Examiners' commentaries 2016–17

CO1102 Mathematics for computing – Zone A

General remarks

Most candidates had a thorough understanding of the syllabus and were able to apply their knowledge and skills in the appropriate context. They were well prepared and demonstrated a good grasp of the subject. A number of candidates showed a limited understanding of the concepts and had difficulty interpreting the questions and answering them fully.

When revising for the examination, it is a good idea to work through the sample paper in the subject guide which has full solutions. You can then compare your answers with those given, and if your approach is very different, you can consider why and perhaps modify your method. You can learn a lot from looking at the notation and wording used in the solution and the way any mathematical definitions or proofs are included.

The way you present your solution may help you clarify the problem and develop the solution, as well as make it easier for the examiners to follow your work. Please try to ensure your answers convey your meaning clearly and correctly and show your working in full, so that the examiners can give marks for the correct method, even if you make an error that means your final solution is incorrect.

It will also help you greatly to work through past papers as part of the revision process so that you are familiar with the type of questions that may arise on each topic, and the knowledge and skills you need to answer them. It may also help to make a list of key points on each chapter as a revision guide, together with typical examination questions.

Comments on specific questions

Question 1

a. This question required candidates to add two binary numbers together, showing all working. When the question specifies working in base 2 that is what should be done. Converting everything to another base to perform the arithmetic, such as base 10, lost marks. Marks were also lost if candidates did not show carries in their solution, as indicated in the question.

b. i. The definition of a rational number is one of the form p/q where p and q are integers and $q \neq 0$. It is not sufficient just to say that a rational number is a fraction.

   ii. Examples could be 3/2 or 5/4, or any fraction in the range.

   iii. Numbers such as $\pi/2$ or $\sqrt{2}$, which are not rational, are needed.

c. The binary number 101.011 has an integer part equivalent to 5 in base 10 and a bicimal part equivalent to 1/4+1/8: the numbers after the point. This becomes 0.24 +0.125 in base 10. Thus, the answer is 5.375 and leaving this as a fraction lost part marks.

d. This is a standard question which requires you to learn the method, which is to take say x to be the number 3.4242... and, as the repeating block is length two, make

$100x = 342.4242...$

Subtracting x from $100x$ gives $99x = 339$ or $x = 339/99$, which simplifies to 113/33 for full marks. It is important that you understand the significance of the repeating block and indicate it by an ellipsis or another accepted notation. Marks were lost by those who abbreviated x to 3.4242 and $100x$ to 342.42 or some such inaccurate numbers.

Question 2

a. i. This requires a standard Venn diagram showing three sets intersecting in the most general way, with eight separate regions.

ii. These eight regions are shaded if there is a 1 in the corresponding row of the table and left unshaded if there is a 0 in that row. This gives X as the overall shaded area. As there are four entries with a 1 in the X column, there are four regions shaded to form X. Marks were awarded for a key indicating what the shading represented.

iii. The region Y has three 1s in its membership table – the 001 row, the 010 row and the 011 row. The rest of its entries are 0.

iv. Thus, Y is a proper subset of X and saying $Y \neq X$ is insufficient for full marks.

b. i. The set is given by {-12,-7,-2,3,8,13}.

ii. $\{3^n : n \in Z, 0 \leq n \leq 8\}$ or equivalent values of n.

Question 3

a. A set of values is required for each of the logical compound statements. This is covered in the subject guide and candidates should ensure they can produce the relevant lists. The truth set for p and q is given by {3,9}, the truth set for p or q by the set {1,3,5,6,7,9} and for $\neg p$ and q by {1,5,7}.

b. The statement is represented by $\neg q \rightarrow \neg p$. Here, candidates were asked to translate from the English to the symbolic logical equivalent compound propositions. This skill (and translating back in the other direction) needs to be practised during revision.

c. A truth table is required with columns for $p; q; \neg p; \neg q; \neg p \rightarrow \neg q;$ and finally $q \rightarrow p$. The resulting column entries for both sides of the equation should be equal, leading us to conclude that "Since the columns are identical, both sides of the equation are equivalent", or some such statement. Not making a concluding statement to justify what the columns mean lost part marks.

d. The truth set is given by {2, 3, 4, 6, 8, 9, 10}.

a. The contrapositive of the given statement is "If n is odd, then the last digit of n is not 4". A number of candidates got this the wrong way around. Some wrote down the contrapositive symbolically, usually in terms of p and q. However, to obtain full marks, they also needed to define both p and q in words.

Question 4

a. This question required candidates to evaluate powers of 2: $2^0 = 1$, $2^3 = 8$, $2^{-1} = 1/2$ and $2^{1/2} = \sqrt{2}$. If candidates gave a decimal value for $\sqrt{2}$, they needed make it clear that it is approximate, or only correct to so many decimal places.

a. Solving $2^x = 100$ is done by taking logs to base 2 of both sides giving log (2^x) = log 100, thus x = log 100 and by change of base, say to base 10, x = log 100/log 2 = 6.64 correct to two decimal places.

a. i. The function is one-to-one since if $f(a) = f(b)$ then $2^a = 2^b$ and this is only true if $a = b$, which means each element in the range has only one pre-image and the function is thus one-to-one.

   ii. The function is not onto as there are elements of the co-domain such as negative numbers, which are not in the range.

   Be careful not to confuse the co-domain and the range. Here, the co-domain is the real numbers and the range is the set of positive real numbers. Simply saying that the range is not equal to the co-domain is not specific enough and for full marks examples of numbers in the co-domain which have no images in the range were required. Whether the function is one-to-one and/or onto is a question that frequently arises, and is well worth preparing for. You need a clear understanding of the concepts of domain, range and co-domain, as well as the ability to find and interpret these according to the example.

b. A function has an inverse if it is both one-to-one and onto.

c. This requires you to demonstrate you know when a function has an inverse and to find this inverse and define it fully. You need to give the inverse function in algebraic terms, and its domain and co-domain. Many candidates missed this latter part of the definition and lost marks.

a. Here, $g^{-1}(x)$ = log (x) to base 10. Its domain is the positive real numbers, R^+, and range, and co-domain the real number R.

Question 5

a. i. Most candidates calculated the first four terms of the sequence correctly by substituting 1, 2, 3 and 4 for n, in the formula $3n + 5$.

b. ii. The recurrence relation for this sequence is of the form $u_{(n + 1)} = u_1 + 3$ and $u_1 = 8$.

c. Candidates were then required to perform a proof by induction on the sum of the terms of this sequence. There is a standard procedure involved in such a proof which breaks down into several stages. You should know these and have practised beforehand; although the individual proofs differ, their structure is the same.

d. The question involved knowledge and understanding of Sigma notation. The sum required recognition that the terms in the sum are of the form $3r + 5$, where r goes from 2 to 671. (This is found by solving $3r + 5 = 2018$.) The induction proved above assumes r goes from 1 to $n = 671$. So, we can use the formula found in b. and subtract the first term which is 8. Thus, the sum is given by $\{3(671)^2 + 13(671)\}/2 - 8$ which gives 679715.

Question 6

a. There are a few basic definitions in each chapter of the subject guide which are key to understanding the concepts in that section. You should note and learn them as part of the revision process. These include knowing what properties a graph must have if it is simple and what is meant by the degree of a vertex of a graph. The concept of isomorphism is fundamental to graph theory and is regularly included in the examination. So you should ensure you have a good understanding of it and can construct and identify graphs which are either isomorphic or non-isomorphic, and give clear, specific reasons why. To say, "Graphs are non-isomorphic as there is no one-to-one correspondence function between them" is insufficient as a justification.

Many candidates answered by saying that a graph is simple if it has no loops or cycles, rather than having no loops or parallel edges, which is the correct solution. This made it difficult to draw a simple graph with degree sequence 3, 3, 3, 3, 2, 2 in part c.

b. i. and ii. The graphs were well drawn by many candidates.

c. i., ii. and iii. There were various possible solutions which did not need to have the same degree sequence as that drawn in i., whereas the graph for iii. did. Some candidates demonstrated little knowledge of this topic and were unable to count vertices or find the degree of a vertex or the degree sequence of a graph.

d. The number of edges in the graph is given by $(7 \times 2n)/2 = 7n$.

Question 7

a. i. The graph *M* was drawn accurately by many. Some incorrectly drew it as a digraph.

ii. and iii. Spanning trees were required. These are trees which contain all the vertices of the graph but no cycles. Minimum edges are removed from the graph until it has no cycles (or loops or multiple edges), but retains its connectivity. Some candidates did not understand what was meant by a tree or a spanning tree. It is quite possible to find two spanning trees for *M* which are non-isomorphic, and again specific reasons are required in ii. for why they are non-isomorphic (though not in iii.).

You should revise the formal definitions for reflexivity, symmetry and transitivity before the examination and you should be able to reproduce them, with appropriate counter examples where required.

b. i. The relation is not reflexive, since for any graph it is not non-isomorphic to itself.

ii. It is symmetric, since if graph *A* is non-isomorphic to graph *B*, then it follows that graph B is non-isomorphic to graph *A*.

iii. This is true for all graphs *A* and *B*. An example is found in T_1 and T_2 in ii.

The relation is not transitive and a counter example is found in a. with the trees T_1, T_2 and T_3.

Question 8

a. Tree diagrams allow us to see all the possible outcomes of an event and calculate their probability. Each branch in a tree diagram represents a possible outcome, and the probabilities on each section of the branch are multiplied together to give the probability of the outcome for that branch. This is covered in the subject guide and you should study it carefully when revising.

In this example, the root of the tree is the first match and there are three possible outcomes so it has three branches from it, representing a win for *A*, a draw and a win for *B*. The next node is the second match and again there are three possible outcomes. There are nine possible overall results with their associated probabilities. Some of these results give the same outcome: for example, there are three branches with the outcome *A* wins the tournament. The probability that *A* wins is given by finding the probabilities for each branch and then summing these three to give the overall probability that A wins the tournament. Likewise, for a draw or a win for *B*.

b. These need to be calculated:

i. A wins in three of these with probabilities of 1/9 + 1/18 + 1/18 = 2/9;

ii. B wins in three of these with probabilities of 1/12 + 1/12 + 1/4 = 5/12;

iii. Three outcomes result in a draw with a probability of 13/36.

These nine outcomes cover all possibilities and their total probability adds up to 1.

Question 9

a. i. A ternary tree is defined. The number of nodes on level six is 3^6.

ii. For internal nodes on a tree, the nodes on the last level are not counted. So, if the tree is of height h, the number of nodes on the last level counted is 3^(h-1). The total number of nodes is thus the number on levels 0 to h-1 which is 1 + 3 + 3^2 + 3^3 +...+ 3^(h-1) which is given by the expression.

iii. This sum must be greater than or equal to 1000 which is the ceiling of log_3 1001. Some candidates took logs to base 2 instead of base 3 when performing this calculation. A manual calculation of the sum can also be done showing that

1 + 3 + 9 + 27 + 81 + 243 + 729 <1000 so the height must be 7.

b. i. Marks were lost where answers were unclear or insufficient. A unique 8-bit binary string is allocated to each subset by assigning a 0 if the element is absent and a 1 if it is present in the subset **and** these symbols are put in the position the element is in.

ii. The string is 10101000 and the subset is {x_2, x_4, x_6, x_8}.

iii. The total number of subsets is 2^8.

Question 10

a. i. This required candidates to multiply two matrices together, leaving the unknown x and y as part of the elements in the resulting matrix. This gives two different results for AB and BA. When two matrices are equal, each of the corresponding elements are equal.

ii. Thus, comparing the elements in row 1, column 1 in both gives $x = x$ which is trivial; for the elements in row 1, column 2 we have $2y = 2x$ which gives $y = x$. Similarly, equating elements in row 2, column 1 and column 2 results in $y = x$, which is the relation required.

b. i. This is a standard question involving Gaussian elimination. First, candidates were asked to write down the augmented matrix which is standard bookwork, requiring care as the co-efficient of y in the first equation is zero, so the first row is

2 0 2 : 8.

ii. In the Gaussian elimination, marks were given for methods where the working was clear. Many candidates lost a mark as they did not fully reduce the matrix to 1 with 1s on the leading diagonal, but left other numbers there. Row echelon form is needed. It is helpful if the row operations employed are clearly labelled and the order of transformations is shown, so any errors can be worked through and credit given.

Examiners' commentaries 2016–17

CO1102 Mathematics for computing – Zone B

General remarks

The overall performance on this paper showed that most of the candidates had a thorough understanding of the syllabus and could apply their knowledge and skills in the appropriate context. They were well prepared and demonstrated a good grasp of the subject. A number of candidates showed a limited understanding of the concepts and had difficulty interpreting the questions and answering them fully.

When revising for the examination, it is a good idea to work through the sample paper in the subject guide which has full solutions. You can then compare your answers with those given, and if your approach is very different then you can consider why and perhaps modify your method. You can learn a lot from looking at the notation and wording used in the solution and the way any mathematical definitions or proofs are included.

The way you present your solution may help you clarify the problem and develop the solution, as well as make it easier for the examiners to follow your work. Please try to ensure your answers convey your meaning clearly and correctly and show your working in full, so that the examiners can give marks for the correct method, even if you make an error that means your final solution is incorrect.

It will also help you greatly to work through other past papers as part of the revision process so that you are familiar with the type of questions that may arise on each topic, and the knowledge and skills you need to answer them. It may also help to make a list of key points on each chapter as a revision guide, together with typical examination questions.

Comments on specific questions

Question 1

a. This question required candidates to add two binary numbers together, showing all working. When the question specifies working in base 2 that is what should be done. Converting everything to another base to perform the arithmetic, such as base 10, lost marks. Marks were also lost if candidates did not show carries in their solution, as indicated in the question.

b. i. The definition of a rational number is one of the form p/q where p and q are integers and $q \neq 0$. It is not sufficient just to say a rational number is a fraction.

   ii. Examples could be 7/2 or 13/4, or any fraction in the range.

   iii. Numbers such as π or $2\sqrt{3}$, which are not rational, are needed.

c. The binary number 110.101 has an integer part equivalent to 6 in base 10 and a bicimal part equivalent to $1/2 + 1/8$: the numbers after the point. This becomes 0.50 +0.125 in base 10. Thus, the answer is 6.625 and leaving this as a fraction lost part marks.

d. This is a standard question which requires you to learn the method which is to take say x to be the number 1.1515... and, as the repeating block is length two, make

$100x = 115.1515...$

Subtracting x from $100x$ gives $99x = 114$ or $x = 114/99$, which simplifies to 38/33 for full marks. It is important that you understand the significance of the repeating block and indicate it by an ellipsis or another accepted notation. Marks were lost by those who abbreviated x to 1.1515 and $100x$ to 115.15 or some such inaccurate numbers.

Question 2

a. i. This requires a standard Venn diagram showing three sets intersecting in the most general way, with eight separate regions.

ii. These eight regions are shaded if there is a 1 in the corresponding row of the table and left unshaded if there is a 0 in that row. This gives X as the overall shaded area. As there are four entries with a 1 in the X column, there are four regions shaded to form X. Marks were awarded for a key indicating what the shading represented.

iii. The region Y has three 1s in its membership table – the 001 row, the 010 row and the 011 row. The rest of its entries are 0.

iv. Thus, Y is a proper subset of X and saying $Y \neq X$ is insufficient for full marks in iv.

b. i. The set is given by {-14, -11, -8, -5, -2, 1}.

ii. $\{5^n : n \in Z, 0 \leq n \leq 6\}$ or equivalent values of n.

Question 3

a. A set of values is required for each of the logical compound statements. This covered in the subject guide and you should ensure you can produce the relevant lists. The truth set for p and q is given by {15}, the truth set for p or q by the set {11, 12, 13, 15, 17, 18, 19} and for $\neg p$ and q by {11, 13, 17, 19}.

b. The statement is represented by $\neg q \rightarrow \neg p$. Here, candidates were asked to translate from the English to the symbolic logical equivalent compound propositions. You need to practise this skill (and translating back in the other direction) during revision.

c. A truth table is required with columns for p, q, $q \rightarrow p$, $\neg p$, $\neg q$, and $\neg p \rightarrow \neg q$. The resulting column entries for both sides of the equation should be equal, leading us to conclude that "Since the columns are identical, both sides of the equation are equivalent", or some such statement. Not making a concluding statement to justify what the columns meant losing part marks.

d. The truth set is given by {12, 14, 15, 16, 18, 20}.

e. The contrapositive of the given statement is "If n is odd, then the last digit of n is not 4". A number of candidates got this the wrong way around. Some candidates wrote down the contrapositive symbolically, usually in terms of p and q. However, to obtain full marks, they also needed to define both p and q in words.

Question 4

a. This question required candidates to evaluate powers of 3: 3 ^ 0 = 1, 3 ^ 4 = 81, 3 ^ -1 = 1/3 and 3 ^ 1/2 = √3. If you give a decimal value for √3, you should make it clear that it is approximate, or only correct to so many decimal places.

b. Solving $3 \wedge x = 100$ is done by taking logs to base 3 of both sides giving $\log (3 \wedge x) = \log 100$, thus $x = \log 100$ and by change of base, say to base 10, $x = \log 100/\log 3 = 4.19$ correct to two decimal places.

c. i. The function is one-to-one since if $f(a) = f(b)$ then $3 \wedge a = 3 \wedge b$ and this is only true if $a = b$, which means each element in the range has only one pre-image and the function is thus one-to-one.

ii. The function is not onto as there are elements of the co-domain such as negative numbers, which are not in the range.

Be careful not to confuse the co-domain and the range. Here, the co-domain is the real numbers and the range is the set of positive real numbers. Simply saying that the range is not equal to the co-domain is not specific enough and for full marks examples of numbers in the co-domain which have no images in the range were required. Whether the function is one-to-one and/or onto is a question which frequently arises, and is well worth preparing for. You need a clear understanding of the concepts of domain, range and co-domain, as well as the ability to find and interpret these according to the example.

d. A function has an inverse if it is both one-to-one and onto.

e. This requires you to demonstrate you know when a function has an inverse and to find this inverse and define it fully. You need to give the inverse function in algebraic terms, and its domain and co-domain. Many candidates missed this latter part of the definition and lost marks.

Here, $g \wedge \text{-}1(x) = \log (x)$ to base 10. Its domain is the positive real numbers, $R \wedge +$, and range, and co-domain the real number R.

Question 5

a. Most candidates calculated the first four terms of the sequence correctly by substituting 1, 2, 3 and 4 for n, in the formula $5n + 3$.

ii. The recurrence relation for this sequence is of the form $u\_(n + 1) = u\_1 + 5$ and $u\_1 = 8$.

b. Candidates were then required to perform a proof by induction on the sum of the terms of this sequence. There is a standard procedure involved in such a proof which breaks down into several stages. You should know these and have practised beforehand; although the individual proofs differ, their structure is the same.

c. The question involved knowledge and understanding of Sigma notation. The sum required recognition that the terms in the sum are of the form $5r + 3$, where r goes from 2 to 403. (This is found by solving $2018 = 5r + 3$.) The induction just proved above assumes r goes from 1 to $n = 403$. So, we can use the formula found in b. and subtract the first term which is 8. Thus, the sum is given by {5(403) ^ 2 + 11(403)}/2 -8 which gives 408231.

Question 6

a. There are a few basic definitions in each chapter of the subject guide which are key to understanding the concepts in that section. You should note and learn them as part of the revision process. These include knowing what properties a graph must have if it is simple, and what is meant by the degree of a vertex of a graph. The concept of isomorphism is fundamental to graph theory and is regularly included in the examination. So you should ensure you have a good understanding of it and can construct and identify graphs which are either isomorphic or non-isomorphic, and give clear, specific reasons why. To say, "Graphs are non-isomorphic as there is no one-to-one correspondence function between them" is insufficient as a justification.

Many candidates answered by saying that a graph is simple if it has no loops or cycles, rather than having no loops or parallel edges, which is the correct solution. This made it difficult to draw a simple graph with degree sequence 3, 3, 3, 3, 2, 2 in c.

b. i. and ii. The graphs were well drawn by many candidates.

c. i., ii. and iii. There were various possible solutions which did not need to have the same degree sequence as that drawn in i., whereas the graph for iii. did. Some candidates demonstrated little knowledge of this topic and were unable to count vertices or find the degree of a vertex or the degree sequence of a graph.

d. The number of edges in the graph is given by $(5 \times 2n)/2 = 5n$.

Question 7

a. i. The graph M was drawn accurately by many. Some candidates incorrectly drew it as a digraph.

ii. and iii. Spanning trees were required. These are trees which contain all the vertices of the graph but no cycles. Minimum edges are removed from the graph until it has no cycles (or loops or multiple edges), but retains its connectivity. Some candidates did not understand what was meant by a tree or a spanning tree. It is quite possible to find two spanning trees for M which are non-isomorphic and again specific reasons are required in ii. for why they are non-isomorphic (though not in iii.).

You should revise the formal definitions for reflexivity, symmetry and transitivity before the examination and you should be able to reproduce them, with appropriate counter examples where required.

b. i. The relation is not reflexive, since for any graph it is not non-isomorphic to itself.

ii. It is symmetric, since if graph A is non-isomorphic to graph B then it follows that graph B is non-isomorphic to graph A. This is true for all graphs A and B. An example is found in T_1 and T_2 in ii.

iii. The relation is not transitive and a counter example is in a. with the trees T_1, T_2 and T_3.

Question 8

a. Tree diagrams allow us to see all the possible outcomes of an event and calculate their probability. Each branch in a tree diagram represents a possible outcome, and the probabilities on each section of the branch are multiplied together to give the probability of the outcome for that branch. This is covered in the subject guide and you should study it carefully when revising.

In this example, the root of the tree is the first match and there are three possible outcomes so it has three branches from it, representing a win for A, a draw and a win for B. The next node is the second match and again there are three possible outcomes. There are nine possible overall results with their associated probabilities. Some of these results give the same outcome: for example, there are three branches with the outcome A wins the tournament. The probability that A wins is given by finding the probabilities for each branch and then summing these three to give the overall probability that A wins the tournament. Likewise, for a draw or a win for B.

b. These need to be calculated:

i. A wins in three of these with probabilities of $1/4 + 1/20 + 1/20 = 7/20$;

ii. B wins in three of these with probabilities of $1/25 + 1/25 + 4/25 = 6/25$;

iii. Three outcomes result in a draw with a probability of 41/100.

These nine outcomes cover all possibilities and their total probability adds up to 1.

Question 9

a. i. A ternary tree is defined. The number of nodes on level five is $3^5 = 243$.

ii. For internal nodes on a tree, the nodes on the last level are not counted. So, if the tree is of height h, the number of nodes on the last level counted is $3^{(h-1)}$. The total number of nodes thus the number on levels 0 to h-1 which is $1 + 3 + 3^2 + 3^3 + \ldots + 3^{(h-1)}$ which is given by the expression.

iii. This sum must be greater than or equal to 1000 which is the ceiling of $\log_3 1001$. Some candidates took logs to base 2 instead of base 3 when performing this calculation. A manual calculation of the sum can also be done showing that

$1 + 3 + 9 + 27 + 81 + 243 + 729 < 1000$ so the height must be 7.

b. i. Marks were lost where answers were unclear or insufficient. A unique 8-bit binary string is allocated to each subset by assigning a 0 if the element is absent and a 1 if it is present in the subset and these symbols are put in the position the element is in.

ii. The string is 00010101 and the subset is $\{x_1, x_3, x_5, x_7\}$.

iii. The total number of subsets is 2^8.

Question 10

a. i. This required candidates to multiply two matrices together, leaving the unknown x and y as part of the elements in the resulting matrix. This gives two different results for AB and BA. When two matrices are equal, each of the corresponding elements are equal.

ii. Thus, comparing the elements in row 1, column 1 in both gives $x = x$ which is trivial; for the elements in row 1, column 2, we have $2y = 2x$ which gives $y = x$. Similarly, equating elements in row 2, column 1 and column 2 results in $3x = 3y$ or $x = y$, which is the relation required.

b. i. This is a standard question involving Gaussian elimination. First, candidates were asked to write down the augmented matrix which is standard bookwork and they needed to be careful as the co-efficient of y in the first equation is zero, so the first row is

2 0 2 : 8.

ii. In the Gaussian elimination, marks were given for methods where the working was clear. Many candidates lost a mark as they did not fully reduce the matrix to 1 with 1s on the leading diagonal, but left other numbers there. Row echelon form is needed. It is helpful if the row operations employed are clearly labelled and the order of transformations is shown, so any errors can be worked through and credit given.