

CS 230 : Discrete Computational Structures  
**Spring Semester, 2019**  
ASSIGNMENT #9  
**Due Date:** Thursday, April 18

**Suggested Reading:** Rosen Section 2.5; LLM Chapter 7.1

These are the problems that you need to turn in. For more practice, you are encouraged to work on the other problems. **Always explain your answers and show your reasoning.**

1. [12 Pts] Show that the following sets are countably infinite, by defining a bijection between  $\mathcal{N}$  (or  $\mathcal{Z}^+$ ) and that set. You do not need to prove that your function is bijective.
  - (a) [6 Pts] the set of integers divisible by 7
  - (b) [6 Pts]  $A \times \mathcal{Z}^+$  where  $A = \{2, 3\}$
2. [7 Pts] Argue that if  $A \subseteq B$ , and  $A$  is uncountable then  $B$  is uncountable.
3. [8 Pts] Prove that the set of functions from  $\mathcal{N}$  to  $\mathcal{N}$  is uncountable, by using a diagonalization argument.
4. [14 Pts] Determine whether the following sets are countable or uncountable. Prove your answer.
  - (a) the set of real numbers with decimal representation consisting of all 9's (9.99 and 99.999... are such numbers).
  - (b) the set of real numbers with decimal representation consisting of 8's and 9's
5. [9 Pts] Give an example of two uncountable sets  $A$  and  $B$  (along with a justification) such that  $A - B$  is
  - (a) finite (b) countably infinite (c) uncountably infinite

For more practice, you are encouraged to work on other problems like the ones below. You can find more problems in the textbook.

1. Argue that the set of all finite strings over the alphabet  $\Sigma$ , where  $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, /\}$ , is countable. Use this to argue that the set of positive rationals is countable.  
*Hint: Represent any positive rational as a finite string.*
2. Show that a countably infinite union of countably infinite sets is countably infinite.