Model Predictive Control

Leonard Mikša

January 31, 2025

Implementation on a linear pneumatic muscle

Problems and issues

- 3 state variables
- steady-state error - unable to compare with LQR and assess the validity
- the error dependent on $\mathbf{Q}$ matrix and p
- approx. linear error ratio with respect to p changes
- nonlinear error ratio with respect to Q changes (unable to even measure)

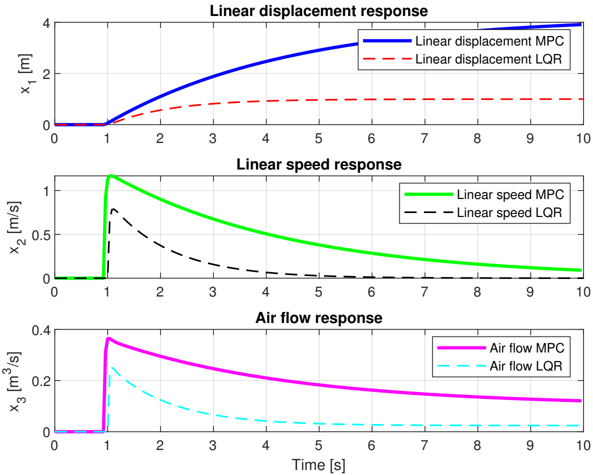

Figure: $Q_{LQR} = 1,\ Q_{MPC} = 1,\ p = 10,\ m = 3$

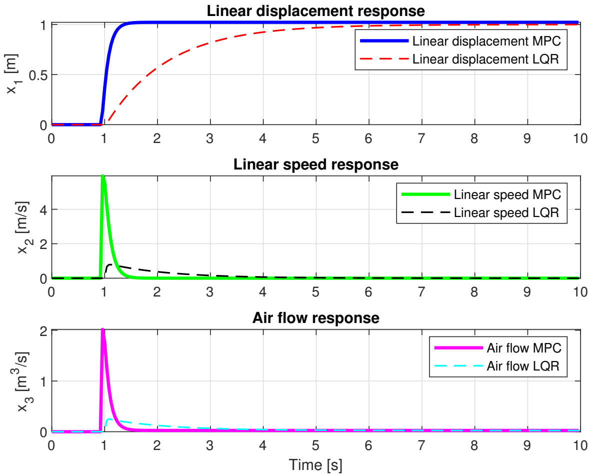

Figure: $Q_{LQR} = 1,\ Q_{MPC} = 100,\ p = 10,\ m = 3$

Implementation on a linear pneumatic muscle

Simulation results

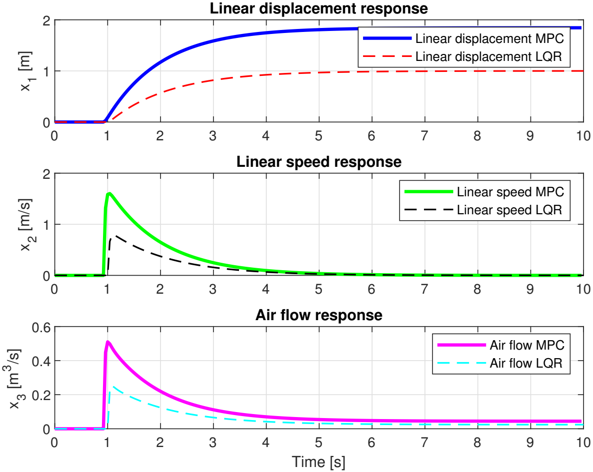

Figure: $Q_{LQR} = 1,\ Q_{MPC} = 1,\ p = 40,\ m = 3$

Implementation on a linear pneumatic muscle

Further analysis

- verify and adjust system parameters
- study interactions between changes in $\mathbf{Q}$, p, m, and T_s
- deep analysis of the code and MPC loop with regard to literature and papers

- continuous LQR $\longrightarrow$ discrete LQR
- elimination of the discretization error
- corrections were insignificant

```
Ksystem = dlqr(Ad, Bd, Q, R);        %dlqr instead of lqr
```

- state-space model correction
- missing coupling term between the airflow state (x_3) and the force (f)
- the force depends on the air pressure, affected by the airflow
- f **shouldn't be treated as an independent input**

$$\dot{x}_2(t) = \frac{1}{m} f(t) - \frac{b}{m} x_2(t) - \frac{k}{m} x_1(t) \tag{1}$$

$$\dot{x}_2(t) = \frac{\gamma A}{m} x_3(t) - \frac{b}{m} x_2(t) - \frac{k}{m} x_1(t) \tag{2}$$

- γ is a proportionality constant of the muscle, linking the airflow with the air pressure ($P = \gamma x_3$)
- A is effective cross-sectional area of the muscle

Model predictive control

Corrections

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{q}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{k}{m} & -\frac{b}{m} & 0 \\ 0 & 0 & -\frac{1}{\tau} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \\ \frac{K}{\tau} \end{bmatrix} \begin{bmatrix} 0 \\ f(t) \\ u(t) \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{q}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{k}{m} & -\frac{b}{m} & \frac{\gamma A}{m} \\ 0 & 0 & -\frac{1}{\tau} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{K}{\tau} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ u(t) \end{bmatrix}$$

- 1 input $\longrightarrow$ voltage applied to the valve
- insignificant corrections in responses

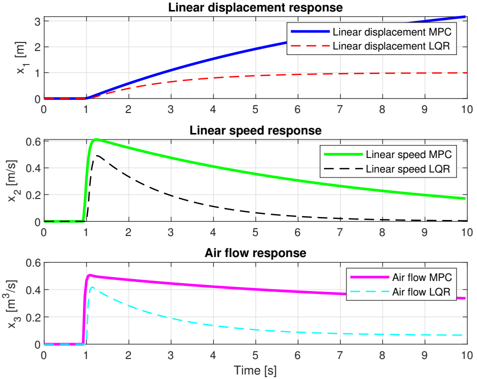

Figure: $Q_{LQR} = 1,\ Q_{MPC} = 1,\ p = 10,\ m = 3$

- **natural responses match at** $\approx p = 40$
- **forced responses mismatch**
- if we define the steady-state MPC error as

$$SS_{MPC} = e \cdot SS_{LQR} \tag{3}$$

and $SS_{LQR} = 0$, then SS_{MPC} must also be equal to 0

Model predictive control

Corrections

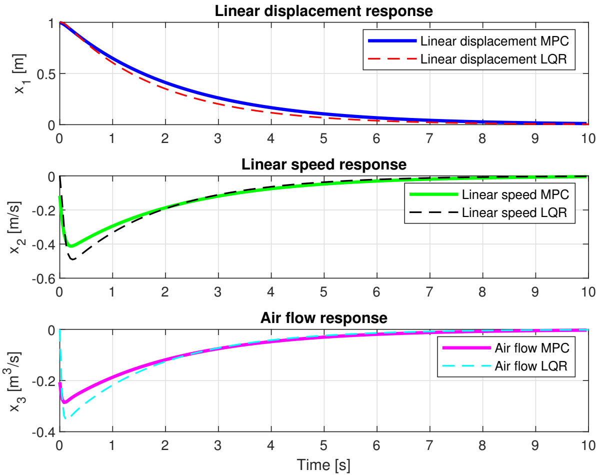

Figure: $Q_{LQR} = 1,\ Q_{MPC} = 1,\ p = 40,\ m = 3$

- valid constraints

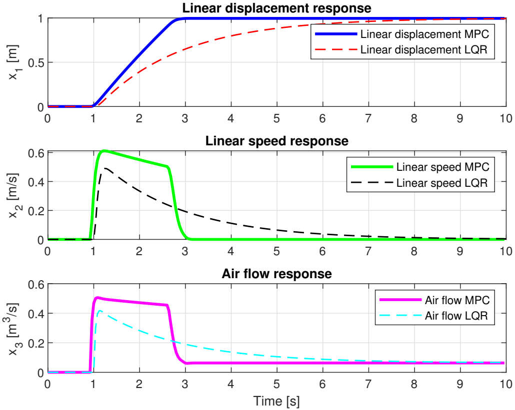

Figure: $Q_{LQR} = 1, \; Q_{MPC} = 1, \; p = 10, \; m = 3$

- MPC with $p \longrightarrow \infty$ approaches LQR

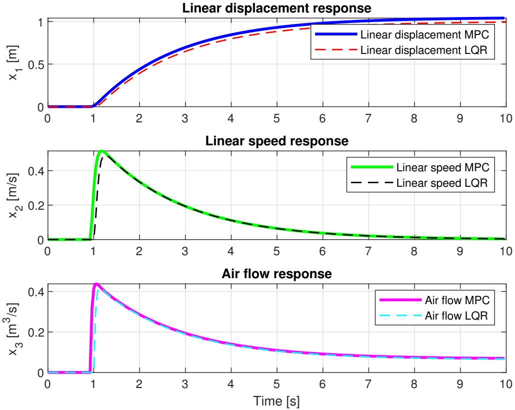

Figure: $Q_{LQR} = 1,\ Q_{MPC} = 1,\ p = 400,\ m = 400$

Implementation on a linear pneumatic muscle

Thank you! Questions?