THIS PAPER IS NOT TO BE REMOVED FROM THE EXAMINATION HALLS

UNIVERSITY OF LONDON **CO0001 ZA**

Diploma Examination

COMPUTING AND INFORMATION SYSTEMS AND CREATIVE COMPUTING

Mathematics for Business

Date and Time: Tuesday 17 May 2016: 10.00 – 13.00

Duration: 3 hours

There are TEN questions on this paper. Candidates should answer **all TEN** questions. All questions carry equal marks and full marks can be obtained for complete answers to **TEN** questions. The mark for each part of a question are indicated at the end of the part in [.] brackets. Graph paper should be provided for this examination.

There are 100 marks available on this paper.

A hand held calculator may be used when answering questions on this paper but it must not be pre-programmed or able to display graphics, text or algebraic equations. The make and type of machine must be stated clearly on the front cover of the answer book.

Graph Paper is provided at the end of this question paper. If used, it must be detached and fastened securely inside the answer booklet.

UL16/0047

1. Given the formula

$$s = ut + \frac{1}{2}at^2$$

(i) Find s if u=10, t=3 and a=5 [2]

(ii) Rearrange the formula to make u the subject. [2]

(iii) Hence find u if s=35, t=2 and a=3.6 [2]

(iv) The variable t represents time and in this context is a positive value. Find t if s=150, u=5 and a=2. [4]

2. (a) Line L passes through the points $(1,8)$ and $(-1,2)$.

(i) Find the equation of L giving your answer in the form $y = mx + c$. [3]

(ii) Does the point $(15,45)$ lie on the line L? You must justify your answer. [2]

(b) Two parcel delivery businesses operate in the same town.

"Dave Delivers" charges a flat fee of $10 plus $0.50 per km to deliver a standard parcel.

"Parcels 4U" charges $1.25 per km to deliver a standard parcel but does not charge a flat fee.

(i) Draw a graph with 0 to 20 km on the x-axis and $ on the y-axis and two lines representing the fee charged by "Dave Delivers" and "Parcels 4U" to deliver a standard parcel. [3]

(ii) Use your graph to find the distance for which Business A and Business B would charge the same delivery fee. [2]

3. Given that the revenue and cost functions of a company are $R(x) = 20 + 12x - 3x^2$ and $C(x) = 83 - 18x$ respectively,

(i) Show that the profit function for the company can be written as $\pi(x) = -3x^2 + 30x - 63$ [2]

(ii) Find the value(s) of x for which $\pi(x) = 0$. i.e. the break-even points. [3]

(iii) Find the value of x that maximises the profit and hence find the maximum profit. [3]

(iv) Sketch a graph of the profit function

$$\pi(x) = -3x^2 + 30x - 63$$

for $0 \leq x \leq 10$ showing clearly where the graph cuts the axes. [2]

4. (a) A firm has the following supply and demand equations where Q is the quantity and P is the price of goods produced.

Supply Equation: $Q = -15 + 6P$
Demand Equation: $Q = 162 - 9P$

(i) Find the value of P which brings equilibrium to the market. [3]

(ii) What is the value of Q when there is equilibrium? [1]

(iii) State the values of P that bring a surplus to the market. [2]

(b) A company's profits are increasing at a constant rate and were \$3.4 million in 2010 and \$4.2 million in 2014. Assuming this trend continues what is the expected profit in 2017? [4]

5. (a) Given the matrices

$$A = \begin{pmatrix} 2 & 1 \\ -3 & 2 \end{pmatrix} \quad B = \begin{pmatrix} 5 & -2 \\ 3 & -1 \\ -2 & 4 \end{pmatrix} \quad C = \begin{pmatrix} -2 & 1 & 0 \\ 6 & -5 & 4 \end{pmatrix}$$

For each of the following expressions, either compute the resulting matrix or state that the matrices in the expression are non-conformable. [6]

(i) $A + B$

(ii) AB

(iii) BA

(iv) $B^T + C$

(b) The market share for two different magazines A and B changes from month to month according to the following matrix equation

$$\begin{pmatrix} A_{t+1} \\ B_{t+1} \end{pmatrix} = \begin{pmatrix} 0.7 & 0.4 \\ 0.3 & 0.6 \end{pmatrix} \begin{pmatrix} A_t \\ B_t \end{pmatrix}$$ where t is the time in months.

Given that $\begin{pmatrix} A_1 \\ B_1 \end{pmatrix} = \begin{pmatrix} 0.62 \\ 0.38 \end{pmatrix}$ find

(i) $\begin{pmatrix} A_2 \\ B_2 \end{pmatrix}$ [2]

(ii) The market share for magazine A after 3 months. [2]

6. A bakery makes two types of biscuit, Chocolate and Ginger. Each batch of chocolate biscuits requires 1 hour to produce and requires 3kg of sugar. Each batch of ginger biscuits requires 1.5 hours to produce and 2kg of sugar. The bakery has 8 hours to produce biscuits and although all the other ingredients are readily available they have only 20kg of sugar.

Chocolate biscuits sell for a profit of $100 per batch and ginger biscuits sell for a profit of $85 per batch.

(i) The problem of finding the number of batches of each type of biscuit the bakery should produce in order to maximise its profit can be modelled as a linear programming problem in two unknowns.

If x and y represent the number of batches of chocolate and ginger batches respectively, then the constraint on time is

$$x + 1.5y \leq 8$$

Write a similar equation showing the constraint on sugar. [1]

(ii) What other constraints on x and y are required? [1]

(iii) Draw a sketch graph to show the feasible region for this problem. [3]

(iv) The profit function for the problem is to maximise $\pi = 100x + 85y$.

Find the co-ordinates of the four corner points of the feasible region. Hence determine the number of batches of each type of biscuits that the bakery should make in order to maximise their profit and find this maximum profit. You may assume that it is feasible to make part of a batch. [5]

7. A factory has a profit function given by

$$\pi(q) = -2q^3 + 8q^2 + 32q - 19$$

where q is the quantity of goods produced.

The profit function has two turning points. One is when $q = -\frac{4}{3}$.

(i) Explain why the value $q = -\frac{4}{3}$ is not meaningful in this context. [1]

(ii) Find the value of q at the other turning point and determine whether it is a maximum or minimum. [7]

(iii) Evaluate $\pi(q)$ at the value of q found in part (ii) and explain the significance of your answer in the context of the question. [2]

8. Differentiate the following functions. You do not need to expand or simplify your answers.

(i) $y = 5x^3 - 2x^2 + 6x - 7$ [2]

(ii) $y = 3(6x - 7)^5$ [2]

(iii) $y = e^x(4x^2 - 6x + 7)$ [3]

(iv) $y = \frac{2x^3+7}{8-x}$ [3]

9. (a) Simplify

(i) $\log_a a^3$ [1]

(ii) $\log_a 6 - \log_a 2$ [1]

(iii) $\left(\frac{1}{\sqrt{x}}\right)^{-6}$ [2]

(b) Solve the equation $240 = 450e^{-x}$ giving your answer correct to 2 decimal places. [2]

(c) Sales, S, of a new toy are expected to grow according to the equation

$$S = 40000(1 - e^{-0.4t})$$

where t is the number of weeks after the launch of the product.

(i) Show that the number of toys sold after five weeks is 34600 correct to the nearest hundred. [1]

(ii) Find the time taken for the sales to reach 30000. [3]

10. (a) Find the following indefinite integrals:

(i) $\int 6x^2 . dx$ [1]

(ii) $\int (4 - 3\sqrt{x+2})\ . dx$ [3]

(b) The function

$$P = q^2 + 15q + 74$$

represents a supply function with market equilibrium occurring when $q = 6$.

(i) Evaluate $\int_0^6 (q^2 + 15q + 74)\, . dq$ showing all of your working. [3]

(ii) Hence calculate the total producer surplus. [3]

END OF PAPER

Candidates using this paper must tie it into their Answer-Books so as to face the answer to the question to which it relates. They must write their number and subject of the paper on every sheet used.

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