THIS PAPER IS NOT TO BE REMOVED FROM THE EXAMINATION HALLS

UNIVERSITY OF LONDON **CO1102 ZA**

BSc, CertHE and Diploma Examination

COMPUTING AND INFORMATION SYSTEMS, CREATIVE COMPUTING AND COMBINED DEGREE SCHEME

Mathematics for Computing

Thursday 10 May 2018 : 10.00 – 13.00

Time allowed: 3 hours

There are **TEN** questions on this paper. Candidates should answer all **TEN** questions. All questions carry equal marks and full marks can be obtained for complete answers to **TEN** questions. The marks for each part of a question are indicated at the end of the part in [.] brackets.

There are 100 marks available on this paper.

A handheld calculator may be used when answering questions on this paper but it must not be pre-programmed or able to display graphics text or algebraic equations. The make and type of machine must be stated clearly on the front cover of the answer book.

Question 1

(a) Convert the decimal number 301 to

i. a binary number;
ii. a hexadecimal number.

[3]

(b) i. Proving your answer, say whether or not the binary number 110.111 is a rational number.
ii. Give an example of an irrational number a where $0 < a < 1$.

[3]

(c) Showing all working, convert the repeating decimal number

$$0.727272...$$

to a fraction in its lowest terms. [4]

Question 2

Given the universal set $\mathbb{Z}$ of integers, let E denote the subset of even integers, A denote the subset of integers divisible by 5 and B the subset of integers divisible by 3.

(a) Use set operations to express the following in terms of the sets E, A, B.

i. The set of odd integers;
ii. the set of integers divisible by 6;
iii. the set of integers with last digit 5.

[3]

(b) Draw separate Venn diagrams to illustrate each of the sets X and Y where $X = E - (A \cup B)$, $Y = (E - A) \cup (E - B)$. Include appropriate keys to the shaded areas in your diagrams. [4]

(c) Give an example of an integer for each of the following three sets:

i. X;

ii. $Y - X$;

iii. $A \cap B \cap E$.

[3]

Question 3

Let p and q be two logical statements.

(a) Give the truth table for each of the compound statements.

i. $p \wedge q$;

ii. $p \vee q$;

iii. $p \leftrightarrow q$.

[3]

(b) Add appropriate extra columns to the truth tables in (a) to prove that the statements $p \leftrightarrow q$ and $(p \wedge q) \vee \neg(p \vee q)$ are logically equivalent. [3]

(c) Design a logic network with inputs p, q and output $p \leftrightarrow q$. Label the diagram carefully, showing input and output at each gate. [4]

Question 4

(a) Let B denote the set of all 5-bit binary strings and let $\mathbb{N} = \{0, 1, 2, 3, ...\}$ be the set of non-negative integers. Let the function SUM : $B \to \mathbb{N}$ be defined as follows:

SUM(S) = the sum of the bits in the string $S \in B$.

i. Find the image under SUM of the string 01011.
ii. Find all strings with an image of 4.
iii. Find the range of SUM.
iv. Say whether SUM is one-to-one, justifying your answer.
v. Say whether SUM is onto, justifying your answer.

[5]

(b) Decide, for each of the following three functions, whether it is invertible. If so give the inverse function and if not, give a reason why no inverse function exists.

i. $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 2x$.
ii. $g : \mathbb{Z} \to \mathbb{Z}$ defined by $g(x) = 2x$.
iii. The function SUM : $B \to \mathbb{N}$, from part (a).

[5]

Question 5

(a) A sequence is defined by the recurrence relation:

$u_{n+2} = 3u_{n+1} - 2u_n$ for $n \geq 0$ and the initial terms $u_0 = 1$ and $u_1 = 2$.

i. Calculate u_2, u_3, u_4 and u_5, showing your working.
ii. Suggest a non-recursive formula for u_n in terms of n. You do not need to prove this formula. [5]

(b) Given the sum $s_n = 1 + 6 + 11 + 16 + 21 + ... + 5n - 4$:

i. For a given $k \geq 1$, express the sum s_{k+1} as a function of the sum s_k.
ii. Prove by induction that:

$$\sum_{r=1}^{n}(5r - 4) = \frac{n(5n - 3)}{2}$$

for all $n \geq 1$.

[5]

Question 6

In a tournament with n players, each player plays against every other player precisely once. Each match results in a win for one of the players and a loss for the other (there are no draws).

(a) The results of a tournament can be modelled by a digraph **D**. Describe this digraph, giving the following information.

i. State carefully what the vertices of **D** represent and what it means when there is an arc in **D** directed from a vertex u to a vertex v.

ii. How many vertices are there in **D**?

iii. How many arcs are there in **D**? [4]

(b) Five players, A, B, C, D, E complete a tournament. The results are as follows: A beat C and D; B beat A, C and D; C beat E; D beat C; E beat A, B and D. Let **D** be the digraph modelling this tournament.

i. Construct the adjacency matrix **A**(**D**) of **D**, explaining briefly how you obtain the elements of the row of **A**(**D**) corresponding to a given vertex of **D**.

ii. What property of this matrix indicates it represents a digraph rather than an undirected graph? [3]

(c) A graph G has vertex set $V(G) = \{u, v, w, x, y, z\}$ and edge set defined by the following adjacency list:

$x : y, u; \quad y : x, z, v; \quad z : y; \quad u : x, v; \quad v : y, u, w; \quad w : v$

i. Draw the graph G.

ii. Draw a simple, connected graph, H, that has the same number of vertices and the same degree sequence as G but is not isomorphic to G. Explain why the graph you have drawn is not isomorphic to G. [3]

Question 7

Let R be the relation on the set $\mathbb{N}$ of non-negative integers defined by aRb if $a - b$ is divisible by 5.

(a) Find the smallest value of $a \in \mathbb{N}$ such that $aR43$. [1]

(b) Find the set of numbers $b \in \mathbb{N}$ such that $0Rb$. [2]

(c) Find all the elements of the set $\{x \in \mathbb{N} : xR3\}$. [2]

(d) Justifying your answer in each case say whether or not the relation R is

i. reflexive;

ii. symmetric;

iii. transitive;

iv. an equivalence relation.

[5]

Question 8

(a) A group of 120 students was surveyed and the numbers claiming to have studied three programming languages, or not, was recorded and the results shown in the following table where:
P is the set of students who said they had studied Python;
C is the set of students who said they had studied C^{++};
J is the set of students who said they had studied Java.

P	C	J	$P \cap C$	$P \cap J$	$C \cap J$	$P \cap C \cap J$
43	47	37	11	8	13	5

i. Taking as universal set the whole group of 120 students, draw a labelled Venn diagram with 8 regions depicting the sets P, C and J. Enter in each of the 8 regions the number of students in that region, ensuring the total sum of your 8 numbers is 120.

ii. Find the probability that a student in this group would claim not to have studied any of these three programming languages. [4]

(b) One of the integers $1, 2, 3, 4, ..., 20$ is selected at random so that each of these integers has an equal chance of being selected. Let A, B, C denote the following events:
A : the integer selected is odd;
B : the integer selected is prime;
C : the integer selected has two digits.
Calculate the probabilities of the following events:

i. A;

ii. B;

iii. C;

iv. $A \cap C$;

v. $B \cap C$;

vi. $A \cup C$.

[6]

Question 9

(a) i. State the two properties a graph must have in order to be a $tree$.

ii. Draw three non-isomorphic trees with five vertices.

iii. Say which of the trees you have drawn in part (ii) are path graphs.

iv. How many edges are there in a path graph with 100 vertices?

[6]

(b) A binary search tree is designed to hold 50 records at its internal nodes.

i. Draw the first three levels of this tree.

ii. What is the height of this tree? [4]

Question 10

(a) Given two matrices **A** and **B** where $\mathbf{A} = \begin{pmatrix} 4 & 1 & 0 \\ 0 & 2 & 3 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 3 & 0 \\ 1 & 5 \\ 2 & 0 \end{pmatrix}$

i. Find **AB**

ii. Consider a random 3×2 matrix **M**. Say, for each of the following matrices, whether they exist, and if they do, give their size: **MA**; **M** + **B**; **BM**.

[5]

(b) A system of equations in three unknowns x, y, z is being solved by Gaussian elimination. The augmented matrix corresponding to this system has been reduced to the following:

$$\begin{pmatrix} 1 & 3 & -2 & : -3 \\ 0 & 1 & 5 & : \ 9 \\ 0 & 2 & 3 & : \ 4 \end{pmatrix}$$

i. Copy down this matrix and circle the entry which represents the *pivot*.

ii. Describe the row operation that is required for the next step in the Gaussian elimination.

iii. Complete the Gaussian elimination to solve the system.

[5]

END OF PAPER