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1. $S(X) = X$ can swim
   $F(X) = X$ can catch fish
   $X =$ all bears

   $\forall x\ (S(x) \wedge F(x))$

   Negation $= \exists x\ \neg(S(x) \wedge F(x))$

   English: There exist bears that can't swim and catch fish.

2. $\forall x(P(x) \rightarrow Q(x))$ is not equivalent to $\forall x\ P(x) \rightarrow \forall x\ Q(x)$
   $X =$ all people
   $P(x) = x$ is comedian
   $Q(x) = x$ is funny
   $\forall x(P(x) \rightarrow Q(x)) =$ all comedians are funny. But not everyone is comedian.
   $\forall x\ P(x) \rightarrow \forall x\ Q(x) =$ If everyone is comedian, then everyone is funny.

3. $S(x) = x$ is a student
   $F(x) = x$ is a faculty member
   $A(x, y)$ x has asked y a question

   (a) $\exists x \forall y\ F(x) \wedge (S(y) \rightarrow \neg A(x, y))$
   (b) $\forall x \exists y\ (F(y) \wedge S(y)) \rightarrow A(x, y)$

4. PREFIX $(x, y) = x$ is a prefix of y
   SUBSTRING $(x, y) = x$ is a substring of y
   NO – 1S(x) = x is empty or a string of 0's

   (a) $\exists y\ (x = yyy)$
   (b) NO – 1S(x) $\wedge \exists y\ (x = yy)$
   (c) $\neg$ (SUBSTRING (0, x) $\wedge$ SUBSTRING (1, x))

5.
   (a) Equal (m, n) = Zero(k) AND A(m, n, k)
   (b) One (n) = $\forall x$ M(x, x, n) x = x . n
   (d) Prime (p) = $\forall x$ NOT(One(x)) AND NOT(Equal(x, p) AND Greater(p, x) AND NOT(M(Zero(k), p, 1/x))

(e) Two(n) = ∀x One(x) AND A(n, x, x)

6.

(a) I = ice

S = snow

C = school is open

E = exam postponed

(1) $I \vee S \rightarrow \neg C$	hyp 1
(2) $\neg C \rightarrow E$	hyp 2
(3) $I \vee S \rightarrow E$	hypothetical syllogism 1,2
(4) $\neg E$	
$\neg (I \vee S)$	Modus tollens 3, 4]

(b) X = people

M = Mary

M(x) = x is Minnesotan

I(x) = x knows how to ice fish

C(x) = x in the class

(1) $C(m) \wedge M(m)$	hyp 1
(2) $\forall x\, M(x) \rightarrow I(x)$	hyp 2
(3) $M(m) \rightarrow I(m)$	universal instantiation 2
(4) $M(m)$	simplification 1
(5) $I(m)$	modes ponens
(6) $C(m)$	simplification 1
(7) $C(m) \wedge I(m)$	conjunction 5, 6
(8) $\exists x\, C(x) \wedge I(x)$	existential generalization 7

(c) X = all bear

S(x) = x is good swimmer

C(x) = x can catch fish

H(x) = x goes hungry

(1) $\forall x\, S(x)$	hyp 1
(2) $S(c)$	universal instantiation 1
(3) $C(x) \rightarrow \neg H(x)$	hyp 2
(4) $\neg C(x) \rightarrow \neg S(x)$	hyp 3
(5) $S(x) \rightarrow C(x)$	Modus tollens 4
(6) $S(c) \rightarrow C(c)$	universal instantiation 5
(7) $C(c) \rightarrow \neg H(c)$	universal instantiation 3
(8) $S(c) \rightarrow \neg H(c)$	hypothetical syllogism 7, 8