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Low-Pass Filter Freqeuncy Response

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1 Introduction

1.1 z-Transform

By applying the bilinear transform to the low-pass frequency response:

$$H(\omega)_I = \frac{1}{1 - i\omega\tau} \tag{1.1}$$

we obtain:

$$V(z) = \left(\frac{1-c}{2}\right)\frac{z+1}{z-c} = 2^{-k-1}\frac{(z+1)}{z-1+2^{-k}} \tag{1.2}$$

where the pole has been placed at $c = 1 - 2^{-k}$.

1.2 Difference equation

By rewriting the z-transform of the system response we get:

$$Y(z)\left(1 - \frac{c}{z}\right) = \left(\frac{1-c}{2}\right)\left(1 + \frac{1}{z}\right)X(z) \tag{1.3}$$

which becomes, in terms of difference equation

$$y[n] = \left(\frac{1-c}{2}\right)(x[n] + x[n-1]) + c \cdot y[n-1] \tag{1.4}$$

which becomes

$$y[n] = (x[n] + x[n-1]) >> (k+1) + y[n-1] - y[n-1] >> k \tag{1.5}$$

1.3 Cut-off frequency

In the limit $T << \tau$ we get

$$c = \frac{1 - T/2\tau}{1 + T/2\tau} \approx (1 - T/2\tau)(1 - T/2\tau) \approx 1 - \frac{T}{\tau} \tag{1.6}$$

which implies $\tau \approx 2^k T$. Therefore, the cut-off frequency f_{3dB} can be approximated as:

$$f_{3dB} = (2\pi\tau)^{-1} \approx \frac{2^{-k}}{2\pi T} = \frac{2^{-k-1}}{\pi} f_s \tag{1.7}$$

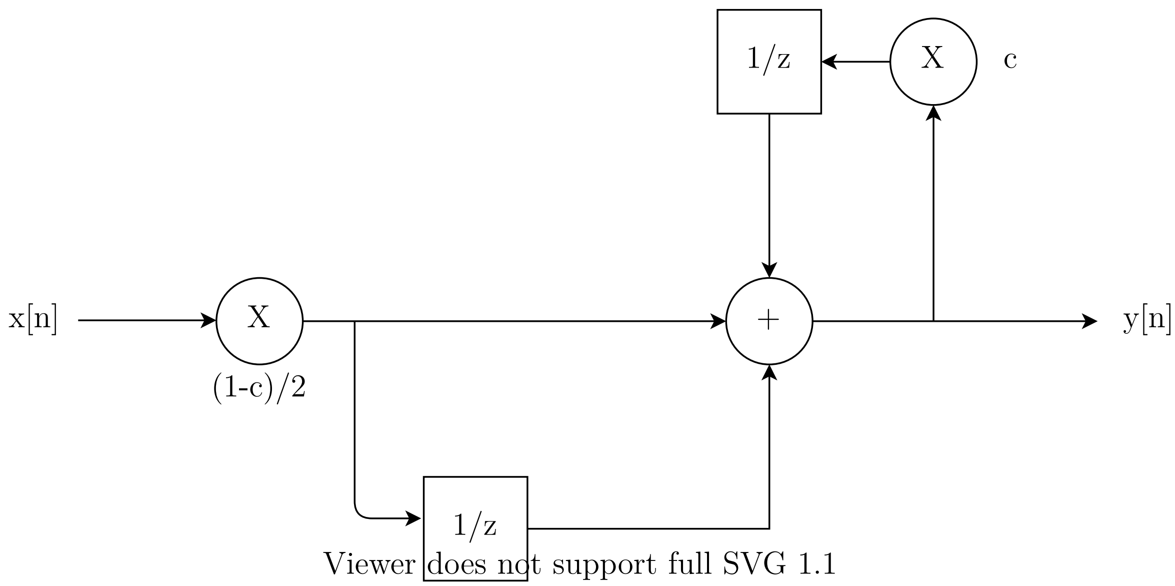

Figure 1: Difference equation Block Diagram. Sorry for the poor quality

1.4 Backward interpretation of Simulation theorem

Starting from $V(z)$ we can get the "real" frequency response of the system:

$$H(\omega)_R = V(z = e^{-i\omega T}) = \frac{2^{-k-1}(1 + e^{-i\omega T})}{e^{-i\omega T} - 1 - 2^{-k}} \tag{1.8}$$

2 Measurement and Comments

The goal provided was to perform a Modulus and Phase measurement of the transfer function of the Simulated Low-Pass Filter. Such measures, along with the Ideal and Real models are shown in (Fig. 2). The parameter value used is $k = 4$.

The experimental data behave correctly for low frequncies. At high frequencies (near 10^4 Hz) the modulus $|H(\omega)|$ begins to diverge from the ideal Low pass (Eq. 1.1) and follows the behaviour of the "real" model obtained using the backward propagation of the simulation theorem (Eq. 1.8).

On the other hand, the phase measures begin to diverge from both the models at high f. A possible explanation is the strong presence of noise in the measures, which makes them less precise and accurate.

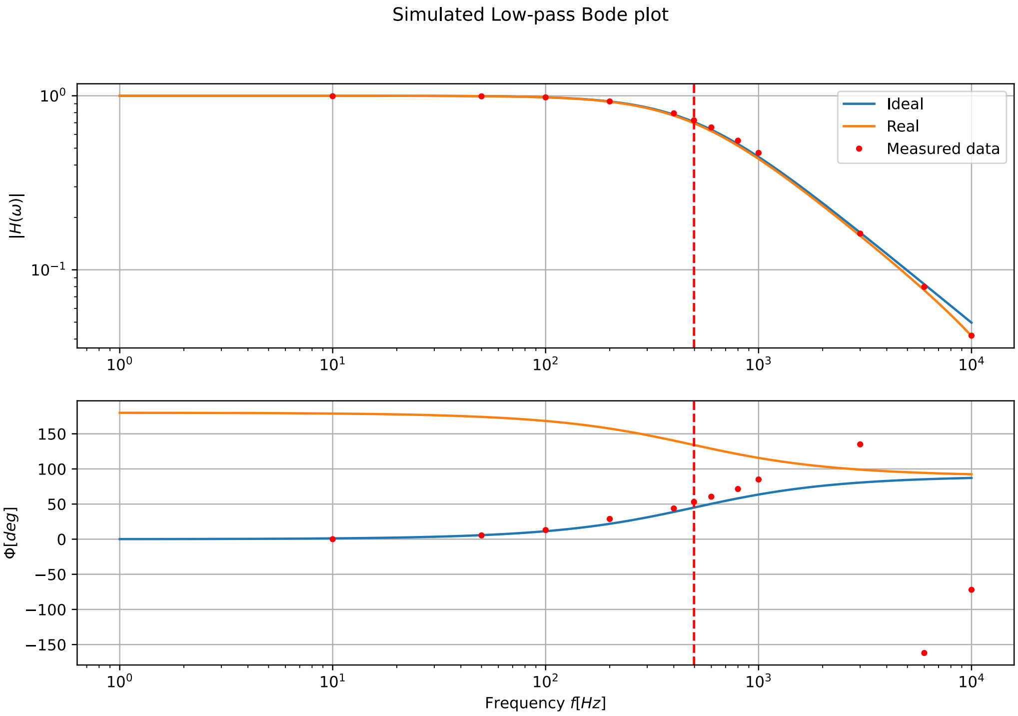

Figure 2: Simulated Low-Pass Filter Bode plot for k=4. Red line is at the predicted $f_{3dB} = 497.36Hz$