1.

a. Base case: $i = 1$
   $4^i = 4$ and $4 \in S$
   Inductive step:
   Assume $4^k \in S\ for\ k\ \in Z^+$
   Prove $4^{k+1} \in S$
   $$4^{k+1} = 4\ x\ 4^k$$
   $4\ and\ 4^k$ both are $\in S$, therefore, $4^{k+1} \in S$ and $A \subseteq S$

b. Base case: $4 \in S$ is proven by inductive definition(1).
   $4 = 4^1\ and\ 1\ is\ \in Z^+$ Therefore, $4 \in A$ is true too.
   Inductive Step:
   Assume x $\in S$ and y $\in S$, and x and y are $\in A$
   Prove xy $\in A$
   $$x = 4^a, y = 4^b$$
   $$xy = (4^a)(4^b)$$
   $$= 4^{a+b}$$
   So xy $\in A$. Therefore $S \subseteq A$

2. (I1) if x ∈ BP then {x} ∈ BP , and (I2) if x ∈ BP and y ∈ BP then xy ∈ BP .
   X = {x}
   If x ∈ BP, which means {x} ∈ BP, By (I1).
   {x} has same amount of { and } which is one for each.
   Therefore, we can say that if x ∈ BP, then x has the same number of { and }.

3. $|l(T)| = |i(T)| + 1$
   Base case: T has only a single vertex. So, there's only 1 root and no vertex.
   Therefore, it satisfies $|l(T)| = |i(T)| + 1$
   Recursive step:
   Let T_1 and T_2 be left root and right root.
   So $|l(T_1)| = |i(T_1)| + 1$ and $|l(T_2)| = |i(T_2)| + 1$
   Vertex $|i(T)| = |i(T_1)| + |i(T_2)| + 1$
   Leaves $|l(T)| = |l(T_1)| + |l(T_2)|$
   $|l(T)| = |i(T_1)| + 1 + |i(T_2)| + 1$
   $|l(T)| = (|i(T_1)| + |i(T_2)| + 1) + 1$
   $|l(T)| = |i(T)| + 1$

4.

a. Base case: (0, 0) $\in L'$
Inductive definition: if (a, b) $\in L'$, then $(a+1, b+1) \in L'$, $(a-1, b-1) \in L'$, $(a, b+4) \in L'$ and $(a, b-4) \in L'$

b. Base case: (a, b) $= (0, 0)$
$a - b = 0 - 0 = 0$ which $\in L$
if (a, b) $\in L'$, then $(a+1, b+1) \in L'$, $(a-1, b-1) \in L'$, $(a, b+4) \in L'$ and $(a, b-4) \in L'$
Since (a, b) $\in L$ and a – b is multiple of 4. So,
$(a+1) - (b+1) = a + 1 - b - 1$
$= a - b = multiple\ of\ 4$
$(a-1) - (b-1) = a - 1 - b + 1$
$= a - b = multiple\ of\ 4$
$(a) - (b+4) = a - b - 4$
$= a - b - 4 = multiple\ of\ 4$
$(a) - (b-4) = a - b + 4$
$= a - b + 4 = multiple\ of\ 4$
All sets we added to L' will always be in L, therefore, $L' \subseteq L$

c. Let (s, t) be an arbitrary element of L, which means (s, t) $\in L$ where x – y is multiple of 4. t can be written as s – 4k for k any positive integer. $(s, s-4k)$

Case 1: s and k are positive. If move $(a+1, b+1)$ for s times and $(a, b-4)$ for k times, then we will get $(s, s-4k)$.
Case 2: s and k are negative. . If move $(a-1, b-1)$ for -s times and $(a, b+4)$ for -k times, then we will get $(s, s-4k)$.
Case 3: s is positive, and k is negative. If move $(a+1, b+1)$ for s times and $(a, b+4)$ for -k times, then we will get $(s, s-4k)$.
Case 4: s is negative, and k is positive If move $(a-1, b-1)$ for -s times and $(a, b-4)$ for k times, then we will get $(s, s-4k)$.

Therefore, we can say that $L \subseteq L'$

d. No, I think my inductive definition is not unambiguous. There is other unique way to produce x using my rules.