

Week 5

Sorting & Big-O (Again!)

EXAM 1!!

6:45 - 7:45pm, Feb. 20th, Troxel 1001 (Sec. A), Hoover 2055 (Sec. B)

Topics:

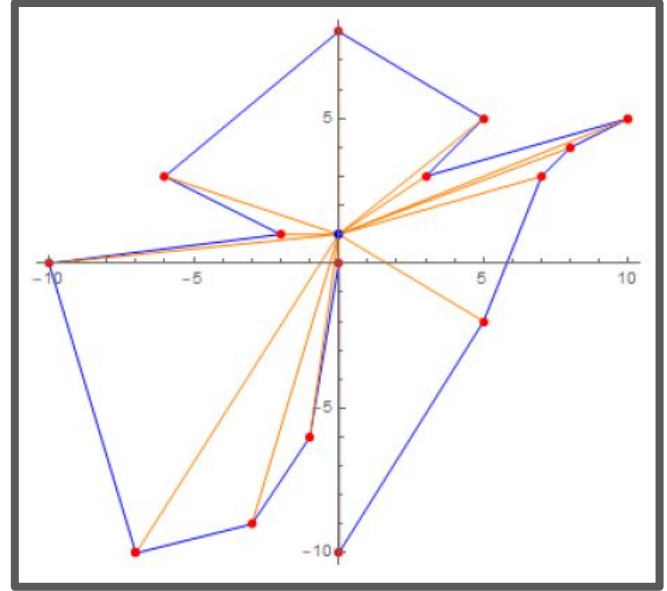
- Inheritance, Polymorphism, Interfaces, Abstract classes, Class hierarchy, Object superclass, Shallow vs. deep equals() and clone(), exception handling (try, catch, throw)
- Time complexity, Big-O notation, algorithm analysis, binary search
- Sorts: Selection, Insertion, Merge, Quick, and their runtimes
- Using the comparator/comparable interface

Project 2: Point Scanning

Due: March 3 (Sunday), midnight

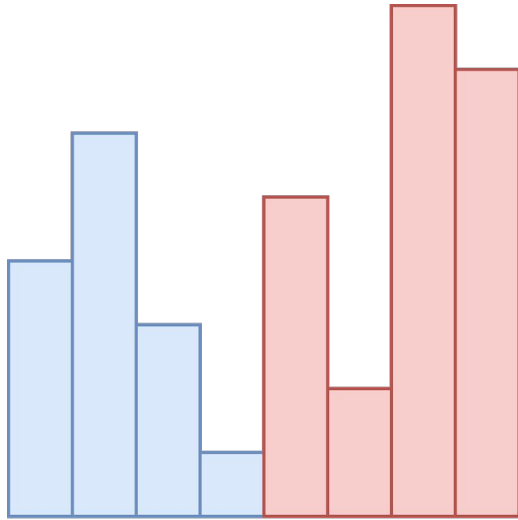
Focuses on:

- Sorting algorithms
- Comparator interface
- Drawing geometry

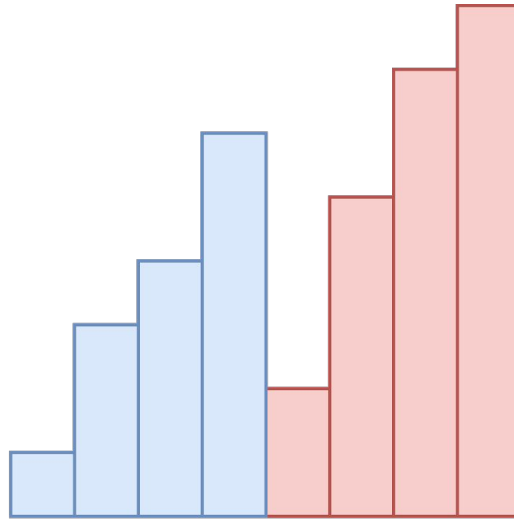


Mergesort

Worst case: $O(n \log n)$
Average case: $O(n \log n)$

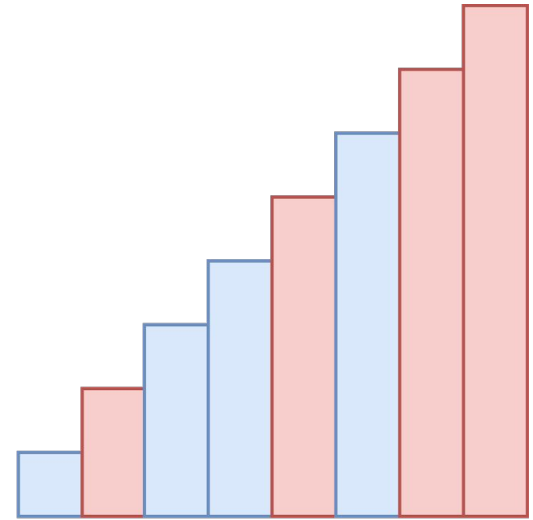


Split array in half



Mergesort

Mergesort

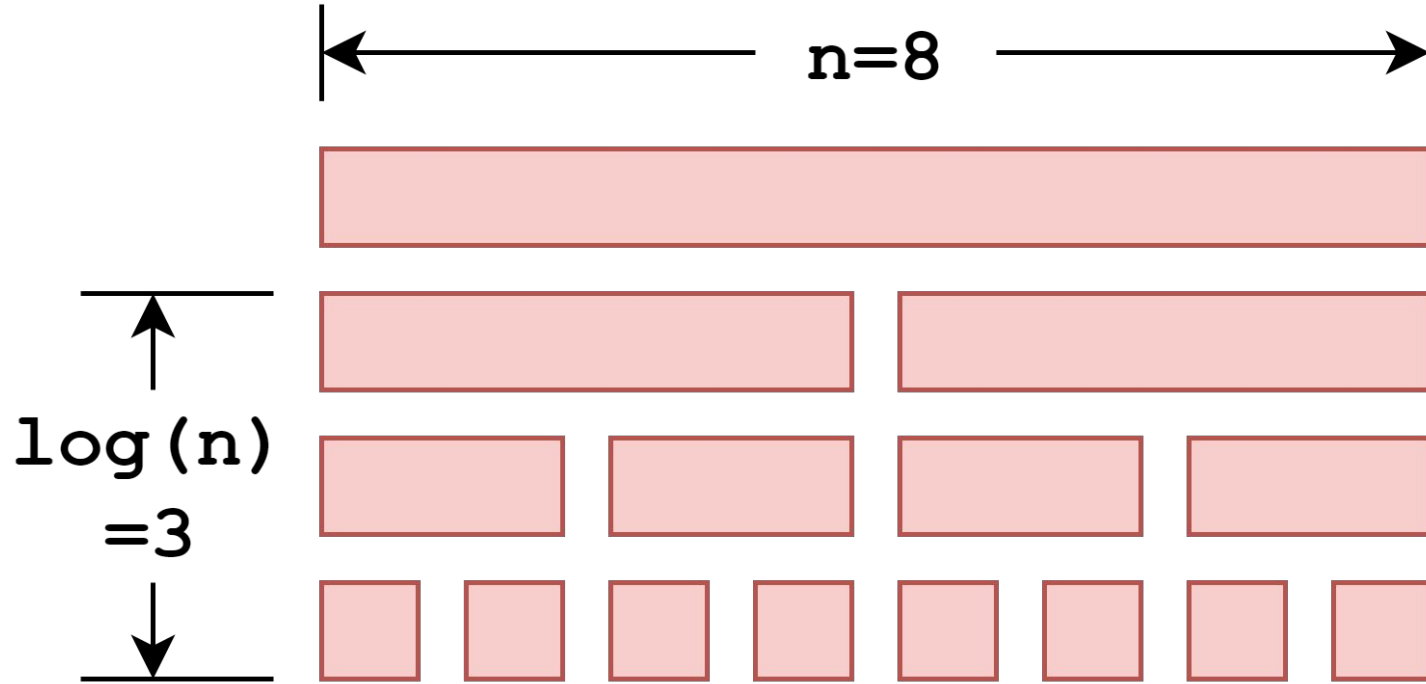


Merge

Mergesort runtime

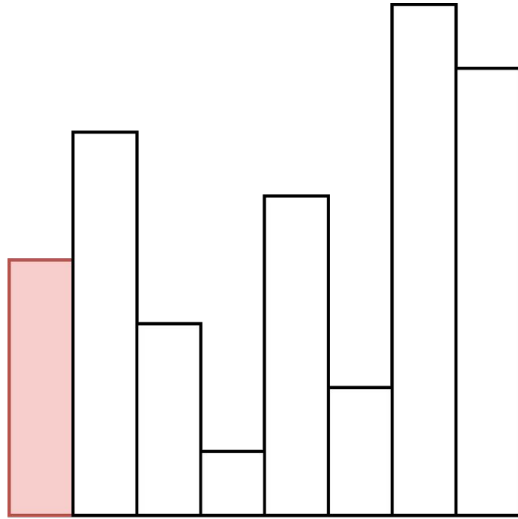
Why does it operate in $O(n \log n)$?

Mergesort runtime

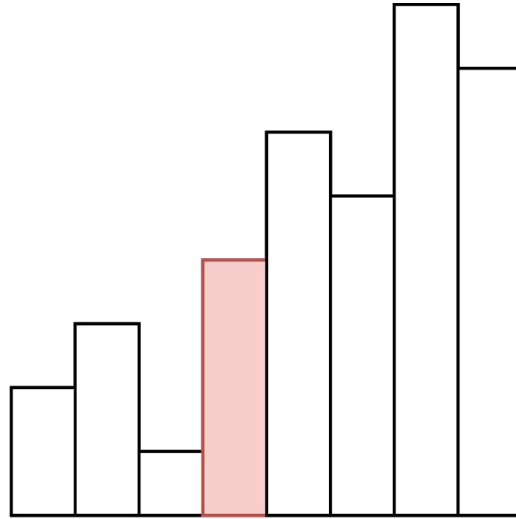


Quicksort

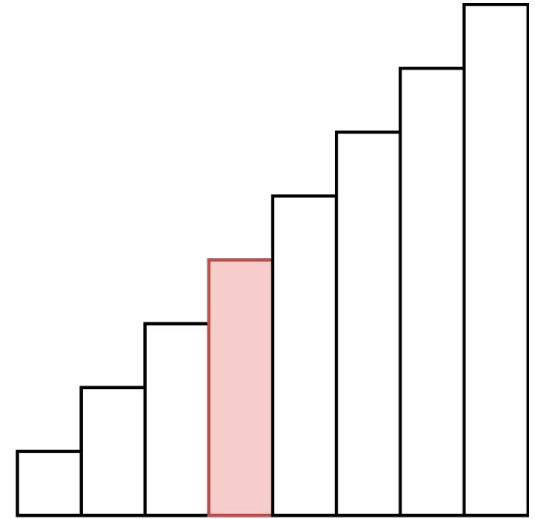
Worst case: $O(n^2)$
Average case: $O(n \log n)$



Select pivot



Partition

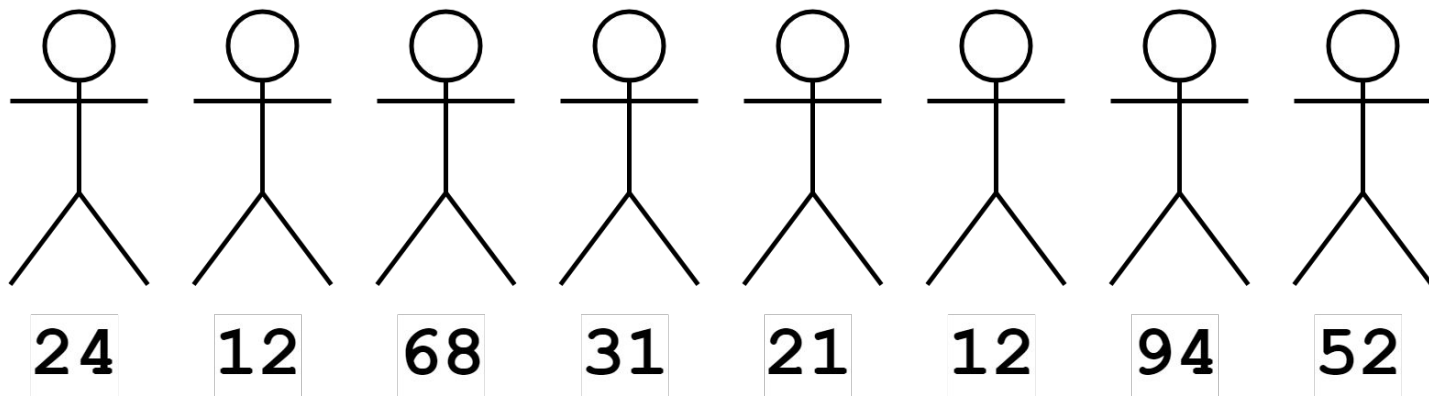


Quicksort

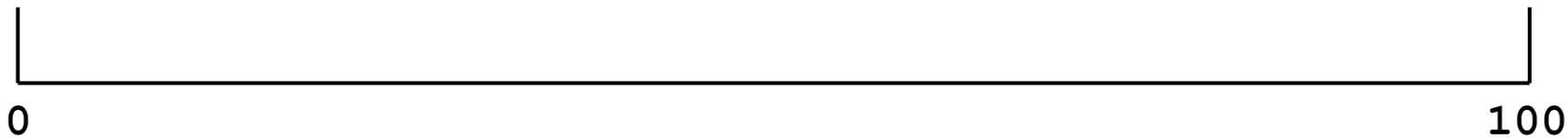
Quicksort

Counting sort

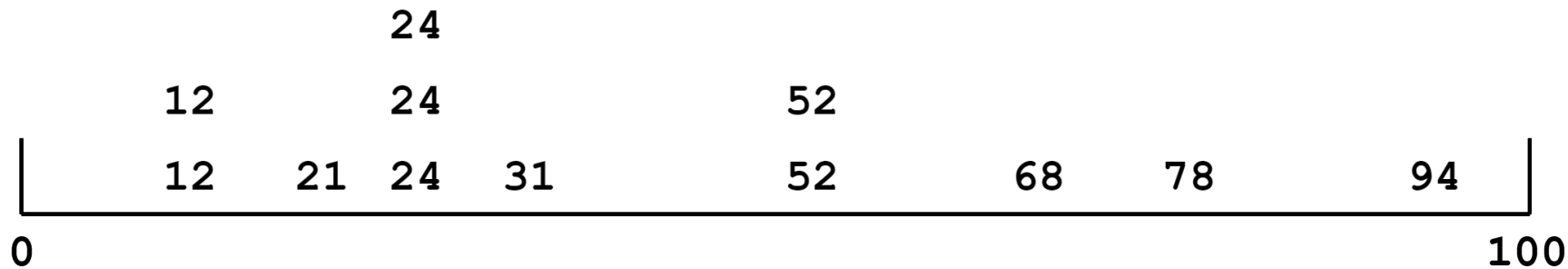
How can we sort a list of people by their age?



Counting sort

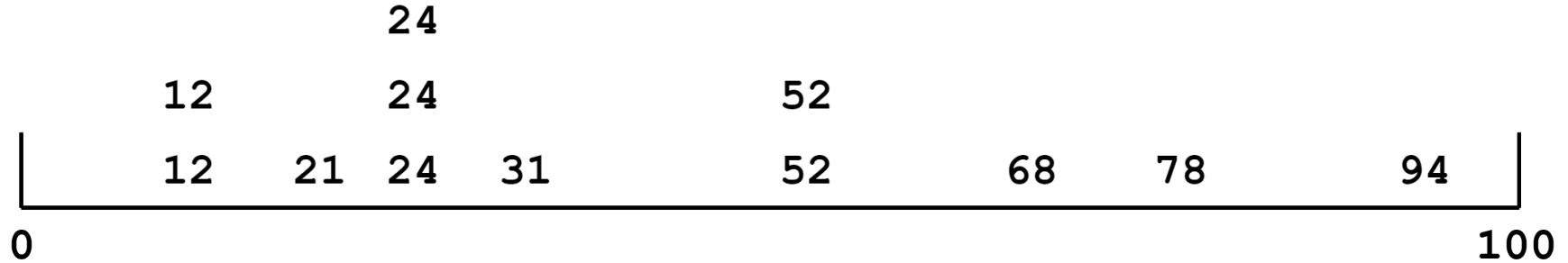


Counting sort



Counting sort

Worst case: $O(r + n)$
Average case: $O(r + n)$



Radix sort

How can we sort people in a phonebook by phone number?

612-749-5598

207-342-5319

805-614-6636

510-313-4327

921-405-2452

492-159-1950

211-539-0927

488-493-2009

821-667-3916

578-574-8029

103-283-7510

397-503-2189

754-710-3985

749-218-3475

903-174-5071

939-457-3908

Worst case: $O(n^2)$
Average case: $O(n \log n)$

Radix sort

Radix sort process:

1. Create “buckets” to store numbers in from 0-9 (like in counting sort)
2. Place numbers in buckets according to least or most significant digit
3. Remove numbers in order from 0 to 9 and repeat with the next significant digit

821-667-3916
578-574-8029
103-283-7510
397-503-2189
754-710-3985
749-218-3475
903-174-5071
939-457-3908

Easier to understand using visualgo.net/en

Comparing Mergesort/Quicksort

Why use Quicksort instead of Mergesort when they both have an average runtime of $O(n \log n)$, but Quicksort at its worst case runs in $O(n^2)$?

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- With a **random pivot**, Quicksort practically never runs in $O(n^2)$
- Quicksort plays better with hardware (Cache locality)
- Uses less space than Mergesort

Comparing runtimes

logn	n	nlogn	n ²
3	10	30	100
10	1,000	10,000	1,000,000
20	1,000,000	20,000,000	1,000,000,000,000
30	1,000,000,000	30,000,000,000	1,000,000,000,000,000,000

Comparing runtimes

Sorting countries by population

of countries: ~200

Range of populations: 10^3 - 10^9

Selection sort	(n^2)	40,000
Mergesort	($n\log n$)	1,500
Counting sort	($r + n$)	10^9
Radix sort	(wn)	1,800

Comparing runtimes

Sorting phone numbers in the US

of phone numbers: $\sim 3 \times 10^8$

Range of phone numbers: $0-10^{10}$

Selection sort	(n^2)	9×10^{16}
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Mergesort	$(n \log n)$	8.4×10^9
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Counting sort	$(r + n)$	10^{10}
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Radix sort	(wn)	3×10^9
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Comparing runtimes

Sorting ages of US citizens

of US citizens: $\sim 3 \times 10^8$

Range of ages: 0-150

Selection sort	(n^2)	9×10^{16}
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Mergesort	($n \log n$)	8.4×10^9
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Counting sort	($r + n$)	10^8
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Radix sort	(wn)	9×10^8
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Sorting Stability

Let us take a small array and sort it

Before: **5 , 9 , 7 , 5**

After: **5 , 5 , 7 , 9**

Sorting Stability

But which 5 is which?

Before: 5, 9, 7, 5

After: 5, 5, 7, 9

Sorting Stability

Stable

5, 9, 7, 5

5, 5, 7, 9

Unstable

5, 9, 7, 5

5, 5, 7, 9