CS 230 : Discrete Computational Structures

Spring Semester, 2019

ASSIGNMENT #8

Due Date: Wednesday, April 3

Suggested Reading: Rosen Sections 5.3; Lehman et al. Chapters 5, 6.1 - 6.2, 7

These are the problems that you need to hand in for grading. Always explain your answers and show your reasoning.

1. **[12 Pts]** Let $S \subseteq \mathcal{Z}^+$ be defined recursively by (1) $4 \in S$ and (2) if $x \in S$ and $y \in S$, then $x \cdot y \in S$. Let $A = \{4^i \mid i \in \mathcal{Z}^+\}$. Prove that
   (a) **[6 Pts]** $A \subseteq S$ by mathematical induction.
   (b) **[6 Pts]** $S \subseteq A$ by structural induction.

2. **[8 Pts]** Let BP, the set of balanced parentheses, be defined recursively by (B) $\epsilon \in BP$, (I1) if $x \in BP$ then $\{x\} \in BP$, and (I2) if $x \in BP$ and $y \in BP$ then $xy \in BP$. For example, $\{\{\}\}\{\} \in BP$. Prove that if $x \in BP$ then x has the same number of $\{$ and $\}$.

3. **[10 Pts]** Given the inductive definition of full binary trees (FBTs), define $I(T)$, the set of internal vertices in tree T, and $L(T)$, the set of leaves in tree T, inductively. Then, use structural induction to prove that for all FBTs T, $|L(T)| = |I(T)| + 1$.

4. **[20 Pts]**
   (a) **[8 Pts]** Let $L = \{(a, b) \mid a, b \in \mathcal{Z}, (a-b) \bmod 4 = 0\}$. Give an inductive definition of L. Let L' be the set obtained by your inductive definition.
   (b) **[6 Pts]** Prove that $L' \subseteq L$.
   (c) **[6 Pts]** Prove that $L \subseteq L'$.
   (d) **Extra Credit [5 Pts]** Is your inductive definition unambiguous? In other words, for any $x \in L$, is there a unique way you can produce x using your rules?

For more practice, you are encouraged to work on other problems, like the ones below.

1. Rosen, Section 5.3: Exercise 8

2. Rosen, Section 5.3: Exercise 11-15

3. Let S be defined by (1) $(0, 0) \in S$, and (2) if $(a, b) \in S$, then $(a, b+5) \in S$, $(a+1, b+4) \in S$ and $(a+2, b+3) \in S$.
   (a) Use structural induction to prove that if $(a, b) \in S$ then 5 divides $a + b$.
   (b) Disprove the converse of the statement above, *i.e.*, show that if $a, b \in \mathcal{N}$, and $a + b$ is divisible by 5, it does not follow that $(a, b) \in S$. Modify the recursive definition of S to make the converse true.

4. Rosen, Section 5.3: Exercise 27-42