THIS PAPER IS NOT TO BE REMOVED FROM THE EXAMINATION HALL

UNIVERSITY OF LONDON

CO1102 ZA

BSc, CertHE and Diploma EXAMINATION

COMPUTING AND INFORMATION SYSTEMS, CREATIVE COMPUTING and COMBINED DEGREE SCHEME

Mathematics for Computing

Friday 10 May 2019: 10.00 – 13.00

Time allowed: 3 hours

DO NOT TURN OVER UNTIL TOLD TO BEGIN

There are **TEN** questions in this paper. Candidates should answer all **TEN** questions. All questions carry equal marks and full marks can be obtained for complete answers to **TEN** questions. The marks for each part of a question are indicated at the end of the part in [.] brackets.

There are 100 marks available on this paper.

A handheld calculator may be used when answering questions on this paper but it must not be pre-programmed or able to display graphics text or algebraic equations. The make and type of machine must be stated clearly on the front cover of the answer book.

Question 1

(a) i. Showing your working, convert the decimal number $(45)_{10}$ to binary.

ii. Explain how to obtain from your answer to (i) the binary representation for the decimal number $(90)_{10}$ without performing a conversion. [3]

(b) Working entirely in binary carry out the following calculations, showing all your working and any carries.

i. $(1011010)_2 + (101101)_2$

ii. $(1011010)_2 - (101101)_2$

[4]

(c) For each of the following numbers, state all of the sets $\mathbb{Z}$, $\mathbb{Q}$ or $\mathbb{R}$ they belong to:

i. $\sqrt{3}$

ii. -6

iii. 0

iv. $\frac{3}{11}$. [2]

(d) The repeating decimal $x = 0.162162162162...$ can be converted to the fraction $\frac{6}{37}$. Explain carefully the FIRST step in this conversion which consists of computing a suitable multiple of x. [1]

Question 2

(a) i. Let A, B, C and X be subsets of a universal set $\mathcal{U}$. Write out and complete the following membership table:

A	B	C	$A \cup C$	$(A \cup C) - B$	X
0	0	0			1
0	0	1			0
0	1	0			1
0	1	1			1
1	0	0			0
1	0	1			0
1	1	0			1
1	1	1			1

ii. Draw a labelled Venn diagram showing A, B and C intersecting in the most general way and shade the region X on it.

iii. Find an expression which defines the set X in terms of A, B and C and set operations.

[6]

(b) Let $A = \{3, 6, 9, 12, ..., 99\}$ and $B = \{2^n + 1 : n \in \mathbb{Z}, 1 \leq n \leq 6\}$ be two subsets of the universal set of integers $\mathbb{Z}$.

i. Describe the set A by the rules of inclusion method.

ii. Describe the set B by the listing method.

iii. Describe the two sets $A \cap B$ and $B - A'$ by the listing method.

[4]

Question 3

Let p and q be the following propositions about an animal:

p : "this animal is a cat";
q : "this animal has a tail".

(a) Express each of the two following compound propositions symbolically by using p, q and appropriate logical symbols.

i. "if this animal is a cat then it has a tail";

ii. "this animal has not got a tail but it is a cat".

[2]

(b) Give the truth table for the statement $q \to p$ and show that it is equivalent to $\neg(\neg p \wedge q)$. [3]

(c) Give the contrapositive of the statement $q \to p$

i. using symbols;

ii. as a statement in words about animals, cats and tails. [2]

(d) Using the equivalence proven in (b), design a logic network with inputs p, q that gives as final output $q \to p$. Label the diagram carefully, showing input and output at each gate. [3]

Question 4

Given any number $x \in \mathbb{R}$, recall that the floor of x is defined as $\lfloor x \rfloor = n$ where n is an integer such that $n \leq x < n + 1$.

(a) Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and let the function $f : A \to \{0, 1, 2, 3\}$ be given by the rule $f(x) = \lfloor \frac{x}{3} \rfloor$.

i. Find $f(5)$.

ii. Find the set of ancestors of 0.

iii. Say whether f is one-to-one, justifying your answer.

iv. Say whether f is onto, justifying your answer.

[4]

(b) Consider the function $g : \{1, 2, 3, ..., 10\} \to \mathbb{Z}$ where $g(n) = \lfloor \frac{n-1}{3} \rfloor$.

i. Find the set of ancestors of 1.

ii. Find the range of g.

iii. Say whether g is invertible, justifying your answer.

[3]

(c) Let $P = \{a, b, c\}$ and $Q = \{1, 2, 3, 4\}$. Draw arrow diagrams for the following functions:

i. a function $f_1 : P \to Q$ that is one-to-one but not onto;

ii. a function $f_2 : Q \to P$ that is onto but not one-to-one;

iii. a function $f_3 : P \to P$ that is both one-to-one and onto.

[3]

Question 5

(a) The terms of a sequence are defined by the formula:

$$u_n = 3n - 2 \text{ for } n \geq 1.$$

i. Calculate u_1, u_2, u_3, and u_4, showing your working.

ii. Find the value of r such that $u_r = 1999$.

iii. Suggest a recurrence relation expressing u_{n+1} in terms of u_n for $n \geq 1$. You do not need to prove this formula.

iv. Use the standard formula

$$\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$$

to find an expression for the sum

$$\sum_{r=1}^{n} (3r - 2)$$

in terms of n.

v. Use the expression found in (iv) to calculate the sum

$$\sum_{r=1}^{667} (3r - 2).$$

[6]

(b) Use the standard formula from part (a) (iv) above to evaluate the following sums:

i. $21 + 22 + 23 + 24 + \ldots + 100$;

ii. $4 + 7 + 10 + \ldots + 100$.

[4]

Question 6

Given the following definitions for graphs:

K_n is the graph on n vertices where each pair of distinct vertices is connected by an edge;

C_n is the graph with vertices $v_1, v_2, v_3, ..., v_n$ and edges

$$\{v_1,v_2\},\ \{v_2,v_3\}, \dots ,\{v_{n-1}, v_n\},\ \{v_n, v_1\};$$

W_n is the graph obtained from C_n by adding an extra vertex, v_{n+1}, and edges from this to each of the original vertices in C_n.

(a) i. Draw K_4, C_4, and W_4.

ii. Giving your answer in terms of n, write down an expression for the number of edges in K_n, C_n, and W_n. [5]

(b) i. Write down the adjacency matrix **A** for K_4

ii. Compute $\mathbf{A}^2$.

iii. Given that a path is an alternating sequence of vertices and edges which are all distinct, use your answer to (b) (ii) to find the total number of paths of length 2 in K_4 which start at v_1 . [5]

Question 7

(a) Consider the set $S = \{m, o, u, s, e\}$ whose elements are the vowels: o, u, e and the consonants: m, s.

i. Suggest how each subset of S could be represented by a unique 5-bit binary string.

ii. Write down the string corresponding to the subset $\{m, s, e\}$ and the subset corresponding to the string 01010.

iii. What is the total number of subsets of S? [5]

(b) R is a relation defined on S as follows:

$$xRy \text{ if } x \text{ and } y \text{ are vowels.}$$

Draw the relationship digraph for R on S and say, with reason, whether this relation is

i. reflexive ii. symmetric iii. transitive. [5]

Question 8

A 3-digit code is made from the digits $1, 2, 3, 4, 5, 6$ and the result recorded as an ordered triple such as $(2, 1, 6)$. Repetitions of digits are not allowed.

(a) Explain why there are 120 different possible codes. [1]

(b) Let A be the outcome that the first digit in the code is even and B the outcome that none of the digits is a 4 or 5. Calculate the number of elements in each of the outcomes A, B and $A \cap B$. [4]

(c) Hence calculate the probability of each of the outcomes A, B, $A \cap B$ and $A \cup B$ occurring. [5]

Question 9

The following matrix shows five American states and an entry of 1 indicates that the states heading that row and column share a common border, whereas a zero entry indicates they do not.

$$\begin{array}{c} \\ \text{Colorado} \\ \text{Kansas} \\ \text{New Mexico} \\ \text{Oklahoma} \\ \text{Texas} \end{array} \begin{array}{c} \begin{array}{ccccc} \text{Colorado} & \text{Kansas} & \text{New Mexico} & \text{Oklahoma} & \text{Texas} \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix} \end{array}$$

(a) Write down the states that share a border with Texas. [1]

(b) Is this matrix symmetric or not? Give an example to show what this means. [2]

(c) Draw a simple graph, G, depicting the information in this matrix. [1]

(d) Explain how the number of edges of the graph can be calculated from the entries in the matrix and find this number. [1]

(e) Draw another graph, H, which has 5 vertices and the same degree sequence as G, but is not isomorphic to it. Give a reason why G and H are not isomorphic. [2]

(f) Draw two non-isomorphic spanning trees for G, and explain why they are non-isomorphic. [3]

Question 10

(a) Given the matrices $\mathbf{P} = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$, $\mathbf{Q} = \begin{pmatrix} 4 & 7 \\ 0 & -1 \end{pmatrix}$, $\mathbf{R} = \begin{pmatrix} a & b \\ -3 & 2 \end{pmatrix}$

i. Find $2\mathbf{P} - \mathbf{Q}$.

ii. Find $\mathbf{PQ}$.

iii. Find a and b such that $\mathbf{PR} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

[5]

(b) i. Write down the augmented matrix for the following system of equations.

$$\begin{aligned} 2x - y + 3z &= 13 \\ x - z &= 1 \\ x + y - z &= 0 \end{aligned}$$

ii. Use Gaussian elimination to solve the system. You should show clearly the row operations you use in this process.

[5]

END OF PAPER