os for the following query?

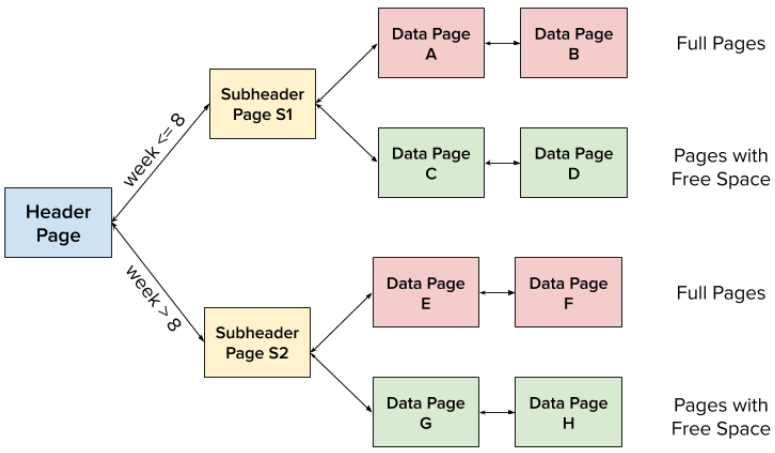
```
SELECT week, topic FROM zoomers WHERE lecture_id > 100 AND lecture_id <= 286;
```

Solution: 6 I/Os. Each page can fit at most 39 records. Since `lecture_id` is a **`PRIMARY KEY`** and **`INTEGER`** there can be at most 186 records that fall in the provided range. It takes at least 5 data pages to store 186 records. The best case is when all 186 records are on the first 5 data pages. Since, the query is a range search on `lecture_id`, which has to be unique, we can guarantee that there are no more matching records in this range. The cost involves reading the first header page and reading the first 5 data pages.

Thinking that students will access lectures based on the `week`, you have the brilliant idea of storing the `zoomers` table as a sorted file based on the `week` column. Unfortunately, The Great California Fires of 2020 obliterated some Zoom datacenters, and the lecture recording on sorted files is lost forever! Without that information you decide to come up with the following design for a partitioned linked list (PLL) heap file:

The header page in the heap file linked list implementation now points to P subheader pages. Each subheader points to a list of full pages and a list of pages with free space. Each subheader and its 2 lists are collectively called a partition; hence there are P partitions. Each record in the file is placed in a partition based on the `week`.

An example PLL heap file with 2 partitions is shown below with the assumption that $week >= 1$ and $week <= 16$. That assumption cannot be made for questions 6-8.



For questions 6-8 assume the following:

> https://www.overleaf.com/project/5f3c264872166c00013e713fThe header page stores 8 byte subheader entries. Each subheader entry is a pair of `week` and a pointer to the corresponding subheader page. P = number of partitions N = number of data pages Each partition has $\frac{N}{P}$ data pages. Assume $\frac{N}{P}$ will always be an integer. Each partition has an equal number of full pages and pages with free space. There are at least 3 pages with free space (and 3 full pages) in each partition The buffer is large enough to hold all data and header pages, and starts empty **for each question**. You can use `ceil(x,y)` to represent $\lceil \frac{x}{y} \rceil$

6. (4 points) What is the worst case I/O cost for inserting a `zoomers` record in terms of P and N? Assume you don't need to check for a duplicate primary key.

Solution: $\lceil \frac{\frac{N}{P}}{2} \rceil + 8$ I/Os
1 I/O to read header, 1 I/O to read subheader. Worst case is inserting to 2nd to last free page and it becomes full. Reading all pages with free space is $\frac{\lceil \frac{N}{P} \rceil}{2}$ I/Os. 2 I/Os to update pointers of previous and next pages with free space. 1 I/O to write page the record was inserted to. 2 I/Os to read and write first full page. 1 I/O to write subheader page after updating its next pointer.

To account for multiple courses sharing the same Zoom account to record lectures, let's add a field for `course_id` to the table and modify the **`PRIMARY KEY`**. The new `zoomers` schema is:

```
CREATE TABLE zoomers (
    lecture_id INTEGER,
    week INTEGER NOT NULL,
    topic VARCHAR[30] NOT NULL,
    num_thanks INTEGER,
    poppin_chat BOOLEAN,
    course_id INTEGER,
    PRIMARY KEY(course_id, lecture_id)
);
```

For questions 7-8, let C = number of courses and assume there are now N data pages **for each course**. Partitions are still based on `week` and each partition now has $\frac{CN}{P}$ data pages, which you can assume is an integer.

7. (3 points) What is the worst case cost in I/Os to perform an equality search on a given `week` and `course_id`? Answer in terms of P, N, and C.

Solution: $2 + \frac{CN}{P}$ I/Os
1 I/O to read the header page and 1 I/O to read the subheader page corresponding to the partition that contains the given `week`. The final $\frac{CN}{P}$ are to scan the entire partition.

8. (4 points) In 2-3 sentences, propose a change to the PLL heap file that minimizes the worst case I/O cost to to perform an equality search on a given `week` and `course_id`. Provide the worst case cost in I/Os in terms of P, N, and C? Your proposal may not use indexes or sorted files.

Solution: The change is to have the header page point to a course header page for each `course_id`. Each course header page acts like a header page and points to subheader pages corresponding to each partition for records with a specific `course_id`.

Using the proposal it costs $3 + \frac{N}{P}$ I/Os. This follows from reading the header page, subheader page for `course_id`, and subheader page for `week`, and then scanning all the data pages in that partition.

4 B+ Trees (25 points)

1. (2 points) Which of the following are always true regarding B+ Trees? **There may be zero, one, or more than one correct answer.**

   A. If $d = 2$, then the root node must have at least two entries.

   B. Suppose we maintain both an Alternative 2 and an Alternative 3 B+ Tree on the primary key of some table. Inserting a value into the Alternative 2 B+ Tree will cost the same number of I/Os as inserting into the Alternative 3 B+ Tree. Assume a leaf node is stored on exactly one page.

   C. Suppose we have two entries A and B that we want to insert into a B+ Tree. Inserting A then B into the B+ Tree results in the same final B+ Tree as inserting B then A.

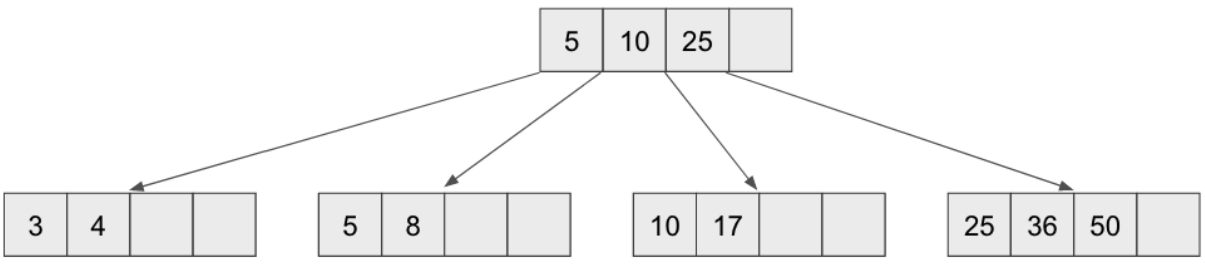
   D. Suppose we want to bulk load a B+ Tree with entries 1 to 10 with a fill factor of $\frac{3}{4}$. The B+ Tree has an order of 2. After bulk loading with 1 to 5, the leftmost leaf node is never touched again.

   Solution:
   A: The root node can have one entry even if $d = 2$.
   B: Since we are building the B+ Tree on a primary key, the Alternative 2 and Alternative 3 B+ Trees are basically the same since we cannot have any duplicates.
   C: Inserting into a B+ Tree is not necessarily commutative when the height of the tree changes.
   D: The leftmost leaf will be filled to the fill factor after inserting 1 to 3. Once the leftmost leaf node is filled to the fill factor, it is never touched again in the bulk loading process.

2. (2 points) Suppose we have a different type of B+ Tree where leaf nodes and inner nodes don't necessarily have the same order. If we have $d = 2$ for inner nodes and $d' = 3$ for leaf nodes, what is the maximum number of records we can store in an Alternative 2 B+ Tree of height 3?

   Solution: The leaf nodes can have a maximum of $2d' = 6$ records, and the maximum fanout for inner nodes is $2d + 1 = 5$. Since we have a Alternative 2 B+ Tree of height 3, the maximum number records we can store is $6 * 5^3 = 750$.

For question 3 and 4, suppose we have the following B+ Tree with order $d = 2$.



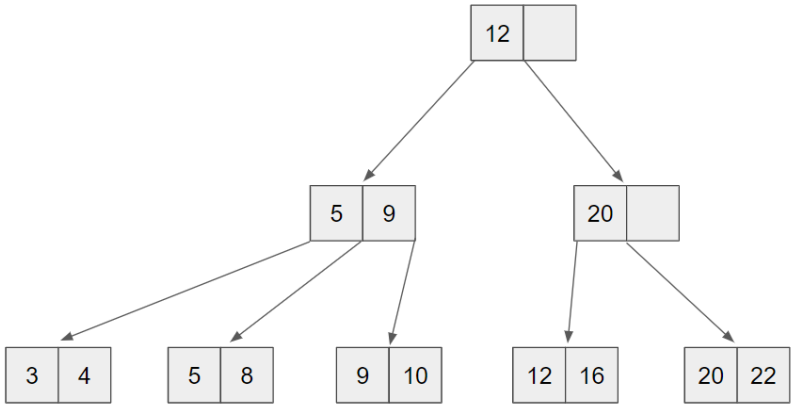
3. (2 points) What is the minimum number of records we can insert to increase the height of the tree?

Solution: 4. Many numbers work but you could insert 51, 52, 53, 54.

4. (3 points) After inserting entries 51 to 100 (inclusive) one by one into the tree, what values are in the rightmost leaf node? Hint: Try inserting a few numbers and see if you can find a pattern.

Solution: 98, 99, 100. If you insert the first few numbers you will realize that 52 causes the tree to split leaving the right most leaf with 50, 51, 52. Then 54 will cause the leaf to split leaving the right most leaf with 52, 53, 54. It seems on even inserts the tree will have the last 3 values in the right most node so after inserting 100, the right most leaf will have 98, 99, 100.

5. (3 points) Suppose we want to bulk load a B+ Tree with order $d = 1$. We want to bulk load 100 records using a fill factor of 1. Is the following B+ Tree a possible intermediate step during our bulk load process? Explain your answer.



Solution: No. In the process of bulk loading those numbers, 12 would never get pushed to be the root. Instead, when 12 is bulk loaded, 9 is pushed up.

For questions 6- 10, suppose we have the following table shown below. Also, assume that our buffer is large enough to hold all pages needed and is empty at the start of each question.

```
CREATE TABLE Restaurants (
    rid INTEGER PRIMARY KEY,
    name VARCHAR,
    type VARCHAR,
    size INTEGER,
    num_ratings INTEGER,
    rating FLOAT
);
```

6. (2 points) Suppose we have a height 3 Alternative 1 B+ Tree with order $d = 2$ on `rid`. What is the minimum number of I/Os it would take to execute the following query (Assume there is at least one matching tuple):

```
UPDATE Restaurants SET num_ratings = 500 WHERE rid = 100;
```

Solution: 5. It takes 3 I/Os to reach the leaf, 1 I/O to read in the leaf, and 1 I/O to write the leaf back to disk.

7. (2 points) Now suppose we have an unclustered Alternative 2 B+ Tree instead of an Alternative 1 B+ Tree. What is the minimum number of I/Os it would take to execute the same query (Assume there is at least one matching tuple)?

Solution: 6. It takes 3 I/Os to reach the leaf, 1 I/O to read in the leaf, and 1 I/O to read in the data page, and 1 I/O to write the data page back to disk. Note that we do not need to write the leaf page back to disk since we are only updating `num_ratings`.

We decide to mix things up by creating an Alternative 2 unclustered B+ Tree with height 3 and order $d = 2$ on (size, num_ratings). As a reminder, for multi-column keys, comparisons are done of the first element in the key, and the second elements are compared to break ties. For problems 8 -10, assume the following:

- There are no duplicate (size, num_ratings) pairs.
- There are exactly 5 records per page in the heap file corresponding to the B+ Tree.
- There are 100 leaf nodes with exactly 3 entries in each leaf node. Each leaf node fits exactly on one page.
- The table has 60 data pages in total.
- The first entry with size=20 appears on the first entry of the 91st leaf node.
- The last entry with num_ratings=500 appears on the last entry of the 86th leaf node.
- None of the index pages are in the buffer pool at the start of each part.

Now, suppose we want to execute the following query:

```
SELECT * FROM Restaurants WHERE size >= 20 AND num_ratings <= 500;
```

8. (3 points) In the worst case what is the number of I/Os it would take to execute the query?

Solution: 43. The useful piece of information is that the 91st leaf contains the first entry with size=20 since our tree is first ordered on size and then ratings. It takes 3 I/Os to reach that leaf. In the worst case we must scan all leaves 91-100 (10 I/Os), and we find that all records in those leaves satisfy our WHERE clause. Since this is an Alternative 2 unclustered index, we need 1 I/O for every matching tuple or 30 I/Os. Thus, our total I/O cost is $3 + 10 + 30 = 43$.

9. (3 points) If we had a clustered index instead of an unclustered index, how many I/Os would it take to execute the query in the worst case scenario?

Solution: 19. We follow a similar procedure as the previous problem in terms of reaching our leaf and scanning our leaves but this time we only need 1 I/O per page of matching tuples. Since our data pages hold 5 tuples we have 30/5 or 6 I/Os to access our clustered data.

(We also accepted 20 if you considered the extreme worst case that the 30 tuples would be spread out across 7 heap pages instead of 6, similar to a vitamin question we asked.)

Now, assume we want to execute the following query:

```
SELECT * FROM Restaurants WHERE size >= 20 OR num_ratings <= 500;
```

10. (3 points) What is the minimum number of I/Os it would take to execute the query? Assume that the index is unclustered for this part.

Solution: 60. Our index is useless as we h ave no guarantee that the tuples we are searching for are ordered in our leaves. Therefore, we need to scan the entire table.

5 Buffer Management (10 points)

1. (2 points) Which of the following statements are true? **There may be zero, one, or more than one correct answer.**
   A. If a page is dirty then it must have at least one pin.
   B. The clock algorithm is used in favor of LRU because it isn't susceptible to sequential flooding.
   C. The buffer manager is only allowed to evict a pinned page if all other pages are also pinned.
   D. If the dirty bit of a page is set the buffer manager should write the page back to disk upon eviction.

> **Solution:**
> A. **False**, a frame can be dirtied but left unpinned after the requester is finished using it.
> B. **False**, the clock algorithm is still susceptible to sequential flooding. It used in favor of LRU because its simpler to implement.
> C. **False**, the buffer manager is never allowed to evict a pinned page under any circumstances.
> D. **True**, this is true.

For questions 2 and 3 assume you have 4 buffer frames, all accesses are unpinned immediately, and empty frames are filled in order. Consider the following access pattern:

A C B D B E A C D B E A C D B

2. (1 point) Which algorithm would be better for this access pattern, LRU or MRU?

> **Solution:** MRU

3. (1 point) Explain your answer to the previous question.

> **Solution:** This workload is essentially sequential flooding. After the first five accesses the pages in order of descending recency are B, D, C, A. To make space for E, A is evicted. Afterwards A is loaded back in evicting C, then C is loaded back in evicting D, etc...
>
> While LRU fairs quite poorly, MRU is able to avoid the debacle by evicting B to make space for E, then successfully getting hits on A, C, and D, putting it ahead of LRU which would get misses on every access after the 5th one.

4. (2 points) Consider the Least Frequently Used (LFU) eviction policy. Whenever a page is loaded into the buffer a 32-bit value representing the page's hit rate is set to 1. Every time the page is requested afterwards the hit rate is incremented. When a page needs to be evicted, this policy chooses the page with the lowest hit rate (breaking ties randomly). Consider the following workload: for one day every month a specific index is used heavily and continuously to perform analysis of a table. The remainder of the month the table is only used occasionally and work is spread out across other tables. Describe why an LFU policy might be problematic here. Assume that the counters never overflow.

Solution: During the hour that the index is used heavily the pages of the index gain extremely high hit counts making them difficult to evict. In the remainder of the month these pages keep their space in the buffer even though they might not be used very often, forcing other pages that may have been more useful to be evicted in their place.

5. (2 points) Describe an adjustment to the LFU policy that can help to mitigate the problem you described in the previous question. There are many possible correct answers to this question, just make sure to retain some aspect of the original policy (i.e. don't say "Just use LRU instead").

Solution: Any solution that addresses the of the index pages going stale in cache is eligible here. The intuitive way would be to factor in the recency of a page's use, for example: introducing a time based decay/decrement to the hit rates, introducing a system similar to clock to estimate the last time a page was referenced. Other options could include: regularly resetting at some interval, introduces an element of randomness that would eventually evict the stale pages, setting a reasonable maximum to the hit rate counter, etc....

6. (2 points) In lecture we mentioned that neural networks can be used to choose which page we should evict next. Imagine that recent breakthroughs in machine learning and Big Data^{TM} have given you access to a neural network that always predicts the OPT policy correctly. The OPT policy chooses a page to evict such that cache hits are maximized and cache misses minimized; it evicts optimally for all access patterns but take awhile to process all that Big Data^{TM}. Briefly explain a situation where you wouldn't want to use this method for buffer management.

Solution: Even if our eviction policy is optimal it doesn't matter much if the time it takes to tell us which page to replace is longer than the time it saves us by avoiding IOs. For example, if the network takes 30 seconds to tell us which page to replace we easily could have completed the work using a different, non-optimal algorithm in that time. Any answer that acknowledges performance decline in factors other than cache hit/miss rates is acceptable here.

Side note: The OPT policy does exist, and is used as a hypothetical benchmark for other cache replacement algorithms, but requires knowledge of the future to work. Given enough parameters, a neural network could learn the OPT policy if it encodes a perfect simulation of your database, although it would almost certainly take a while for it to tell you which page you should evict.

6 Sorting and Hashing (20 points)

Suppose we are trying to do external sorting or hashing on the following relation:

```
CREATE TABLE Students (
    calid INTEGER PRIMARY KEY,
    zipcode VARCHAR NOT NULL,
    department VARCHAR NOT NULL
);
```

This table contains information about 500,000 students, split on disk across 1000 pages. `department` can take on one of 10 distinct values; `zipcode` can be one of 1000 values. You can assume a uniform distribution across departments, i.e., the same number of students are part of each department, and zipcodes, i.e., the same number of students are in each zipcode. We have $B = 20$ pages in our DBMS buffer.

A calculator is recommended for this question.

1. (5 points) Say we wanted to sort the tuples in `Students` based on the `zipcode`.

   How many I/Os would we need for this algorithm?

   How many *runs* would you read into memory across all phases of the algorithm? (Recall that a run is a sorted sequence of blocks. We're not asking about the length of the run, just the number of runs. Do not treat the initial reading-in of the table as reading in any runs.)

   Solution: Read in relation and write out 50 runs of 20 pages each (1000 x 2 units). Read in 19 runs twice, merge, output. Read in 12 runs, merge, output. (1000 x 2 = 2000 I/Os). Read in 3 runs, merge, output (1000 x 2 = 2000 I/Os). Total = 6000 I/Os.

   We read in 19 + 19 + 12 + 3 runs for a total of 53 runs

2. (5 points) Say we wanted to sort the tuples in `Students` based on the `zipcode` once again.

   This time, you are going to derive an improved version of the previous algorithm on the fly! The key insight for this improvement is that we did not fully utilize all of the B pages in the buffer pool during every stage of the algorithm. Briefly explain how you would improve the previous algorithm using this insight. How many I/O units would we need using our improved algorithm?

   Hint: you could use some of the unused pages in the buffer to "start early" on the next phase of the algorithm.

   Solution: Read in relation and write out 50 runs of 20 pages each (1000 x 2 = 2000 I/Os). (a) Read in 19 runs, merge, output (2x). Read in 12 runs + 2 runs from sub-step (a), merge, output. (1000 x2 I/Os. In addition, 4 x 19 x 20 written out and read again). Total = 5520 I/Os.

3. (5 points) Say we wanted to hash the tuples in `Students` based on the `zipcode`.

   How many I/O units would we need for this algorithm?

   You can assume that we use a perfect hash function that divides values up evenly.

Solution: Read in relation and hash into 19 buckets each with up to 53 distinct values and 53 pages (1000 + 19 * 53 = 2007 units). Read in each bucket and hash into 19 buckets each with up to 3 distinct values and 3 pages (19 * 53 + 19 * 19 * 3 = 2090 units). Read in each bucket and sort (19 * 19 * 3 x2 = 2166 units). Total = 2007 + 2090 + 2166 = 6263 units.

4. (5 points) Say we wanted to hash the tuples in `Students` based on the `zipcode` once again.

   Let's once again try to develop a better algorithm for this on the fly! For designing this algorithm, say we have more information: one of the `zipcode`s (e.g., the Berkeley 94705 zipcode) is really popular—with $\frac{3}{5}^{th}$ of the students living in this `zipcode`, while the rest of the students are evenly distributed across the remaining `zipcode`s. Moreover, we can make use of this information in our hash function, i.e., you can design the ideal hash function you want. Briefly explain how you could improve the algorithm using this insight.

   How many I/Os would we need for this algorithm?

   Hint: Consider using a hash function that treats 94705 as a special value.

Solution: Our function will hash all tuples with the zipcode 94705 into one bucket, and evenly divides the rest of the values into the remaining buckets. Thus, the process is: Read in relation and hash into 18+1 (Berkeley) buckets. The 18 non-Berkeley buckets can each have up to 56 distinct values and ceil((1000 * 2/5)/18) = 23 pages, while the Berkeley bucket has 600 pages (1000 (reading) + 600 + (23 * 18) = 1000 + 600 + 414 = 2014 I/Os). Read in Berkeley bucket and write in sequence (600 + 600 = 1200 I/Os) Read in each non-Berkeley bucket of 56 different values and hash into 19 buckets each with up to 3 distinct values and 2 pages (414 + 18*19*2 = 414+684 = 1098 I/Os). Read in each bucket and hash (684 * 2 = 1368 I/Os) Total = 2014 + 1200 + 1098 + 1368 = 5680 I/Os.

Credit was also awarded for answers of 4480 in which case the Berkeley bucket was not read into memory and constructed in hash tables.