ISE-2404 - DOR I
Project

This project is due on May 6th, 2020, the last class of the semester. This is a team project, and teams should consist of between 3 and 4 students (it is highly encouraged that each team should have at least one male student and one female student). The deliverables are:
1. A well written project report, which would be appropriate for the client, included should be a technical appendix with formulations and more details on the analysis;
2. An electronic poster, which can be used to introduce the problems to a client without any knowledge of linear programming.

5pts: formatting,
Long tables in the main text, not type the primal and dual, which are messy, -3

10pts: poster

Testing Kits Production for Coronavirus (COVID-19) in U.S.

Suppose in January, 2020, CDC planned to prepare for COVID-19. It was estimated that the infected population would be 1% of entire population of each state who might need to test for COVID-19. The 1% of population can be found in Table 1. Thus, CDC planned to produce all the testing kits and send them to the states. There are three manufacturing centers, located at different places, which are able to fulfill the orders of CDC. Please find the average travel distance from each state to each manufacturing center in Table 1. Since each manufacturing center has limited capability, their maximum production capacities can be found in Table 2. In addition, according to CDC, the production cost for each testing kit is $1, and the shipping cost of each kit is $0.001 per mile.

**Table 1. 1% Population and three manufacturing center of test kits and their average distances (mile) to each state.**

| | Population | 1% of Population | Center 1 | Center 2 | Center 3 |
|---|---|---|---|---|---|
| Alabama | 4,779,736 | 47798 | 2302 | 228 | 1251 |
| Alaska | 710,231 | 7103 | 2933 | 5221 | 5780 |
| Arizona | 6,392,017 | 63921 | 530 | 2016 | 2904 |
| Arkansas | 2,915,918 | 29160 | 1903 | 630 | 1546 |
| California | 37,253,956 | 372540 | 0 | 2529 | 3372 |
| Colorado | 5,029,196 | 50292 | 979 | 1602 | 2393 |
| Connecticut | 3,574,097 | 35741 | 3263 | 971 | 110 |
| Delaware | 897,934 | 8980 | 3059 | 705 | 366 |
| District of Columbia | 601,723 | 6018 | 2954 | 618 | 462 |
| Florida | 18,801,310 | 188014 | 2644 | 292 | 1221 |
| Georgia | 9,687,653 | 96877 | 2529 | 0 | 1070 |
| Hawaii | 1,360,301 | 13604 | 2795 | 5137 | 6096 |
| Idaho | 1,567,582 | 15676 | 614 | 2276 | 2980 |
| Illinois | 12,830,632 | 128307 | 2109 | 658 | 1263 |
| Indiana | 6,483,802 | 64839 | 2329 | 526 | 1046 |

| | | | | | |
|---|---|---|---|---|---|
| Iowa | 3,046,355 | 30464 | 1846 | 948 | 1538 |
| Kansas | 2,853,118 | 28532 | 1491 | 1093 | 1884 |
| Kentucky | 4,339,367 | 43394 | 2382 | 361 | 1023 |
| Louisiana | 4,533,372 | 45334 | 1963 | 602 | 1629 |
| Maine | 1,328,361 | 13284 | 3535 | 1334 | 285 |
| Maryland | 5,773,552 | 57736 | 2985 | 644 | 432 |
| Massachusetts | 6,547,629 | 65477 | 3372 | 1070 | 0 |
| Michigan | 9,883,640 | 98837 | 2419 | 846 | 1008 |
| Minnesota | 5,303,925 | 53040 | 1886 | 1205 | 1608 |
| Mississippi | 2,967,297 | 29673 | 2111 | 421 | 1423 |
| Missouri | 5,988,927 | 59890 | 1888 | 730 | 1492 |
| Montana | 989,415 | 9895 | 1005 | 2046 | 2650 |
| Nebraska | 1,826,341 | 18264 | 1413 | 1277 | 1971 |
| Nevada | 2,700,551 | 27006 | 267 | 2374 | 3169 |
| New Hampshire | 1,316,470 | 13165 | 3366 | 1117 | 94 |
| New Jersey | 8,791,894 | 87919 | 3119 | 800 | 276 |
| New Mexico | 2,059,179 | 20592 | 977 | 1555 | 2450 |
| New York | 19,378,102 | 193782 | 3070 | 887 | 314 |
| North Carolina | 9,535,483 | 95355 | 2805 | 352 | 720 |
| North Dakota | 672,591 | 6726 | 1520 | 1557 | 2047 |
| Ohio | 11,536,504 | 115366 | 2563 | 544 | 810 |
| Oklahoma | 3,751,351 | 37514 | 1547 | 986 | 1868 |
| Oregon | 3,831,074 | 38311 | 479 | 2670 | 3398 |
| Pennsylvania | 12,702,379 | 127024 | 2907 | 689 | 466 |
| Rhode Island | 1,052,567 | 10526 | 3341 | 1028 | 44 |
| South Carolina | 4,625,364 | 46254 | 2692 | 198 | 876 |
| South Dakota | 814,180 | 8142 | 1453 | 1401 | 1994 |
| Tennessee | 6,346,105 | 63462 | 2305 | 301 | 1139 |
| Texas | 25,145,561 | 251456 | 1470 | 1104 | 2087 |
| Utah | 2,763,885 | 27639 | 585 | 1998 | 2795 |
| Vermont | 625,741 | 6258 | 3291 | 1095 | 175 |
| Virginia | 8,001,024 | 80011 | 2848 | 479 | 598 |
| Washington | 6,724,540 | 67246 | 687 | 2759 | 3434 |
| West Virginia | 1,852,994 | 18530 | 2697 | 456 | 701 |
| Wisconsin | 5,686,986 | 56870 | 2149 | 917 | 1274 |
| Wyoming | 563,626 | 5637 | 979 | 1764 | 2460 |

**Table 2. Capacity of three manufacturing center of test kits.**

| | Center 1 | Center 2 | Center 3 |
|---|---|---|---|
| Capacity | 1,500,000 | 1,200,000 | 1,350,000 |

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Your team has been hired as an operations research consultant to assist CDC in determining the optimal production plan of testing kits to minimize the total costs.

a) Formulate this problem as a linear program and write down its dual.

**Notations:**

Parameter:

$C_j$, the capacity of the manufacturing center of test kits, j = 1, 2, 3

$D_i$, the demand at state i, the 1% of population at state i, i = 1, 2, 3, …, 51

$p$, the production cost for each testing kit, p = 1

$m_{ij}$, the distance(mile) between state i and manufacturing center j

$c_{ij}$, the cost of sending one unit of testing kit between state i and manufacturing center j, $c_{ij} = 0.001 m_{ij}$

Decision Variable:

$x_{ij}$: the number of testing kits sending between state i and manufacturing center j

**Model (primal)** 2.5pts**:**

$$\min \sum_i \sum_j c_{ij}\, x_{ij} + \sum_i \sum_j p\, x_{ij}$$

$$s.t. \sum_j x_{ij} \geq D_i, for\ all\ i \quad (1a)$$

$$\sum_i x_{ij} \leq C_j, for\ all\ j \quad (1b)$$

$$x_{ij} \geq 0, for\ all\ i, j$$

Model explanation:

Objective, minimize the total cost, the first term is the shipping cost, and the second term is the production cost.

Constraint (1a), the sum of testing kits at one state is greater than the demand.

Constraint (1b), the sum of testing kits at one manufacturing center is less than the capacity.

**Dual** 2.5pts**:**

Let $\alpha_i, \beta_j$ be the dual variable of constraint (1a) and constraint (1b) respectively,

$$\max \sum_i \alpha_i D_i + \sum_j \beta_j C_j$$

$$\alpha_i + \beta_j \leq c_{ij} + p, for\ all\ i, j$$

$$\alpha_i \geq 0, for\ all\ i$$

$$\beta_j \leq 0, for\ all\ j$$

b) Solve the model using Excel and clearly state your recommendation to the CDC. 5pts

Solution is in the attachment.

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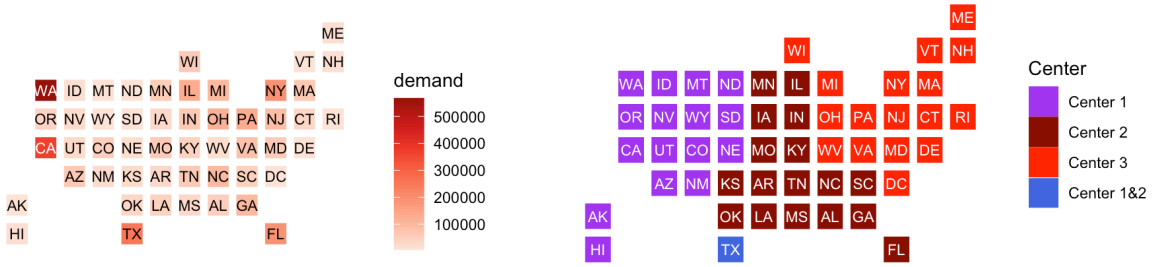
Total cost is 4775913.746

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c) Use the sensitivity report from part (b) to determine how much the **unit shipping cost** (current is $0.001) from Kansas to the manufacturing center 2 can increase (assuming no change in the costs for the other states) before the current optimal solution would no longer be optimal. 10pts

Allowable increase is 0.032 (dollar/unit) or 0.000029277 (dollar/unit/mile)

-3 for 0.033 but no unit

-10 wrong number

d) Use the sensitivity report from part (b) to determine how much the **unit shipping cost** (current is $0.001) from Pennsylvania to the manufacturing center 3 can increase (assuming no change in the costs for the other states) before the current optimal solution would no longer be optimal. 10pts

Allowable increase is 0.589 (dollar/unit) or 0.0012639 (dollar/unit/mile)

-3 for 0.589 but no unit

-10 wrong number

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e) In March, CDC realized that Washington State is in severe situation and would like to order 500,000 testing kits for it. Use the sensitivity report from part (b) to determine whether the current optimal dual solution is still optimal if we increase the number of testing kits of Washington State by 500,000. If your answer is Yes, please state the new optimal cost. Otherwise, please explain. 10pts

Allowable increase 608,043. current optimal solution is still optimal.
Solution is in the attachment.
Total cost is $5619413.75

-5 wrong allowable increase, -5 wrong cost

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f) A foreign company, expert in producing testing kits, offers $300,000 for 1 million testing kits. Since it is a special time, the company agrees to deliver all the test kits to any of manufacturing centers. Use the shadow price from the report obtained in part (b) to determine whether it would be worthwhile to purchase from company. Please explain your choice. 10pts

The shadow price for center 2 is -0.366, 1,000,000 testing kits would cost center 2 $366,000, which is greater than $300,000 offer from the company. Thus, it is worthwhile to purchase from the company instead of producing by the center 2.

Not compare shadow price with 300,000/1million -5pts
Correct answer but wrong reasoning, or wrong conclusion-5pts

Redo-problem: Solution is in the attachment. (6pts each)
(b): total cost is $8060778
(c): Allowable increase is 1.582 (dollar/unit) or 0.0014474 (dollar/unit/mile) (-2pts no unit)
(d): Allowable increase is 0.446 (dollar/unit) or 0.00095708 (dollar/unit/mile) (-2pts no unit)
(e): No. The allowable increase is 84000, which is less than 500,000.
(f): The shadow price for center 1 is -1.246, 1,000,000 testing kits would cost center 1 $1,246,000, which is greater than $300,000 offer from the company. Thus, it is worthwhile to purchase from the company rather producing at center 1. (-6pts if not compare with 300,000/1million, -3pts correct answer but wrong reasoning)

(h) 5pts