CS 230 : Discrete Computational Structures
Spring Semester, 2019
ASSIGNMENT #6
Due Date: Wednesday, March 13

Suggested Reading: Rosen Section 5.1 - 5.2; Lehman et al. Chapter 5.1 - 5.3

These are the problems that you need to turn in. For more practice, you are encouraged to work on the other problems. **Always explain your answers and show your reasoning.**

For Problems 1-5, prove the statements by mathematical induction. Clearly state your basis step and prove it. What is your inductive hypothesis? Prove the inductive step and show clearly where you used the inductive hypothesis.

1. **[6 Pts]** $1^3 + 2^3 + \cdots + n^3 = (n(n+1)/2))^2$, for all $n \in \mathcal{Z}^+$.
2. **[6 Pts]** $1 \cdot 2 + 2 \cdot 3 + \cdots + n(n+1) = n(n+1)(n+2)/3$, for all $n \in \mathcal{Z}^+$.
3. **[6 Pts]** $2 - 2 \cdot 7 + 2 \cdot 7^2 - \cdots + 2(-7)^n = (1 - (-7)^{n+1})/4$, for all $n \in \mathcal{N}$.
4. **[6 Pts]** $1/2n \leq [1 \cdot 3 \cdot 5 \cdots (2n-1)]/[2 \cdot 4 \cdot 6 \cdots 2n]$, for all $n \in \mathcal{Z}^+$.
5. **[6 Pts]** 6 divides $n^3 - n$, for all $n \in \mathcal{N}$.
6. **[12 Pts]** Let $P(n)$ be the statement that n-cent postage can be formed using just 4-cent and 7-cent stamps. Prove that $P(n)$ is true for all $n \geq 18$, using the steps below.
   (a) First, prove $P(n)$ by regular induction. State your basis step and inductive step clearly and prove them.
   (b) Now, prove $P(n)$ by strong induction. Again, state and prove your basis step and inductive step. Your basis step should have multiple cases.
7. **[8 Pts]** Use strong induction to prove that every positive integer n can be expressed as the sum of distinct powers of 2. *Hint: Separately consider the cases when k is odd or even.*

For more practice, you are encouraged to work on other problems, like the ones below.

1. Rosen Section 5.1 Problem 4
2. Rosen Section 5.1 Problem 12
3. Rosen Section 5.1 Problem 31
4. Rosen Section 5.2 Problem 26