Answers to questions in

Lab 1: Filtering operations

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**Instructions**: Complete the lab according to the instructions in the notes and respond to the questions stated below. Keep the answers short and focus on what is essential. Illustrate with figures only when explicitly requested.

Good luck!

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**Question 1**: Repeat this exercise with the coordinates p and q set to (5, 9), (9, 5), (17, 9),

(17, 121), (5, 1) and (125, 1) respectively. What do you observe?

Answers:

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**Question 2**: Explain how a position (p, q) in the Fourier domain will be projected as a sine wave in the spatial domain. Illustrate with a Matlab figure.

Answers:

A specific position (p,q) in the discrete Fourier domain can be projected back as a sinw wave in the spatial domain applying the inversion theorem:

Once Euler’s formula is applied the result obtained is a complex sinusoidal component:

Taking, for example, (p,q) =(5,9) the result obtained when projecting it back in the spatial domain is showed in the figures below.

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**Question 3**: How large is the amplitude? Write down the expression derived from Equation (4) in the notes. Complement the code (variable amplitude) accordingly.

Answers:

From lecture notes (3)

(Inversion theorem),

(Euler’s formula),

(complex module).

The amplitude A of F is given by A = max(|F|), therefore

In the case of only for (u,v)=(p,q) and zero otherwise and then A = 0,000061.

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**Question 4**: How does the direction and length of the sine wave depend on p and q? Write down the explicit expression that can be found in the lecture notes. Complement the code (variable wavelength) accordingly.

Answers:

ADD

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**Question 5**: What happens when we pass the point in the center and either p or q exceeds half the image size? Explain and illustrate graphically with Matlab!

Answers:

By using numpy’s function fft.fftshift() the zero frequency component is shifted to the center of the spectrum by swapping the 1st quadrant with the 3rd and the 2nd with the 4th, therefore the new position of point (p,q) must be computed with respect to the new origin. Given an image of size (N,N) point (p,q) is shifted if it is placed over the center, thus if either p > N/2 or q > N/2. The results are equivalent due to Fourier transform’s periodicity and conjugate symmetry properties.

Immagine che contiene testo, schermata, nero

Descrizione generata automaticamenteImmagine che contiene schermata, testo, nero, design

Descrizione generata automaticamenteImmagine che contiene testo, schermata, nero

Descrizione generata automaticamente

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**Question 6**: What is the purpose of the instructions following the question *What is done by these instructions?* in the code?

Answers:

The purpose of these instructions is to compute the new position of the point associated with coordinates (p,q) in order to obtain the correct relative position to the origin after moving Fhat's zero component to the center of the image by using numpy’s fft.fftshift().

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**Question 7**: Why are these Fourier spectra concentrated to the borders of the images? Can you give a mathematical interpretation? Hint: think of the frequencies in the source image and consider the resulting image as a Fourier transform applied to a 2D function. It might be easier to analyze each dimension separately!

Answers:

Focusing on the first case (horizontal white band) and given that

(1)

we can simplify it given modularity’s property and knowing that only when , thus obtaining

Since the spectrum assumes value equal to 1 when v = 0 and 0 otherwise we can define

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**Question 8**: Why is the logarithm function applied?

Answers:

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**Question 9**: What conclusions can be drawn regarding linearity? From your observations can you derive a mathematical expression in the general case?

Answers:

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**Question 10**: Are there any other ways to compute the last image? Remember what multiplication in Fourier domain equals to in the spatial domain! Perform these alternative computations in practice.

Answers:

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**Question 11**: What conclusions can be drawn from comparing the results with those in the previous exercise? See how the source images have changed and analyze the effects of scaling.

Answers:

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**Question 12**: What can be said about possible similarities and differences? Hint: think of the frequencies and how they are affected by the rotation.

Answers:

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**Question 13**: What information is contained in the phase and in the magnitude of the Fourier transform?

Answers:

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**Question 14**: Show the impulse response and variance for the above-mentioned t-values. What are the variances of your discretized Gaussian kernel for t = 0.1, 0.3, 1.0, 10.0 and

100.0?

Answers:

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**Question 15**: Are the results different from or similar to the estimated variance? How does the result correspond to the ideal continuous case? Lead: think of the relation between spatial and Fourier domains for different values of t.

Answers:

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**Question 16**: Convolve a couple of images with Gaussian functions of different variances (like t = 1.0, 4.0, 16.0, 64.0 and 256.0) and present your results. What effects can you observe?

Answers:

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**Question 17**: What are the positive and negative effects for each type of filter? Describe what you observe and name the effects that you recognize. How do the results depend on the filter parameters? Illustrate with Matlab figure(s).

Answers:

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**Question 18**: What conclusions can you draw from comparing the results of the respective methods?

Answers:

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**Question 19**: What effects do you observe when subsampling the original image and the smoothed variants? Illustrate both filters with the best results found for iteration i = 4.

Answers:

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**Question 20**: What conclusions can you draw regarding the effects of smoothing when combined with subsampling? Hint: think in terms of frequencies and side effects.

Answers:

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