Examiners' commentary 2018-2019
CO1102 Mathematics for computing – Zone A

General remarks

It was clear that most candidates had a thorough understanding of the syllabus and the ability to apply their knowledge and skills in the appropriate context. They were well prepared and demonstrated that they had a good grasp of the subject and had revised well. However, a number of candidates showed a limited understanding of the concepts and had difficulty interpreting the questions and answering them fully.
When revising for the exam it is a good idea to work through the sample paper in the subject guide, which has full solutions. You can then compare your answers with those given, and if your approach is very different then you can consider why and perhaps modify your method. You can learn a lot from looking at the notation and wording used in the solution and how any mathematical definitions or proofs are included. The way you present your solution may help you clarify the problem and develop the solution, as well as making it easier for the examiners to follow your work. Please try to ensure your answers convey your meaning clearly and correctly and show your working in full, so that the examiners can give you marks for correct method, even if you make an error which means your final solution is incorrect. It will help you greatly to work through other past papers as part of the revision process so that you are familiar with the type of questions which may arise on each topic, and the material and skills you need to answer them. It may also help to make a list of key points in each chapter as a revision guide, together with typical exam questions.

Comments on specific questions

Question 1

For part (a) candidates were required to convert the decimal number 45 to base 2 and then use this to obtain the binary representation of the decimal number 90. Converting 45 may be done by the method of repeated division (alternative methods were accepted), to give the solution 101101_2. Candidates needed to realise that 90 is exactly twice 45 so the solution can be found by doubling the binary number 101101_2 and this is easily done by adding a zero to the end, giving 1011010_2, or adding 110011_2 to itself. An explanation of this process was required.

In (b) all the working should be shown, with all carries and adjustments for subtractions clearly indicated, giving the solutions 10000111_2 and 101101_2.

Part (c) tested candidates' knowledge of the sets $\mathbb{Z}$ – the integers, $\mathbb{Q}$ – the rationals and $\mathbb{R}$ – the real numbers and their inter-relationships. Thus $\sqrt{3}$ is an irrational number and belongs only to the set $\mathbb{R}$. The number -6 is a negative integer and belongs to all three sets, as does 0. The fraction $\frac{3}{11}$ is a rational number and belongs to both the sets $\mathbb{Q}$ and $\mathbb{R}$.

Part (d) required candidates to have learned the method, which is to take say x to be the number 0.162162... and, as the repeating block is length three, make $1000x = 162162162...$ Subtracting x from $1000x$ would give $999x = 162$ or $x = 162/999$ which simplifies to $6/37$. However, candidates were not required to carry out the full conversion, just to explain why the multiple $1000x$ is needed.

Question 2

In part (a) i) the membership table for $A \cup C$ is in the subject guide, Vol. 1, section 2.3. The column for $(A \cup C) - B$ is the opposite of that for X as it is the same as X'. Part ii) required a standard Venn diagram showing 3 sets intersecting in the most general way, with 8 separate regions. For $X = (A \cup C) - B$ the region in B is shaded together with the whole of the outside of the three sets. An appropriate key was required for full marks. In iii) $(A \cup C) - B = X'$.

Part (b) i) $A = \{3n : n \in \mathbb{Z}, 1 \leq n \leq 33\}$.
For ii) $B = \{3, 5, 9, 17, 33, 65\}$.
For iii) $A \cap B = \{3, 9, 33\}$ and $B - A' = \{3, 9, 33\}$.

Question 3

In part (a) the expressions are represented symbolically by $p \to q$ and $p \wedge \neg q$.

For part (b) the following columns should be added to the truth table:

$$\neg p; \quad (\neg p \wedge q); \quad \neg(\neg(p \wedge q); \quad q \to p.$$

The resulting column entries for the last two of these expressions should be equal, leading us to conclude that since the columns are identical the expressions are logically equivalent, or some such statement. Not making a concluding statement to justify what the columns mean resulted in loss of marks.

In (c) the contrapositive of the statement $q \to p$ is $\neg p \to \neg q$. In words a statement such as "if this animal is not a cat then it does not have a tail" was required.

In part (d) a logic network should be drawn with input p and q, with p going through a NOT gate and the result $\neg p$ going through an AND gate together with q. The output from this AND gate is $\neg p \wedge q$ and this is put through a final NOT gate giving output $\neg(\neg p \wedge q)$. A NAND gate was used by some candidates instead of the NOT followed by the AND gate, and this was acceptable. For full marks candidates should draw the different gates clearly and label them, as well as showing the relevant outputs.

Question 4

The first part of this question was about a function which takes the floor of a number which has been divided by 3. The floor is the nearest integer to a number such that it is less than or equal to that number. Thus $\lfloor 5 \rfloor = 1$ as 1 is the nearest integer less than or equal to 5/3. The set of ancestors of 0 in ii) is $\{1, 2\}$ as these are the only numbers in the domain with image 0. For iii) the function is not one to one since, for example $\lfloor 1 \rfloor = \lfloor 2 \rfloor$ from part ii), so not every element of the domain has a unique image. The function is onto as the range is equal to the co-domain which equals $\{0, 1, 2, 3\}$. Be careful not to confuse the co-domain and the range. Whether or not a function is one to one and/or onto is a question which arises almost every year and is well worth preparing for. A clear understanding of the concepts of domain, range and co-domain is necessary as well as the ability to find and interpret these according to the particular example.

Part (b) involved a similar floor function to that in part (a), however the co-domain is now $\mathbb{Z}$ rather than $\{0, 1, 2, 3\}$. Thus in i) the ancestors of 1 are $\{4, 5, 6\}$.
In ii) the range of $g = \{0, 1, 2, 3\}$.
Part iii) required candidates to demonstrate that they know when a function has an inverse, that is when it is both one to one AND onto. This function is neither. The range is $\{0, 1, 2, 3\}$ and the co-domain is $\mathbb{Z}$ so they are not equal. There are some elements of $\mathbb{Z}$ which are not in the range, such as $4, 5, 6, ...$

For part (c) arrow diagrams should be drawn as shown in the subject guide, Vol, 1, Chapter 4. For i), a diagram with arrows from each of a, b, c to $1, 2, 3$ is sufficient, with no arrow leading to 4. For ii) an arrow from $1, 2$ to a, b together with arrows from both 3 and 4 to c meets the requirements. In iii) a single arrow from each of a, b, c to a, b, c meets the conditions.

Question 5

Part (a) i) The majority of students calculated the first four terms of the sequence correctly by substituting $1, 2, 3$ and 4 for n, in the formula. Thus $u_1 = 3.1 - 2 = 1$; $u_2 = 3 \times 2 - 2 = 4$; $u_3 = 3.3 - 2 = 7$; $u_4 = 3.4 - 2 = 10$.
For ii) find n such that $3n - 2 = 1999$ giving $n = 667$. For iii) the recurrence relation is $u_{n+1} = u_n + 3$ and $u_1 = 1$. Many candidates missed the second part of this definition which is an essential part of it.
For iv) $\sum_{r=1}^{n}(3r-2) = 3\sum_{r=1}^{n} r - 2\sum_{r=1}^{n} 1$. We can now use the given formula for the first part of this expression to give $3\frac{n(n+1)}{2}$. The second part of the expression, $2\sum_{r=1}^{n} 1$, has been replaced by $2n$. This simplifies to $\frac{3n(n+1)}{2} - 2n$ or $\frac{3n^2+3n-4n}{2} = \frac{3n^2-n}{2}$.
For part v) simply substitute 667 for n in the result from part iv) to give the solution 667000.

Part (b) i) was given by $\sum_{r=1}^{100}(r) - \sum_{r=1}^{20}(r)$ and using the standard formula with first $n = 100$ then $n = 20$ and subtracting the second result of 210 from the first of 5050 to give the final answer 4840.
In part (b) ii) the sum $\sum_{r=1}^{33}(3r+1)$ is to be evaluated. Breaking this sum into parts gives $3\sum_{r=1}^{33} r + \sum_{r=1}^{33} 1$. The standard formula in the first part of this results in 1683 and the second part is 33. The sum of these gives the final answer of 1716.

Question 6

In part (a) i) the instructions for drawing these graphs were often not followed accurately, resulting in incorrect graphs. K_4 has four vertices and from each vertex there are three edges going to each of the other vertices. This is in the subject guide Vol. 1 section 5.1, page 73. C_4 can be drawn as a square with each of its four vertices at one of the corners and edges the sides of the square. W_4 is like K_4 but has one extra vertex (making five in total) which is connected to each of the original four vertices.
For ii) K_n has $\frac{n(n-1)}{2}$ edges, C_n has n edges and W_n has $2n$ edges.

In (b) the matrix $\mathbf{A}$ is:

$$\begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

The matrix $\mathbf{A}^2$ is given by:

$$\begin{pmatrix} 3 & 2 & 2 & 2 \\ 2 & 3 & 2 & 2 \\ 2 & 2 & 3 & 2 \\ 2 & 2 & 2 & 3 \end{pmatrix}$$

So for paths of length two from v_1 we look at the first row of this matrix and see that there are 6 paths if we discount the 3 paths from v_1 to itself.

Question 7

For part (a) i) a 5-bit binary string represents the subsets of the set S using a string of zeroes and ones. A one is used to show the presence of one of the elements and a zero to show the absence of the element. The position of the digit in the 5-bit binary string is significant. The first bit represents the element m, the second bit the element o and so on with the last bit representing the letter e. In ii) The subset $\{m, s, e\}$ is represented by the string 10011 and the subset corresponding to the string 01010 is $\{o, s\}$.
In iii) there are $2^5 = 32$ subsets of S.
For part (b) the digraph needs to show one of the letters o, u, e on each of three vertices which are all connected to each other by an arc in both directions and also a loop at each vertex. There are two more vertices at m and s which are not connected to any other vertices or themselves.
Thus in i) the relation is not reflexive as, for example, m is not related to itself.
In ii) the relation is symmetric since, whenever an element x is related to an element y, it is also the case that y is related to x, for all elements x and y in S.
In iii) the relation is transitive since, whenever X is related to y and y is related to z, then it is also the case that x is related to z, for all elements x, y and z in S.

Question 8

In (a) there are $6 \times 5 \times 4 = 120$ possible codes. Or $\dfrac{6!}{3!}$.
For part (b) $|A| = 60$. $|B| = 24$ and $|A \cap B| = 12$.
Thus for (c) we use the fact that $P(X) = \dfrac{|X|}{|S|}$.
$P(A) = 60/120 = 1/2, P(B) = 24/120 = 1/5, P(A \cap B) = 12/120 = 1/10$ and
$P(A \cup B) = P(A) + P(B) - P(A \cup B) = 72/120 = 3/5$.

Question 9

There are a few basic definitions in each chapter of the study guide which are key to understanding the concepts in that section. These should be noted and learnt as part of the revision process. These include: knowing what properties a graph must have if it is simple; what is meant by the degree of a vertex of a graph; how the number of edges of a graph may be found from its matrix representation; what properties a graph must have in order for it to be a tree and what is meant by a spanning tree. The concept of isomorphism is fundamental to graph theory and regularly included in the exam so candidates should ensure they have a good understanding of it and can construct and identify graphs which are either isomorphic or non-isomorphic, and give clear, specific reasons why. To say graphs are non-isomorphic as there is no one to one correspondence function between them is not sufficient as a justification.

For part (a) the states New Mexico and Oklahoma share a border with Texas.
In (b) the matrix is symmetric as the elements in each row are the same as the entries in the corresponding column. This reflects the fact that if state x shares a border with state y it is the case that the state y must share a border with the state x.
For (c) the graph is the same as graph H in the subject guide Vol 1, p.79, question 6.
For (d) the number of edges in the graph is half the sum of the entries in the matrix $= 14/2 = 7$.
In part (e) a non-simple graph may be drawn with a multiple edge (but no loops) as long as the degree sequence of $4, 3, 3, 2, 2$ is the same.
For (f) two spanning trees of the graph are shown in the subject guide Vol.2, p. 32, Example 3.1 the sixth and seventh tree in Figure 3.1. They are non-isomorphic as one is a path graph and one has a vertex of degree 3 which the path graph does not have.

Question 10

Part (a) (i) required candidates to multiply the matrix $\mathbf{P}$ by 2 and then subtract $\mathbf{Q}$ to give the solution:

$$\begin{pmatrix} 0 & -5 \\ 6 & 3 \end{pmatrix}$$

For ii) they needed to multiply the two matrices together, in the order given to obtain the solution

$$\begin{pmatrix} 8 & 13 \\ 12 & 19 \end{pmatrix}$$

For iii) candidates needed to multiply $\mathbf{P}$ by $\mathbf{R}$ and equate each of the elements in the result to the corresponding element in the identity matrix, giving $2a - 3 = 1, \quad 2b + 2 = 0, \quad 3a - 6 = 0, \quad 3b + 4 = 1$. Solving any two of these gives $a = 2,\ b = -1$.

Part (b) was a standard question involving Gaussian elimination. Firstly candidates were asked to copy down the augmented matrix. Care should be taken to include a zero for the second row where the y coefficient is so the equation $x - z = 1$ will read $x + 0y - z = 1$. It is a good idea to swap row 1 with row 2 or row 3 as there will then be a pivot of 1 in the first entry in the matrix. If we swap row 1 with row 3 to give the first row and take the second row as old row 3 - row 2 and the new row 3 as the original row 1 - twice row 2 we obtain the matrix:

$$\begin{pmatrix} 1 & 1 & -1 & : 0 \\ 0 & 1 & 0 & : -1 \\ 0 & -1 & 5 & : 11 \end{pmatrix}$$

Finally we reach row-echelon form by leaving the first and second row of the above matrix and making row 3 equal to row 2 + row 3 all divided by 5:

$$\begin{pmatrix} 1 & 1 & -1 & : 0 \\ 0 & 1 & 0 & : -1 \\ 0 & 0 & 1 & : 2 \end{pmatrix}$$

In the Gaussian elimination marks were given for method where the working was clear. Many candidates lost a mark as they did not fully reduce the matrix to one with ones on the leading diagonal, but left other numbers in the third row. Row-echelon form is needed. It is helpful if the row operations employed are clearly labelled and the order of transformations is shown, so any errors can be worked through and credit given. The final solution is $(3, -1, 2)$.