Examiners' commentaries 2015–2016

CO0001 Mathematics for business – Zone A

General remarks

The examination paper was based on the material in the subject guide. Candidates who had properly understood this material were able to complete the examination in the given time and to a high standard. Although some of the questions were purely numerical and tested skills such as differentiation and integration, most questions contained parts designed to test whether candidates really understood what they were doing and why; that is, how the mathematics related to real-life problems. Some candidates were unable to explain the significance of certain variables, or what their answer meant, thereby betraying a lack of real understanding. In some cases, candidates were unable to apply methods that they had correctly used in other parts of the examination if the method was not explicitly stated or obvious. For example, most candidates were able to solve the quadratic equation in Question 3, but were unable to solve the simpler quadratic equation in Question 1 since it first required rearranging. Unfortunately, this again demonstrated a lack of proper understanding. The following are comments on each individual question.

Comments on specific questions

Question 1

Most candidates were able to use substitution and to re-arrange the given formula correctly to gain the first six marks on this question. However, a common mistake was to apply the power 2 in the term $\frac{1}{2}at^2$ to the whole term, incorrectly calculating $(\frac{1}{2}at)^2$ rather than the correct $\frac{1}{2} \times a \times t^2$.This is an application of BODMAS (the order of operations), a fundamental concept of mathematics that you must master if you are to progress.

The final part of the question was not well answered with few candidates realising that you must solve the quadratic equation $0 = t^2 + 5t - 150$ in order to find the value of t. The quadratic can be solved by factorising or by using the quadratic formula to give the solutions $t = 10$ or $t = -15$, but since t represents time and is positive, the final answer is $t = 10$ only.

Question 2

Many candidates were able to find the gradient using the formula $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-8}{-1-1} = 3$ and then substitute this value for m into the equation to find the question of the line, which was

The given point (15, 45) does not lie on this line because the equation $y = 3x + 5$ is not satisfied when $x = 15$, $y = 45$ In this case, the left-hand side of the equation is equal to 45, but the right hand side is equal to $3(15) + 5 = 50$. Most candidates stated that the point did not lie on the line, but not all were able to clearly explain why.

Although most candidates had worked correctly with straight line graphs in part (a), not many were able to relate this knowledge to part (b). 'A flat fee of \$10 plus \$0.50 per km' translates to the line $y = 0.5x + 10$.

Similarly, a charge of \$1.25 per km with no flat fee translates to the line $y = 0.5x + 10$. Therefore, the graph for this question should have shown two straight lines, both with a positive gradient, one cutting the y-axis at 10 and the other with a steeper gradient passing through the origin.

Candidates who had drawn the graph correctly could easily read off the point of intersection to answer part (ii).

Question 3

This question was generally well answered, with the exception of part (iv). Candidates were able to apply BODMAS correctly here (possibly helped by the fact that the answer was given) in order to obtain the required profit function. Most were then able to solve the quadratic using either

factorisation or the quadratic formula to find the break-even points at $x = 3$ and $x = 7$. Since the equation is a quadratic, the graph is symmetrical, so the maximum profit will occur at the mid-point of the two break-even points; that is, when $x = \frac{3+7}{2} = 5$ and then $\pi(5) = -3(5)^2 + 30(5) - 63 = 12$, so the maximum profit is 12.

Although many candidates answered parts (i), (ii) and (iii) well, they were unable to sketch the graph of the function. However, this is now a relatively easy task since the break-even points at $x = 3$ and $x = 7$ tell us the roots (x-axis intercepts) and the maximum point at (5,12) tells us where the curve turns. Joining up these three co-ordinates with a smooth symmetrical curve, which continues below the x-axis, completes the graph.

Question 4

Equilibrium occurs when supply equals demand. Therefore, to answer Question 4, it was necessary to set the two given equations equal to each other and then solve to find first P and then Q. Equilibrium occurs when $P = 11.8$ and $Q = 55.8$. Most candidates were able to find these values.

Surplus occurs when supply is greater than demand. Looking at the two given equations for supply and demand, you can see that as P increases, supply will increase but demand will decrease. The two equations are equal when $P = 11.8$ and so *supply* > *demand* when $P > 11.8$. Thus, the values of P which bring a surplus to the market are $P > 11.8$. Fewer candidates were able to answer this part of the question.

Part (b) was generally quite well answered. Since the profits are increasing at a constant rate, it is only necessary to work out by how much they are increasing per year (\$0.2 million) and then apply this for a further three years. This gave a final answer of \$4.8 million.

Question 5

This is a straightforward question for candidates who know the rules of matrix addition and multiplication. In order to add matrices, they must have the same dimensions; that is, the same number of rows and columns. Since A is of dimensions 2×2 and B is of dimensions 3×2 it is not possible to add A and B , so $A + B$ is nonconformable.

We can only multiply matrices if the number of columns in the first equals the number of rows in the second. Hence AB is nonconformable since A has two columns but B has three rows. However, BA is conformable since B has two columns and A has two rows. The result will be a matrix of dimensions 3×2.

In part (iv), B^T means the transpose of matrix B. To find B^T swap the rows and columns so that $B^T = \begin{pmatrix} 5 & 3 & -2 \\ -2 & -1 & 4 \end{pmatrix}$. This matrix has dimensions 2×3, which is the same as matix C, so $B^T + C$ is conformable and results in another 2×3 matrix.

Part (b) is an application of matrix multiplication. $\begin{pmatrix} A_2 \\ B_2 \end{pmatrix}$ is found by multiplying $\begin{pmatrix} 0.7 & 0.4 \\ 0.3 & 0.6 \end{pmatrix}\begin{pmatrix} 0.62 \\ 0.38 \end{pmatrix}$ which gives $\begin{pmatrix} A_2 \\ B_2 \end{pmatrix} = \begin{pmatrix} 0.586 \\ 0.414 \end{pmatrix}$.

The market share for magazine A after three months is the value of Therefore, it is necessary to calculate $A_3 = 0.7 \times 0.586 + 0.4 \times 0.414 = 0.5758$. Working out the matrix $\begin{pmatrix} A_3 \\ B_3 \end{pmatrix} = \begin{pmatrix} 0.5758 \\ 0.4242 \end{pmatrix}$ was an acceptable method, but to gain full marks candidates had to clearly state that the market share for magazine A after three months was 0.5758 or equivalent; that is, relating their matrix answer to the question.

Question 6

Responses to this question were disappointing as it is a straightforward linear programming problem. Most candidates were able to answer part (i) correctly, stating that $3x + 2y \leq 20$. However, few were able to answer part (ii), which asked for the other constraints required. This was referring to the nonnegativity constraints $x \geq 0$, $y \geq 0$, which we must always include when defining a linear programming problem, otherwise the feasible region would be unbounded. Unfortunately, some candidates started talking about constraints unrelated to the problem and were then unable to continue with the question.

Candidates who did attempt to draw a graph showing the feasible region generally did a good job, and were also able to find the corners of the feasible region either using their graph or simultaneous equations to find the point of intersection of the two constraint lines. Evaluating the given profit function at each of the corners of the feasible region gives the solution that making 5.6 batches of chocolate and 1.6 batches of ginger biscuits maximises the profit at $696. Candidates who used their graph to find the point of intersection may not have found the exact solution, but could still gain full credit as long as their conclusion was consistent with their working, and the values of x and y used were actually inside the feasible region; that is, they satisfied all of the constraints.

Question 7

In answer to part (i), most candidates said that profit would be negative if $q = -\frac{4}{3}$ and this is why $q = -\frac{4}{3}$ is not meaningful. This is incorrect, since it is possible (although undesirable) to have a negative profit (also called a loss). However, it is not possible to make a negative amount of something, and since q is the quantity of goods produced it is not possible to have a negative value of q.

Most candidates correctly attempted to find the other turning point by differentiating the profit function, setting the result equal to zero and then using the quadratic formula to find $q = 4$. To determine whether the turning point at $q = 4$ is a maximum or a minimum, it is necessary to differentiate again to find $\pi''(x)$ and evaluate at $x = 4$. If $\pi''(4) > 0$ then the turning point is a minimum, but if $\pi''(4) < 0$, as is the case in this question, then the turning point is a maximum. It is also possible to determine the nature of the turning point by either sketching the graph or comparing coordinates either side of the turning point. Most candidates used the double differentiation method.

The final part required candidates to evaluate $\pi(4)$, which should have given the answer 109, and to explain that the value 109 is the maximum profit that can be made.

Question 8

This question simply tested whether or not candidates could apply the different methods of differentiation included in the course syllabus.

Part (i) was a straightforward term by term differentiation, which most candidates were able to do.

Part (ii) was an application of the chain rule, which most candidates attempted.

Part (iii) was an application of the product rule (if $y = uv$ *then* $y' = u'v + uv'$), which also required candidates to know that the differential of e^x is also e^x. Again, most candidates attempted this question.

Part (iv) was an application of the quotient rule $if\ y = \frac{u}{v}$ then $y' = \frac{u'v - uv'}{v^2}$. Many candidates attempted this but got the formula wrong, putting the two terms in the numerator the wrong way around. This obviously led to an incorrect answer, although some marks were gained for the attempt.

Question 9

This question was generally not well answered, which showed that few candidates had understood the importance of logs and how to work with them. Learning the laws of indices and logs (see p.29, Volume 2 of the subject guide) would enable all candidates to answer this question.

For part (i) $\log_a a^3 = 3 \log_a a = 3 \times 1 = 3$ using the power law of logs and the fact that $\log_a a = 1$.

For part (ii) use the division law of logs to simplify $\log_a 6 - \log_a 2 = \log_a ({}^6/{}_2) = \log_a 3$.

For part (iii) you need to know that $\sqrt{x} = x^{\frac{1}{2}}$, and that a negative power means 'one over'; that is, $x^{-n} = \frac{1}{x^n}$. In this question, we have $\left(\frac{1}{\sqrt{x}}\right)^{-6} = \left(\frac{1}{x^{\frac{1}{2}}}\right)^{-6} = \left(x^{\frac{1}{2}}\right)^{6} = x^3$.

To find the value of an unknown when it is a power in an equation we usually use logs (that is why logs are so useful and important). Hence, to solve part (b), first divide both sides by 450 and then take *ln* on both sides (*ln* is shorthand for log to the base *e*). This gives $\ln\frac{240}{450} = \ln e^{-x}$, so $-x = \ln\frac{240}{450}$ since $\ln e^{-x} = -x \ln e = -x$; therefore,

$x = -\ln\frac{240}{450} = 0.63\ (2\ d.p.)$.

For part (c) most candidates were able to substitute $t = 5$ into the given exponential equation to show that S= 34,600 to the nearest 100.

To answer the final part, it is again necessary to take *lns*, having first rearranged the equation to $e^{-0.4t} = 0.25$. Thus $-0.4t = ln\ 0.25$ and so $t = 3.4657$. Therefore, it takes 3.5 weeks for sales to reach 30,000.

Question 10

This question tested candidates' understanding of integration, and whether they could apply integration to find definite integrals and relate this to an area of a graph.

Most candidates were able to correctly integrate part (i) to obtain $2x^3 + c$, although not all remembered to include the constant of integration 'c' in their answer.

Part (ii) proved more of a challenge, requiring candidates to first rewrite the given expression using indices notation before using the backward chain rule to integrate. Thus, we have

$\int 4 - 3(x+2)^{\frac{1}{2}}.dx = 4x - 2(x+2)^{\frac{3}{2}} + c.$

Most candidates were able to attempt part (b) by integrating the given function and then evaluating it between the limits of 0 and 6 to obtain 786.

Only the most capable candidates were able to find the producer surplus for the final part of this question. The producer surplus relates to the part of the graph above the curve of the supply function bounded by the rectangle with coordinates $(0,0)(P_0,0)(P_0,Q_0)(Q_0,0)$ (see p.66, Volume 2 of the subject guide). Here $Q_0 = 6$ and $P_0 = 6^2 + 15(6) + 74 = 200$. Thus, the area of the rectangle is $6 \times 200 = 1{,}200$ and the area under the curve is 786 (found in part (i)). Hence, the producer surplus is equal to 1,200 – 786 = 414.