CS 230 : Discrete Computational Structures
Spring Semester, 2019
ASSIGNMENT #11 [**Extra Credit**]
Due Date: Friday, May 3

Suggested Reading: Rosen Sections 6.4 - 6.5, CLRS Chapter on Graphs

These are the problems that you need to turn in. Always explain your answers and show your reasoning. **Spend time giving a complete solution. You will be graded based on how well you explain your answers. Just correct answers will not be enough!**

1. [**5 Pts**] Prove, using a combinatorial argument, that $C(m+n,2) = C(m,2) + C(n,2) + mn$, where $m, n \geq 2$. To make your combinatorial argument, describe a problem that both the lhs and rhs expressions count.

2. [**8 Pts**] Prove, (a) using a combinatorial argument, and (b) using an algebraic proof, that $C(n,2)C(n-2,k-2) = C(n,k)C(k,2)$.

3. [**6 Pts**] A cookie shop sells 5 different kinds of cookies. How many different ways are there to choose 16 cookies if (a) you pick at least two of each? (b) you pick at least 4 oatmeal cookies and at most 4 chocolate chip cookies?

4. [**9 Pts**] How many solutions are there to the equation $x_1 + x_2 + x_3 + x_4 = 20$, where x_i is a non-negative integer, for all i, if (a) there are no restrictions? (b) $x_1 > 1$, $x_2 > 2$, $x_3 > 3$, $x_4 > 4$? (c) $x_1 > 4$ and $x_3 < 5$?

5. [**9 Pts**] How many ways are there to split 30 people into three committees of 5 people each and five committees of 3 people each if (a) all eight committees have different tasks? (b) all eight committees have the same task? (c) the three 5-member committees and two of the 3-member committees are all given the same task while the remaining three 3-member committees are not given any task yet?

6. [**6 Pts**] How many ways are there to pack 6 different books into 6 identical boxes with no restrictions placed on how many can go in a box (some boxes can be empty)? What if the books are identical?

7. [**6 Pts**] How many ways can we place 12 books on a bookcase with 5 shelves if the books are (a) indistinguishable copies (b) all distinct? Note that the position of the books on the shelves matter.

8. [**6 Pts**] Consider a graph G that has 7 vertices with degrees of 5, 4, 3, 3, 2, 2, 1. How many edges does G have? Explain.

For more practice, you are encouraged to work on other problems, like the ones below. Remember that there will be graph problems in the exam, so make sure you go over problems on graphs!

1. How many integers between 1000 and 9999 inclusive contain (a) at least one 0 and at least one 1, (b) at least one 0, at least one 1 and at least one 2? Solve part (a) using the Inclusion-Exclusion Principle for two sets, and part (b) using the Inclusion-Exclusion Principle for three sets: $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$.

2. Prove that a graph is a tree if and only if it is acyclic but adding any edge will create a cycle.

3. Prove by induction that a complete binary tree of height h has 2^h leaves. Use the inductive definition of complete binary trees.

4. If G is a simple graph with n vertices and $n-1$ edges, (a) is G connected? (b) is G acyclic? For each question, if *yes*, give a short justification. If *no*, give a counterexample.