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1. (a) Proof by contrapositive: $F \circ G$ and G one-to-one don't ensure F is.
   As a counter-example, let $A = \{1\}$, $B = \{1, 2\}$, $C = \{1\}$, and $g : A \rightarrow B$ where $g(1) = 1$ and $f : B \rightarrow C$ where $f(1) = f(2) = 1$. Then $F \circ G : A \rightarrow C$ is defined by $(F \circ G)(1) = 1$. This map is a bijection from A = f1g to C = f1g, so is one-to-one. However, F is not one-to-one, since $F(1) = F(2) = 1$.

   (b) Suppose $x, y \in A$ and $g(x) = g(y)$. Therefore $F \circ G (x) = F \circ G (y) \rightarrow x = y$ because $F \circ G$ is one-to-one

2. (a) it is anti-reflexive. We cannot have $(1, -1)$ because $(1, -1) \ngeq 0$
   it is symmetric. $xy = yx \in R_1$
   it is transitive. We have $(x, y) \in R_1$ and $(y, z) \in R_1$
   if $xy \geq 0$, both x and $y \geq 0$. So all x, y, z are ≥ 0.
   Therefore, it follows that $(x, z) \in R_1$

   (b) It is anti-reflexive. We cannot have $(2, 2)$ because it doesn't satisfy the statement $x = 2y$.
   It is anti-symmetric. we can have $(4, 2)$ but we cannot have $(2, 4)$
   It is non-transitive. If we have $(x, y) = (4, 2)$ and $(y, z) = (2, 1)$, $(x, z) = (4, 1)$ but it doesn't satisfy the statement $x = 2y$ as $(4) \neq 2(1)$

3. (a) It is reflexive. Because $0 = 0 \rightarrow a - a = b - b \rightarrow ((a, b),(a, b)) \in R$
   It is symmetric. $((a, b),(c, d)) \rightarrow a - c = b - d \rightarrow c - a = d - b \rightarrow ((c, d),(a, b))$
   It is transitive. Have $((a, b),(c, d)) \rightarrow a - c = b - d$, $((c, d),(e, f)) \rightarrow c - e = d - f$.
   Then, $(a - c) + (c - e) = (b - d)+(d - f) \rightarrow a - e = b - f \rightarrow ((a, b),(e, f))$

   (b) $f(x, y) = x - y$. Then we have: $f(a, b) = f(c, d) \rightarrow a - b = c - d$
   $\rightarrow a - c = b - d$
   $\rightarrow ((a, b),(c, d))$

   (c) $((a, b), (1, 1)) \rightarrow a - 1 = b - 1 \rightarrow a = b \rightarrow$ The class is $\{(a, a)\}$
   $\rightarrow$ 2 elements: $(2, 2)$ and $(3, 3)$

   (d) There are infinite classes with infinite number of elements. Each class is the set of tuples $\{(a, b)\}$ where the difference between a and b is constant.

4. (a) It is reflexive because $(a, a) \in R_4$.
   It is symmetric because $(a, b) \in R_4 = (b, a) \in R_4$
   It is transitive because if a and b is in the same building, b and c is in the same building, then a and c is in the same building.
   Therefore, it is an equivalent relation.

   $R_4 = \{(a, b) \mid \text{lives in the same building}\}$

   If R is the refinement of S. Subset or refinement of same building or same unit or same floor.

   (b) It is reflexive because $(a, a) \in R_5$.
   It is symmetric because $(a, b) \in R_5 = (b, a) \in R_5$
   It is transitive because $(a = b) = (b = c)$. a and b graduated from the same high school, b and c graduated from the same high school, so a and c graduated from the same high school.
   Therefore, it is an equivalent relation.

   $R_5 = \{(a, b) \mid \text{graduated from the same high school}\}$

   If R is the refinement of S. Subset or refinement of same high school, same college, or same year.

5. (a) it is reflexive → $x/x = 1 \in Z$
   It is anti-symmetric → x/y does not always mean y/x unless $x = y$
   It is transitive → if we have $(x, y) = (1, 2)$ and $(y, z) = (2, 3)$ then we have $(x, z) = (1, 3)$. $x/z = 1/3$ which is $\in Z$.

   $(x, y) \in R_6$ if and only if $x/z \in Z$ is partial ordered set.

   (b)It is reflexive → $x - x = 0 \in Z$
   It is symmetric → $x - y \in Z$ → $-(x-y) \in Z$ → $y - x \in Z$
   It is transitive → have $x - y \in Z$ & $y - z \in Z$, $x - z$

   $$= x - y + y - z$$
   $$=(x - y) +(y - z) \in Z$$

   $(x, y) \in R_7$ if and only if $x - y \in Z$ is equivalent relation
   $[2]_{R7} = Z^+$ and $[\pi]_{R7} = \{ \pi - 3 + n \mid n \in N\}$