# CS 230 : Discrete Computational Structures

**Spring Semester, 2019**

ASSIGNMENT #5

**Due Date:** Wednesday, March 6

**Suggested Reading:** Rosen Sections 2.3, 9.1 and 9.5; Lehman et al. Chapter 4.3, 4.4, 10.6 and 10.10

These are the problems that you need to hand in for grading. Always explain your answers and show your reasoning.

1. [**10 Pts**] Let $g$ be a total function from $A$ to $B$ and $f$ be a total function from $B$ to $C$.
   (a) If $f \circ g$ is one-to-one, then is $f$ one-to-one? Prove or give a counter-example.
   (b) If $f \circ g$ is one-to-one, then is $g$ one-to-one? Prove or give a counter-example.

2. [**10 Pts**] For each of these relations decide whether it is reflexive, anti-reflexive, symmetric, anti-symmetric and transitive. Justify your answers. $R_1$ and $R_2$ are over the set of real numbers.
   (a) $(x, y) \in R_1$ if and only if $xy \geq 0$
   (b) $(x, y) \in R_2$ if and only if $x = 2y$

3. [**8 Pts**] Let $R_3$ be the relation on $\mathcal{Z}^+ \times \mathcal{Z}^+$ where $((a, b), (c, d)) \in R_3$ if and only if $a + d = b + c$.
   (a) Prove that $R_3$ is an equivalence relation.
   (b) Define a function $f$ such that $f(a, b) = f(c, d)$ if and only if $((a, b), (c, d)) \in R_3$.
   (c) Define the equivalence class containing $(1, 1)$.
   (d) Describe the equivalence classes. How many classes are there and how many elements in each class?

4. [**10 Pts**] Prove that these relations on the set of all ISU students are equivalence relations. Describe the equivalence classes. Now, describe new equivalence relations which are refinements of the relations given.
   (a) $(a, b) \in R_4$ if and only if $a$ and $b$ live in the same building
   (b) $(a, b) \in R_5$ if and only if $a$ and $b$ graduated from the same high school

5. [**12 Pts**] Consider the following relations on the set of positive real numbers. One is an equivalence relation and the other is a partial order. Which is which? For the equivalence relation, describe the equivalence classes. What is the equivalence class of 2? of $\pi$? Justify your answers.
   (a) $(x, y) \in R_6$ if and only if $x/y \in \mathcal{Z}$
   (b) $(x, y) \in R_7$ if and only if $x - y \in \mathcal{Z}$