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ORIGINAL PAPER

Bilevel optimization to deal with demand response in power grids: models, methods and challenges

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Abstract
This paper presents a review of selected models, methods, and challenges associated with the use of bilevel optimization in problems that involve consumers' demand response arising in the power sector. The main formulations and concepts of bilevel optimization are presented. The importance of demand response as a "dispatchable" resource in the evolution of power networks to smart grids is emphasized. The hierarchical nature of the interaction between decision-makers controlling different sets of variables in several problems involving demand response is highlighted, which establishes bilevel optimization as an adequate approach to decision support. The main concepts and solution approaches to those problems are underlined, in the context of the theoretical, methodological, and computational issues associated with bilevel optimization.

Keywords Bilevel optimization · Demand response · Power grids · Power systems

Mathematics Subject Classification 90-02 (Research exposition (monographs, survey articles) pertaining to operations research mathematical programming) · 90C26 (Nonconvex programming, global optimization) · 90C90 (Applications of mathematical programming)

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1 Introduction

Bilevel optimization (BLO) models enable to formulate problems involving non-cooperative hierarchical decision processes. These models have their roots in the leader–follower duopoly model presented by Stackelberg in his book "Market Structure and Equilibrium", first published in German in 1934 (von Stackelberg 2011). For this reason, the solution of a BLO problem is also called Stackelberg equilibrium, although this type of problem had been introduced in the mathematical programming community about 40 years late with the work of Bracken and McGill (1973). In a Stackelberg equilibrium, the leader forms a conjecture about the follower's reaction and acts in such a way that the ensuing the follower's behavior provides the leader with an advantage.

In a BLO model, the leader (upper level—UL) and the follower (lower level—LL) decision-makers control different sets of variables and have, in general, objective functions displaying some antagonism and being subject to interdependent constraints (i.e., involving variables of both levels). The LL problem belongs to the constraint set of the UL problem; the BLO problem is the UL decision-maker's problem. Decisions are made in a sequential manner: the leader establishes the values for his variables, thus restricting the follower's options; the follower reacts by selecting a solution to optimize his objective function in the feasible region restricted by the instantiation of the leader's decision variables. However, the leader should consider the follower's reaction in his decision process, since it affects the UL objective function value and possibly the solution feasibility. Two reference books for BLO are (Bard 1998) and (Dempe 2002), in which theoretical results, algorithms and applications are presented. Dempe et al. (2015) discuss linear, convex, and mixed-integer BLO problems, as well as the reduction of BLO problems to a single level, also presenting applications in energy systems—a natural gas cash-out problem, an equilibrium problem in a mixed oligopoly and a toll assignment problem, including numerical experiments.

Although the present paper focus on the bilevel single-leader single-follower case, these models can be generalized to the multiple-leader multiple-follower equilibrium, in which there is more than one leader deciding in the first stage, which is affected by the reactions of multiple followers and the other leaders' decisions; the followers make their decisions considering the other followers' decisions, which can be modeled as a Nash game parameterized by the leaders' decisions. Also, multi-level problems can be considered, in which a series of optimization problems should be solved in a predetermined sequence pertaining to a hierarchical structure.

BLO models have been extensively used in the power sector to address several problems, including strategic bidding and market clearing, participation of storage systems in energy and reserve markets, microgrid power and reserve capacity planning, energy management of combined heat and power microgrids, energy allocation mechanism with grid constraints, bidding and offer strategies of electric vehicle (EV) fleets, power generation investment expansion planning, transmission/distribution networks investment planning, bidding of flexible load aggregators in system-level service, day-ahead energy and reserve markets, analysis of the vulnerability of power systems under attacks, and pricing problems consisting of the design of time-differentiated tariff schemes to induce changes in energy consumption patterns (see also Pozo et al. 2017).

Multiple trends have contributed for a more active role of (residential, commercial, or industrial) consumers in the management of power systems. Deregulation opened the generation and retail segments to competition, with different types of wholesale and retail markets balancing supply and demand while keeping the grids as regulated monopolies. Due to factors such as regulation uncertainties, the investment in grid assets may not be enough to cope with aging equipment and/or growing demand, which may cause a strain in existing lines and equipment (e.g., transformers). Climate change has led to increasingly extreme weather phenomena, with more frequent heat and cold waves that require further climatization needs. Activating power plants just to meet very high peak demand of short duration is very expensive and has significant environmental impacts. Environmental concerns and economy decarbonization induce a growing share of renewable generation of intermittent nature, namely based on wind and solar photovoltaics (PV). The progressive electrification of the transportation sector puts an extra burden on the grid requiring adjusted planning and management to cope with the increasing implementation of (fast and super fast) EV battery chargers. The deployment of pervasive sensing and control equipment in the grid, including smart meters at the customers' premises, enables acquiring vast amounts of data that can be used for a more efficient grid management and offering tariffs more adequate to the consumption profiles. Consumers are becoming *prosumers* (producing and consuming energy) and *prosumagers* (also owning storage equipment), who may produce their own electricity, draw from the grid, store, and feed to the grid. In this energy transition context, the paradigm is changing from "supply follows load" to "load follows supply". The designation *demand response* refers to schemes that may induce voluntary changes in the habitual consumption patterns reacting to economic signals, including time-differentiated energy prices or incentive payments according to wholesale market prices, renewable energy availability, and/or grid conditions. Therefore, supporting decisions in several settings in the power sector should capture the reactive nature of demand response through the optimization of demand side resources embodying consumer empowerment. This can be accomplished by BLO models whenever leader–follower decisions are at stake as, for instance, the design of time-of-use tariffs subject to the consumer's reaction by means of rescheduling load operation.

The interest of BLO models to deal with problems in the power sector in which demand response plays a role has been addressed in this Introduction. The main concepts and approaches of BLO, including some pitfalls of approximate algorithms, are presented in Sect. 2. Section 3 summarizes how demand response has been included in optimization models for several applications of optimization in the power sector. A review of recent selected BLO models and solution techniques in problems in which demand response is a relevant component of the decision process is offered in Sect. 4. Conclusions and challenges ahead are discussed in Sect. 5.

2 Bilevel optimization

2.1 Formulation

A BLO problem can be formulated as in (1), where x represents the leader-controlled decision variables and y the follower-controlled decision variables:

$$\max_{x \in X} F(x, y) \tag{1}$$

$$\text{s.t. } G(x, y) \leq 0$$

$$y \in \arg\max_{y \in Y} \{f(x, y) : g(x, y) \leq 0\}$$

$X \subset \mathbb{R}^{n_1}$ and $Y \subset \mathbb{R}^{n_2}$ are compact sets and place additional constraints on variables, such as upper and lower bounds; n_1 is the number of UL variables and n_2 is the number of LL variables. $F(x, y)$ and $f(x, y)$ are the leader's and the follower's objective functions, respectively.

The follower optimizes his objective function $f(x, y)$ after the decision variables $x \in X$ are set by the leader, i.e. the LL objective function is optimized for an instantiation of x. However, the leader's decision is implicitly affected by the follower's reaction. The follower's feasible region for a given x is $Y(x) = \{y \in Y : g(x, y) \leq 0\}$ and the corresponding follower's rational reaction set is

$$\Psi(x) = \left\{ y \in \mathbb{R}^{n_2} : y \in \arg\max_{y \in Y(x)} f(x, y) \right\}. \tag{2}$$

The feasible set of the BLO problem, generally called inducible region, is $IR = \{(x, y) : x \in X, G(x, y) \leq 0, y \in \Psi(x)\}$. The BLO problem (1) is equivalent to optimizing the leader's objective F over IR. Solving a BLO problem is very challenging from theoretical, methodological, and computational perspectives, since it is intrinsically non-convex and even the linear BL problem is NP-hard (Dempe, 2002).

2.2 Solution concepts

Figure 1 illustrates these concepts for a linear BLO problem with two decision variables: x is controlled by the leader, y is controlled by the follower, and S is the set of all constraints (in this example, no UL constraints $G(x, y) \leq 0$ involving the LL variables y exist). In a linear BLO problem, once the leader selects a value for x, the corresponding term in $f(x, y)$ becomes constant and can be removed from the problem; so, the follower's objective function can be expressed as $f(y)$ only. For each value of x, the follower selects the value of y that optimizes his objective function $f(y)$. Therefore, the inducible region, where the feasible solutions to the BLO problem are located, is $IR = [\text{AE}] \cup [\text{ED}]$. The optimal solution to the BLO problem is point D that maximizes $F(x, y)$ in IR.

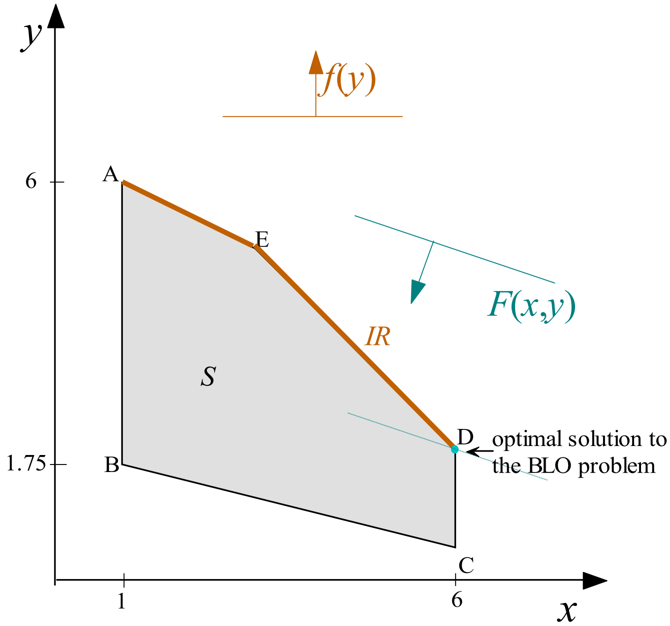

Fig. 1 Inducible region and optimal solution to a linear BLO problem

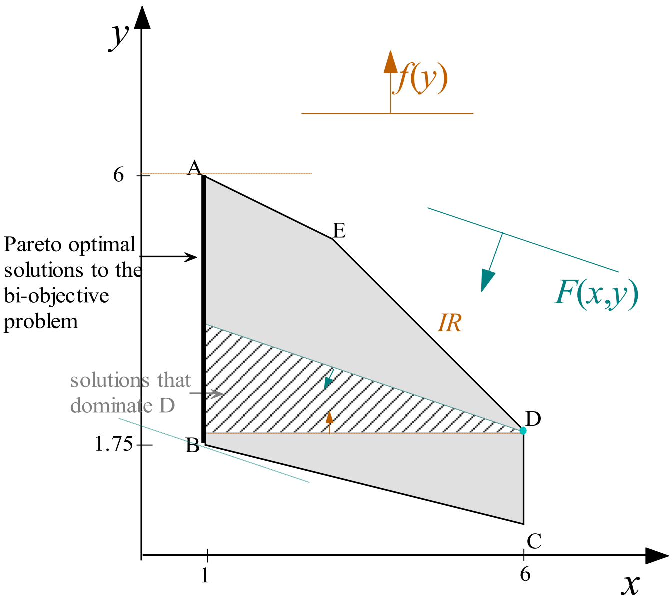

Fig. 2 Bi-objective vs. BLO problems

Note that the optimal solution to a BLO problem is not, in general, a Pareto optimal (efficient) solution to the bi-objective problem in which the leader's and the follower's objective functions are considered at the same level, i.e. cooperation would exist. This issue is illustrated in Fig. 2 for the same BLO problem as in Fig. 1. The Pareto optimal solutions to the bi-objective problem defined by the maximization of $F(x, y)$ and $f(y)$ over S lie on the edge [AB]. The set of all feasible solutions to the bi-objective problem that dominate the optimal solution to the BLO problem (point D) is displayed with the hatched pattern. In this region, better values than the ones in D can be obtained for both UL and LL objective functions. Therefore, the cooperation between the leader and the follower would be beneficial for both decision-makers with respect to the optimal decision in the hierarchical non-cooperative setting.

However, several real-world problems involve sequential non-cooperative decisions for which BLO models are adequate. An example in the energy sector is the setting of appropriate price signals to induce behavior changes in the time, and possibly magnitude, of electricity use by (different types of) consumers.

Note that a solution is feasible to the BLO problem only if it is optimal to the LL problem. However, this may not be guaranteed if approximate approaches (such as metaheuristics) are used to solve this problem or the (exact) solver cannot reach optimality due, for instance, the combinatorial complexity of the LL problem after the instantiation of the UL variables. In this case, the results may be misleading in the sense that apparently better, but infeasible, solutions can be given.

Whenever alternative optimal solutions exist for the LL problem for a given x, the follower may choose the one that is the best or the worst (or any intermediate one) for the leader's objective function. Therefore, the leader may obtain different solutions with different objective function values ranging from the optimistic solution, assuming that the follower's choice among his alternative optimal solutions is the one leading to the best outcome for the leader, to the pessimistic solution, assuming that the follower's choice is the solution with the worst outcome for the leader. It is often assumed that the follower breaks ties (among his multiple optima for a given leader's decision) in favor of the leader, presuming that the leader has some means to influence the follower's decision. This is the optimistic formulation of the BLO problem, whose optimal solution is easier to calculate. However, if this assumption is not possible and the leader is risk averse, he may want to choose the pessimistic optimal solution (which is more difficult to obtain), thus bounding the damage resulting from an unfavorable selection made by the follower. While in the optimistic formulation, the optimization of the leader's objective function of (1) may be taken with respect to x and y, i.e., $\max_{x \in X, y \in Y} F(x, y)$, the pessimistic formulation is a max–min problem, i.e., $\max_{x \in X} \min_{y \in Y} F(x, y)$.

If multiple objective functions are considered at the LL (e.g., the electricity consumer wants to minimize electricity costs and minimize the discomfort of rescheduling the appliances to benefit from the time-differentiated prices established by the retailer to maximize his profit), a different decision issue is at stake at the LL problem. That is, for each leader's decision, the follower has to choose a solution among an efficient solution set displaying distinct trade-offs between his objectives (e.g., enduring some discomfort to decrease costs). Therefore, it may be difficult for the leader to anticipate the follower's decision, as well as to influence this decision in some way. The leader may adopt a more optimistic or a more pessimistic attitude according to his expectation that the follower will decide in a more or less favorable way for the leader's objective. An optimistic/pessimistic attitude assumes that the follower's decision, within his efficient solution set for a given instantiation of the UL variables, is the best/worst for the leader. In most real-world problems, it is not reasonable to assume that the follower would always choose his efficient solution that gives the best solution for the leader. For instance, this would mean that the consumer would choose the solution that minimizes discomfort regardless of cost so that the leader's profit would be maximum, which is seldom realistic. This BLO problem with a single-objective function at the UL and multiple objective functions at the LL is generally designated in the literature as semi-vectorial BLO (SV-BLO). For this type of problems, Alves and Antunes (2018) proposed the concepts of deceiving, rewarding and moderate solutions, considering the optimistic/pessimistic leader's attitude in selecting x and the follower's choices within his efficient solution set. The deceiving solution is obtained when the leader adopts an optimistic attitude when setting his decision variables and the follower's decision is the most unfavorable. The rewarding solution is obtained when the leader adopts a pessimistic attitude and the follower selects the most beneficial solution for the leader. The optimistic/deceiving and the pessimistic/rewarding solutions frame the extreme solutions the leader can obtain, which offer relevant insights for decision support. A moderate solution can be defined as a solution that gives the highest expected value for $F(x, y)$ considering an optimism/pessimism index (e.g., probabilities of the follower's decision being in favor or against the interests of the leader).

The BLO problem becomes even more complicated when multiple objective functions exist at both levels. The aim is generally to find the UL Pareto optimal front, which is easier to determine when the multiple objective LL problem is transformed into a single objective problem by considering that the follower's preferences are known such that a LL utility function can be developed. Alves et al. (2019) proposed the concepts of optimistic Pareto front (consisting of all feasible solutions belonging to the inducible region such that no other solution in this region dominates them) and pessimistic Pareto front (based on the concept of "most dominated" solutions for the leader). For a formal definition of these solutions, please see Alves et al. (2019).

2.3 Methodological approaches

Different methodological approaches have been developed to address BLO problems, which may be broadly categorized as classical and metaheuristics-based approaches. The classical approaches include single-level problem reformulation using the Karush–Kuhn–Tucker (KKT) conditions (or primal–dual reformulation for linear problems), descent and penalty function methods, optimal value function reformulations and enumeration techniques in linear problems.

The first approach consists of replacing the LL problem by its KKT conditions, which provide the necessary and sufficient conditions for optimality if the LL problem is a convex optimization problem in the y variables for fixed x, and add this set of constraints (the equilibrium constraint set) to the UL problem. Under the convexity assumption, the optimistic formulation of the BL problem and the problem resulting from the KKT conditions are equivalent (Dempe and Dutta 2012). The resulting single-level optimization model is nonlinear, called a mathematical program with equilibrium constraints (MPEC), due to the complementarity conditions (in pricing problems, the product of price and quantity variables is another source of nonlinearity). If the UL and the LL problems are linear, the complementarity constraints can be replaced by linear constraints with binary variables, thus transforming the MPEC into a mixed-integer linear programming (MILP) problem (Fortuny-Amat and McCarl 1981). The optimal solution can be then obtained using a general MIP solver, usually with a significant computational effort.

For the following linear BLO problem with $x \in \mathbb{R}^{n_1}$, $y \in \mathbb{R}^{n_2}$, m_1 UL constraints and m_2 LL constraints

$$
\begin{aligned}
&\max_x F(x, y) = c_1 x + d_1 y \\
&\text{s.t.}\, A_1 x + B_1 y \le b_1 \\
&\quad x \ge 0 \\
&\quad \max_y f(y) = d_2 y \\
&\quad s.t.\, A_2 x + B_2 y \le b_2 \\
&\qquad\qquad y \ge 0
\end{aligned}
\tag{3}
$$

the resulting MPEC, being λ the dual variables of the LL constraints, is

$$
\begin{aligned}
&\max_{x,y} F(x, y) = c_1 x + d_1 y \\
&\text{s.t.}\, A_1 x + B_1 y \le b_1 \\
&\quad A_2 x + B_2 y \le b_2 \\
&\quad \lambda B_2 \ge d_2 \\
&\quad \lambda(b_2 - A_2 x - B_2 y) = 0 \\
&\quad y(\lambda B_2 - d_2) = 0 \\
&\quad x \ge 0, y \ge 0, \lambda \ge 0.
\end{aligned}
\tag{4}
$$

The nonlinear constraints of this MPEC can be linearized by introducing additional binary variables. Considering u the binary variables associated with $\lambda(b_2 - A_2x - B_2y) = 0$ and v the binary variables associated with $y(\lambda B_2 - d_2) = 0$, these constraints of problem (4) are transformed into the MILP formulation (5), where M is a large positive number and e is a vector of 1s of appropriate dimension:

$$
\begin{aligned}
& b_2 - A_2x - B_2y \leq Mu \\
& \lambda \leq M(e - u) \\
& \lambda B_2 - d_2 \leq Mv \\
& y \leq M(e - v) \\
& u \in \{0, 1\}^{m_2}, v \in \{0, 1\}^{n_2}.
\end{aligned}
\tag{5}
$$

The transformed MILP has $n_1 + n_2 + m_2$ continuous variables, $n_2 + m_2$ binary variables, and $m_1 + 3m_2 + 3n_2$ constraints. Therefore, computing the optimal solution to this model using a MILP solver may impose a very significant computational effort. Moreover, the big-Ms may be difficult to determine in some problems, leading to computational difficulties. Pineda and Morales (2019) showed that the usual trial-and-error procedure to tune the big-Ms may lead to highly sub-optimal solutions, encouraging the use of more sophisticated techniques to tune accurately the values of the big-Ms to solve linear BLO problems (Pineda et al. 2018).

Other solution approaches to deal with the KKT transformation include the use of a branch-and-bound strategy or penalty function methods to deal with the complementarity constraints.

Another way to reformulate a BLO model with a linear LL problem into a single level is to replace the LL problem by its primal and dual constraints and enforcing the strong duality by equating the primal and dual objective functions (Garcia-Herreros et al. 2016). This reformulation yields a nonlinear problem, the nonlinearity arising from the product of UL variables and LL dual variables in the dual objective function. These nonlinear components can be linearized approximately using the McCormick (1976) envelopes, which become exact when the UL variables are binary.

The approaches based on descent and penalty function methods enable to compute stationary points and local minima. Gradient descent methods are iterative methods that define descent (for minimization) directions using gradient information to improve the UL objective function while keeping the solutions feasible. Finding the descent direction can be quite challenging, requiring solving auxiliary problems. Penalty methods consist of solving a nonlinear programming problem approximating the original one, incorporating a penalty function associated with the violation of constraints or certain optimality conditions (e.g., the gap between the primal and dual solutions of the follower's problem is used as a penalty term in the leader's problem), which is solved iteratively. Under certain conditions, this process leads to a sequence of approximate solutions converging to the optimal solution, as the penalty function algorithm introduced by White and Anandalingam (1993) which finds an optimal solution to the linear BLO problem.

There are other methodological approaches that attempt to iteratively approximate the *optimal value function* of the LL problem, i.e. $\varphi(x) = \max\{f(x,y) : y \in Y, g(x,y) \leq 0\}$. The BLO problem (1) can be equivalently replaced by

$$\max\{F(x,y) : x \in X, G(x,y) \leq 0, y \in Y, g(x,y) \leq 0, f(x,y) \geq \varphi(x)\}. \tag{6}$$

The optimal value function approaches are based on the use of formulation (6) with approximations of $\varphi(x)$. Two algorithms representative of this approach for problems with discrete variables are the ones proposed by Mitsos (2010) and Lozano and Smith (2017), which obtain a sequence of lower and upper bounds converging to the optimal objective value. These algorithms solve a relaxation of the BLO problem with disjunctive constraints generated from successive optimal solutions to the LL problem to obtain upper bounds. Feasible solutions are used to establish lower bounds. The subproblems to be solved in the operational framework of these algorithms are solved by adequate methods, namely by calling state-of-the-art mathematical programming solvers. In particular, in the application to general mixed-integer nonlinear (MINL) BLO problems, it is assumed that the functions in the formulations of the UL and LL problems satisfy the requirements of the MINL programming solvers to be used.

Some methods are devoted to BLO problems with special features, including linear/nonlinear objective functions, having just continuous or just integer variables in one or both levels, UL variables not appearing in the LL problem, the functions of integer UL variables in the LL constraints being also integer valued, etc.

For linear BLO problems, there are different approaches that involve some form of vertex enumeration in the context of the simplex method. The approaches built on the exploration of vertices in this type of problems are based on the property that only vertices of the constraint region (comprising all the constraints of the UL and LL problems) need to be considered for the computation of the optimal (optimistic) solution to a linear BL problem. A popular method in this category is the *k*-th *best* algorithm from Bialas and Karwan (1984).

The methodological and computational difficulties to solve BLO problems led to the development of metaheuristic and hybrid approaches, namely population-based ones such as evolutionary algorithms, particle swarm optimization, and differential evolution. In these approaches, metaheuristics perform the search at both levels, possibly using specific features of the problems to improve the search breadth capability, or they are combined with classical methods (e.g., a MILP solver) to obtain (optimal) solutions to the LL problem for each instantiation of the UL variables. Exact solvers guarantee the optimality of the LL solution for each UL variable setting (provided the MIP gap is zero, which may be not possible for difficult LL problems), which is not ensured in pure metaheuristic approaches. The population-based algorithms (pure or hybrid metaheuristics), in which the LL optimization problem is solved for each and every UL member, are called nested methods by Sinha et al. (2018). Other metaheuristic algorithms have also been developed using different schemes, for instance to deal with the problem reformulation after applying the KKT conditions of the LL to reduce the BLO problem to a single-level problem.

The extension of BLO models to account for the time dimension as well as considering uncertainty in several parameters (arising for instance, in equilibrium problems among different agents in open markets) further complicates the development of BL algorithmic approaches, which should be able to deal with multi-period stochastic models where equilibrium under uncertainty is sought in each node of the multi-period scenario tree. Techniques as the stochastic nested decomposition matheuristic proposed by Escudero et al. (2020) to deal with a network expansion planning may be useful in those problems.

Surveys on classical methods can be found in Vicente and Calamai (1994), Bard (1998), Dempe (2002), Colson et al. (2005), Colson et al. (2007), Dempe et al. (2015), Sinha et al. (2018). The latter work presents a recent review on classical and evolutionary approaches to (single and multi-objective) BLO.

3 Modeling demand response

The efficient management of network assets, including the deferral of infrastructure investments, the need to accommodate larger shares of variable renewable generation and the deployment of smart meters offering bidirectional communication capabilities, has led to an increasing active role of end users and demand resources. According to the U.S. Department of Energy, the concept of demand response (DR) encompasses "changes in electric usage by end-use customers from their normal consumption patterns in response to changes in the price of electricity over time, or to incentive payments designed to induce lower electricity use at times of high wholesale market prices or when system reliability is jeopardized" (US Department of Energy 2006). That is, DR materializes the flexibility consumers generally have in the operation of their loads regarding the timing of operation and the level of quality of the energy service being provided. Eurelectric (2014) defines flexibility as "the modification of (…) consumption patterns in reaction to an external signal (price signal or activation) to provide a service within the energy system. The parameters used to characterize flexibility in electricity include: the amount of power modulation, the duration, the rate of change, the response time, the location etc.". DR is expected to bring benefits for all stakeholders involved, contributing to improve the global system efficiency, lower peak generation costs, facilitate the penetration of renewable sources, reduce network losses and delay network reinforcement investments, while offering economic benefits to end users by exploiting the flexibility they may have in the operation of some of their appliances in response to prices or event (e.g., emergency) notifications. Active DR programs play a key role to improve power system reliability and market efficiency. Load modulation may be called for by a utility company, such as an independent system (grid) operator (ISO), transmission system operator (TSO), distribution system operator (DSO), retailer, or third-party aggregators. Aggregators are emerging as relevant entities in power system management, leveraging and monetizing the consumption flexibility of small consumers through the participation in system and energy/reserve markets.

DR is generally categorized in two broad groups: *price-based* and *incentive-based*.

Price-based schemes offer time-differentiated rates, which may depend on the time of the day, day-of-week, or season. The possibly significant magnitude of price variation is expected to lead consumers to voluntary adapt the timing of load operation to make the most of lower price periods. These rates may consist of time-of-use (ToU) pricing, critical peak pricing, peak load pricing, real-time pricing and variations thereof. The simplest ToU tariff considers just static peak and off-peak prices, being contracted for long periods (e.g., 1 year) so customers know the prices well in advance, which in principle are attractive in off-peak and comparatively high in peak demand hours. Consumers can, therefore, adapt their consumption patterns accordingly. This type of ToU tariffs may, however, be more complicated: present more price levels (e.g., one for each hour), thus enabling to better adapt retail prices to wholesale market prices helping to mitigate their spikes, renewable energy availability, and grid status (e.g., congestion); and be dynamic, for instance, announced 1 day or 2 days ahead. Critical peak pricing is used to anticipate high demand peaks, in general with a maximum number of events and event duration, in which the ratio of on-peak to off-peak price is higher than in a ToU scheme. Real-time pricing is the limit situation, in which prices generally reflect the ones in the wholesale market with very frequent variations possibly with significant price differences along the day, which may be announced within very short notice (e.g., 15 min). This pricing scheme may be advantageous for commercial and industrial consumers able to adjust operations to make the most of lower price periods that may involve premium/discount associated with consumption above/below a baseline, which requires advanced energy management and communication systems to optimize real-time energy usage in face of forward prices. Other variations of ToU may exist, such as block-and-swing pricing, which consists of a combination of fixed and real-time pricing—a certain amount of load is subject to fixed prices and additional load changes with market prices.

Incentive-based schemes include direct load control, interruptible load contracts, peak time rebate, demand bidding/buy back, emergency programs, capacity, and ancillary service markets. The incentives offered to end users, in addition to retail electricity rates, may be fixed or time varying for the actions required. Direct load control programs have been primarily directed for residential and small service consumers through signals that remotely cycle the operation of thermostatic-controlled loads (for which the interruption of supply during short periods does not jeopardize the energy service provided due to some thermal inertia). Several types of interruptible service (curtailable load) contracts have been used since the 1980s, namely for large commercial and industrial consumers able to reduce at least a certain amount of load per event within a short period notification (half hour to two hours), involving rewards/penalties and constraints on the number of total interruption hours, guaranteed minimum load curtailment and payments for availability. Peak time rebates allow participant consumers to receive a rebate payment for load reduction below an estimated baseline, otherwise they pay the contracted price. Demand bidding /

buy-back programs allow (large) consumers to offer load reduction bids in wholesale markets, whose price competes with the market price. DR resources may bid for some ancillary service products, such as synchronous and non-synchronous reserve, through load curtailment for a given period. Combinations of these schemes can be used (e.g., emergency DR), which may involve direct load control and interruptible load. At large, incentive-based schemes account for the vast majority of peak load reduction among the array of DR schemes, namely for reliability triggered events.

Several configurations of these DR mechanisms exist, involving different types of contracts and corresponding rewards and penalties, being voluntary or mandatory, having opting-in or opting-out conditions. The Federal Energy Management Program developed profiles of DR and time variable pricing programs throughout the U.S. (https://www.energy.gov/eere/femp/demand-response-and-time-variable-pricing-programs). It should be noticed that ToU tariffs and DR programs may lead to benefiting non-participant consumers with consumption patterns more fitted with the prices, while participants may be penalized from not being able to shift base consumption away from peak price periods. Also, a negative impact on low-income consumers may exist, since they cannot generally reduce or shift demand. These issues should be accounted for when designing DR programs, namely by incorporating them into optimization models.

Vardakas et al. (2015) offer an extensive review of DR programs in smart grids, classifying them in three categories based on control mechanisms (centralized vs. decentralized), motivations offered (price vs. incentive based) and decision variables (task scheduling vs. energy management). Jordehi (2019) makes a review of research works on DR mainly from the perspective of the optimization algorithms used.

Several single- and multi-objective models to optimize DR, from the consumer's point of view, have been proposed in the scientific literature, namely in power systems and operations research / optimization journals. The objective functions in those models encompass economic, technical and quality of service concerns, such as minimizing the electricity cost, minimizing peak power, maximizing the utility (i.e., level of comfort/satisfaction) associated with energy consumption (a proxy for energy services delivered), minimizing the discomfort associated with changing the habitual time slots of appliance operation (e.g., laundry or dishwasher) or having a control variable (e.g., indoor or water temperature, state of charge of the EV battery) outside user-defined comfort thresholds. The discomfort may be measured in physical quantities (e.g., delay in load operation beyond the expected time, period the load operates outside those thresholds regarding time or temperature, or both) or may be monetized and aggregated in an overall cost function (which implies assessing, for instance, the cost of having the indoor temperature one degree above or below those thresholds). A quadratic utility function is typically used, which displays linear decreasing marginal benefit associated with energy consumption. Some models also integrate the optimization of specific DR actions with local microgeneration (in general, photovoltaic panels) and storage (static batteries or the EV battery operating in grid to vehicle—G2V—or/and vehicle to grid—V2G—modes). Moreover, the possibility of selling back energy to the grid or participating in ancillary markets (e.g., reserve or frequency regulation) is also considered.

In addition to the consumer's perspective, according to the type of application envisaged, DR optimization models can also be seen in the perspective of the grid operator (DSO, ISO, and TSO), the retailer, or a third-party aggregator. In these cases, objective functions to be considered include the maximization of social welfare (which may be broadly defined as end user's utility minus the procurement capacity cost and day-ahead/real-time balancing energy costs, and may also include storage operational costs; Deng et al. 2015), maximization of economic benefit, minimization of network losses, maximization of the system reliability (e.g., voltage control), minimization of the load diagram peak-to-average ratio (as a proxy for grid efficiency and stability), maximization of the use of renewable energy resources, etc. Special attention has been devoted to DR in the framework of optimizing grid-connected/islanded microgrid operation, in which the integrated optimization of all energy resources is of utmost importance.

In a residential setting, controllable appliances can be classified as shiftable (loads having an operation cycle associated with each program that cannot be interrupted once initiated; e.g., laundry machine or dishwasher), interruptible (loads whose operation can be interrupted as long as a given amount of energy is supplied during a specified time slot and possibly can be charged at different power levels; e.g., the battery of an EV), and thermostatically controlled (being switched on/off or possibly being supplied at a fraction of the nominal power according to the parameterization of a thermostat; e.g., air conditioning system). In general, lighting, oven, refrigerator, etc., are not deemed for control and constitute the base (uncontrollable) load. Therefore, optimization models should consider a realistic physical-based modelling of appliances and not resort to excessive simplifications. For instance, it is rather simplistic to just consider that an appliance should be supplied with a given amount of energy for service completion, as it happens in some models, without modelling the actual operation cycles. Since peak demand is a crucial issue in the optimization problem, it is not realistic to consider a load diagram as the one depicted in Fig. 3b when the actual operation load diagram is the one in Fig. 3a, although the amount of energy consumed is the same.

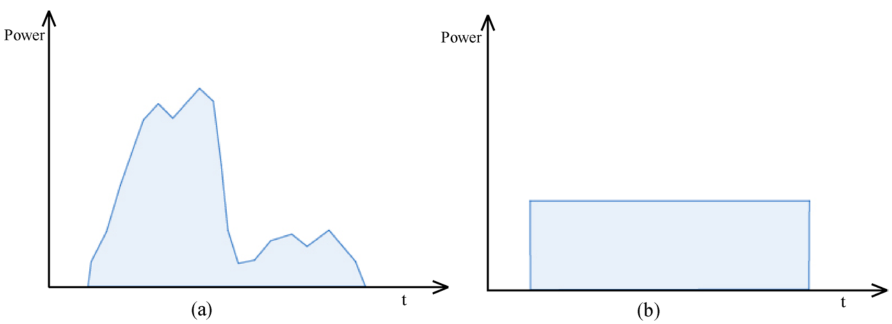

Fig. 3 Load diagram associated with real operation and simplification considering that the same amount of energy is supplied

Therefore, realistic load characterizations are necessary to model DR accurately. Approaches that rely on the existence of a utility function representing the end user's benefit/satisfaction of consuming a certain amount of energy for the provision of energy services, which is rather difficult to elicit, are not able to capture consumer's decisions at appliance level, considering their typical operation patterns and information about the time slots preferred for operation. Modelling approaches using a penalty function associated with the deviation from the "optimal" consumption point suffer from the same drawbacks. However, the physical-based modelling of load operation comes at the cost of increasing the combinatorial complexity of the models, requiring many auxiliary binary variables and constraints, which may turn impracticable obtaining an optimal solution (for single objective models) or Pareto optimal solutions (for multiple objective models) in an acceptable computation time.

The next section offers a review of selected problems and optimization models in the power sector where DR plays a relevant role, which is captured by means of BLO approaches. The context of the problem, the identification of the leader and the follower as well as their concerns operationalized through their objective functions, and the techniques used to obtain solutions are described.

4 Bilevel optimization models including demand response

4.1 Designing ToU tariffs–retailer-consumer interaction

In retail market pricing problems, the retailer (in general operating in a competitive environment) aims to design a ToU tariff structure to be offered to consumers to maximize profit (revenue obtained with selling energy minus cost of acquiring energy through bilateral contracts and/or in day-ahead/balancing markets). The consumers, who are willing to adopt this type of tariff because they consider they can manage their flexibility in appliance utilization, react by resetting/rescheduling (shiftable, interruptible and thermostatic) appliance operation to lower priced periods to minimize the electricity bill and/or to minimize the dissatisfaction (or maximizing comfort regarding preferences and requirements). This dissatisfaction objective function may encompass postponing or anticipating the operation of some appliances regarding most habitual periods, having indoor temperature below/above a comfort threshold, not reaching the desired state of charge of the EV battery, etc. Consumer's decisions may be assisted by a home energy management system endowed with optimization algorithms parameterized with user's preferences and requirements. Therefore, the retailer is the leader, who decides first by announcing the ToU prices, and the consumer is the follower who decides his load operation in face of those prices. Depending on the configuration of the power sector, the interaction can refer to the relationships between a utility company and a demand response aggregator.

Zugno et al. (2013) developed a BLO model in which the UL problem consists of determining the dynamic prices to maximize the retailer's expected market profits (revenues of selling energy to consumers, minus the cost of purchasing energy at the spot market and the cost/profit of purchasing/selling up/down-regulation power). Constraints include lower and upper bounds on prices and an average price as a surrogate for market competition (otherwise the retailer would set the prices at the upper bound to maximize profits). The consumer's objective is a utility function considering the cost of energy procurement and the discomfort for deviating from the reference temperature (a flexible heating load is considered). Stochastic prices, weather data and must serve load are considered. The BLO model is transformed into a single-level MILP problem using the KKT conditions, which are linear except for the complementarity conditions that are linearized (cf. Sect. 2.3).

Afsar et al. (2016) presented a BLO model in which the energy provider (the leader) aims to determine hourly prices to maximize revenue minus a penalty associated with peak consumption. The consumers (the followers) aim to minimize overall disutility consisting of the electricity bill and an inconvenience cost (which is proportional to the length of the delay in starting load operation to profit from lower prices and inversely proportional to the width of the desired operation time window). The complementarity constraints of the KKT conditions of the LL problem are linearized to yield an equivalent MILP formulation.

Soares et al. (2020) proposed a comprehensive BLO model in which the retailer establishes ToU prices (UL variables) to maximize the profit and a cluster of consumers (with the same consumption profile) reacts to these prices by determining the operation of the controllable loads to minimize the electricity bill and a monetized discomfort term associated with the indoor temperature deviations. The consumer optimization model encompasses the operation of shiftable, interruptible, and thermostatic loads considering their physical characteristics. The BLO model is dealt with a hybrid approach: a PSO algorithm searches for UL solutions and calls an exact MILP solver to address the LL problem for each price instantiation. The inclusion of the thermostatic load in the LL problem imposes a higher effort, being impossible to solve it to optimality with a certain computational budget. Since a sub-optimal LL solution is infeasible to the BLO problem, a procedure is developed to compute good estimates of bounds for the UL objective function, thus providing the retailer further information to make sounder decisions.

Alves et al. (2020) compared two BLO models to design ToU tariffs with profit maximization as the retailer's objective function, and energy and power costs associated with the operation of shiftable and interruptible appliances as the consumer's objective function. Whereas in the first model, the periods in which prices apply are pre-defined and the aim is to determine the price values, in the second model, the periods and prices are decision variables, thus leading to a very large search space for the UL problem due to the number of combinations of periods–prices. For the latter model, a hybrid approach was developed combining a GA for the UL search, using specific encoding as well as crossover and mutation operators to make the most of the physical features of the problem, with a MILP solver to obtain optimal solutions to the LL problem.

This type of models can be extended for multiple players at one of both levels and for more than two levels. In some cases, a Nash equilibrium is sought between multiple players at the same level.

Meng et al. (2018) considered a BLO model in a smart grid context where an electricity retailer serves three different types of customers (with a home energy management system coupled with smart meters, with smart meters only and without smart meters). The retailer aims to decide day-ahead dynamic prices to maximize profit and customers (displaying different price responsiveness) adjust their energy use to minimize costs. Interruptible, non-interruptible, and curtailable appliances are considered. The consumption scheduling of interruptible and non-interruptible appliances is modeled as integer programming problems, while that of curtailable appliances is formulated as a linear program. The BLO problem is tackled by a GA to coordinate the solution of the UL and LL problems.

Soares et al. (2019) presented a BLO model considering a single leader (the retailer), who aims to establish a ToU pricing scheme to maximize the profit, and multiple independent followers (clusters of residential consumers) with different ownership rate of (interruptible and shiftable) appliances and different consumption patterns, who aim to determine appliance scheduling to minimize their electricity bill. Contracted power constraints are considered at the UL problem to avoid undesirable consumption peaks. Two approaches based on GA and PSO are proposed to perform the UL search, then calling a MILP solver to tackle the LL problem.

Kovács (2019) developed a multi-follower model, in which the retailer defines ToU tariffs to maximize profit and followers are groups of prosumers who respond by scheduling controllable loads and defining the battery charging/discharging strategy to minimize the cost of electricity and maximize their utility (as an aggregate objective function). The algorithmic approach consists of a primal–dual reformulation of the linear LL problem to convert the BLO problem into a single-level quadratically constrained quadratic program. The nonlinearities arise in the leader's objective function and in the optimality constraint derived from the primal–dual reformulation. The nonlinear problems are solved by iteratively constructing local linear approximations of the original problem. The author concludes that this approach outperforms methods based on the problem reformulation using the KKT conditions, regarding both solution quality and computational efficiency on practically relevant problem sizes.

Aussel et al. (2020) proposed a trilevel multi-leader multi-follower model for load shifting induced by ToU pricing. The energy supplier defines time-differentiated prices to which the consumers adapt by shifting their loads, either directly through local agents or indirectly through aggregators. These two latter players are in an intermediate level between suppliers, at the UL, and consumers, at the LL. The tri-level problem is transformed into a single-level optimization problem with complementarity constraints. First, the consumers' problems are appended to the aggregators' problems to obtain a BLO reformulation, which is then tackled by replacing the followers' problems by their KKT conditions in the leader's problem.

Luo et al. (2020) developed an energy scheduling model for a trilevel integrated energy system consisting of one electricity utility company and one natural gas utility company (UL), multiple smart energy hubs that can produce electricity and heat (middle level) and multiple consumers (LL). The utility companies and the hubs aim to maximize profits, whereas the end users aim to maximize their utility with the amount of energy (electricity and gas) consumed. A decentralized algorithm is developed to determine the energy prices in the market, in which participants decide their operation strategies according to the announced electricity prices.

Feng et al. (2020b) modeled the Stackelberg relationship between a profit-maximizing retailer (leader) and the strategic consumers (followers) in an incentive-compatible market as a BLO problem. The retailer aims to maximize its profits by providing diverse types of price schemes, considering the revenue from consumers, the acquisition costs in forward contracts and day-ahead markets, and the loss risk using the conditional value at risk. Individual consumer's preferences are modeled using a utility function representing consumer's satisfaction (a concave function to indicate decreasing marginal utility). Consumers with similar preferences are clustered to be offered the same price scheme, thereby reducing the number of choices. Linear transformations are used to account for nonlinear terms and simplify the conditional value at risk function.

Although most models encompass the consumer's comfort in an overall utility objective function associated with energy consumption levels and energy service provision, the explicit consideration of economic and quality of service objective functions enables to exploit the trade-offs between these evaluation aspects of different appliance operation decisions. Alves and Antunes (2018) proposed a model in which the retailer (UL) establishes ToU prices to maximize profits and the consumer (LL) responds by selecting, under that price setting, a load scheduling decision leading to a Pareto optimal solution balancing the minimization of the electricity bill and the minimization of the dissatisfaction associated with postponing or anticipating load operation to different periods in face of routines and preferences. The model is tackled using a hybrid approach consisting of a GA for the UL problem and an exact solver to solve surrogate scalar problems (i.e., combining both objective functions) at the LL. Since there are multiple objective functions at the LL, a set of "extreme" solutions is computed—the optimistic, pessimistic, deceiving, and rewarding solutions mentioned above. Each of these solutions results from the retailer's optimistic/pessimistic position and the possible consumer's reaction more or less favorable to the retailer.

4.2 Impact of demand response on network and generation planning

BLO models are also useful to take into account the impact of DR on network and generation planning. ToU tariffs may be established just for the network access component of the price to minimize peak load. In general, network access tariffs are defined by the energy regulator and are paid by all electricity consumers regardless of the retailer. In addition to tariffs associated with the use of the distribution grid and the transmission grid, these tariffs may also encompass a global system use tariff, which include political costs for different purposes (e.g. promotion of energy efficiency programs, feed-in tariffs for renewable generation, etc.). Since the periods of present tariffs may not adhere to the actual consumption levels, and, therefore, to the levels of utilization of the distribution network, DSOs strive for the regulator to allow giving stronger price signals associated with the high grid cost in the periods of higher consumption considering distributed generation and power flows.

Thus, well-designed dynamic ToU tariffs contribute to reduce losses in networks at all voltage levels, with impacts on deferring investments on network equipment and on the secondary reserve markets.

Zhang et al. (2016) developed an integrated generation–transmission expansion planning model considering DR impacts with potential limits and operation requirements. The UL deals with the planning problem, formulated as a MILP model whose objective function is the minimization of the overall cost consisting of investment in generation and transmission expansion, operation costs of the generation units and cost of carbon emissions. The LL is a unit commitment problem including the dispatch of load curtailment and load shifting induced by DR to find the optimal schedule for daily operation, which is formulated as a nonlinear model with the objective of minimizing the overall cost including fuel cost, generators' start-up cost and shut-down cost, and incentive cost of DR (which is aimed at lowering the peak load with a positive impact on system planning).

Asensio et al. (2017) presented a BLO model for the integrated distribution network and renewable energy expansion planning considering short-term real-time pricing DR, which is modeled as elastic demand functions calibrated by load levels. The distribution network and generation planner (UL decision-maker) aims to minimize the generation and network investment cost to meet demand, including investment, maintenance, energy purchase from substations and distributed generation production, and unserved power. Constraints relate to power flow and generation limits. At the LL problem, consumers aim to minimize overall cost in face of time-varying prices. This problem is linear since, in the objective function, the DR component only depends on the price difference between adjacent load levels, and power balance equations and DR constraints are linear. The substation prices are determined at the UL, which are the linkage between both levels, being then used to define the maximum and minimum amount of shiftable load for every load level. The BLO model is recast as a MILP using the KKT conditions of the LL problem appended to the UL problem.

4.3 Demand response in electric mobility

The increasing trend of electric mobility creates challenging problems to the power sector where DR has a relevant role, which can be modeled as BLO problems. Strategical (location, sizing, etc.) and operational (charging/discharging schedules, etc.) problems are at stake as the load profile in distribution networks is significantly changed because of EV, with impacts on the grid reliability. Moreover, EV are valuable to make a better management of larger shares of intermittent generation based on renewable sources by exploiting the possible flexibility of charging profiles associated with mobility patterns. Energy exchanges (in G2V and V2G modes) between the grid and clusters of plug-in EV involve pricing decisions to maximize profits while vehicles' owners choose the battery charging strategies to minimize their costs and/or maximize their benefits (e.g., availability of the mobility service, sales to the grid).

Yoon et al. (2016) presented a single-leader multi-follower BLO model for at home EV charging considering DR. The electricity retailer is the leader and the EV owners are the followers. The retailer establishes the prices aiming to maximize profit subject to EV charging requirements. The consumers' demand is flexible and shaped according to the electricity price according to a utility function, which expresses the degree of satisfaction when an appliance charges a certain amount of electricity in an hour following a quadratic function with linearly decreasing marginal satisfaction. The model seeks to obtain a balance between a minimum generation cost solution and an equal-charging scheme, which depends on the weighting factor for the utility function of each consumer. The optimum policy aims at minimizing the costs only, whereas the equal-charging policy attempts to charge electricity at an equal rate throughout a given period.

Li and Li (2019) proposed a BLO scheduling approach for isolated microgrids with renewable generation considering DR provided by EV under real-time pricing. In the UL problem, the decision-maker is the microgrid manager aiming to minimize the net operating cost, which includes the fuel cost of microturbines and the cost of spinning reserve provided by the microturbines and energy storage systems. In the LL problem, the EV owners aim to minimize the charging cost. A hybrid solution algorithm called JAYA interior point method is developed to solve the model through an iterative process between levels.

Sadati et al. (2019) developed a BLO model for operational scheduling of a distribution company with an EV parking lot and renewable (PV and wind) energy sources. The parking lot is able to sell energy to the grid. The UL objective function is the maximization of the company's profit including the cost of power purchased from the wholesale market. The LL problem aims to maximize the profit of the parking lot owner. The model is reformulated as a nonlinear single-level problem by applying the KKT conditions to the LL problem, which is then linearized. Uncertainties as the length of the stay of EV in the parking lot, the initial state-of-charge of EV and power generation of wind and PV units are considered through a set of scenarios. Price-based, incentive-based, and combined DR programs are considered with different arrangements of renewable generation and smart charging/discharging of EV.

Salyani et al. (2019) proposed a BLO for planning distributed generators and EV parking lots considering DR under uncertainty. The model has one leader (the distribution company) and multiple followers (the consumers). The leader aims to planning the distribution network (siting and sizing of microturbines and parking lots) to maximize its payoff. The followers are interested in adopting the DR program to maximize their own utility functions representing the perceived benefit of consumption during the day. The cost component of the leader's objective function is associated with microturbines investment, maintenance and operation costs and the parking lots' capital, repair and maintenance costs, as well as the energy purchasing cost. The distribution company pays EV owners for V2G operation. The benefit component relates to the energy sold to DR participating and nonparticipating consumers, with dynamic energy selling prices set for 24 h with DR participating consumers shifting their demand schedule according to their utility function. The load shifting performed by responsive loads results in decreasing the overall planning cost. A predictor corrector proximal multiplier iterative algorithm has been used to compute solutions.

Rui et al. (2019) considered a distributed charging model based on day-ahead optimal internal price for an EV charging station powered by the grid and PV, which is devoted to industrial and commercial workplaces. The charging station operator is the leader aiming to maximize profit and the users are the followers aiming to minimize their individual costs. Users send their charging demand information to the charging station one day-ahead, according to price and their energy needs. A real-time billing strategy is proposed, which takes into account forecasting errors in PV generation and charging arrangements. The LL problem is represented by a constrained nonlinear model. The UL problem consists of determining the prices to maximize the profit including the revenue from EV users, subsidy to distributed PV generation, revenue/expenditure of selling/buying energy to/from the grid. Differential evolution and particle swarm optimization metaheuristics have been used for obtaining solutions.

4.4 Demand response in microgrid planning and operation

Microgrids are gaining a growing importance in power systems, interconnecting distributed generation, namely based on renewable sources, energy storage systems, as well as controllable loads within defined electrical boundaries. Microgrids are a single controllable entity with respect to the main grid and can have grid-connected or autonomous (islanded) modes of operation. BLO models are useful to model the hierarchical relation between different stakeholders related to planning and operational problems in microgrids, including the interaction with the upstream grid system operators and the participation in different types of markets.

Asimakopoulou et al. (2013) investigated the impact of the participation of demand resources in the energy market via an energy service provider, acting as an intermediary between the retail and the wholesale markets that manages several microgrids comprising controllable loads and dispatchable distributed generation units. Part of the microgrid load is served by the distributed generation units that submit production bids (LL problem) and the rest is served by a central production unit (UL problem) participating in the wholesale market. At the LL, the energy service provider seeks the optimal combination of generation mix (distributed generation and central production unit) and load curtailment to maximize the profit. At the UL, the central production unit decides upon the profit margin to minimize costs, considering the optimal response of the energy service provider to the market prices. The BLO problem is transformed into a single-level problem by replacing the LL convex programming problem with the KKT conditions, which is then tackled using a nonlinear solver.

Alipour et al. (2018) developed a multi-follower BLO approach to deal with energy management in a combined heat and power (CHP) microgrid in grid-connect mode. The leader is the microgrid operator who aims to maximize profit associated with forecasted demand considering DR programs as well as participation in day-ahead and real-time markets. The followers are the CHP operators who aim to maximize profits obtained from the thermal and electrical energy sales. Network operation constraints as bus voltage magnitude and line flow limits are considered. Both UL and LL problems are formulated as a stochastic two-stage problem, to capture the uncertain nature of consumers' loads, wind generation, and real-time market price. The BLO model of each CHP operator is transformed into an MPEC, replacing each LL problem by its KKT conditions, whose nonlinear complementarity terms are then linearized.

Quashie et al. (2018) presented a BLO model for a coupled microgrid power and reserve capacity planning problem. The UL decision-maker is the microgrid planner whose goal is to optimize the design configuration and power output of distributed energy resources to minimize planning and operational cost. The dispatch set points also include the hourly energy available for DR (as a dispatchable resource). The LL decision-maker is the DSO, who aims to maximize the capacity of flexible reserve resources to ensure reliable power supply. The LL problem is linear and a primal–dual reformulation is used to recast the BLO model as a MPEC, which is then transformed into a MILP model by linearizing the nonlinear terms.

Li et al. (2018) developed a BLO model to determine the optimal day-ahead scheduling between of an isolated microgrid and EV battery swapping stations in multi-stakeholder scenarios. The aim is promoting the participation of these stations in regulating the isolated microgrid operation. A real-time pricing mechanism based on DR is designed considering dynamic supply–demand relationships between the microgrid and the stations. The UL problem consists of the minimization of the isolated microgrid net costs associated with scheduling schemes, comprising the charge/discharge cost, the spinning reserve cost, the fuel cost of microturbine units, and the cost of reserve provided by the storage system. The LL problem is related with the maximization of the profits of the stations associated with charge/discharge scheduling under real-time pricing schemes determined by DR in the UL decisions. The LL objective function comprises charge/discharge costs, swapping incomes, battery depreciation costs and reserve incomes. A hybrid algorithm is proposed, called JAYA branch and bound. This approach combines a real/integer-coded JAYA algorithm to deal with the UL problem and a branch and bound algorithm to address the LL problem, which alternate iterations between the two levels to provide the solution to the BLO problem.

Haghifam et al. (2020) presented a multi-follower BLO framework for the operational scheduling of smart distribution networks. The leader is the DSO aiming to minimize operating costs, subject to active and reactive power balance, voltage limits, and line power constraints. The followers are the demand response aggregator (DRA) and microgrid owner (MGO) aiming to maximize the respective profits. The DSO should determine the optimal amount of power purchased from the upstream grid and exchanged with the DRA and the MGO. At the LL, the DRA should determine the optimal amount of power exchanged with the DSO and the customers, whereas the MGO should determine the optimal amount of power exchanged with the DSO and the operation point of the energy resources (CHP, wind turbines, photovoltaics, and storage systems). The BLO model is converted into a linear single-level problem using the KKT conditions and the linearization of its complementarity components.

4.5 Market participation of demand response

The participation of DR in different types of markets has potential advantages due to its capability to respond to grid operator requests, including in spot markets in which electricity is traded for the day-ahead, intraday and balancing markets with a typical time scale of one hour, flexibility markets in which flexibility is activated in (near) real time, and reserve markets in which flexibility capacity is traded for larger time horizons (e.g., several days). Demand side flexibility may be used for up-regulation (reduce load, which implies assessing the revenue from selling to the market vs. the cost associated with loss of comfort) or down-regulation (increase load, which may be only profitable with very low or even negative market prices). Aggregators play the role of intermediaries between small consumers and the system operators and/or the market, enabling to exploit the small consumers' flexibility potential and offering increased flexibility volumes to the market. BLO models are useful to capture the market feedback mechanism, in general considering the LL problem as a market-clearing problem, whereas the UL problem relates to some type of more strategic decisions.

Asimakopoulou et al. (2015) proposed a BLO model for supporting the decision-making process of an aggregator, who is the leader that decides the price signals, in face of the reaction of distributed energy resources, whose interaction is determined by a bilateral contract. The UL problem involves determining the optimal pricing scheme for the energy bought from/sold to the local resources, as well as determining the energy volumes from the wholesale market and the charging/discharging of the energy storage system. The UL objective function is the energy supply net cost (expenses associated with buying energy from the wholesale market and the local production units and for remunerating the curtailable loads minus revenues from selling energy to the consumers with flexible loads). In the LL problem, consumers (with load bids and curtailable loads) and producers (with production bids) decide price/quantity pairs to maximize benefits. The BLO model is transformed into a single-level model adding the KKT conditions of the LL linear problem to the UL problem, then linearizing bilinear products thus resulting in a MILP problem.

Wei et al. (2015) considered a retail market composed of two stages in which the retailer (who also owns storage facilities) acts as an intermediary between the wholesale energy market (being a price taker) and consumers in the retail market (being a price-maker). In the first stage, the retailer decides retail prices to maximize profits and end users decide their consumption patterns to minimize costs. In the second stage, the retailer manages the operation of storage units and the energy contracts in the energy market after the consumers decide their demand patterns in the first stage, being the energy dispatch modeled as a linear max–min problem associated with the worst-case realization of market prices. The KKT conditions are used to reformulate the consumers' problems, which are then linearized.

Saez-Gallego et al. (2016) presented a market-bidding problem of a pool of price-responsive consumers. The price response of flexible loads is captured by means of a stepwise marginal utility function, maximum load pick-up and drop-off rates, and maximum and minimum power consumption, in a similar approach to the energy offers made by power generators. Therefore, the electricity price is the result of a competitive market-clearing process and not a retailer's or aggregator' decision variable, thus consumers are exposed to the wholesale market price. The aggregator is the UL decision-maker, who aims to determine the parameters of the market bid relative to the aggregated pool of consumers, to minimize the estimated absolute value of the prediction error (i.e., the optimal consumption resulting from this problem should be as close as possible to the measured consumption in terms of a certain norm). In the LL problem, the price response of the pool of consumers is modeled in the form of a market bid, parameterized by the UL variables, to maximize consumers' welfare (difference between the total utility and the total payment). Since this problem is linear, it is recast using the KKT conditions to develop a single-level problem.

Mahmoudi et al. (2016) developed a BLO model, in which the leader is a wind power producer who aims to decide the offers in the day-ahead market and the price to offer to the aggregator by the DR product. The followers are related with the strategic behavior of the producer in the day-ahead market modeled through the market clearing process (volume and price) and the aggregator behavior modeled through a revenue function associated with selling DR to the wind power producer, other competitors and the day-ahead market. The overall problem is a stochastic MPEC in which wind generation and imbalance prices are uncertain, which is transformed into a single-level problem by replacing the LL problems with their KKT conditions.

In the BLO model proposed by Sekizaki et al. (2016), the leader (retailer) determines the day-ahead prices to maximize the expected profit under uncertainty associated with spot and real-time prices. The followers (residential, commercial, and industrial consumers) then schedule their loads according to those prices offered to minimize the sum of the purchasing and the disutility cost associated with the load suppressed. A cost component of the retailer's objective function is the network tariff which is determined by the distribution line loss. A GA is used to obtain approximate solutions to the BLO problem.

Jia et al. (2018) developed a BLO model for optimal bidding of a flexible load aggregator (encompassing distributed storage systems, electric vehicles, and thermostatic loads) in day-ahead energy and reserve markets. In the UL problem, the aggregator aims to maximize profit, whereas in the LL problem, the independent system operator aims to determine the bidding of generation companies and the aggregator to clear the market maximizing social welfare. The LL problem is replaced with its KKT conditions and the BLO is transformed into a single-level MPEC, which is solved with a MILP solver.

Feng et al. (2020a) proposed a BLO model to optimize the transactive price signal representing the impact of wholesale market locational marginal prices on retail customers' DR participation. The electricity utility company at the UL determines the optimal day-ahead strategy for bidding in the wholesale electricity market using the offers by demand response aggregators (DRA). The optimal price signal is used to dispatch the DR resources, also considering energy procurement from distributed energy resources, to maximize the worst-case realization of its payoff (utility minus costs) in face of uncertain wholesale market prices. At the LL, each DRA adjusts its electricity consumption using the transactive price signals set by the utility company, competing with the other DRA to maximize its payoff function with respect to operation constraints (to form a Nash equilibrium). The BLO model is transformed into a mixed integer quadratically constrained programming model using the KKT conditions. The linearization of the bilinear terms in the KKT conditions is dealt with the McCormick relaxation (McCormick, 1976) and big-M disjunctive constraints.

Bruninx et al. (2020) formulated a BLO problem for the strategic participation of a price-making aggregator in the day-ahead electricity market (Stackelberg game) and the interaction of the aggregator with its consumers. These DR providers may display real-time deviations from an expected DR load profile modeled as chance constraints (Stackelberg or Nash bargaining game). The aggregator at the UL maximizes the difference between the revenue from the consumers and the cost of procuring electricity in the wholesale market. Consumers at the LL minimize cost. The interaction between the aggregator and the market is also modeled as a BLO model, in which the aggregator is the leader bidding in the market and the market operator is the follower aiming to maximize the total surplus with respect to the bids and offers of the market participants. The problems are recast as MPEC and the nonlinearities in the complementary slackness conditions are linearized.

5 Conclusions

This paper presented a review of selected models and methods, as well as considerations regarding the application of bilevel optimization in problems involving consumers' demand response. It is expected that, in the evolution of power networks to smart grids, demand-side resources become increasingly responsive to dynamic pricing schemes, allowing for a better utilization of supply availability, network infrastructures, and demand flexibility. Bilevel optimization models are adequate to deal with the leader–follower structure of the interaction between stakeholders controlling different sets of variables, capturing the reactive nature of demand response in different settings. However, there is no interest in considering as a BLO model actual decision contexts in which cooperation is possible between the decision-makers, since it is advantageous for all stakeholders if a single-level multi-objective model can be adopted instead. Some papers in the literature are misleading in this regard, since they consider different decision levels (sometimes associated with the same entity) and decisions may even be sequential, but no leader–follower structure is actually at stake because the follower's decision does not influence the leader's outcome.

A review of selected applications focusing on model structure and solution techniques in problems in which demand response is a relevant component addressed the design of time-of-use tariffs dealing with the retailer–consumer interaction, the impact on network and generation planning, as well as in microgrid planning and operation, electric mobility, and forms of market participation.

BLO problems are very difficult to solve due to its inherent nonconvexity. The solution technique most widely used consists of the reformulation of the BLO problem as a single-level problem by replacing the linear lower level problem with its KKT conditions leading to an mathematical problem with equilibrium constraints, which can be transformed into a mixed integer linear programming by means of additional auxiliary binary variables and constraints to linearize the nonlinear terms. For the reformulation to be valid, big-M constants should be carefully chosen to be large enough for not excluding the optimal solution from the feasible space, but too large values may lead to poor computational efficiency in the solution by a MILP solver. The reformulation using the KKT conditions assumes an optimistic approach and the computation of the pessimistic solution is even more challenging. In the case of lower level problems with integer variables, the KKT optimality conditions cannot be used directly to make the usual reformulation as a single-level problem. In this case, decomposition techniques involving the iterative solution of master and slave problems may be of help, for instance using cuts (fixing the value of integer variables) enabling the use of the KKT conditions in the slave problems and the ensuing reformulation as a single-level problem.

Whenever multiple objective functions exist in the lower level problem, the uncertainty of the follower's decision is at stake and should be duly taken into account. In several contexts, the scalarization of the lower level problem by assuming the existence of a utility function may not be realistic. Therefore, it is necessary to identify solutions capturing the different leader's optimistic/pessimistic attitude and the follower's favorable/unfavorable response to the leader's decision.

A solution is feasible to the BLO problem only if it is optimal (efficient) for a single (multiple) objective lower level problem. However, it may be difficult to guarantee that solutions are indeed optimal (efficient) when using approximation algorithms, such as metaheuristics, to deal with strongly nonlinear or combinatorial problems. That is, apparently better solutions to the BLO problem may just result from approximate (even of good quality) solutions to the true optimal (efficient) solution(s). Whenever this type of approaches are required due to the characteristics of the BLO problem, the possibility of using an exact solver to deal with the lower level problem for each instantiation of the upper levels variables should be assessed, i.e. coordinating metaheuristics for the upper level search with exact mathematical programming algorithms to solve the lower level problem.

The changes underway in the power sector associated with the energy transition will continue to offer a fertile ground for the application of BLO models and algorithms. The empowerment of *prosumagers*, namely in the framework of emerging energy communities, is expected to give demand response a growing valuable role in the overall system efficiency at different segments of the whole value chain. BLO models are well suited to address the hierarchical nature of design and policy decision and operational decisions, possibly involving different (leaders/followers) stakeholders with potential conflicting interests. Novel and challenging applications as, for instance, managing congestion by exploiting the flexibility associated with the charge/discharge of electric vehicle batteries or enabling flexibility load aggregators to participate in ancillary services or capacity markets require innovative models and algorithmic approaches.

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