1. Fibonacci numbers = 0, 1, 1, 2, 3, …
   Base: $F_1^2 = 0$
   Inductive step:
   Assume $f_1^2+f_2^2+...+f_n^2 = f_n f_{n+1}$
   Prove $f_1^2+f_2^2+...+f_n^2 + f_{n+1}^2 = f_{n+1} f_{n+2}$
   LHS = $f_1^2+f_2^2+...+f_n^2 + f_{n+1}^2$ By IH
   $= f_n f_{n+1} + f_{n+1}^2$
   $= f_{n+1} (f_n + f_{n+1})$
   $= f_{n+1} f_{n+2}$ by Fibonacci definition
   = RHS

2. Base n = 0
   State Machine is in state 0 after 0 steps.

   Induction step:
   Assume that k is divisible by 4 if and only if state machine is in state 0 after k steps.
   Prove that (k+1) is divisible by 4 if and only if state machine is in state 0 k steps.

   K mod 4 ≡ 0
   State machine is in state 0 after k steps, by IH. So, it will be in state 1 or 2 after k+1 step. Since k is divisible by 4, k+1 would have a remainder when divided by 4, so (k+1) assertion holds.

3. P(1) and P(2) are true
   a. $p(k) \rightarrow p(k+3)$
      $p(1+3) \rightarrow p(4)$
      $p(2+3) \rightarrow p(5)$
      $p(4+3) \rightarrow p(7)$
      $p(5+3) \rightarrow p(8)$

      Therefore, p(k) is true when k is 1, 2, 4, 5, 7 and any other number that is not multiple of 3.

   b. $p(k) \rightarrow p(k+2)$
      $p(1+2) \rightarrow p(3)$
      $p(2+2) \rightarrow p(4)$
      $p(3+2) \rightarrow p(5)$
      $p(4+2) \rightarrow p(6)$
      $p(5+2) \rightarrow p(7)$
      $p(7+2) \rightarrow p(9)$

      Therefore, p(k) is true when k is 1, 2, 3, 4, and any other positive integers.

4. Start (0,0)
   Steps (-1,+3), (+2,-2) and (+4,0)
   So, robot can only be at position (x, y) which $|x| + |y|$ will always divisible by 4

   Base case:
   P(0) = 0 which is divisible by 4

   Induction step:
   Case 1: (-1, +3)
   $|-1| + |3| = 4$ and it is divisible by 4
   Case 2: (+2, +2)
   $|2| + |2| = 4$ and it is divisible by 4
   Case 3: (+4, 0)
   $|4| + |0| = 4$ and it is divisible by 4

   (2,0)
   $|2| + |0| = 2$ and it is not divisible by 4

   Therefore, robot will never get to (2, 0) position.

5. Assume $S = \{1,2,3,4, \dots, k, k+1\}$
   Let k be an element of S.
   P(k) is true if k is a power of 2 and false otherwise.
   Let $k + 1$ be an element of S too.
   We know that k+1 will always be greater than k and p(k+1) is true because all integers of S are true.