1.

a. $S = \{ 7k \mid k \in N\}$
So S is subset of N and S is infinite.
Since N is countably infinite and S is infinite subset of N, and we know that infinite subset of a countably infinite set is countably infinite.
So S is also countably infinite.

b. $A = \{ 2, 3\}$
$B = A * Z^+$
$= B_1 \cup B_2$ where $B_1 = \{2\} * Z^+$ and $B_2 = \{3\} * Z^+$
We know that Z^+ is countably infinite and the function $B_1 \rightarrow Z$ is a bijection.
So B_1 is also countably infinite. Same thing goes for B_2.
We know that countable U countable is also countable.
$B_1 \cup B_2$ is countable and Z^+ is countable infinite,
Therefore, B is countable infinite.

2. Assuming that B is not uncountable, then B is finite set. And since A is a subset of B, therefore A is also a finite set. Because element in A must be in B. So, if B is countable, A is also countable. By contrapositive, it's true that if A is uncountable, then B is uncountable.

3. $F_{N \to N}$ is uncountable.
Suppose for contradiction, $F_{N \to N}$ is countable.
Where , $F_{N \to N} = \{ F_1, F_2, F_3, \dots \}$
Let $B = \{ i \mid i \notin F_{N \to N}\}$
Some $B \in F_{N \to N}$, so $\exists j$ where $B = F_{N \to N}$
There are 2 cases:
Case 1: $j \in B$
$j \in B => j \notin F_{N \to N}$
$=> j \notin$ B by definition of B

Case 2: $j \notin$ B
$j \notin$ B $=> j \in F_{N \to N}$
$=> j \in B$
So $F_{N \to N}$ is uncountable.

4.

a.

	0	1	2	3	And so on
0		0.9	0.99	0.999	…
1	9	9.9	9.99	9.999	…
2	99	99.9	99.99	99.999	…
3	999	999.9	999.99	999.999	…
And so on	…	…	…	…	…

Therefore, this set is countable as it can be counting off diagonally.

b. Suppose S is uncountable set of real numbers [0, 1] where decimal representation consisting of 8's and 9's
0.888 where there's infinite number of 8 after the decimals, same goes to 0.999
By contradiction, suppose S is countable where S it has element s1, s2, s3 and so on.
//not complete

5.

a. Let A = set of R numbers of (0, 1] and B = set of R numbers of (0, 1)
A – B = {1}
Therefore A-B is a finite set.

b. Let A = set of R number (0, 1) union Z^+ and B = set of R number of (0, 1)
A – B = Z^+
Therefore, A – B is countably infinite.

c. Let A = set of R number (0, 1) and B = set of R numbers (1, 2)
A – B = A as there are no common elements.
Therefore, A – B is uncountably infinite.