1. F

2.

    a. F
    b. $C(n,2)C(n-2,k-2) = C(n,k)C(k,2)$

    $$LHS = \frac{n!}{2!\,(n-2)!} * \frac{(n-2)!}{(k-2)!\,(n-2-k+2)!}$$
    $$= \frac{n!}{2!\,(k-2)!\,(n-k)!}$$
    $$= \frac{n!}{k!\,(n-k)!} * \frac{k!}{2!\,(k-2)!}$$
    $$= C(n,k)C(k,2) = RHS$$

3.

    a. Must take at least two of each means, 10 cookies are fixed, because $5 * 2$ And we left with 6 cookies to choose from.

    $$\binom{6+5-1}{6} = \frac{10!}{4!\,6!}$$

    b. Sum of all cases of chocolate chip cookies.

    (1) 0 chocolate $16 - 4\ oatmeal = 12$
    $$\binom{12+4-1}{12} = \frac{15!}{3!\,12!}$$
    (2) 1 chocolate $16 - 4\ oatmeal - 1\ chocolate = 11$
    $$\binom{11+4-1}{11} = \frac{14!}{3!\,11!}$$
    (3) 2 chocolate $16 - 4\ oatmeal - 2\ chocolate = 10$
    $$\binom{10+4-1}{10} = \frac{13!}{3!\,10!}$$
    (4) 3 chocolate $16 - 4\ oatmeal - 3\ chocolate = 9$
    $$\binom{11+4-1}{11} = \frac{12!}{3!\,9!}$$
    (5) 4 chocolate $16 - 4\ oatmeal - 4\ chocolate = 8$
    $$\binom{11+4-1}{11} = \frac{11!}{3!\,8!}$$
    $$Ans = \frac{15!}{3!\,12!} + \frac{14!}{3!\,11!} + \frac{13!}{3!\,10!} + \frac{12!}{3!\,9!} + \frac{11!}{3!\,8!}$$

4.

a. $\binom{20+4-1}{20} = \frac{23!}{3!20!}$

b. $x1 + x2 + x3 + x4 = 20$

X1 is reduced by 1 because possibility to get 1 is removed.
X2 is reduced by 2 because possibility to get 1 and 2 are removed
X3 is reduced by 3 because possibility to get 1, 2, 3 are removed
X4 is reduced by 4 because possibility to get 1, 2, 3, 4 are removed

$20 - 1 - 2 - 3 - 4 = 10$

$\binom{10+4-1}{10} = \frac{13!}{3!\,10!}$

c. $20 - 4 = 16$ Because we let x3 $>$ 4. Then we reduce the possibility of x3 $>$ 4 to get x3 $<$ 5

$$\frac{23!}{3!\,20!} - \binom{16+4-1}{16} = \frac{23!}{3!\,20!} - \frac{19!}{3!\,6!}$$

Then we add it by x1 $>$ 4

$$\frac{23!}{3!\,20!} - \frac{19!}{3!\,6!} + \binom{16+4-1}{16} = \frac{23!}{3!\,20!}$$

5.

a. $\frac{30!}{5!5!5!3!3!3!3!3!}$ Because all of them are distinct.

b. $\frac{30!}{5!5!5!3!3!3!3!3!*5!*3!}$ It needs to divide by 3! and 5! Because the 3 groups of 5 and 5 groups of 3 is not distinct.

c. $\frac{30!}{5!5!5!3!3!3!3!3!*3!*2!*3!}$ It needs to divide by 3!, 2!, 3! Because first 3! is there are 3 groups of 5 doing the same task, 2! Because there are 2 groups of 3 doing the same task, and last 3! Because the 3 groups of 3 is not doing any task.

6. 6-0-0-0-0-0 – 1 way
   5-1-0-0-0-0 – $\binom{6}{5} = 6$
   4-2-0-0-0-0 – $\binom{6}{4}\binom{4}{2} = 15 * 6$
   4-1-1-0-0-0 – $\binom{6}{4} = 15$
   3-3-0-0-0-0 – $\binom{6}{3} * \frac{1}{2} = \frac{20}{2}$
   3-2-1-0-0-0 – $\binom{6}{3}\binom{3}{2} = 20 * 3$
   3-1-1-1-0-0 – $\binom{6}{3}\binom{3}{1}\binom{2}{1} * \frac{1}{3!} = 20 * 3 * 2 * \frac{1}{3*2}$
   2-2-2-0-0-0 – $\binom{6}{2}\binom{4}{2} * \frac{1}{3!} = 15 * 6 * \frac{1}{3*2}$
   2-2-1-1-0-0 – $\binom{6}{2}\binom{4}{2}\binom{2}{1} * \frac{1}{2} * \frac{1}{2} = 15 * 6 * 2 * \frac{1}{2} * \frac{1}{2}$
   2-1-1-1-1-0 – $\binom{6}{2}\binom{4}{1}\binom{3}{1}\binom{2}{1} * \frac{1}{4!} = 15 * 4 * 3 * 2 * \frac{1}{4!}$
   1-1-1-1-1-1 – 1 way
   If books are different, there's $1 + 6 + 90 + 15 + 10 + 60 + 20 + 15 + 45 + 15 + 1$ ways $= 278$ ways

   If books are identical, then there's only 11 ways to put because there's only 11 combinations

7. 
   a. $\binom{12+5-1}{12} = \frac{16!}{4!12!}$
   b. $\binom{12+5-1}{12}12! = \frac{16!}{4!}$ need to times by 12! Because the 12 books can be put in every different position.

8. Adding all degrees $= 5 + 4 + 3 + 3 + 2 + 2 + 1 = 20$
   Edge $= \frac{20}{2} = 10$