Wei Xian, Lim 901579699

1. Let k = first integer, k + 1 = second integer, k + 2 = third integer
   $k + (k + 1) + (k + 2) = 3k + 3 = 3(k + 1)$
   Therefore $3(k + 1)$ is always divisible by 3

2. Let k = all integer
   Even = 2k
   Odd = 2k + 1

   $P = 2k + 1 \quad (1)$
   $p^2 = (2k + 1)(2k + 1) = 4k^2 + 4k + 1 \quad (2)$
   $p^3 = (4k^2 + 4k + 1)(2k + 1) = 8k^3 + 12k^2 + 6k + 1$
   $= 2(4k^3 + 6k^2 + 3k) + 1 \quad (3)$

   $P = 2k \quad (1)$
   $p^2 = 4k^2 \quad (2)$
   $p^3 = 8k^3 = 2(4k^3) \quad (3)$

   Therefore $p^3$ will be odd if and only if p is odd. And $p^3$ is even if and only if p is even.

3. Let x, y, z be rational and let a, b, c, d, e and f be integers.
   $X = \frac{a}{b}, \quad y = \frac{c}{d}, \quad z = \frac{e}{f}$

   For b, d , f $\neq 0$

   $x + yz = \frac{a}{b} + \frac{c}{d}\left(\frac{e}{f}\right) = \frac{a}{b} + \frac{ce}{df}$
   $= \frac{adf + bce}{bdf}$
   Since a, b, c, e, d, f is integer and $d \neq 0, f \neq 0, b \neq 0$
   So x + yz is rational.

4. Let xyz = rational and x, y is rational
   $z = \frac{xyz}{xy} \quad Let\ x = \frac{a}{b}, \quad y = \frac{c}{d}, \quad xyz = \frac{e}{f}$

   $z = xyz \times \frac{1}{xy}$
   $z = \frac{e}{f} \times \frac{bd}{ac}$

$z = \frac{ebd}{fac}$

Since a, b, c, d, e, f is integer, and

Therefore, if xyz is rational, then z is rational. So z is irrational when xyz is irrational.

You cannot use direct proof because there is no way of showing irrational number.

5. If 3n + 11 is even, then n is odd

   Let n be even.

   n = 2k

   3n + 11 = 3(2k) + 11

   = 6k + 11

   = 2(3k +5) + 1

   Therefore, if 3n + 11 is even, then n is odd.

6. Proof by contradiction: she has to schedule at most 6 on same day.
   So each day gets 6 lessons, and there are 7 days in a week. So 6 x 7 = 42
   So the statement is proven because she only have 40 lessons over the week.

7. Proof by contradiction: Assuming square root of 7 is rational.
   $\sqrt{7} = \frac{a}{b}$
   For a, b is integer, and in reduced form with no common factors. B ≠ 0
   $7 = \frac{a^2}{b^2}$
   $7b^2 = a^2$
   Since left side is divisible by 7, so right side will also be divisible by 7. So, we can let a = 7k for k ≠ 0.
   $7b^2 = (7k)^2$
   $7b^2 = 49k^2$
   $b^2 = 7k^2$
   $B^2$ is multiple of 7, which means b is also multiple of 7
   But a and b should not have common factor. Therefore, it contradicts with the assumption and square root of 7 is irrational

8. X is irrational and y is non-zero rational but $x^y$ is rational
   Let x = $\sqrt{2}$ because we know that $\sqrt{2}$ is irrational and y = 2

$$x^y = \sqrt{2}^2$$

$$x^y = (2^{\frac{1}{2}})^2$$

$$x^y = 2$$

   Therefore, it is proven that when x is irrational and y is non-zero, $x^y$ is rational.