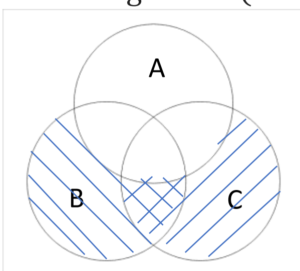
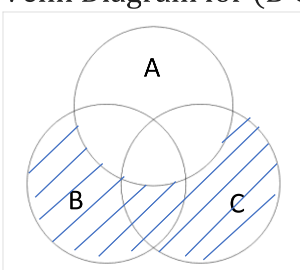
1. .A ⊆ B ←→ { x ∈ A → x ∈ B}
   So, C ⊆ D ←→ { x ∈ C → x ∈ D}
   We want to show A x C ⊆ B x D.
   So, (y, z) ∈ A x C → (y, z) ∈ B x D
   (y, z) ∈ A x C implies that {y ∈ A} Λ {y ∈ C}
   Which means {y ∈ B} Λ {y ∈ D} because A ⊆ B and C ⊆ D
   Therefore, it's proved that A x C ⊆ B x D

2. Venn Diagram of (B – A) ∪ (C – A)
   

   Venn Diagram for (B ∪ C) - A
   

3. (B – A) ∪ (C – A) = (B ∪ C) – A
   <u>Iff argument</u>
   Iff (B – A) ∪ (C – A)
   Iff x ∈ (B – A) V x ∈ (C – A) Definition of Union
   Iff (x ∈ B ∩ x ∈ $\bar{A}$) V (x ∈ C ∩ x ∈ $\bar{A}$) Definition of intersection
   Iff (x ∈ B V x ∈ C) ∩ x ∈ $\bar{A}$
   Iff (x ∈ (B V C)) ∩ x ∈ $\bar{A}$
   Iff (B ∪ C) – A

   <u>Logical equivalences</u>
   (B ∪ C) – A
   $(B \cup C) \cap \bar{A}$
   $(B \cap \bar{A}) \cap (C \cap \bar{A})$
   (B – A) ∪ (C – A)

4. (a) if A – C = B – C then A = B

    A = {1, 2, 3}

    B = {2, 3}

    C = {1}

    A – C = {2, 3}

    B – C = {2, 3}

   Therefore, the statement is wrong because A – C and B – C both share the same elements but A ≠ B.

   (b) if A ∩ C = B ∩ C then A = B

    A = {1, 2, 3}

    B = {2, 3, 4}

    C = {2, 3}

    A ∩ C = {2, 3}

    B ∩ C = {2, 3}

   Therefore, the statement is wrong because A ∩ C and B ∩ C both share the same elements but A ≠ B.

5. Prove that if A − C = B − C and A ∩ C = B ∩ C then A = B.
   To proof that A - C = B - C and A ∩ C = B ∩ C then A = B. Let A and B be sets. Let's try to prove that if $x \in B$, then $x \in A$. We can assume that $x \in B$, and to conclude that $x \in A$. There are two cases now which are either $x \in C \, or \, x \notin C$.

   Case 1

   If $x \in$ C, we assume that $x \in B$, $x \in B \cap C$. Since A ∩ C = B ∩ C we can conclude that $x \in A \cap C$. Hence $x \in A \, and \, x \in C$. Therefore, $x \in A$.

   Case 2

   If $x \notin C$, we assume that $x \in B$, $x \in B - C$. Since A − C = B − C we can conclude that $x \in A - C$. Hence $x \in A \, and \, x \notin C$. Therefore, $x \in A$.

   Both of these cases show that x ∈ B → x ∈ C. Hence, $B \subseteq A$

   Let's assume the other way, if $x \in A$, then $x \in B$. With the two cases now, which are either $x \in C \, or \, x \notin C$.

   Case 1

   If $x \in$ C, we assume that $x \in A$, $x \in A \cap C$. Since A ∩ C = B ∩ C we can conclude that $x \in B \cap C$. Hence $x \in B \, and \, x \in C$. Therefore, $x \in B$.

   Case 2

If $x \notin C$, we assume that $x \in A, x \in A - C$. Since A − C = B − C we can conclude that $x \in B - C$. Hence $x \in B$ and $x \notin C$. Therefore, $x \in B$.

Both of these cases show that x ∈ A → x ∈ B. Hence, $A \subseteq B$

We have shown that $A = B$, by the definition of equivalency which is If x ∈ B → x ∈ C. (This proves B ⊆ A). If x ∈ A → x ∈ B . (This proves A ⊆ B).

6. $\overline{A \cup \mathrm{B}} = \bar{A} \cap \bar{B}$

$A \cup (\mathrm{B} \cap C) = (A \cup \mathrm{B}) \cap (A \cup \mathrm{C})$

$Assume\ \exists x(\mathrm{B} \cap \mathrm{C})$

$= x \in A\ V\ (x \in B\ \Lambda\ x \in C)$

$= (x \in A\ V\ x \in B)\ \Lambda(x \in A\ V\ x \in C)$

$=\ x \in (A\ V\ B)\ \Lambda\ x \in (A\ V\ C)$

$= x \in (A\ \cup\ B)\ \cap\ x \in (A\ \cup\ C)$

$=\ x \in\ A\ \cup (\mathrm{B}\ \cap\ C)$

$= A \cup (B \cap C) \subseteq (\mathrm{A} \cup \mathrm{B}) \cap (\mathrm{A} \cup \mathrm{C})$ (1)

$Assume\ \exists x\big((\mathrm{A} \cup \mathrm{B}) \cap (\mathrm{A} \cup \mathrm{C})\big)$

$= x \in (\mathrm{A} \cup \mathrm{B}) \cap (\mathrm{A} \cup \mathrm{C})$

$= x \in (\mathrm{A}\ V\ \mathrm{B})\ \Lambda\ x \in (\mathrm{A}\ V\ \mathrm{C})$

$= \big((x \in A)\ V\ (x \in B)\big)\Lambda\ \big((x \in A) V\ (x \in C)\big)$

$= (\ x \in A)\ V\ (x \in B\ \Lambda\ C)$

$= (\ x \in A)\ \cup\ (x \in B\ \cap\ C)$

$= x \in \big(A\ \cup (B\ \cap\ C)\big)$

$= x \in (A \cup B) \cap (A \cup C)$

$= (A \cup B) \cap (A \cup C)\ \subseteq\ \big(A\ \cup (B\ \cap\ C)\big)$ (2)

$=\ A \cup (B \cap C) \subseteq (\mathrm{A} \cup \mathrm{B}) \cap (\mathrm{A} \cup \mathrm{C})$ (1) (2)

$\overline{A \cup \mathrm{B}} = \bar{A} \cap \bar{B}$

Q.E.D

7. f: $Z^+ \rightarrow Z^+$

f(n) = 5n + 12

f(y) = 5y + 12

f(n) = f(y)
5n + 12 = 5y + 12
5n = 5y
n = y
Therefore, it is one-to-one

f(n) = 5n + 12
Let 5n + 12 = 0
5n = -12
n = $-\frac{12}{5}$
Therefore, f is not onto because domain and co-domain supposed to be positive integers and n is not positive nor integer in this case.

8. f(m, n) = 3mn for f: R x R → R
$a = 3mn$
$m = \dfrac{a}{3n}$
$a = 3\left(\dfrac{a}{3n}\right)n$
$a = a$
Therefore, it is onto because inverse of it gives 1 value only

f(m, n) = 3mn
f(-2, -2) = 3(-2)(-2) = 12
f(2, 2) = 3(2)(2) = 12

f(-2, -2) and f(2, 2) both have different values of m and n but both the equations have the same result. There, it is not one-to-one.