Coursework reports 2013–2014

CO0001 Mathematics for business

Coursework assignment 1

Question 1

This question involved finding the geometric mean of a set of numbers. In parts (a), and (b) i. and ii., there were two numbers concerned and many candidates were able to find the square root of the product of the two numbers. The question specified the solution be given to two decimal places which meant candidates needed to work throughout to three or more decimal places and then correct back at the very end.

In (b) iii., three numbers were involved and candidates had therefore to take the cube root of the product of all three numbers, again working throughout to at least three decimal places and correcting back to two decimal places for the final solution.

In (c) one of the three numbers was unknown and if represented by a symbol such as x an equation could be set up and solved for x: $6.9.x = 8^3$ or $54x = 512$ which could then be solved to find x. Many candidates were able to do the first two parts of this question successfully. Some candidates did not use the ∏ button on their calculator but a less accurate version of ∏ such as 3.14. It is necessary to be as accurate as possible and be familiar with the facilities of the calculator you are using.

Question 2

In this question a consumption function is given in terms of the variable X which represents income. Candidates were asked to sketch the graph of this function, showing the points where the graph meets the axis. This is easily done by setting first $X = 0$ to give C = 20, and then setting $C = 0$ and rearranging the equation to give $X = 50$. The two points thus found are (0, 20) and (50, 0) if X is on the horizontal axis and C on the vertical axis. Another point can be found as a check such as $X = 10$, $C = 24$. The line joining the two intercepts should pass through the extra point too, otherwise candidates need to check their calculations.

In part (b) a point on the savings line and its gradient were given and candidates were asked to find the equation of the savings function. This is a common question that candidates should be familiar with and be able to calculate using a standard method for finding the equation of a line of the form $y = mx + c$. In this particular case we have a line of the form $S = 0.6\,x + c$ and c is found by substituting in the given point (10, –14), that is $S = –14$ and $x = 10$. Thus we have $–14 = 0.6\ .10 + c$ and this gives c = –20. Unfortunately, some candidates did not realise that the 'c' in this formula refers to the point where the line cuts the vertical axis and confused it with the consumption function of part (a).

Part (c) required taking the two linear equations for consumption and saving and adding them to give X.

Question 3

In the first part of this question, candidates were asked to find the profit function. This is done by subtracting costs from revenue: $25q - 0.5\ q\hat{\ }2$ $-(q+ 88)$. It is important to subtract the whole cost function from the revenue. This gives the required expression for profit in terms of q.

In part (b) i., the break–even points are found by setting the profit equal to zero and solving the quadratic to find the two values of q which make the profit zero. This may be done using the formula for finding the roots of a quadratic equation. As the coefficient $q\hat{\ }2$ is –0.5 this may be simplified by first multiplying through by –2: $q\hat{\ }2 - 48\ q + 176 = 0$. This will represent –2 times profit which has the same roots as the original profit function.

The value of q which gives maximum profit occurs half way between the roots or alternatively may be found by differentiating the profit function with respect to q and setting this derivative to zero and solving for q.

The maximum profit itself is found by substituting this value of q into the original profit equation (not the one multiplied by –2).

Question 4

In part (a) the left–hand side of the equation should be multiplied out first and then the terms in x gathered together to give $-6 = 3\ x$. Dividing both sides by 3 gives $-2 = x$.

Alternatively, from the given equation both sides may be divided through by 2 to give $x - 3 = 2.5\ x$. Then gathering the terms in x together gives $-3 = 1.5\ x$ and by dividing both sides by 1.5 the same result is obtained.

In part (b) both sides of the equation should be multiplied by x to give $5 = 20\ x\hat{\ }2$ and then dividing by 20 gives $0.25 = x\hat{\ }2$. Taking the square root of both sides gives $x = 0.5$ or -0.5. Some candidates did not give the negative solution.

In part (c) multiplying both sides by x gives a quadratic equation in x: $x(3 - x) = 2$ which when multiplied out and rearranged gives $0 = x\hat{\ }2 - 3\ x + 2$. This factorises into $(x - 2)(x - 1) = 0$, giving the solutions $x = 2$, $x = 1$.

Question 5

In (a) the point $p = 120$, $q = 30$ should satisfy all of the inequality constraints if it lies in the feasible region. It does not satisfy the first constraint as $5\ p + q = 630$ which is not less than 600. Therefore the point does not lie in the feasible region. Some such justification was required.

To find the corners of the feasible region in part (b) the points where the lines $5\ p + q = 600$ and

$p + q = 160$ cut the axes must be found and sketched. Also the point where these two lines intersect needs to be calculated and this may be done by solving the two equations simultaneously to give

$p = 110$ and $q = 50$. The four corners are thus (0,0) (160,0), (110,50) and (0,120). Substituting these points into the profit function gives maximum profit of 960 at (0,120).

Coursework assignment 2

Question 1

Part (a) of this question involved finding various composite functions of *f(x)* and *g(x)*. Candidates were not required to simplify the resulting functions so it was not a test of algebraic manipulation but rather of understanding how functions operate and how they can be combined. Some candidates confuse *g(f(x)),* where *f* is applied first to give an input of $x + 2$ into the function *g* which results in *(x+2)^2,* with *f(g(x))* where *g* is applied first and then input into the function *f.*

In the second part of this question candidates were asked to calculate the derivative of *y* with respect to *x* for three functions. The first two require knowledge of the chain rule and the third may be most simply evaluated applying laws of indices to give *y = x^-2* and a derivative of -2*x^-3* with no need to simplify further.

Question 2

The dimensions of *A* are 3 x 2 and of *B,* 2 x 3.

In part (b) and ((c) i. and ii. represent valid matrices. In i we have a 2 x 3 added to a 2 x 3 which is possible and gives a 2 x 3 matrix. In ii we have a 2 x 3 multiplied by a 3 x 1 which again is possible and results in a 2 x 1 matrix.

It was notable that a considerable number of candidates could not carry out a matrix multiplication correctly and had not learnt the method for doing so.

Question 3

The first part of this question asked candidates to write the two given series using Sigma notation, a general term and appropriate upper and lower limits. Thus they needed to work out the number of terms in the series and whether it was a GP or another type of series. (a) was a GP with first term $a = 5$, common ration $r = 2$. The number of terms can be found by setting the general term *ar^(n - 1)* = 1280, and solving for n to give $n = 9$. The upper limit is then 9 and the lower limit is 1 with a general term of 5.2^ *(n - 1)*. It is possible to have a different general term which requires different upper and lower limits to produce the given series.

In (b), which is also a GP, the first term is $a = 1200$ and the common ration $r = 0.1$. Again, n may be found by solving the equation *ar^(n - 1)* = 0.00012 to give $n = 8$.

The sums of these series can be found by substituting the relevant values of a, r and n into the formula for the sum of a geometric progression.

Question 4

Stationary points of a function may be found by setting the first derivative equal to zero. Here

dy/dx = 3x^2 – 6 x + 3 and when equated to zero can be divided throughout by 3 and then factorised into (x – 1)^2 = 0. So there is only one stationary point of this cubic function at x = 1, and it is a point of inflexion, though candidates were not required to state this. The corresponding y co-ordinate is found by substituting x = 1 into the original equation to give y = 8.

Part (b) requires division of the original function by $(x + 1)$ or multiplying out the linear and quadratic equation and equating the co-efficients of x^3, x^2, x and the constant term to find b and c.

In part (c) setting $f(x) = (x + 1)(x^2 - 4x + 7)$ equal to zero to find the roots we see that there is a root at $x = -1$ from the linear factor. However, applying the formula to the quadratic we see that it has no real roots as its determinant is negative and we cannot take the square root of a negative number. So the only real root of the original cubic equation is $x = -1$.

Question 5

In (a) i., the indefinite integral with respect to x is $x^2/2 - 2x$. Then the upper and lower limits are calculated and the second result subtracted from the first. Candidates should be aware of the integral notation and the use of square brackets and placing of the limits after them. In ii. the indefinite integral is $\ln(x-2)$ and again the limits should be substituted appropriately to evaluate the solution.

In part (b) a knowledge of the graph of $y = x^3$ is needed to ascertain whether any of the area is below the x-axis and should therefore be separately calculated and then added. In this case the area is all above the axis and can be represented by one expression with upper limit of 4 and lower limit of 0. Candidates were not asked to evaluate this integral, just to write down an appropriate expression for it.