Introduction to Data Science

(Lecture 10)

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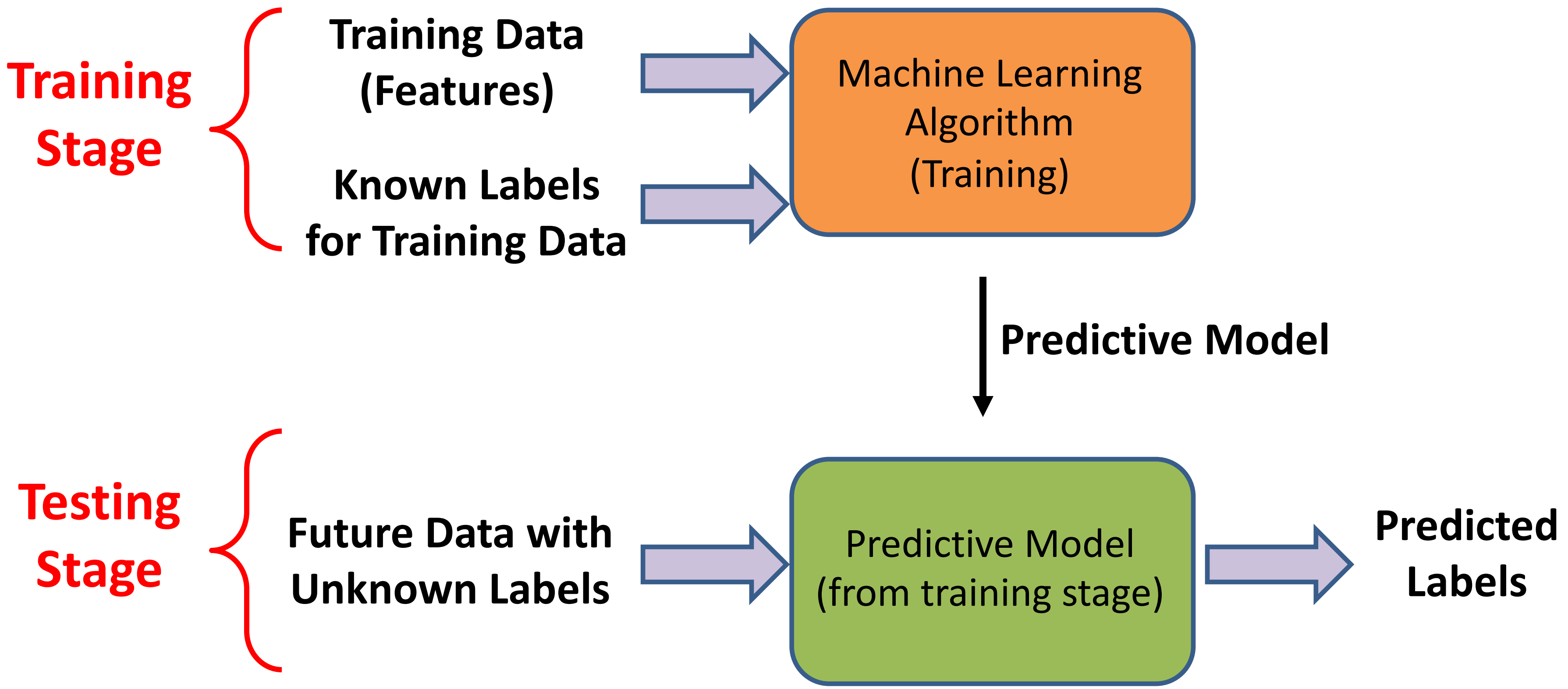
Linear Regression

Review: Supervised Learning: Learning from labeled Data



Review: Two main approaches in supervised learning

- **Classification**: Predicts a discrete valued output.
  - Labels are discrete (categorical)
  - Labels can be binary (e.g., rainy/sunny, spam/non-spam,)
    or non-binary (e.g., rainy/sunny/cloudy, Setosa/Versicolor/Virginica)

- **Regression:** Predicts a continuous valued output.
  - Labels are continuous, e.g., stock price, housing price
  - Can define 'closeness' when comparing prediction with true values

Regression Examples

- Predicting a ***company's future stock price*** using its profit and other financial information.
- Predicting ***housing price*** based on the property information (size, # rooms, …).
- Predicting ***annual rainfall*** based on the weather information.
- Predicting a ***risk of granting a credit card*** based on financial and personal information.
- **…**

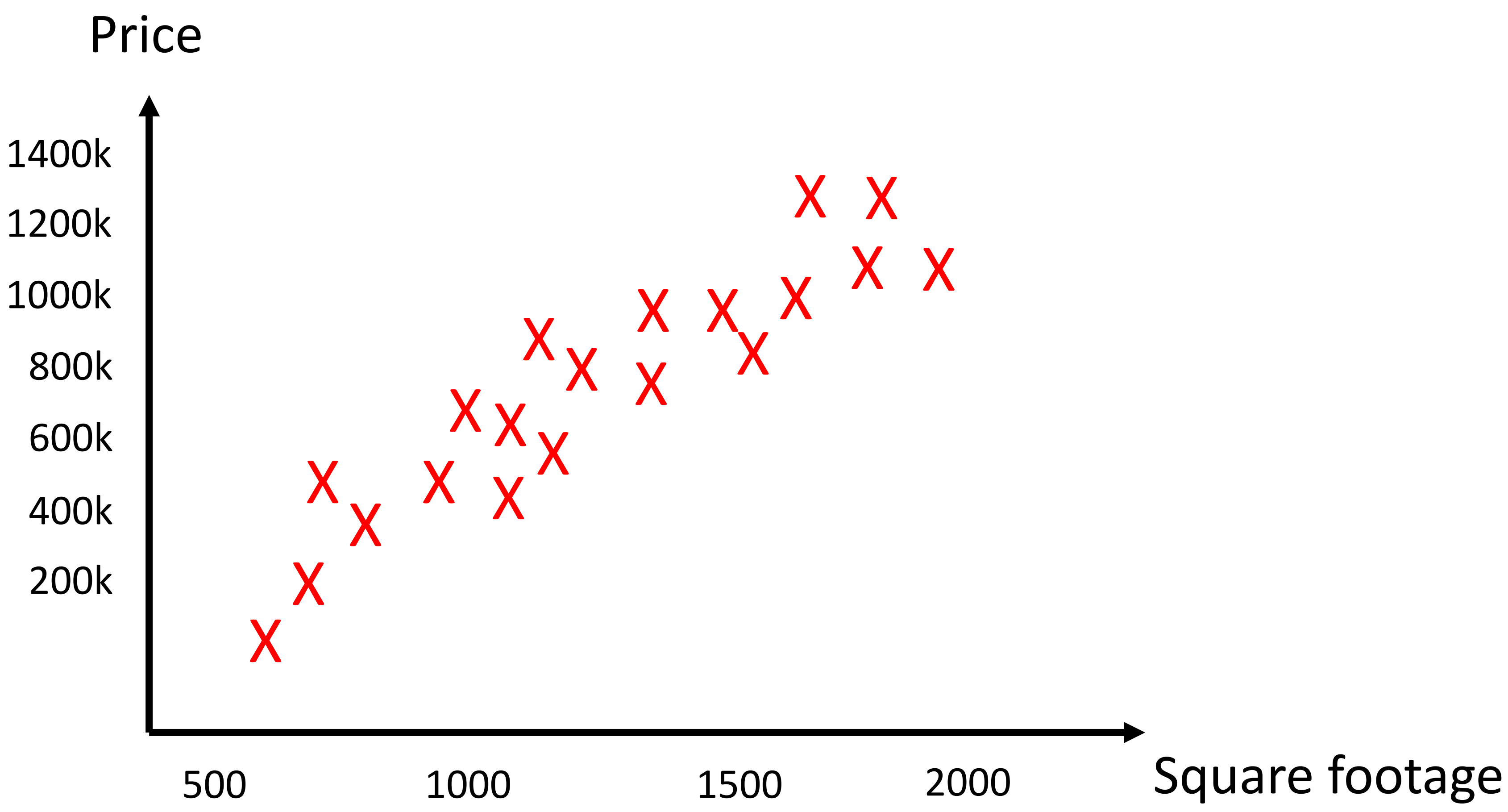
Special Case

- Let’s start with a simple, special case:

  Linear Regression with **single input** variable (**one feature**).

  It is also called “***Univariate Linear Regression.***”

Regression Example: Housing Price



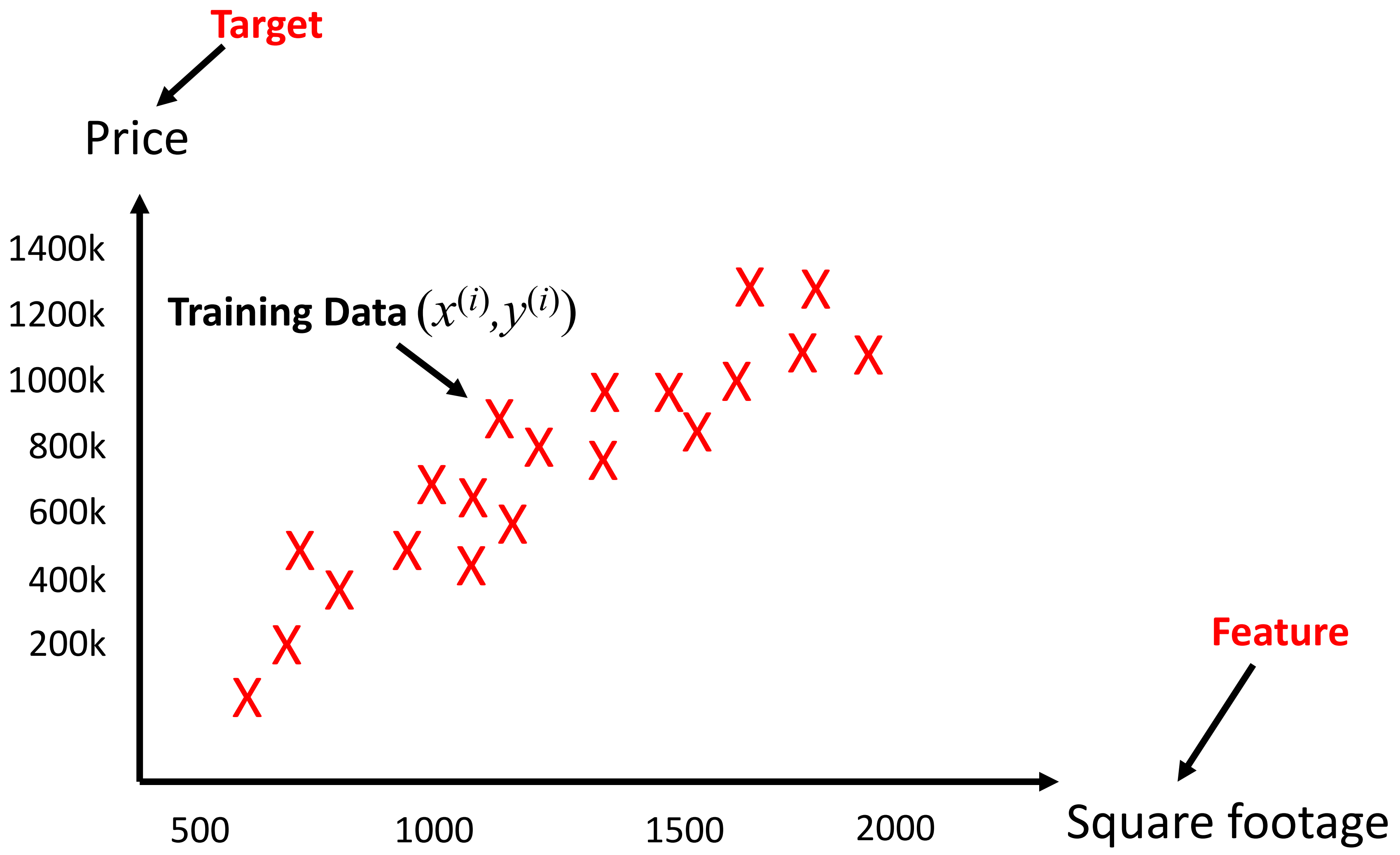
Training Dataset (past sales records)

Important Notation:

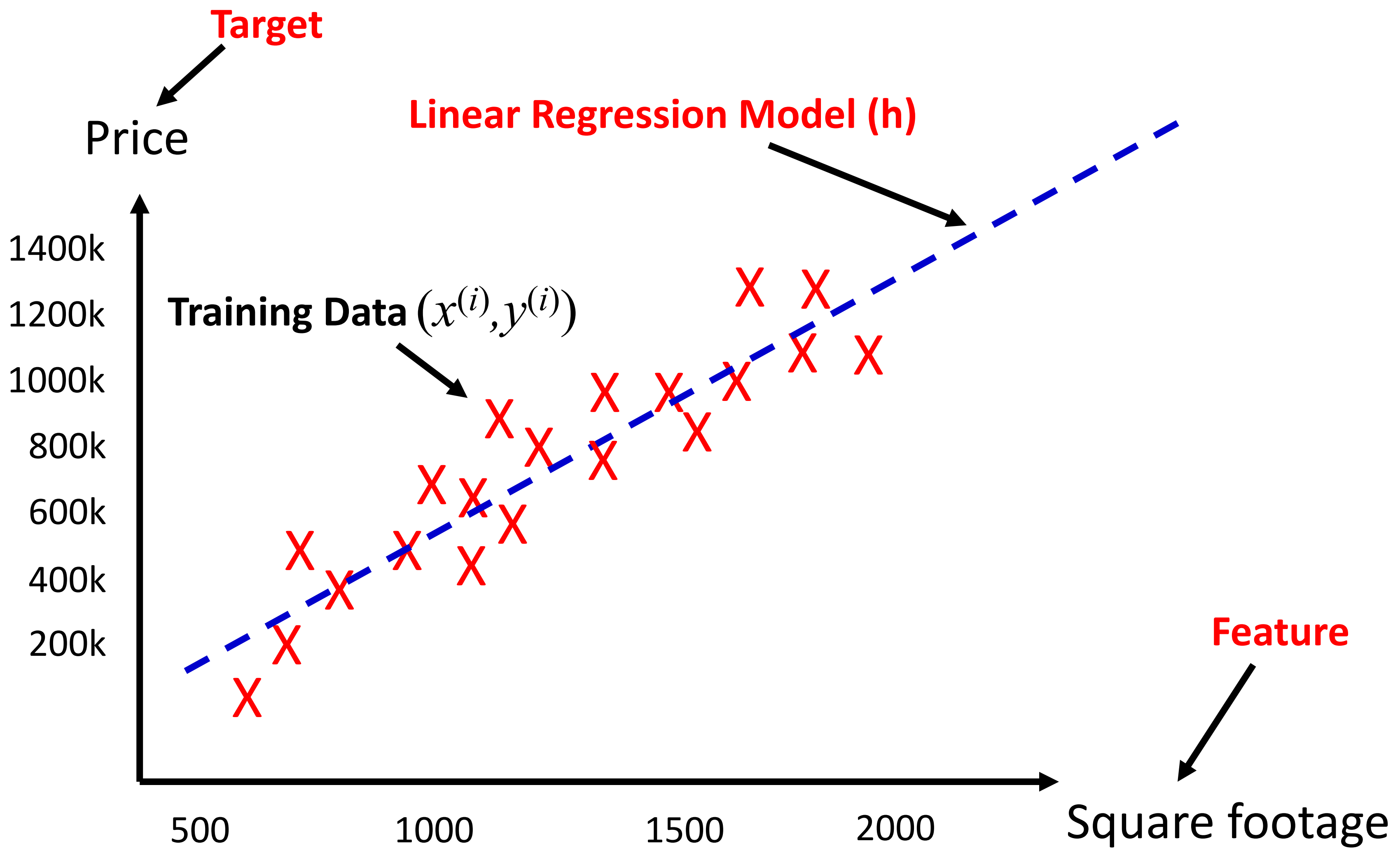
- $\boldsymbol{x}$'s = "input" variable (**features**)
- y's = "output" variable (**target**)
- $(\boldsymbol{x},y)$ = one training sample
- $(\boldsymbol{x}^{(i)},y^{(i)})$ = i^{th} training sample (**Note**: (i) is index not exponent!)
- m = Number of training examples

Size in feet^2 (***x***)	Price (*y*)
1000	410K
1200	600K
1230	620K
1340	645K
…	…

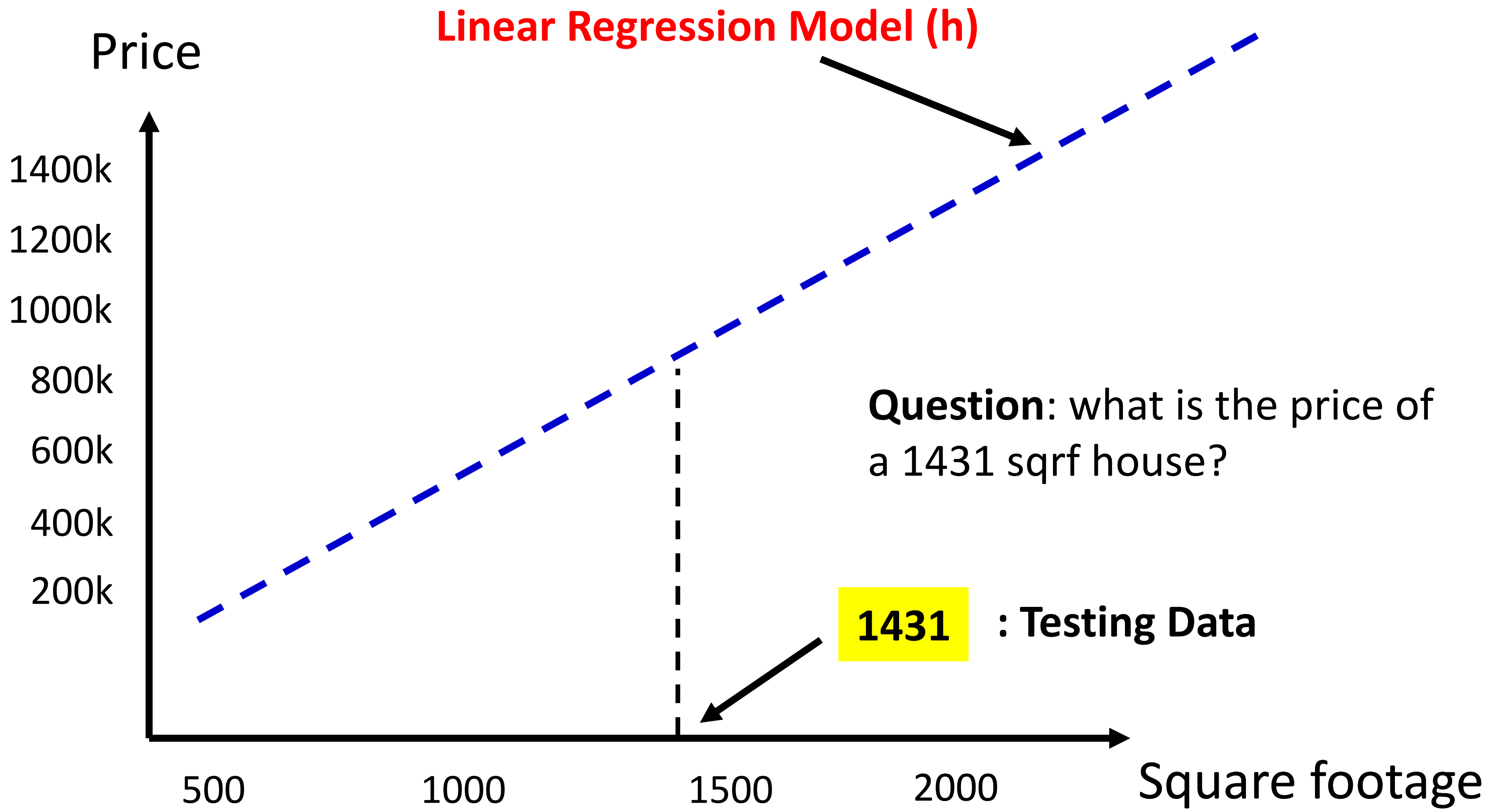
Regression Example: Housing Price



Regression Example: Housing Price

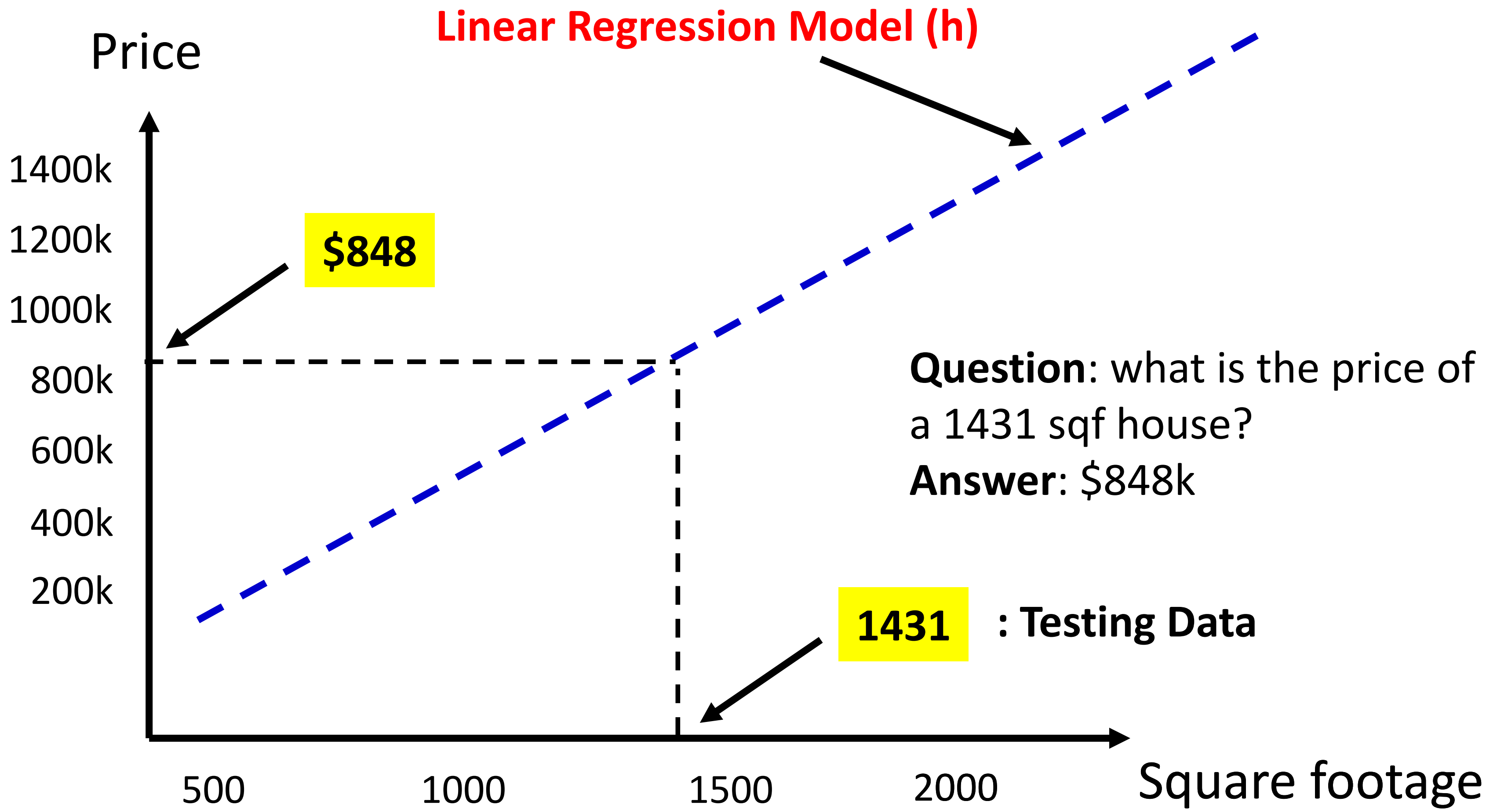


Regression Example: Housing Price

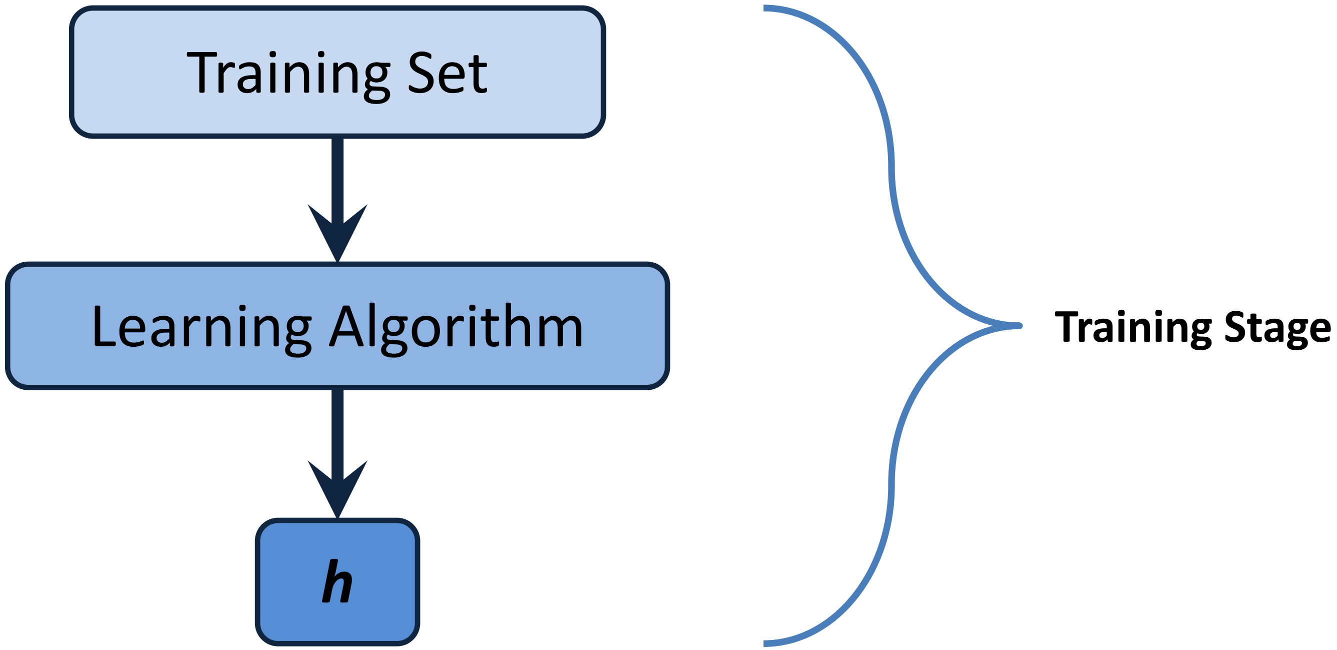


Question: what is the price of a 1431 sqrf house?

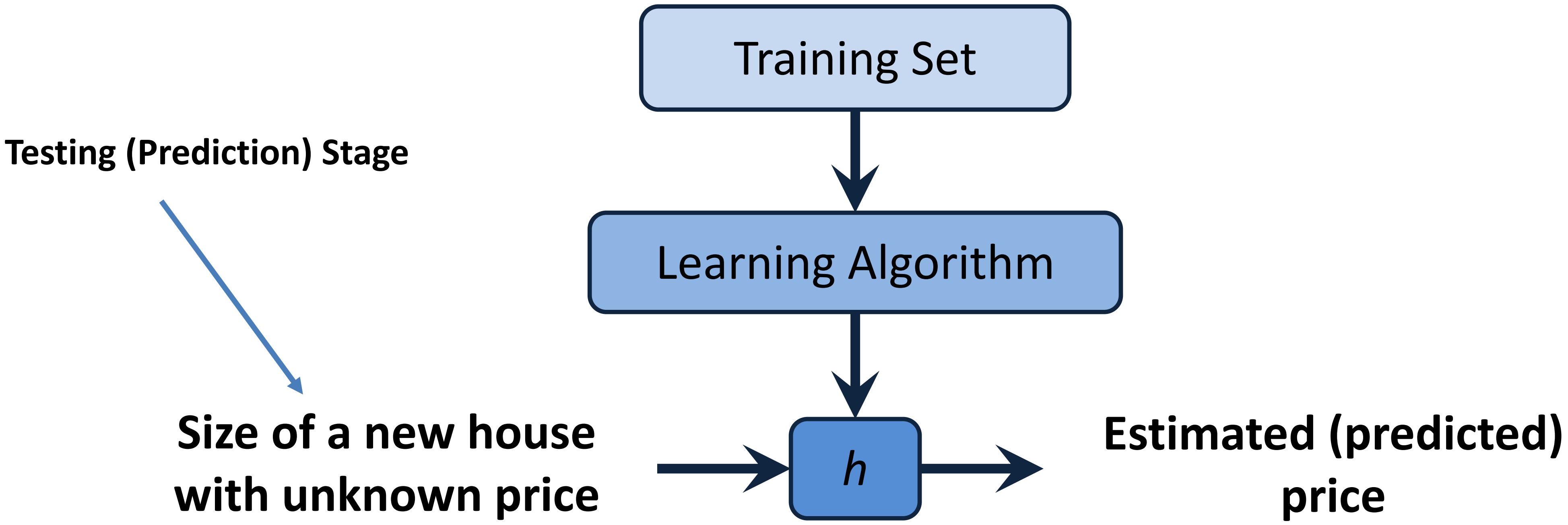
Regression Example: Housing Price



Question: what is the price of a 1431 sqf house?
Answer: $848k



- *"h"* is the <u>Regression Model</u> (Predictive Model) built by machine learning algorithm based on the training set. It will be later used to predict or estimate the "target" (e.g. price) on new observations.



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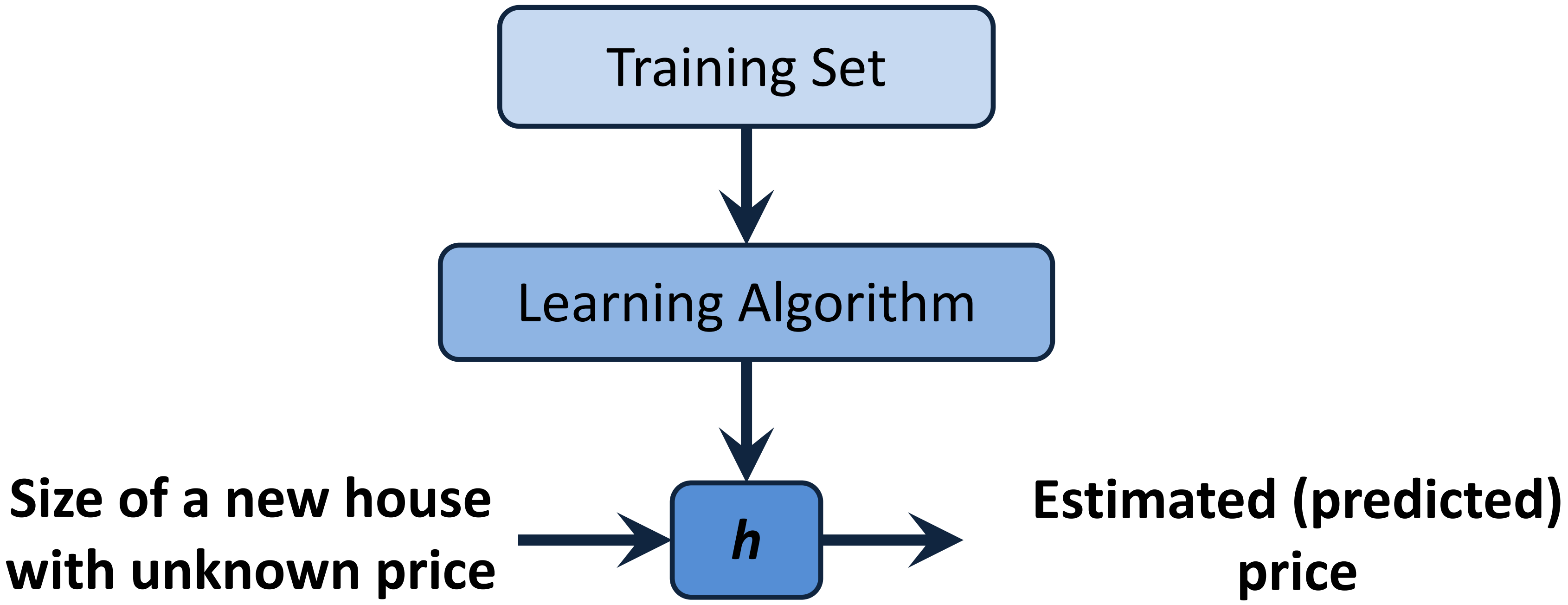


- In the case of **Linear Regression** with **One Variable**, the Regression model (h) will be a line:

$$h_\theta(x) = \theta_0 + \theta_1 x$$

- In the case of Linear regression with one variable, the Regression model (h) will be a line:

$$h_\theta(x) = \theta_0 + \theta_1 x$$

- This line should be the **Best Fit** to the **training samples**!

How to find the Regression Model?

How to find the Regression Model*

Training Sets (x,y)
(past sales records):

Size in feet^2 (*x*)	Price (*y*)
1000	410K
1200	600K
1230	620K
1340	645K
…	…

Regression Model (Hypothesis): $h_\theta(x) = \theta_0 + \theta_1 x$

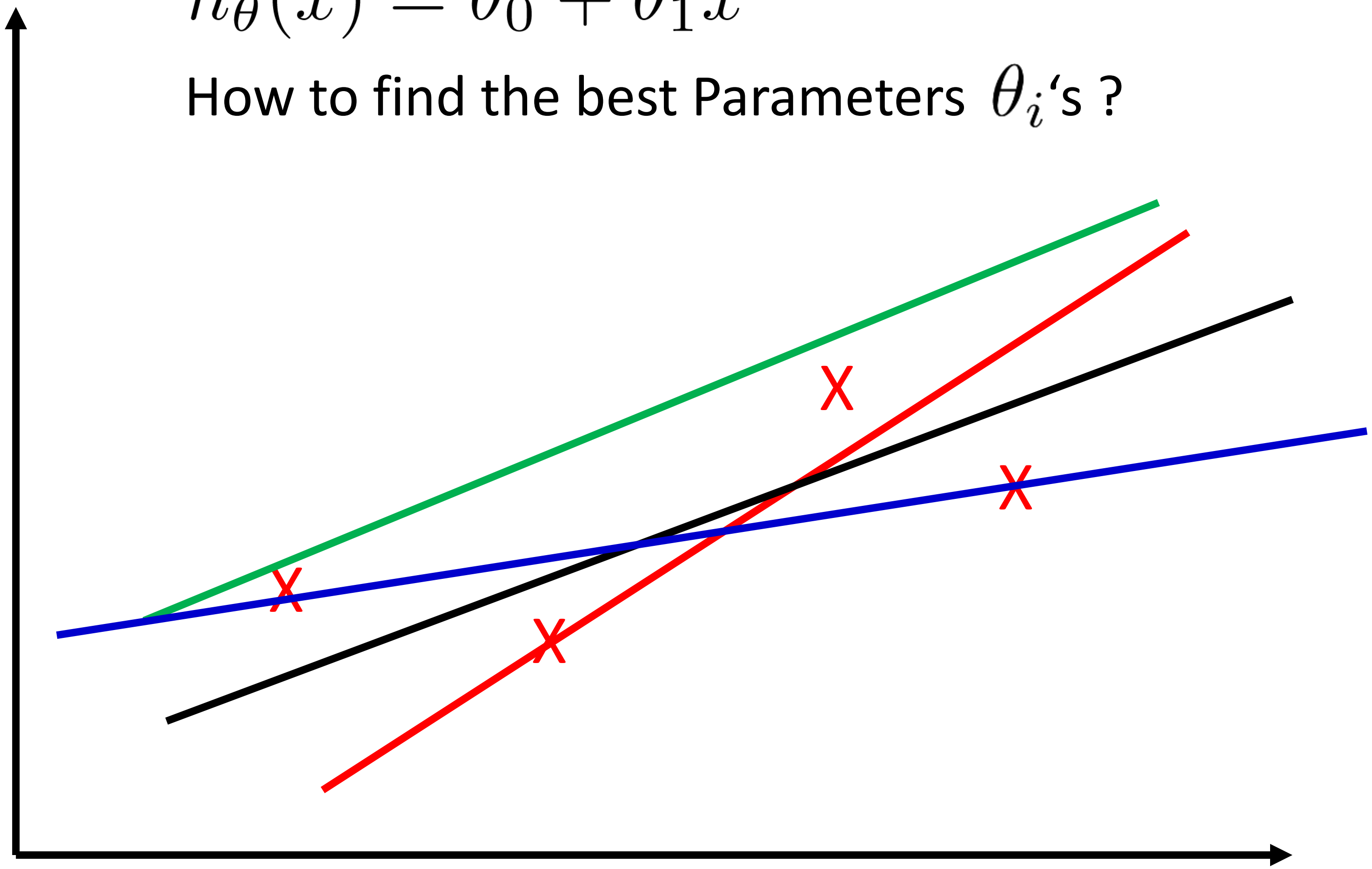
Question: How to find the best fit line? In other word, How to find the Parameters θ_0, θ_1 that correspond to the best line?

* Reference: Andrew Ng, Machine Learning, Stanford University.

How to Select the Parameters: A Simple Example

$$h_\theta(x) = \theta_0 + \theta_1 x$$

How to find the best Parameters θ_i‘s ?



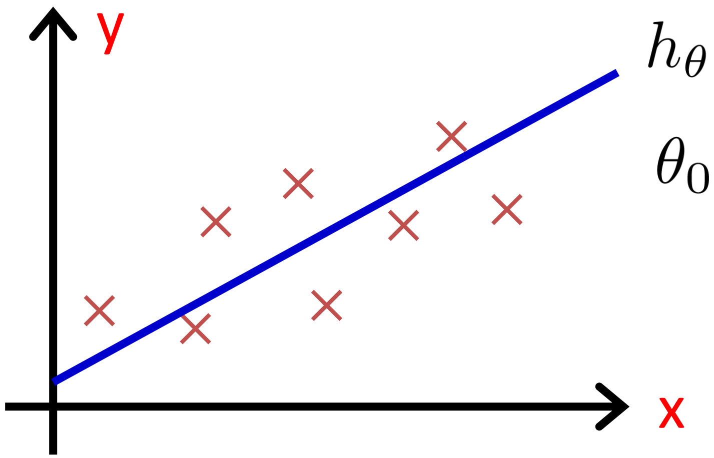
How to Select the Parameters

- How to find the best Parameters θ_i's ?

Answer: We should find the **Best Fit** to the training samples. To do that, we have to Choose θ_0, θ_1 so that $h_\theta(x)$ (the blue line) is as close as possible to all y 's (actual red points) for **ALL training examples** (x, y)

Training Set:

Size in feet^2 (*x*)	Price (*y*)
1000	410K
1200	600K
1230	620K
1340	645K
…	…

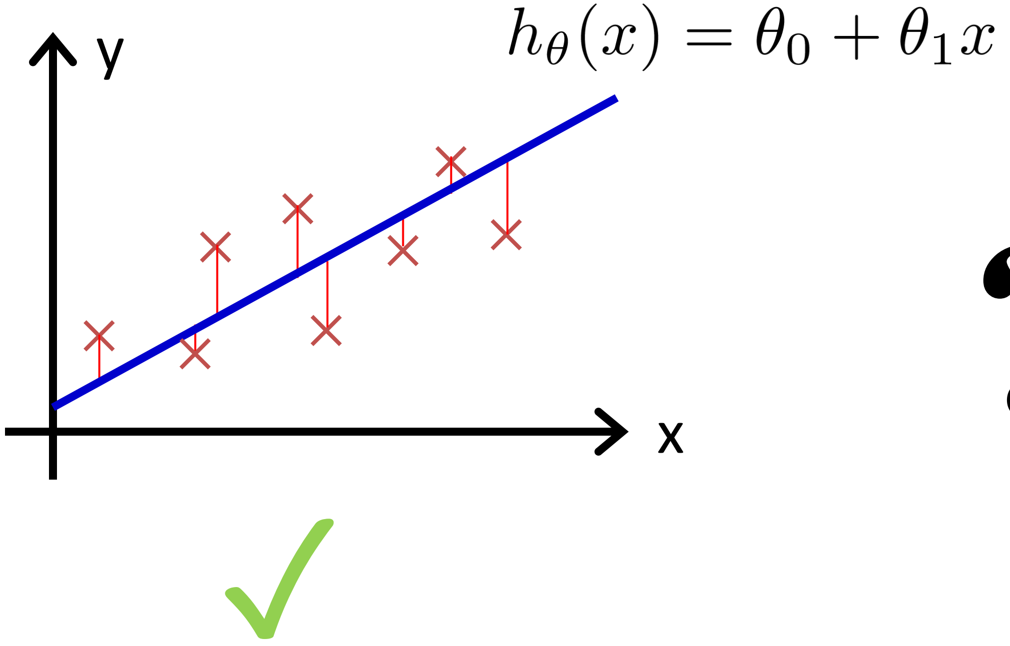


$$h_\theta(x) = \theta_0 + \theta_1 x$$

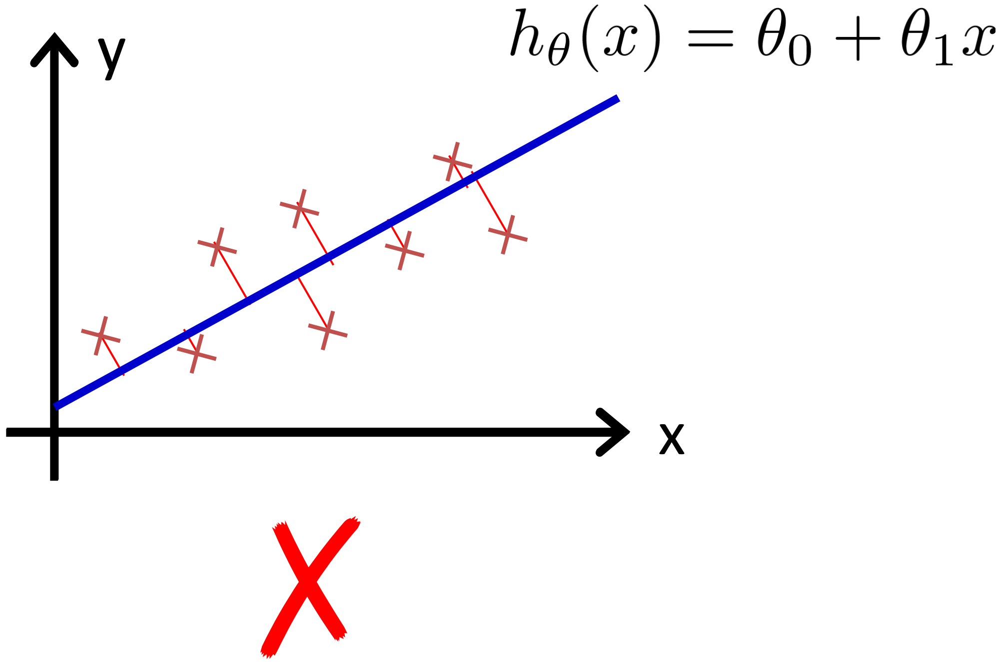
θ_0, θ_1 =?

Which one?

- **Question:** Which **distance** should be minimum to find the **best fit** line? The distance to the line Or the vertical distance?



?

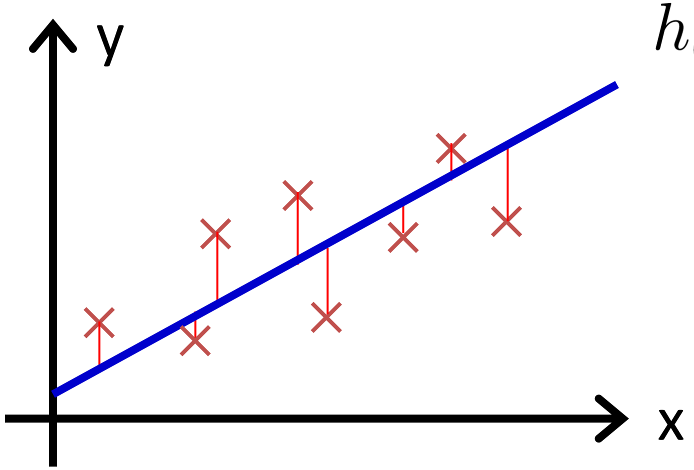


How to Select the Parameters

- **Answer**: Choose θ_0, θ_1 so that $h_\theta(x)$ is as close as possible to y for all training examples (x, y)
- In other word, we have to minimize the differences between "prediction $h(x^{(i)})$ " and "actual value y" in **training set** to find the best "fit" to our training data.

Training Set:

Size in feet^2 (*x*)	Price (*y*)
1000	410K
1200	600K
1230	620K
1340	645K
…	…



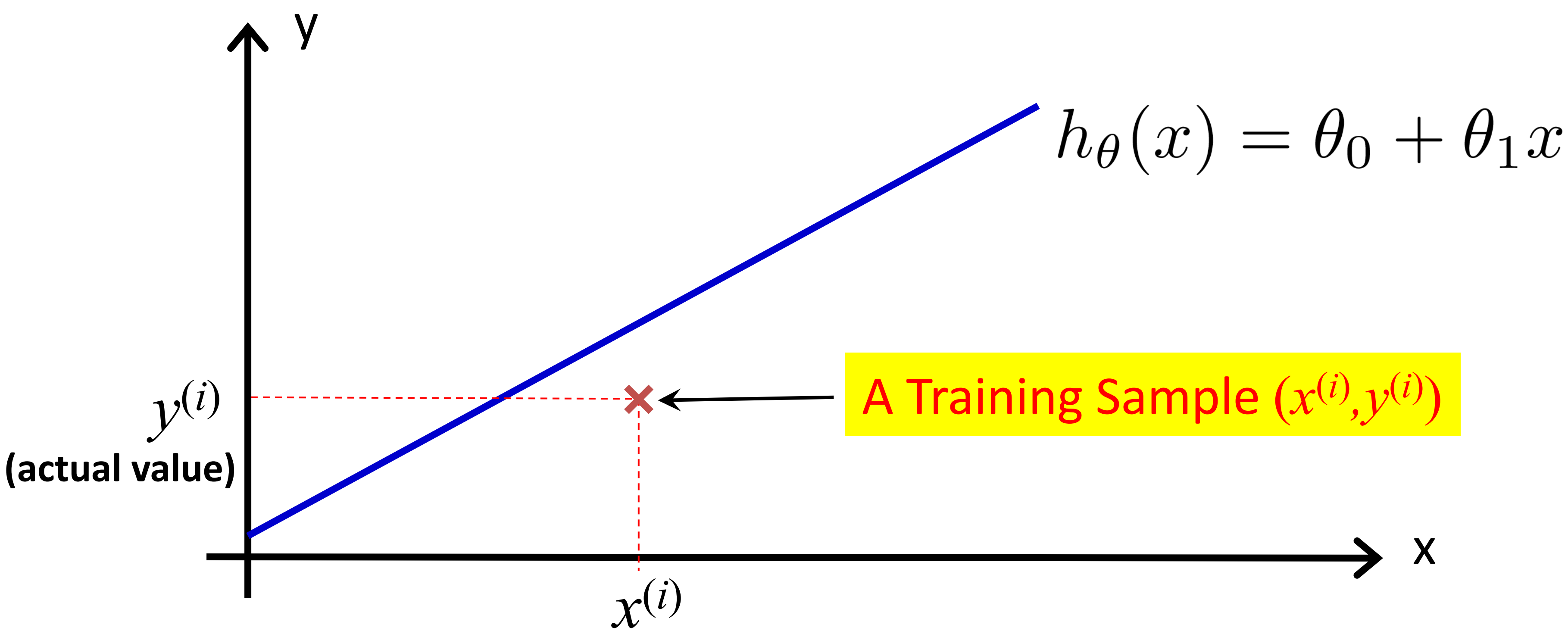
$$h_\theta(x) = \theta_0 + \theta_1 x$$

θ_0, θ_1 =?

How to Select the Parameters

Training Set: $\{(x^{(1)},y^{(1)}), (x^{(2)},y^{(2)}), \ldots, (x^{(m)},y^{(m)})\}$

Note: (i) is index!



How to Select the Parameters

Training Set: $\{(x^{(1)},y^{(1)}), (x^{(2)},y^{(2)}), \ldots, (x^{(m)},y^{(m)})\}$

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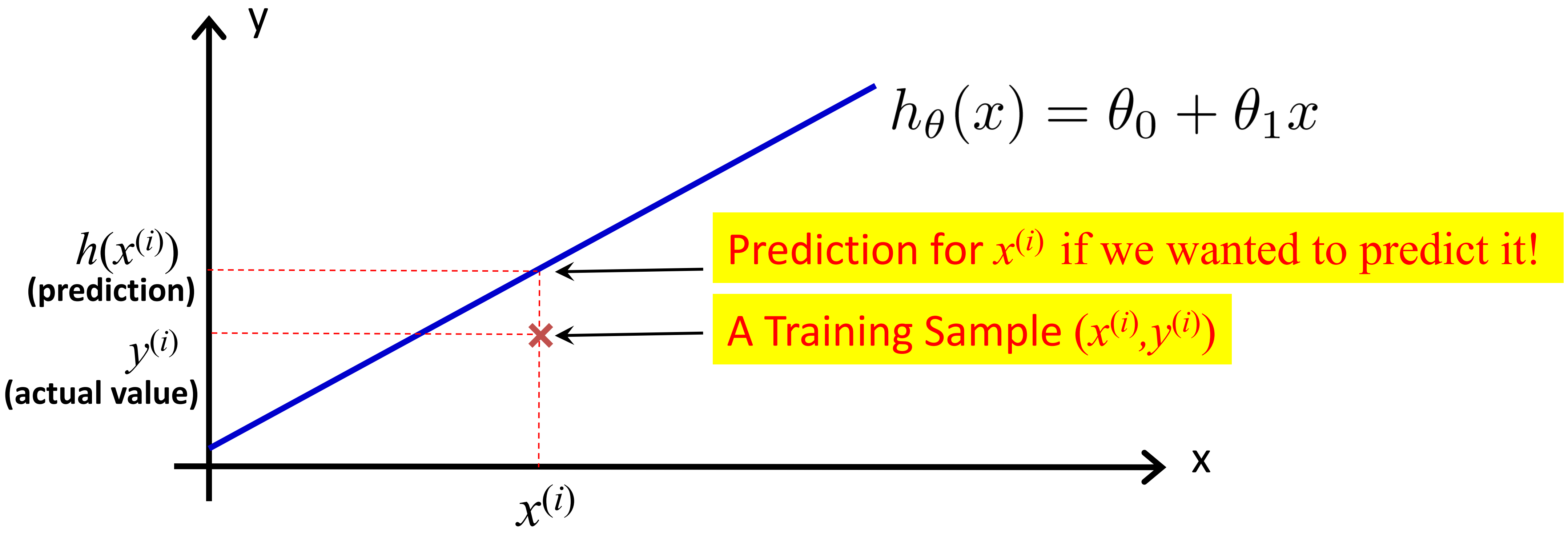
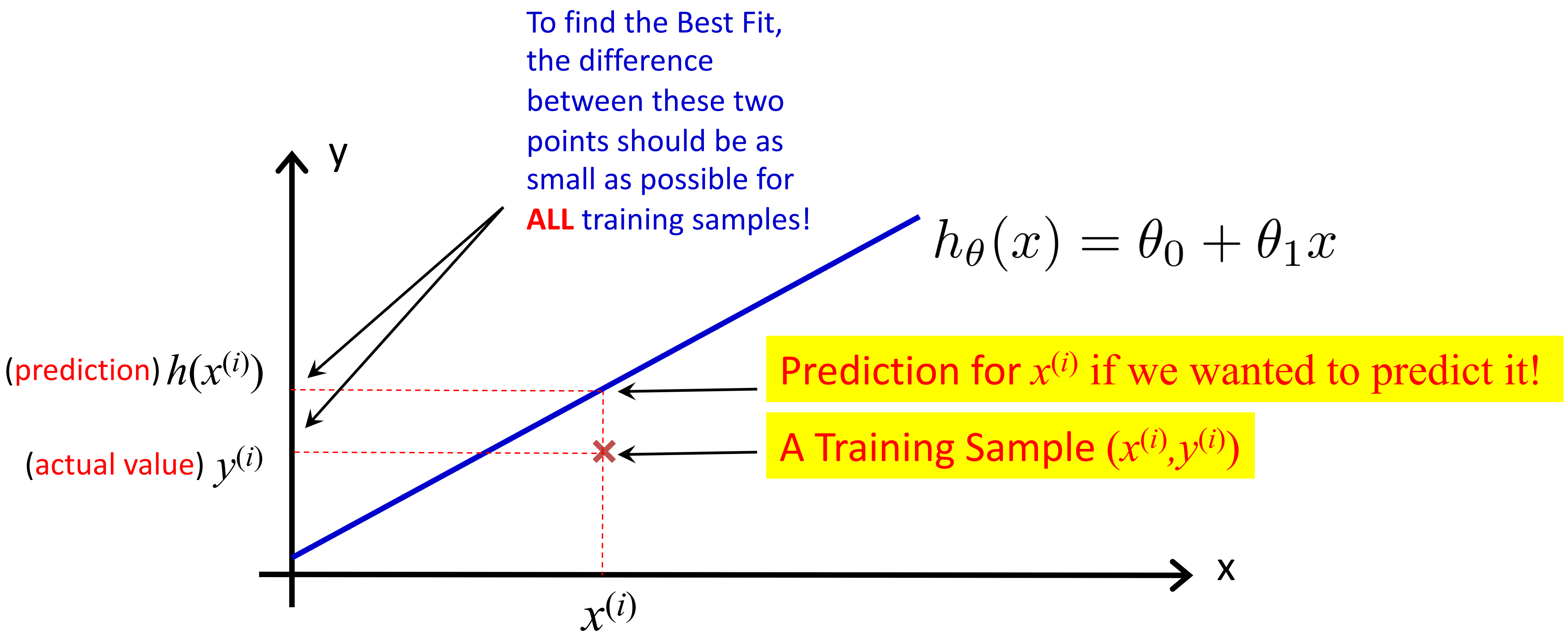


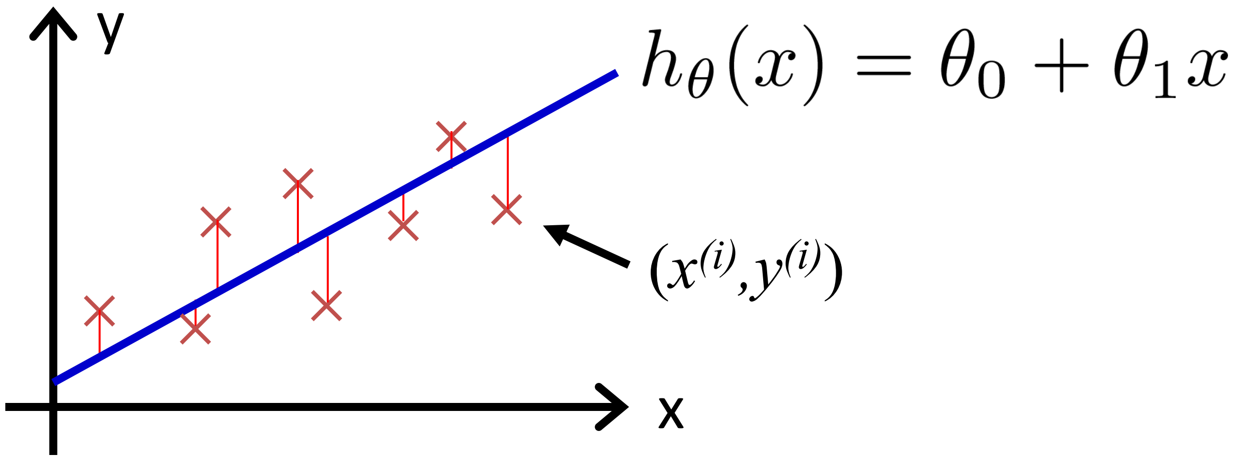
- Thus, we just need to minimize $(h(x^{(i)}) - y^{(i)})^2$ for **every** training sample *i.*
- *Question: Why squared?*



Let's define a **Cost Function** $\boldsymbol{J(\theta_{0,}\ \theta_1)}$ as the summation (or average) of **all** differences between "**training samples**" and the **"regression line"**.

Cost Function: $$J(\theta_0, \theta_1) = \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

Thus, the Best Fit Line is the line with minimum Cost Function $\boldsymbol{J(\theta_{0,}\ \theta_1)}$.

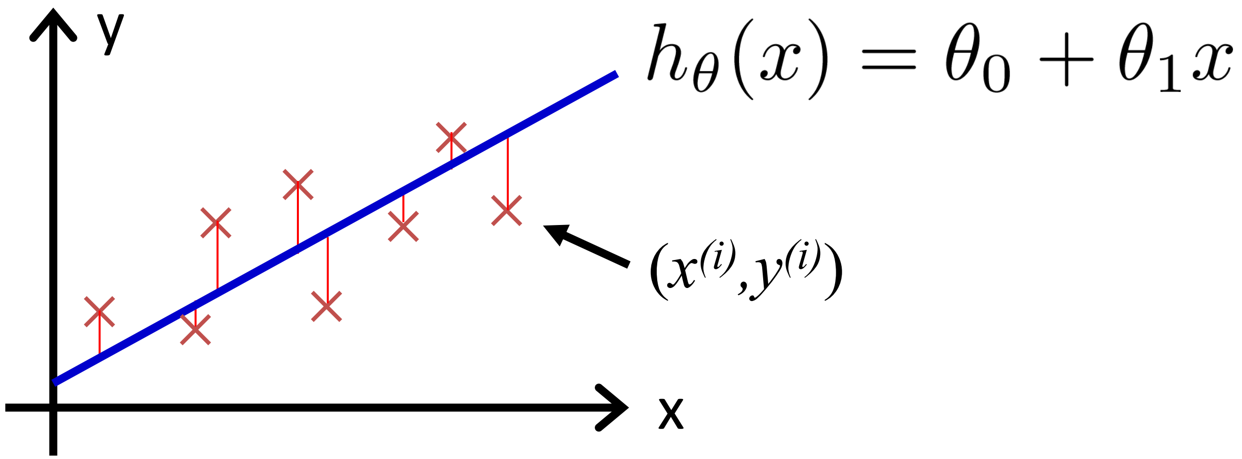


Training Set: $\{(x^{(1)},y^{(1)}), (x^{(2)},y^{(2)}), \ldots, (x^{(m)},y^{(m)})\}$

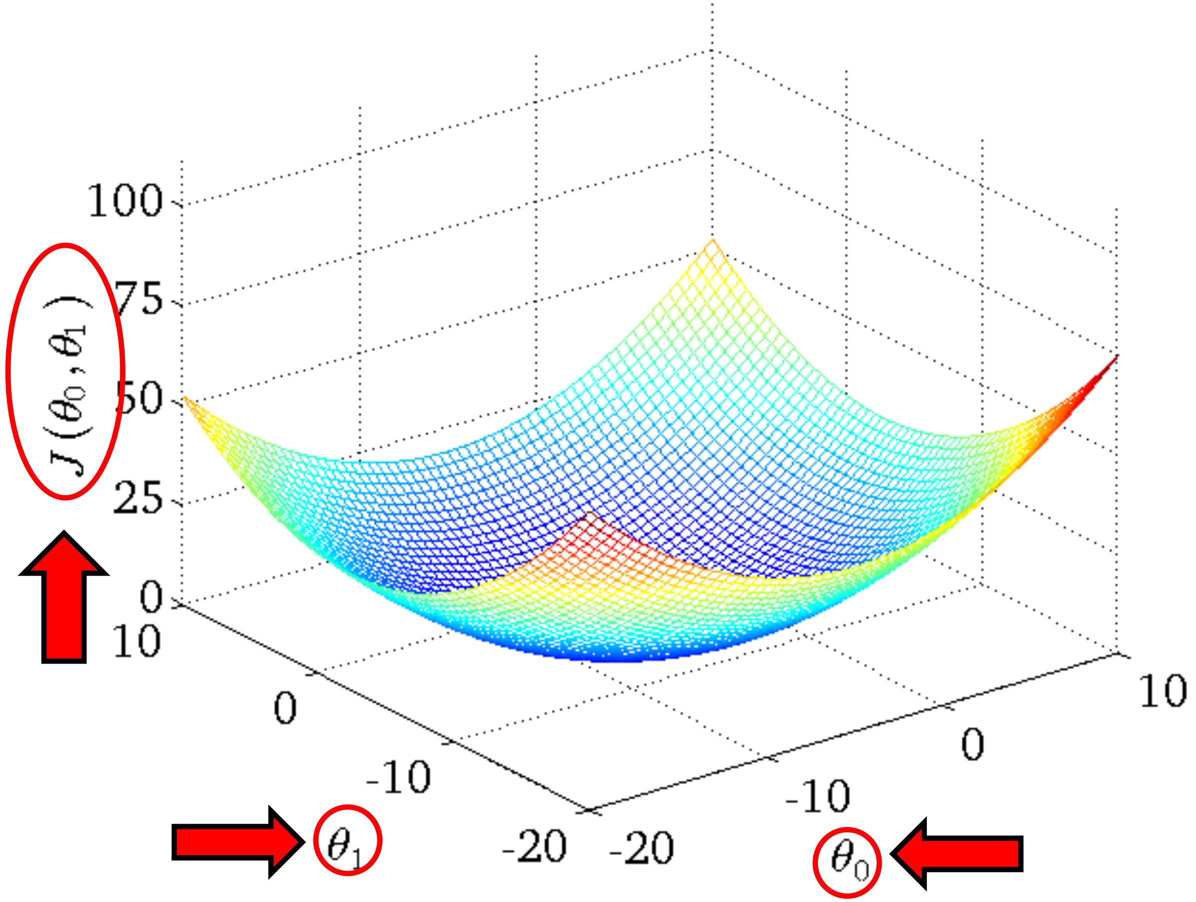
Cost Function: $$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

Goal: To find the best parameters $\boldsymbol{\theta_0}, \boldsymbol{\theta_1}$ that minimizes the **Cost Function** $\boldsymbol{J(\theta_0, \theta_1)}$:

$$\underset{\theta_0, \theta_1}{\text{minimize}}\ J(\theta_0, \theta_1)$$



- It turns out that the **Cost Function** $J(\theta_0, \theta_1)$ is a Convex (Bowl-Shaped) function. So, it has a global minimum! Example:



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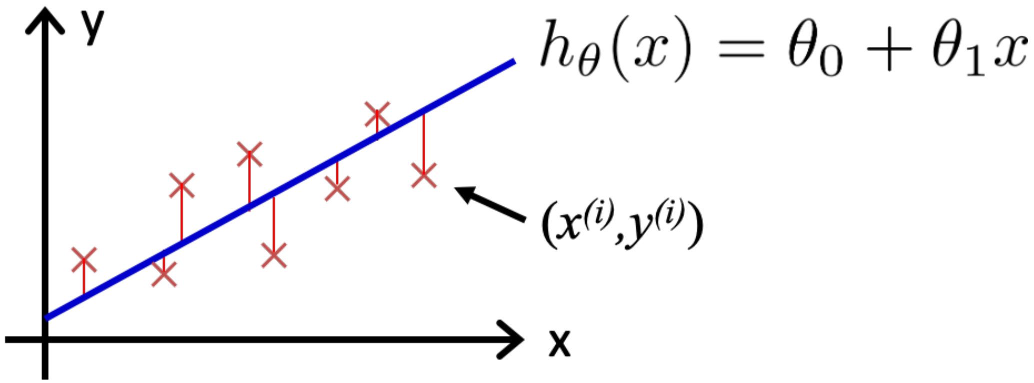
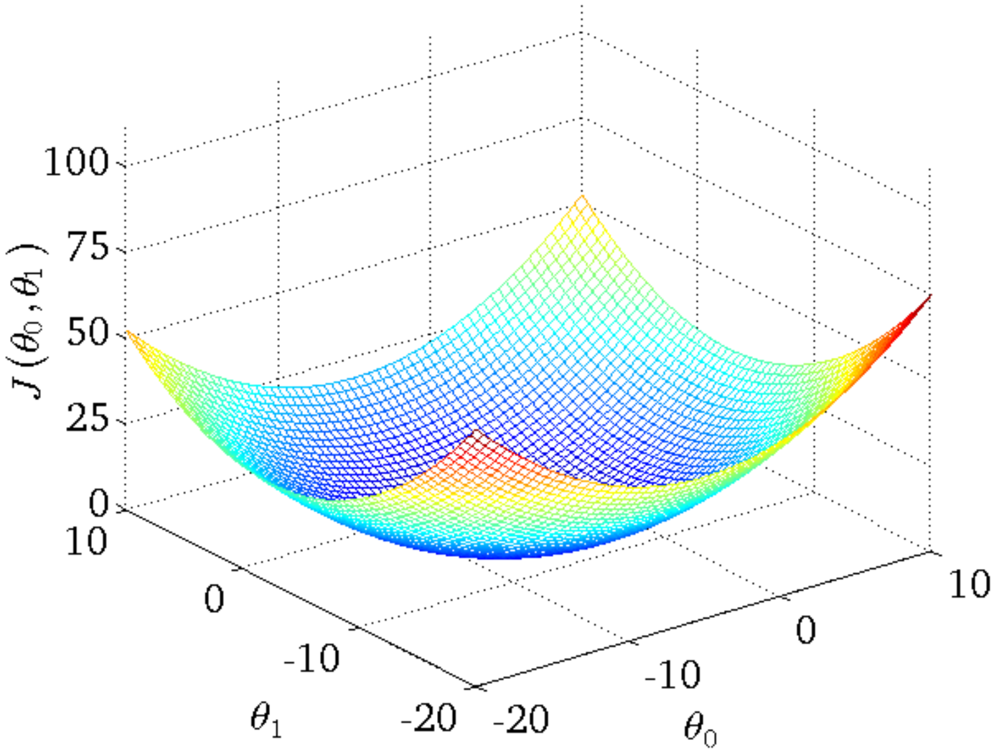
Regression Model: $h_\theta(x) = \theta_0 + \theta_1 x$

Cost Function: $J(\theta_0, \theta_1) = \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$

Goal: $\underset{\theta_0, \theta_1}{\text{minimize}}\ J(\theta_0, \theta_1)$





Thank You!

Questions?