ISE 2404 DOR I Project

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Part I

a) **Linear Program and Dual**

Variables: X_ij, where i is the state (Demand (D): i=1, 2, 3, … 51), that requires X_ij number of testing kits from the manufacturing center j (Supply (S): j = A, B, C). In this way, there will be a total of 51*3 = 153 variables. All variables are non-negative. C_ij is the cost per kit associated with each X_ij, see Excel for full C_ij for variables. Calculation of C_ij is in worksheet ¨Costs Data"

$$Min: \ Z \ = \sum_{i \, \varepsilon \, D} \sum_{j \, \varepsilon \, S} C_{ij} X_{ij} \ + \ 1 * \sum_{i \, \varepsilon \, D} \sum_{j \, \varepsilon \, S} X_{ij}$$

The first part of the equation corresponds to the shipping cost, represented by C_ij.This is calculated by multiplying the average distance from state i to manufacturing center j with the shipping cost which is $0.001 per mile and kit. The second part of the equation corresponds to the production cost which is $ 1, and is the same for all variables. In the end we have:

$$Min: \ Z \ = \sum_{i \, \varepsilon \, D} \sum_{j \, \varepsilon \, S} (C_{ij} + 1) X_{ij}$$

For instance:

$$Min: Z \ = 2.302\, X_{1A} + 0.228 X_{1B} + 1.251 X_{1C} + ... \ + 0.979 X_{51A} + 1.764_{51B} \ + \ 2.46 X_{51C}$$

The first 51 constraints will represent the minimum requirement of testing kits for each state. This will be represented by:

$$\sum_{j \, \varepsilon \, S} X_{ij} \geq D_{i\,,} \ \forall \, i \, \varepsilon \, D$$

For instance:

$$Constraint \ \ 1: \ X_{1A} + X_{1B} + X_{1C} \geq 47\ 798 \ (Alabama)$$
$$Constraint \ \ 2: \ X_{2A} + X_{2B} + X_{2C} \geq 7\ 103 \ (Alaska)$$
$$Constraint \ \ 3: \ X_{3A} + X_{3B} + X_{3C} \geq 63\ 921 \ (Arizona)$$

The last 3 constraints will represent the maximum amount supplied of testing kits for each manufacturing center. This will be represented by:

$$\sum_{i\,\varepsilon\,D} X_{ij} \geq S_j\,,\ \forall j\ \varepsilon\ S$$

For instance:

$$Constraint\ \ 52:\ X_{1A} + X_{2A} + X_{3A} +\ \ldots\ +\ X_{51A} \leq 1\ 500\ 000\ (MC\ 1)$$
$$Constraint\ \ 53:\ X_{1B} + X_{2B} + X_{3B} +\ \ldots\ +\ X_{51B} \leq 1\ 200\ 000\ (MC\ 2)$$
$$Constraint\ \ 54:\ X_{1C} + X_{2C} + X_{3C} +\ \ldots\ +\ X_{51C} \leq 1\ 350\ 000\ (MC\ 3)$$

Finally:

$$X_{ij} \geq 0,\ for\ all\ i\ and\ all\ j$$

Dual: Using the information above and the same variables, we can get the Dual Linear Program

$$Max:\ Z\ =\ \sum_{i\,\varepsilon\,D} Y_i D_i\ +\ \sum_{j\,\varepsilon\,S} Y_j S_j$$

Constraints:

$$Y_i + Y_j \leq C_{ij},\ \ \forall i\ \varepsilon\ D,\ \ \forall j\ \varepsilon\ S$$

In the primal we got 153 variables, now we will get 153 constraints. Finally:

$$Y_i,\ Y_j \geq 0,\ for\ all\ i\ and\ all\ j$$

b) Excel Solution and Recommendation

After putting all of this into excel, we get the objective value as follows. The Linear Program is in the worksheet ¨Linear Program Part 1¨. Optimal Solution for all variables is included in appendix 1.

Objective Value: Z = $ 4 775 913.75

c) Use the sensitivity report from part (b) to determine how much the <u>unit shipping cost</u> (current is $0.001) from Kansas to the manufacturing center 2 can

increase (assuming no change in the costs for the other states) before the current optimal solution would no longer be optimal.

All information of Sensitivity Analysis is available in

From original data:

	Population	1% of Population	Center 1	Center 2	Center 3
Kansas	2,853,118	28532	1491	1093	1884

From sensitivity report:

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
BR3	x17	28532	0	2.093	0.032	2.459

$$Current\ Cost\ =\ \$2.093$$
$$Maximum\ Cost\ =\ Current\ Cost\ +\ Allowable\ Increase$$
$$=\ \$2.093\ +\ \$0.032$$
$$=\ \$2.125$$
$$\$2.125\ =\ production\ cost\ +\ shipping\ cost$$
$$=\ \$1\ +\ shipping\ cost$$
$$Shipping\ Cost\ =\$1.125$$

$$Shipping\ Cost\ per\ mile\ with\ new\ cost\ =\ 1.125\ /\ 1094\ =\$\ 0.00102834$$

$$Allowable\ Increase\ of\ unit\ shipping\ cost\ =\$0.00102834\ -\ \$0.001$$

= $0.002834 (final answer)

d) Use the sensitivity report from part (b) to determine how much the <u>unit shipping cost</u> (current is $0.001) from Pennsylvania to the manufacturing center 3 can increase (assuming no change in the costs for the other states) before the current optimal solution would no longer be optimal.

From original data:

	Population	1% of Population	Center 1	Center 2	Center 3

Pennsylvania	12,702,379	127024	2907	689	466

From sensitivity report:

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
EM3	x39	127024	0	1.466	0.589	1.466

$$Current\ Cost\ = \ \$1.466$$
$$Maximum\ cost\ = \ Current\ cost\ +\ Allowable\ Increase$$
$$= \ \$1.466\ +\ \$0.589$$
$$= \ \$2.055$$
$$\$2.055\ = \ Production\ cost\ +\ Shipping\ cost$$
$$= \ \$1\ +\ Shipping\ cost$$
$$Shipping\ Cost\ = \$1.055$$

$$Shipping\ Cost\ per\ mile\ with\ new\ cost\ = \ 1.055\ /\ 466\ \ = \$\ 0.00226$$
$$Allowable\ Increase\ of\ unit\ shipping\ cost\ = \$0.00226\ -\ \$0.001$$

= $0.00126 (final answer)

e) In March, CDC realized that Washington State is in a severe situation and would like to order 500,000 testing kits for it. Use the sensitivity report from part (b) to determine whether the current optimal dual solution is still optimal if we increase the number of testing kits of Washington State by 500,000. If your answer is Yes, please state the new optimal cost. Otherwise, please explain.

Cell	Name	Final Value	Shadow Price	Objective Coefficient	Allowable Increase	Allowable Decrease
EZ54	Constraint 46	6258	1.175	6258	354476	6258
EZ55	Constraint 47	80011	1.598	80011	354476	80011
EZ56	Constraint 48	67246	1.687	67246	608043	67246

According to the Sensitivity Analysis, the Allowable increase is 608043 for Washington State (Constraint 48) before the Dual Solution changes. Since 500 000 < 608043, we are within range to increase the number of testing kits without changing our Dual Solution. This means we can calculate the new objective Value in the following way:

$$Z\ = 4\ 775\ 913\ +\ 1.687 * (500\ 000)\ \ = 5\ 619\ 413.75\ (New\ Optimal\ Cost)$$

f) **A foreign company, expert in producing testing kits, offers $300,000 for 1 million testing kits. Since it is a special time, the company agrees to deliver all the test kits to any of the manufacturing centers. Use the shadow price from the report obtained in part (b) to determine whether it would be worthwhile to purchase from the company. Please explain your choice.**

First Iteration:

Cell	Name	Final Value	Shadow Price	Objective Coefficient	Allowable Increase	Allowable Decrease
EZ60	Constraint 52	891957	0	1500000	1E+30	608043
EZ61	Constraint 53	1200000	-0.366	1200000	139363	112093
EZ62	Constraint 54	995524	0	1350000	1E+30	354476

Unit Cost from foreign company = $300 000 / 1 000 000 = $0.3 per kit

Since the shadow price for manufacturing center 1 and 3 is 0, which is less than 0.3, we conclude that there is no reason to send testing kits to those centers. However, in the case of Manufacturing Center 2, for each testing kit that we add we are reducing our cost by $0.366 dollars (-0.366), which is what we want. Also, $0.366 > $0.3 so what we save is more than our cost per testing kit. But, we will only save 0.366 dollars until we reach the allowable increase of 139363, and the foreign company is asking us to buy 1 million kits.

Second iteration:

Cell	Name	Final Value	Shadow Price	Objective Coefficient	Allowable Increase	Allowable Decrease
EZ60	Constraint 52	752594	0	1500000	1E+30	747406
EZ61	Constraint 53	1339364	-0.357	1339364	56869	1
EZ62	Constraint 54	995523	0	1350000	1E+30	354477

In the second iteration, we see that if we purchase an additional testing kit, we will save $0.357, until we reach 56869. Since, 0.357 > 0.3, this is still good. However, we have not yet reached 1 000 000 testing kits.

Third Iteration:

Cell	Name	Final Value	Shadow Price	Objective Coefficient	Allowable Increase	Allowable Decrease
EZ60	Constraint 52	752594	0	1500000	1E+30	747406
EZ61	Constraint 53	1396234	-0.266	1396234	115365	1
EZ62	Constraint 54	938653	0	1350000	1E+30	411347

In the third iteration, we start saving $0.266 per each additional kit we order. However, our unit cost is $0.3, which is more than what we save ($0.266) if we purchase a kit. Then, it would only be worthwhile for the additional first 139 363 + 56 869 = 196232 testing kits, but 196 232 < 1 000 000. Since by this point we have not reached 1 million test kits, we conclude that it is not worthwhile to purchase from the foreign company.

Part 2

g) Please redo Parts (b)-(f) with the updated data and re-answer their corresponding questions.

Linear Program and Dual

Variables: X_ij, where i is the state (Demand (D): i=1, 2, 3, … 51), that requires X_ij number of testing kits from the manufacturing center j (Supply (S): j = A, B, C). In this way, there will be a total of 51*3 = 153 variables. All variables are non-negative.

$$Min:\ Z\ =\ \sum_{i\,\varepsilon\,D}\ \sum_{j\,\varepsilon\,S} C_{ij}X_{ij}\ +\ 1.5*\sum_{i\,\varepsilon\,D}\ \sum_{j\,\varepsilon\,S} X_{ij}$$

The first part of the equation corresponds to the shipping cost, represented by C_ij.This is calculated by multiplying the average distance from state i to manufacturing center j with the shipping cost which is $0.002 per mile and kit. The second part of the equation corresponds to the production cost which is $ 1.5, and is the same for all variables. In the end we have:

$$Min:\ Z\ =\ \sum_{i\,\varepsilon\,D}\ \sum_{j\,\varepsilon\,S} (C_{ij}+1.5)X_{ij}$$

For instance:

$$Min: Z\ = 6.104X_{1A} + 1.956X_{1B} + 4.002X_{1C} + \ldots\ + 3.458X_{51A} + 5.028X_{51B}\ +\ 6.42X_{51C}$$

The first 51 constraints will represent the minimum requirement of testing kits for each state. This will be represented by:

$$\sum_{j \varepsilon S} X_{ij} \geq D_{i\,,}\ \forall i\ \varepsilon\ D$$

For instance:

$$Constraint\ \ 1:\ X_{1A} + X_{1B} + X_{1C} \geq 9\ 000\ (Alabama)$$
$$Constraint\ \ 2:\ X_{2A} + X_{2B} + X_{2C} \geq 1\ 500\ (Alaska)$$
$$Constraint\ \ 3:\ X_{3A} + X_{3B} + X_{3C} \geq 15\ 000\ (Arizona)$$

The last 3 constraints will represent the maximum amount supplied of testing kits for each manufacturing center. This will be represented by:

$$\sum_{i \varepsilon D} X_{ij} \geq S_{j\,,}\ \forall j\ \varepsilon\ S$$

For instance:

$$Constraint\ \ 52:\ X_{1A} + X_{2A} + X_{3A} +\ \ldots\ +\ X_{51A} \leq 1\ 500\ 000\ (MC\ 1)$$
$$Constraint\ \ 53:\ X_{1B} + X_{2B} + X_{3B} +\ \ldots\ +\ X_{51B} \leq 1\ 200\ 000\ (MC\ 2)$$
$$Constraint\ \ 54:\ X_{1C} + X_{2C} + X_{3C} +\ \ldots\ +\ X_{51C} \leq 1\ 350\ 000\ (MC\ 3)$$

Finally:

$$X_{ij} \geq 0,\ for\ all\ i\ and\ all\ j$$

Dual: Using the information above and the same variables, we can get the Dual Linear Program

We use the same variables as before: i= 1,2 ...51 and j = A, B, C; D_i is the demand from node i, and S_j is the supply from node j.

$$Max:\ Z\ =\ \sum_{i \varepsilon D} Y_i D_i\ +\ \sum_{j \varepsilon S} Y_j S_j$$

Constraints:

$$Y_i + Y_j \leq C_{ij},\ \ \forall i\ \varepsilon\ D,\ \ \forall j\ \varepsilon\ S$$

In the primal we got 153 variables, now we will get 153 constraints. Finally:

$Y_i,\ Y_j \geq 0,\ for\ all\ i\ and\ all\ j$

Excel Solution and Recommendation

After putting all of this into excel, we get the objective value as follows. Optimal Solution for all variables is included in appendix X.

Objective Value: Z = $ 8 060 778

Use the sensitivity report from part (b) to determine how much the unit shipping cost (current is $0.002) from Kansas to the manufacturing center 2 can increase (assuming no change in the costs for the other states) before the current optimal solution would no longer be optimal.

From original data:

			Population	New Data	1% of Population	Center 1	Center 2	Center 3
16	**Iowa**		3,046,355	25500	30464	1846	948	1538
17	**Kansas**		2,853,118	9000	28532	1491	1093	1884

From sensitivity report:

	Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
16	BQ3	B	25500	0	3.396	1.18	3.396
17	BR3	B	9000	0	3.686	1.582	3.686
18	BS3	B	21000	0	2.222	1.324	2.222

$$
\begin{aligned}
Current\ Cost &= \$3.686 \\
Maximum\ Cost &= Current\ Cost + Allowable\ Increase \\
&= \$3.686 + \$1.582 \\
&= \$5.268
\end{aligned}
$$

$$
\begin{aligned}
\$5.268 &= production\ cost + shipping\ cost \\
&= \$1.5 + shipping\ cost \\
Shipping\ Cost &= \$3.768
\end{aligned}
$$

$$Shipping\ Cost\ per\ mile\ with\ new\ cost = 5.268 / 1093 = \$\ 0.00344739$$

$$Allowable\ Increase\ of\ unit\ shipping\ cost = \$0.00344739 - \$0.002$$

= $0.00145 (final answer)

Use the sensitivity report from part (b) to determine how much the unit shipping cost (current is $0.002) from Pennsylvania to the manufacturing center 3 can increase (assuming no change in the costs for the other states) before the current optimal solution would no longer be optimal.

From original data:

			Population	New Data	1% of Population	Center 1	Center 2	Center 3
39	Pennsylvania		12,702,379	61500	127024	2907	689	466

From sensitivity report:

	Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
39	EM3	C	61500	0	2.432	0.446	2.432

$Current\ Cost\ =\ \$2.432$

$Maximum\ cost\ =\ Current\ cost\ +\ Allowable\ Increase$

$=\ \$2.432\ +\ \$0.446$

$=\ \$2.878$

$\$2.878\ =\ Production\ cost\ +\ Shipping\ cost$

$=\ \$1.5\ +\ Shipping\ cost$

$Shipping\ Cost\ = \$1.378$

$Shipping\ Cost\ per\ mile\ with\ new\ cost\ =\ 1.055\ /\ 466\ = \$\ 0.002957$

$Allowable\ Increase\ of\ unit\ shipping\ cost\ = \$0.002957\ -\ \$0.002$

= $0.00096 (final answer)

In March, CDC realized that Washington State is in a severe situation and would like to order 500,000 testing kits for it. Use the sensitivity report from part (b) to determine whether the current optimal dual solution is still optimal if we increase the number of testing kits of Washington State by 500,000. If your answer is Yes, please state the new optimal cost. Otherwise, please explain.

Cell	Name	Final Value	Shadow Price	Objective Coefficient	Allowable Increase	Allowable Decrease
EZ56	.Washington	852000	4.12	852000	84000	31500

According to the Sensitivity Analysis, the Allowable increase is 84000 for Washington State before the Dual Solution changes. Since 500 000 > 84000, we are not within range to increase the number of testing kits. The current optimal dual solution will no longer be optimal if we add 500 000 extra testing kits to Washington State.

A foreign company, expert in producing testing kits, offers $300,000 for 1 million testing kits. Since it is a special time, the company agrees to deliver all the test kits to any of the manufacturing centers. Use the shadow price from the report obtained in part (b) to determine whether it would be worthwhile to purchase from the company. Please explain your choice.

Cell	Name	Final Value	Shadow Price	Objective Coefficient	Allowable Increase	Allowable Decrease
EZ60	MC1	1500000	-1.246	1500000	31500	84000
EZ61	MC2	777000	0	1200000	1E+30	423000
EZ62	MC3	1056000	0	1350000	1E+30	294000

Unit Cost from foreign company = $300 000 / 1 000 000 = $0.3 per kit

Since the shadow price for manufacturing center 2 and 3 is 0, which is less than 0.3, we conclude that there is no reason to send testing kits to those centers. However, in the case of Manufacturing Center 1, for each testing kit that we add we are reducing our cost by $1.246 dollars (-1.246), which is what we want. Also, $1.246 > $0.3 so what we save is more than our cost per testing kit. But, we will only save 1.246 dollars until we reach the allowable increase of 31 500 testing kits, and the foreign company is asking us to buy 1 million kits.

Cell	Name	Final Value	Shadow Price	Objective Coefficient	Allowable Increase	Allowable Decrease
EZ60	MC1	1531501	-1.156	1531501	14999	1
EZ61	MC2	745499	0	1200000	1E+30	454501
EZ62	MC3	1056000	0	1350000	1E+30	294000

In the second iteration, we see that if we purchase an additional testing kit, we will save $1.156, until we add 14 999 additional testing kits. Since, 1.156 > 0.3, this is still good. However, we have not yet reached 1 000 000 testing kits.

Cell	Name	Final Value	Shadow Price	Objective Coefficient	Allowable Increase	Allowable Decrease
EZ60	MC1	1546501	-0.074	1546501	1499	1
EZ61	MC2	730499	0	1200000	1E+30	469501
EZ62	MC3	1056000	0	1350000	1E+30	294000

In the third iteration, we start saving \$0.074 per each additional kit we order. However, our unit cost is \$0.3, which is more than what we save (\$0.074) if we purchase a kit. Then, it would only be worthwhile for the additional first 31 500 + 14 999 = 46 499 testing kits, but 46 499 < 1 000 000. Since by this point we have not reached 1 million test kits, we conclude that it is not worthwhile to purchase from the foreign company.

PART C

h) Also, please remark which datasets (i.e., Table 1 vs Table 3) make more sense. You might want to check the (https://covidtracking.com/data/) to see the current infected population of each state and number of testing cases.

In terms of comparison to the actual data on the confirmed cases of Coronavirus in each state and the number of tests that have been performed so far, Table 1 more accurately represents the distribution of people that have been tested, while Table 3 is far closer to the distribution of confirmed cases. However, neither table was very close to the distribution of confirmed cases at all, and since the data is being used to plan the distribution of kits to test for Coronavirus, Table 1's estimates make much more sense to use.

District of Columbia:
Table 1 Estimate = 6018
Table 3 Estimate = 15000
Positive Cases = 5322
Total Tests with Results = 24329
Table 1 Cases % Error = $100 * \frac{6018-5322}{5322} = +13.07\%$
Table 1 Testing % Error = $100 * \frac{6018-24329}{24329} = -75.62\%$
Table 3 Cases % Error = $100 * \frac{15000-5322}{5322} = -72.11\%$
Table 3 Testing % Error = $100 * \frac{15000-24329}{24329} = -38.35\%$

Appendix:

1. Optimal number of testing kits sent from Manufacturing center (J= A,B, C) to State (i=1, 2, 3, … 51)

x1	A	0
x2	A	7103
x3	A	63921
x4	A	0
x5	A	372540
x6	A	50292
x7	A	0
x8	A	0
x9	A	0
x10	A	0
x11	A	0
x12	A	13604
x13	A	15676
x14	A	0
x15	A	0
x16	A	0
x17	A	0
x18	A	0
x19	A	0
x20	A	0
x21	A	0
x22	A	0
x23	A	0
x24	A	0
x25	A	0
x26	A	0

x27	A	9895
x28	A	18264
x29	A	27006
x30	A	0
x31	A	0
x32	A	20592
x33	A	0
x34	A	0
x35	A	6726
x36	A	0
x37	A	0
x38	A	38311
x39	A	0
x40	A	0
x41	A	0
x42	A	8142
x43	A	0
x44	A	139363
x45	A	27639
x46	A	0
x47	A	0
x48	A	67246
x49	A	0
x50	A	0
x51	A	5637
x1	B	47798
x2	B	0
x3	B	0
x4	B	29160

x5	B	0
x6	B	0
x7	B	0
x8	B	0
x9	B	0
x10	B	188014
x11	B	96877
x12	B	0
x13	B	0
x14	B	128307
x15	B	64839
x16	B	30464
x17	B	28532
x18	B	43394
x19	B	45334
x20	B	0
x21	B	0
x22	B	0
x23	B	0
x24	B	53040
x25	B	29673
x26	B	59890
x27	B	0
x28	B	0
x29	B	0
x30	B	0
x31	B	0
x32	B	0
x33	B	0

x34	B	95355
x35	B	0
x36	B	0
x37	B	37514
x38	B	0
x39	B	0
x40	B	0
x41	B	46254
x42	B	0
x43	B	63462
x44	B	112093
x45	B	0
x46	B	0
x47	B	0
x48	B	0
x49	B	0
x50	B	0
x51	B	0
x1	C	0
x2	C	0
x3	C	0
x4	C	0
x5	C	0
x6	C	0
x7	C	35741
x8	C	8980
x9	C	6018
x10	C	0
x11	C	0

x12	C	0
x13	C	0
x14	C	0
x15	C	0
x16	C	0
x17	C	0
x18	C	0
x19	C	0
x20	C	13284
x21	C	57736
x22	C	65477
x23	C	98837
x24	C	0
x25	C	0
x26	C	0
x27	C	0
x28	C	0
x29	C	0
x30	C	13165
x31	C	87919
x32	C	0
x33	C	193782
x34	C	0
x35	C	0
x36	C	115366
x37	C	0
x38	C	0
x39	C	127024
x40	C	10526

x41	C	0
x42	C	0
x43	C	0
x44	C	0
x45	C	0
x46	C	6258
x47	C	80011
x48	C	0
x49	C	18530
x50	C	56870
x51	C	0

2.