THIS PAPER IS NOT TO BE REMOVED FROM THE EXAMINATION HALLS

UNIVERSITY OF LONDON **CO1102 ZB**

BSc and Diploma Examination

COMPUTING AND INFORMATION SYSTEMS, CREATIVE COMPUTING AND COMBINED DEGREE SCHEME

Mathematics for Computing

Date and Time: Tuesday 10 May 2016 : 10.00 - 13.00

Duration: 3 hours

There are TEN questions in this paper. Candidates should answer all **TEN** questions. All questions carry equal marks and full marks can be obtained for complete answers to **TEN** questions. The marks for each part of a question are indicated at the end of the part in [.] brackets.

There are 100 marks available on this paper.

A hand held calculator may be used when answering questions on this paper but it must not be pre-programmed or able to display graphics text or algebraic equations. The make and type of machine must be stated clearly on the front cover of the answer book.

Question 1

(a) Working in binary and showing all carries, compute $(110011)_2 + (1011)_2$. [2]

(b) Consider the integer s defined by

$$s = \sum_{i=0}^{8} 2^{2i}.$$

Showing your working, express s and $2s$ in

i. binary notation;

ii. hexadecimal notation. [5]

(c) Showing your working, express the repeating decimal

$$0.353535353535\ldots$$

as a rational number in its simplest form. [3]

Question 2

Let B denote the set of all 9-bit binary strings and consider the function $\varphi : B \to \mathbb{N}$ defined by

$$\varphi(s) = \text{ the sum of all bits in the binary string s.}$$

(a) Give the cardinality of the set B. [1]

(b) i. Compute $\varphi(001001000)$.

ii. Give the *range* of φ.

iii. Find the number of strings s in B with $\varphi(s) = 2$ and explain why this shows that the function φ is *not* one-to-one. [4]

(c) A computer program generates a random 9-bit binary string. Justifying your answers, find the probability that

i. the string contains precisely four 0s;

ii. the string has an equal number of 0s and 1s;

iii. the string has more 0s than it has 1s. [5]

Question 3

(a) When is a positive integer p said to be a prime? [2]

(b) Express the integer 5040 as a product of its prime factors, using power notation for repeated factors. [2]

(c) Justifying your answer, say whether each of the following two propositions are true or false.

i. If $x = n^2 + 2n$ for some positive integer n, then n and $n + 2$ are factors of x.

ii. The number $n^2 + n + 41$ is a prime for all positive integers n. [4]

(d) Give the contrapositive of the following proposition concerning an integer p.

"If p is odd and $p > 3$ then $p + 1$ and $p - 1$ are not primes"

[2]

Question 4

(a) Consider the relation R on the set $\{1, 2, 3, \ldots, 10\}$ defined by

$$aRb \text{ if and only if 6 is a factor of } a - b.$$

Justifying your answers, say whether R is

i. symmetric;

ii. reflexive;

iii. transitive. [7]

(b) Consider the relation R' on the set $\{1, 2, 3, \ldots, 10\}$ defined by

$$aR'b \text{ if and only if 6 is a factor of } a + b.$$

Show that R' is neither reflexive, nor transitive. [3]

Question 5

(a) Let A, B and C be subsets of a universal set U and consider the two sets $X = (A \cup C) \cap (B \cup C)$ and $Y = A \cap B$.

i. Draw a labelled Venn diagram depicting the sets A, B and C in such a way that they divide U into 8 disjoint regions, and shade the two regions corresponding to X and Y. [3]

ii. Construct a membership table which shows that $(X - C) \subseteq Y$. [3]

(b) Give the set $A = \{a \in \mathbb{Z} \mid (3a+1)(a+2)(a-7) = 0\}$ by the listing method. [2]

(c) Give the set $B = \{-14, -9, -4, 1, 6, 11, 16, \ldots, 61\}$ by using rules of inclusion. [2]

Question 6

(a) Suppose that it is given that a graph G has degree sequence $4, 3, 3, 2, 2, 2$.

i. Explain why this information is not sufficient to enable us to draw G.

ii. Justifying your answer, find the number of vertices in G.

iii. Justifying your answer, find the number of edges in G. [4]

(b) Let G be the simple graph on the vertex set $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ with adjacency matrix

$$A = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}.$$

i. Draw G.

ii. Find a 6-cycle in G.

iii. Construct a graph H, which contains a 6-cycle and has the same degree sequence as G, but is non-isomorphic to G. Explain why the two graphs are not isomorphic. [6]

Question 7

(a) Explain why the number of edges in a simple graph G is precisely half the sum of the degrees of the vertices of G. [2]

(b) What properties must a graph satisfy in order for it to be a *tree*? [2]

(c) How many edges are there in a tree on n vertices? [1]

(d) Justifying your answer, say whether it is possible to construct a tree on 19 vertices in which every vertex has degree 1 or 3. [2]

(e) A binary search tree T is designed to store an ordered list of 21 records at its internal nodes.

i. Which record is stored at the root of T?

ii. Which records are stored at level 1 of T? [3]

Question 8

(a) Showing all your working, find the simplest possible form of the following two expressions.

i. $4 \cdot 2^n + 2^{n+2}$;

ii. $\log_2(\sqrt{2^x})$. [3]

(b) Consider the function $f\colon \mathbb{R} \to \mathbb{R}$ agiven by

$$f(x) = 3x^2 - 4x + 1.$$

i. Compute $f(2)$ and $f(f(2))$. [2]

ii. Find all the pre-images (ancestors) of 0 under f. [2]

iii. Show that the function $h\colon \mathbb{R} \to \mathbb{R}$ given by the rule

$$h(x) = f(f(x))$$

is $O(x^4)$. [3]

Question 9

A sequence is defined for $n \geq 0$ by the recurrence relation

$$a_{n+1} = 3a_n + 1$$

and the initial term $a_0 = 2$.

(a) Use the recurrence relation to calculate a_1, a_2, a_3 and a_4. [4]

(b) Prove by induction that $a_n > 3^n$ for all $n \geq 0$. [6]

Question 10

(a) Consider the two matrices

$$M = \begin{pmatrix} 1 & 3 & 5 \\ 0 & 1 & 2 \\ 2 & 1 & 1 \end{pmatrix} \text{ and } N = \begin{pmatrix} -1 & 2 & 1 \\ 4 & -9 & -2 \\ -2 & 5 & 1 \end{pmatrix}.$$

i. Showing your working, compute NM. [2]

ii. Show for all 3×1 matrices $\mathbf{x}$ and $\mathbf{y}$ that if $M\mathbf{x} = \mathbf{y}$, then $\mathbf{x} = N\mathbf{y}$. [3]

(b) Write the following system of equations as a matrix equation $A\mathbf{x} = \mathbf{b}$.

$$\begin{aligned} x + 3y + 5z &= 1 \\ y + 2z &= 1 \\ 2x + y + z &= 3. \end{aligned}$$

[2]

(c) Solve the system of equations from part (b). [3]

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