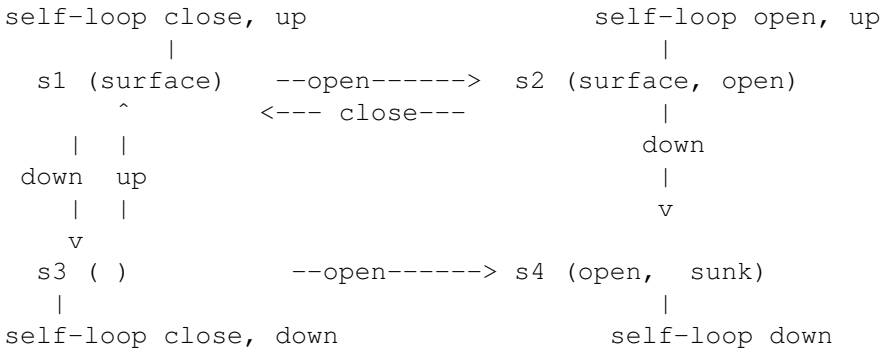
INFOMLSAI Logics for Safe AI
Coursework 1
Solutions

Tasks that can be done in Week 1 (w/c 24 April)

W1-1 Submarine state transition system (everybody had a better picture in their submissions, but just for completeness sake).



W1-2 Now define the system above formally as in the definition of state transition system on slide 5 (specify $St, \longrightarrow, \mathcal{V}$).

$St = \{s1, s2, s3, s4\}$

$s1 \longrightarrow s1, s1 \longrightarrow s2, s2 \longrightarrow s1, s2 \longrightarrow s2, s1 \longrightarrow s3, s3 \longrightarrow s1, s3 \longrightarrow s3, s2 \longrightarrow s4, s4 \longrightarrow s2, s3 \longrightarrow s4, s4 \longrightarrow s3, s4 \longrightarrow s4.$

$V(\mathsf{surface}) = \{s1, s2\}, V(\mathsf{open}) = \{s2, s4\}, V(\mathsf{sunk}) = \{s4\}.$

Tasks that can be done during Week 2 (w/c 1 May)

W2-1 Consider the state q in your state transition system where the submarine is on the surface and open and sunk are false. Express in CTL: on all paths, always, the submarine is not sunk. Is this property true in q? Explain your answer with the reference to CTL truth definitions.

Answer $AG\neg\mathsf{sunk}$ is false in $s1$. It would have been true in $s1$ if $\neg\mathsf{sunk}$ was true in all states on all paths starting in $s1$. Here is an example of a path where it is false in one of the states: $s1, s2, s4, s4, ...$

W2-2 Express in CTL: there exists a path where in some future state the submarine is not on the surface and not sunk, and until that state, it holds that the submarine was also not sunk. Is this property true in state q described above? Explain your answer with the reference to CTL truth definitions.

Answer $E(\neg\mathsf{sunk}\, U\, (\neg\mathsf{surface}\wedge\neg\mathsf{sunk}))$. It is true on the path $s1, s2, ...$ because at index 1 (in $s2$), $\neg\mathsf{surface}\wedge\neg\mathsf{sunk}$ is true, and on all indices before that (which is 0, $s1$) $\neg\mathsf{sunk}$ is true.

W2-3 *Answer*: Yes, there is a difference, and a formula which is always true on infinite paths (but false on some finite paths) is $EX\,\mathsf{true}$ (or $EX(p \vee \neg p)$).

W2-4 See ispl. Obviously multiple correct solutions were possible.

W2-5 See ispl

W2-6 See ispl

W2-7 The CTL truth definition for $E\varphi\, U\, \psi$ does not require φ to hold in the state satisfying ψ, but only in preceding states. Consider an alternative definition $M, q \models E\varphi UU\psi$ iff there exists a path λ from q such that for some $i \geq 0$, $M, \lambda[i] \models \psi$ and for all j, $0 \leq j \leq i$, $M, \lambda[j] \models \varphi$.

Answer: Same as for EU, but in the initialisation, Q_2 is assigned $[\psi \wedge \varphi]_M$ rather than just $[\psi]_M$. (1 point)