**UNIVERSITY OF LONDON**                                  **CO1102 ZB**

**BSc, CertHE and Diploma Examination**

**COMPUTING AND INFORMATION SYSTEMS, CREATIVE COMPUTING AND COMBINED DEGREE SCHEME**

**Mathematics for Computing**

Thursday 10 May 2018 :      10.00 – 13.00

Time allowed:               3 hours

There are **TEN** questions on this paper. Candidates should answer all **TEN** questions. All questions carry equal marks and full marks can be obtained for complete answers to **TEN** questions. The marks for each part of a question are indicated at the end of the part in [.] brackets.

There are 100 marks available on this paper.

A handheld calculator may be used when answering questions on this paper but it must not be pre-programmed or able to display graphics text or algebraic equations. The make and type of machine must be stated clearly on the front cover of the answer book.

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**Question 1**

(a) Convert the decimal number 365 to

    i. a binary number;

    ii. a hexadecimal number.

[3]

(b)  i. Proving your answer, say whether or not the binary number $101.101$ is a rational number.

    ii. Give an example of an irrational number $a$ where $2 < a < 3$.

[3]

(c) Showing all working, convert the repeating decimal number

$$0.454545...$$

to a fraction in its lowest terms.

[4]

**Question 2**

Given the universal set $\mathbb{Z}$ of integers, let $E$ denote the subset of even integers, $A$ denote the subset of integers divisible by 5 and $B$ the subset of integers divisible by $3$.

(a) Use set operations to express the following in terms of the sets $E$, $A$, $B$.

    i. The set of odd integers;

    ii. the set of integers divisible by $6$;

    iii. the set of integers with last digit $5$.

[3]

(b) Draw separate Venn diagrams to illustrate each of the sets $X$ and $Y$ where $X = E - (A \cap B)$, $Y = (E - A) \cap (E - B)$. Include appropriate keys to the shaded areas in your diagrams. [4]

(c) Give an example of an integer for each of the following three sets:

  i. $Y$;

  ii. $X - Y$;

  iii. $A \cap B \cap E$.

[3]

## Question 3

Let $p$ and $q$ be two logical statements.

(a) Give the truth table for each of the compound statements.

  i. $p \wedge q$;

  ii. $p \vee q$;

  iii. $p \leftrightarrow q$ .

[3]

(b) Add appropriate extra columns to the truth tables in (a) to prove that the statements $p \leftrightarrow q$ and $(p \wedge q) \vee \neg (p \vee q)$ are logically equivalent. [3]

(c) Design a logic network with inputs $p, q$ and output $p \leftrightarrow q$. Label the diagram carefully, showing input and output at each gate. [4]

**Question 4**

(a) Let $B$ denote the set of all 5-bit binary strings and let $\mathbb{N} = \{0, 1, 2, 3, ...\}$ be the set of non-negative integers. Let the function SUM : $B \to \mathbb{N}$ be defined as follows:

SUM($S$) = the sum of the bits in the string $S \in B$.

   i. Find the image under SUM of the string 10111.
   ii. Find all strings with an image of 1.
   iii. Find the range of SUM.
   iv. Say whether SUM is one-to-one, justifying your answer.
   v. Say whether SUM is onto, justifying your answer.

[5]

(b) Decide, for each of the following three functions, whether it is invertible. If so give the inverse function and if not, give a reason why no inverse function exists.

   i. $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 3x$.
   ii. $g : \mathbb{Z} \to \mathbb{Z}$ defined by $g(x) = 3x$.
   iii. The function SUM : $B \to \mathbb{N}$, from part (a).

[5]

**Question 5**

(a) A sequence is defined by the recurrence relation:

$u_{n+2} = 3u_{n+1} - 2u_n$ for $n \geq 0$ and the initial terms $u_0 = 1$ and $u_1 = 2$.

   i. Calculate $u_2, u_3, u_4$ and $u_5$, showing your working.
   ii. Suggest a non-recursive formula for $u_n$ in terms of $n$. You do not need to prove this formula.

[5]

(b) Given the sum $s_n = 1 + 8 + 15 + 22 + 29 + ... + 7n - 6$ :

   i. For a given $k \geq 1$, express the sum $s_{k+1}$ as a function of the sum $s_k$.
   ii. Prove by induction that:

$$\sum_{r=1}^{n}(7r - 6) = \frac{n(7n - 5)}{2}$$

for all $n \geq 1$.

[5]

**Question 6**

In a tournament with $n$ players, each player plays against every other player precisely once. Each match results in a win for one of the players and a loss for the other (there are no draws).

(a) The results of a tournament can be modelled by a digraph **D**. Describe this digraph, giving the following information.

    i. State carefully what the vertices of **D** represent and what it means when there is an arc in **D** directed from a vertex $u$ to a vertex $v$.

    ii. How many vertices are there in **D**?

    iii. How many arcs are there in **D**?

[4]

(b) Five players, $A, B, C, D, E$ complete a tournament. The results are as follows: $A$ beat $B$ and $E$; $B$ beat $E$; $C$ beat $A$, $B$ and $D$; $D$ beat $A$, $B$ and $E$; $E$ beat $C$. Let **D** be the digraph modelling this tournament.

    i. Construct the adjacency matrix **A(D)** of **D**, explaining briefly how you obtain the elements of the row of **A(D)** corresponding to a given vertex of **D**.

    ii. What property of this matrix indicates it represents a digraph rather than an undirected graph?

[3]

(c) A graph $G$ has vertex set $V(G) = \{u, v, w, x, y, z\}$ and edge set defined by the following adjacency list:

    $x : y, v; \quad y : x, z; \quad z : y; \quad u : y, v; \quad v : x, u, w; \quad w : v$

    i. Draw the graph $G$.

    ii. Draw a simple, connected graph, $H$, that has the same number of vertices and the same degree sequence as $G$ but is not isomorphic to $G$. Explain why the graph you have drawn is not isomorphic to $G$.

[3]

**Question 7**

Let $R$ be the relation on the set $\mathbb{N}$ of non-negative integers defined by $aRb$ if $a - b$ is divisible by 4.

(a) Find the smallest value $a \in \mathbb{N}$ such that $aR31$. [1]

(b) Find the set of values of $b \in \mathbb{N}$ such that $0Rb$. [2]

(c) Find all the elements of the set $\{x \in \mathbb{N} : xR3\}$. [2]

(d) Justifying your answer in each case, say whether or not the relation $R$ is

    i. reflexive;

    ii. symmetric;

    iii. transitive;

    iv. an equivalence relation.

[5]

**Question 8**

(a) A group of 120 students was surveyed and the numbers claiming to have studied three programming languages, or not, was recorded and the results shown in the following table where:
P is the set of students who said they had studied Python;
C is the set of students who said they had studied C$^{++}$;
J is the set of students who said they had studied Java.

| P | C | J | P∩C | P∩J | C∩J | P∩C∩J |
|---|---|---|-----|-----|-----|-------|
| 47 | 45 | 39 | 15 | 11 | 12 | 7 |

    i. Taking as universal set the whole group of 120 students, draw a labelled Venn diagram with 8 regions depicting the sets P, C and J. Enter in each of the 8 regions the number of students in that region, ensuring the total sum of your 8 numbers is 120.

    ii. Find the probability that a student in this group would claim not to have studied any of these three programming languages. [4]

(b) One of the integers $1, 2, 3, 4, ..., 20$ is selected at random so that each of these integers has an equal chance of being selected. Let $A, B, C$ denote the following events:

$A$ : the integer selected is even;

$B$ : the integer selected is prime;

$C$ : the integer selected has one digit.

Calculate the probabilities of the following events:

  i. $A$;

  ii. $B$;

  iii. $C$;

  iv. $A \cap C$;

  v. $B \cap C$;

  vi. $A \cup C$.

[6]

## Question 9

(a)  i. State the two properties a graph must have in order to be a *tree*.

  ii. Draw three non-isomorphic trees with five vertices.

  iii. Say which of the trees you have drawn in part (ii) are path graphs.

  iv. How many edges are there in a path graph with 100 vertices?

[6]

(b) A binary search tree is designed to hold 60 records at its internal nodes.

  i. Draw the first three levels of this tree.

  ii. What is the height of this tree?

[4]

**Question 10**

(a) Given two matrices **A** and **B** where $\mathbf{A} = \begin{pmatrix} 0 & 4 & 1 \\ 2 & 1 & 3 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 0 & 3 \\ 5 & 1 \\ 0 & 2 \end{pmatrix}$

    i. Find **AB**

    ii. Consider a random $3 \times 2$ matrix **M**. Say for each of the following three matrices, whether they exist, and if they do, give their size:
**MA**; **M** + **B**; **BM**.

[5]

(b) A system of equations in three unknowns $x, y, z$ is being solved by Gaussian elimination. The augmented matrix corresponding to this system has been reduced to the following:

$$\begin{pmatrix} 1 & 3 & -2 & : & -7 \\ 0 & 1 & 5 & : & 14 \\ 0 & 2 & 3 & : & 7 \end{pmatrix}$$

    i. Copy down this matrix and circle the entry which represents the *pivot*.

    ii. Describe the row operation that is required for the next step in the Gaussian elimination.

    iii. Complete the Gaussian elimination to solve the system.

[5]

**END OF PAPER**