

1. 9 colors, 5 sizes, striped or solid, long or short sleeve
 - (a) $9 * 5 * 2 * 2$
 - (b) (Red, gold) + the rest of the color
 $(2 * 5 * 1 * 1) + (7 * 5 * 2 * 2)$

2.
 - (a) $26 * 26 * 26 * 26$ because the letter can be reused after each letter and there's 26 different letters.
 - (b) $4 * 25 * 24 * 23$, 4 because K can be in any positions, and $25 * 24 * 23$ because the letter used before cannot be used again.
 - (c) $4 * 26 * 26 * 26$, 4 because K can be in any position, and $26 * 26 * 26$ because K can be reused.

3. $D_5 = \text{divisible by 5}$ and $D_7 = \text{divisible by 7}$, $S = \{1, \dots, 99999\}$, $T = \{1, \dots, 9999\}$
 $T - S = \{1000, \dots, 99999\}$
 $|D_5 \cup D_7| = |D_5| + |D_7| - |D_5 \cap D_7|$
 $= (19999 - 1999) + (14285 - 1428) - (2857 - 285)$

4.
 - (a) $8!$
 Because we can consider Ann, Beth, and Chris as one because the order must be Ann-Beth-Chris order. So $(7 + 1)!$
 - (b) $8! * 3!$
 $3!$ Because Ann, Beth and Chris can be in any order among 3 of them. And $8!$ Because we can take Ann, Beth and Chris as a whole because they have to stand next to each other. So $(7 + 1)!$

5.
 - (a) 9^{10} for function A to B and 10^9 for function B to A
 - (b) 2^{9*10}
 because each element in set A can link to any of the element in set B and theirs is 9 elements in set A and 10 elements in set B.
 - (c) $\frac{10!}{(10-9)!}$ functions
 because if all the element from A that is linked must be excluded
 - (d) $\frac{C(10,2)*C(9,2)*9!}{2!} + C(10,3) * 9!$ functions
 because onto functions must have all element in B linked to A.

6.
 - (a) $C(8,5) * 7!$,
 $C(8,5)$ because only choose 5 out of 8 people as mom and dad is chosen already.
 And $7!$ to shuffle all 7 people in all position.
 - (b) (*mum in*) + (*dad in*)

$$C(8, 6) * 7! + C(8, 6) * 7!$$

If mom or dad is in already, then we need to choose 6 out of 8 people because if mom is in, dad is not and vice versa.

7.

(a) $K = 10, r = 5$

$$N \geq (5 - 1)10 + 1$$

At least 41 tickets to get 5 same movie ticket.

(b) 5 to 50 tickets because assuming the you took the first 45 tickets and all of it is not Avengers ticket or you are lucky and first 5 tickets are Avengers ticket.

(c) Problem A requires Pigeonhole Principle

8. F

9.

(a) $C(7, 3) = 35$

(b) $C(7, 0) + C(7, 1) + C(7, 2) + C(7, 3) = 1 + 7 + 21 + 35 = 64$

(c) $2^7 - C(7, 0) - C(7, 1) - C(7, 2) = 128 - 1 - 7 - 21 = 99$

10.

(a) $2 * C(20, 7)$

(b) *Total – only man*

$$C(22, 8) - C(10, 8) =$$

(c) *Total – only man – only woman*

$$C(22, 8) - C(10, 8) - C(12, 8) =$$