CS 230 : Discrete Computational Structures
Spring Semester, 2019
STRONG INDUCTION EXAMPLE

Let $P(n)$ be the statement that n-cent postage can be formed using just 4-cent and 5-cent stamps. Prove that $P(n)$ is true for all $n \geq 12$, using the steps below.

1. First, prove $P(n)$ by regular induction. State your basis step and inductive step clearly and prove them.

   Basis step: $P(12)$*: 12 cents can be made of three 4-cent stamps as* $3(4) = 12$.

   Inductive step: Assume $P(k)$ *for* $k \geq 12$. *So,* k *cents can be made of only 4-cent and 5-cent stamps. To prove* $P(k) \rightarrow P(k+1)$, *we will break it into cases.*

   Case 1: k *contains a 4-cent stamp. Then we can remove the 4-cent stamp and replace it with a 5-cent stamps as* $k - 4 + 5 = k + 1$.

   Case 2: k *contains no 4-cent stamps, so* k *only contains 5-cent stamps, implying that* k *is a multiple of 5. Since* $k \geq 12$, *it follows that* $k \geq 15$ *since 15 is the smallest multiple of 5 that is greater or equal to 12. Therefore, at least three 5-cent stamps are used. Then three 5-cent stamps can be removed and replaced with four 4-cent stamps as* $k - 3(5) + 4(4) = k + 1$.

   Thus, $P(k) \rightarrow P(k+1)$.

2. Now, prove $P(n)$ by strong induction. Again, state and prove your basis step and inductive step. Your basis step should have multiple cases.

   Basis steps:
   $P(12)$*: 12 cents can be made of three 4-cent stamps as* $3(4) = 12$.
   $P(13)$*: 13 cents can be made from two 4-cent stamps and one 5-cent stamp as* $2(4) + 5 = 13$.
   $P(14)$*: 14 cents can be made from one 4-cent stamp and two 5-cent stamps as* $4 + 2(5) = 14$.
   $P(15)$*: 15 cents can be made from three 5-cent stamps as* $3(5) = 15$.

   Strong Inductive Step: For $k \geq 15$, *assume that* $P(j)$ *is true for all* j *where* $12 \leq j \leq k$. *We prove* $P(k+1)$. *Since* $k \geq 15$, *we have* $k - 3 \geq 12$, *so* $P(k-3)$ *is true. By adding another 4-cent stamp to* $k - 3$ *cent postage, we have postage for* $k - 3 + 4 = k + 1$. *So,* $P(k+1)$ *is true.*