Examiners' reports 2014–15

CO3352 Operations research and combinatorial optimisation – Zone A

Comments on specific questions

Question 1

This question is based on the given simple graph G.

a. This part of the question asks for justifications of three facts about the maximum degree, Hamilton cycle and maximum path length of G. Most candidates gave a good answer for all the facts.

b. There are four sub-questions about the graph colouring in this part of the question. Most candidates answered only the first sub-question and skipped the rest.

   i. An easy approach to this part of the question is to draw the diagram showing the vertex colours.

   ii. A good answer to this sub-question would simply include a trace of the three-colouring algorithm on the graph and demonstrate how the algorithm works.

   iii. An answer of Yes/No should be given first. An explanation is all that is required next.

   iv. There are two sub-questions in this part. You should answer them individually. It is necessary to know the Greedy algorithm well before answering both sub-questions.

c. This part of the question is about matching. Most candidates who attempted this part of the question gave a good answer.

   i. Good reasons are expected in this part to explain why the whole set of vertices cannot be matchable when applying the greedy algorithm.

   ii. A good answer to this part of the question includes a trace of the greedy algorithm, the values of the total weight and the matching in the form of an edge set.

Question 2

This question is about the matroid.

a. This part of the question includes three sub-questions. You should answer them individually adopting the part numbers. Parts (i) and (ii) are straightforward and part (iii) requires some reasoning. Most candidates could answer this part of the question well.

b. An easy approach to answer this part of the question is to follow the instructions and to answer each of the sub-questions individually. Most candidates could answer some of the sub-questions but not all of them. A common error was apparently due to a lack of knowledge of effective techniques of confirming independence, and students lost marks unnecessarily.

   i. This part tests the relationship between independence and matrix vector linear independence.

ii. This part requires knowledge about the relationship between the matrix rank and the row rank of the matrix.

iii. This part asks for the representation of the matrix. A good answer would show all the working steps.

d. There are two sub-questions in this part and you are expected to answer the sub-questions individually. A good way to answer this part of the question would be to first adopt the corresponding question numbers precisely and then give a correct graph for each sub-question. An incidence matrix is also expected for the second graph. Many candidates did well on this part of the question.

Question 3

This question was not well answered in comparison to answers to the other questions. All parts are based on the given 6-vertex simple graph.

a. A good answer to this part of the question would simply include two sets of edges that follow the requirements. Unfortunately, many candidates did not answer this part well.

b. This part of the question asked for an appropriate explanation and justification. This part was answered well by many candidates.

c. The answer to this part of the question required knowledge of several important concepts, including the maximum-length path and the spanning tree of G, as well as the cycle matroid of G, and the maximum independent sets in the matroid. This part of the question was not well answered.

d. Two sub-questions are included in this part of the question and you should answer them individually.

i. The answer to this sub-question would simply be a maximum independent set in the cycle matroid of G' as required.

ii. A good answer to this sub-question would be a brief explanation.

e. This part of the question requires a good knowledge of the matrix diagram. There are three sub-questions in this part and you should answer them individually.

i. A good answer to the first sub-question would consist of the two required incidence matrices.

ii. The answer to the second subsection would be the computation of the Binet-Cauchy product and the result.

iii. A good answer to the third sub-question would include not only the computational result but also all the working steps. Unfortunately, this part of the question was poorly answered. Candidates on average lost nearly half of the marks available.

Question 4

This question addresses a network problem.

Few candidates answered this question, but those who attempted the question gave an excellent answer to some parts of it.

a. A straightforward approach would be to give the required graph G.

b. There are two sub-questions in this part, and you should answer them individually. The first sub-question requires an explanation about the vertex cover of the graph G in part a. The second sub-question asks for the example of two specific vertex covers for G; namely, the minimum, and the **minimal** but not the mininum vertex covers.

c. This part of the question requires knowledge of the application of an integer linear programme. There are four sub-questions in this part and you should answer them individually.

   i. This part of the sub-question requires an explanation of the meaning of the given constraints.

   ii. A good answer to this sub question would be a complete set of the constraints as required.

   iii. The answer to this sub-question would include a suitable linear objective function and the constraints in the form of a matrix.

   iv. This part of the question includes two sub-questions and you should answer them in two distinct sections. The first section should include an appropriate explanation of purpose as required. The second section should include an example in which a solution may satisfy the constraints mathematically but can be meaningless from a practical point of view.

Question 5

This question is about the application of linear programming.

i. This part of the question requires the standard set of linear functions for solving the given problem. Most candidates answered this part well.

ii. An easy way to answer this part of the question would be to show that, under an equal value K for W, X, Y and Z, whether or not the constraints are satisfied.

iii. Few candidates answered this part of the question incorrectly.

iv. A good answer to this part of the question would be the dual programme as required. This part of the question was answered well by most candidates.

v. Few candidates answered this part of the question correctly. However, the answer to this part of the question is not that difficult. The critical point is to first identify the optimal value from the diagram; and secondly, to explain the meaning of the intersection of lines for two constraints. Finally, the value of the intersection point and of the dual programme should be given.

vi. A good answer to this part of the question would consist of two sections. The first section should include an explanation on how the dual programme optimal value can be used to assess the primal programme optimal value. The second section should include a calculation to determine the values of W, X, Y and Z under the optimal value. Few candidates answered this part of the question but those who did answered very well.