CS 230 : Discrete Computational Structures

Spring Semester, 2019

ASSIGNMENT #7

Due Date: Thursday, March 28

Suggested Reading: Rosen Sections 5.2 - 5.3; Lehman et al. Chapters 5, 6.1 - 6.3

These are the problems that you need to turn in. For more practice, you are encouraged to work on other problems. **Always explain your answers and show your reasoning.**

1. [**10 Pts**] Prove that $f_1^2 + f_2^2 + \cdots + f_n^2 = f_n f_{n+1}$, for all positive integers n, where f_i are the Fibonacci numbers.

2. [**12 Pts**] Consider the following state machine with five states, labeled 0, 1, 2, 3, 4 and 5. The start state is 0. The transitions are $0 \to 1$, $0 \to 2$, $1 \to 3$, $2 \to 3$, $3 \to 4$, $3 \to 5$, $4 \to 0$ and $5 \to 0$.

   Prove that if we take n steps in the state machine we will end up in state 0 if and only if n is divisible by 3. Argue that to prove the statement above by induction, we first have to *strengthen the induction hypothesis*. State the strengthened hypothesis and prove it.

3. [**8 Pts**] Suppose $P(1)$ and $P(2)$ are true. Determine for what values of n, $P(n)$ is true if

   (a) for every positive integer k, if $P(k)$ is true then $P(k + 3)$ is true.

   (b) for every positive integer k, if $P(k)$ is true then $P(k + 2)$ is true.

4. [**12 Pts**] A robot wanders around a 2-dimensional grid. He starts out at (0,0) and can take the following steps: (-1,+3), (+2,-2) and (+4,0). Define a state machine for this problem. Then, define a Preserved Invariant and prove that the robot will never get to (2,0).

5. [**8 Pts**] Suppose $P(n)$ is true for every positive integer n that is a power of 2. Also, suppose that $P(k + 1) \to P(k)$ for all positive integers k. Now, prove that $P(n)$ is true for all positive integers.

For more practice, you are encouraged to work on other problems, like the ones in the textbook.

1. Rosen, Section 5.2: Exercise 16

2. Rosen, Section 5.3: Exercise 16

3. LLM Problem 6.3

4. LLM Problem 6.4