Examiners' commentary 2017–2018

CO1102 Mathematics for computing – Zone A

General remarks

These scripts showed that most of the candidates had a thorough understanding of the syllabus and the ability to apply their knowledge and skills in the appropriate context. They were well prepared and demonstrated that they had a good grasp of the subject and had revised well. A number of candidates showed a limited understanding of the concepts and had difficulty interpreting the questions and answering them fully.

When revising for the examination it is a good idea to work through the sample paper in the subject guide, which has full solutions. You can then compare your answers with those given, and if your approach is very different then you can consider why and perhaps modify your method. You can learn a lot from looking at the notation and wording used in the solution and the way any mathematical definitions or proofs are included. The way you present your solution may help you clarify the problem and develop the solution, as well as making it easier for the examiners to follow your work. Please try to ensure your answers convey your meaning clearly and correctly and show your working in full, so that the examiners can give you marks for correct method, even if you make an error which means your final solution is incorrect. It will help you greatly to work through other past papers as part of the revision process so that you are familiar with the type of questions which may arise on each topic, and the material and skills you need to answer them. It may also help to make a list of key points in each chapter as a revision guide, together with typical examination questions.

Comments on specific questions

Question 1

a. This question required candidates to convert the decimal number 301 first to base 2 and then to base 16. This may be done by the method of repeated division or alternative methods were accepted to give the solution 100101101_2. To convert the number to base 16, just group the binary number in blocks of four from the right-hand side and convert each block. Thus 1101 converts to D, 0010 converts to 2 and the first block requires three zeroes in front to become 0001, so the number in base 16 is $12D_{16}$.

b. i. The binary number 110.111_2 is equivalent to:

$6+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}$ or $\frac{55}{8}$ in base 10. The definition of a rational number is that it has the form p/q where p and q are integers and $q \neq 0$. Thus $\frac{55}{8}$ is such a number and is rational. It is not sufficient to just say a rational number is a fraction, or a terminating decimal, in this case 6.875.

ii. Examples could be $\frac{\pi}{4}$ or $\sqrt{\frac{2}{2}}$.

c. This is a standard question that requires candidates to have learnt the method, which is to take say x to be the number 0.7272... and, as the repeating block is length two, make 100x = 72.7272... Subtracting x from 100x gives 99x = 72 or x = 72/99 which simplifies to 8/11 for full marks. It is important that candidates understand the significance of the repeating block and indicate this repetition by ellipsis or other accepted notation. Marks were lost by those who abbreviated x to 0.7272 and 100x to 72.72 or some such inaccurate numbers.

Question 2

a. i. The set of odd integers is denoted by E_t or $Z – E$.

ii. The set of integers divisible by 6 is denoted by $E \cap B$.

iii. The set of integers with last digit 5 by $A – E$.

b. This required a standard Venn diagram showing three sets intersecting in the most general way, with eight separate regions. For $X = E – (A \cup B)$ the region in E which is not in either A or B or both is shaded. An appropriate key must be included for full marks. For $Y = (E – A) \cup (E – B)$ the whole of E except the region in $A \cap B \cap E$ is shaded and again a clear key to the shading must be given.

c. i. $2 \in X$ or any even number which is **not** a multiple of 3 or 5 or both.

ii. $6 \in (Y – X)$ or any multiple of 6 or 10 but not both, such as 30.

iii. $30 \in (A \cap B \cap E)$ or any $30n : n \in Z$.

Question 3

In part (a) and (b) a truth table is required with columns for:

p; q; $p \wedge q$; $p \vee q$; $p \rightarrow q$; $q \rightarrow p$; $p \leftrightarrow q$.

b. The following columns should be added to the truth table:

$\neg(p \vee q)$; $(p \wedge q) \vee (\neg(p \vee q)$.

The resulting column entries for the last of these expressions should be equal, leading us to conclude that since the columns are identical the expressions are logically equivalent, or some such statement. Not making a concluding statement to justify what the columns mean results in loss of marks.

c. A logic network should be drawn with input p and q, both going through an OR gate and an AND gate. The output from the OR Gate is $p \vee q$ and this is put though a NOT gate giving output $\neg(p \vee q)$. This, together with the output from the AND box, $p \wedge q$, go through the final OR gate to give the final output $(p \wedge q) \vee (\neg(p \vee q))$. A NOR gate was used by some candidates instead of the OR followed by the NOT gate, and this was acceptable. For full marks candidates should draw the different gates clearly and label them, as well as showing the relevant outputs.

Question 4

a. The first part of this question was about a function which added up the number of digits in a 5-bit binary string. Thus SUM (10101) = 3 and SUM (01111) = 4. Some candidates misinterpreted this function as a conversion from base 2 to base 10 of the 5-digit inputs, for which some follow through marks were awarded.

i. Gives the image of 01011 = 3.

ii. Has solution 01111, 10111, 11011, 11101, 11110.

iii. The range of SUM is {0, 1, 2, 3, 4, 5}.

iv. The function is not one to one since, for example SUM (01111) and SUM (10111) = 4, so not every element of the domain has a unique image.

v. The function is not onto as there are elements of the co-domain such as the numbers greater than 5, which are not in the range.

Be careful not to confuse the co-domain and the range. Here the co-domain is the set of integers {0, 1, 2, 3, ...} and the range is the set {0, 1, 2, 3, 4, 5}. Simply saying that the range is not equal to the co-domain is not specific enough for full marks, and an example of a number in the co-domain which has no image in the range is required. Whether or not the function is one to one and/or onto is a question which arises almost every year and is well worth preparing for. A clear understanding of the concepts of domain, range and co-domain is necessary as well as the ability to find and interpret these according to the particular example.

b. Candidates were required to demonstrate they know when a function has an inverse and to be able to find this inverse and define it fully. This requires giving the inverse function in algebraic terms and also its domain and co-domain. Many candidates missed this latter part of the definition and lost marks. A function is invertible and has an inverse if it is both one to one and onto.

i. $f^{-1}(x) = x/2$ exists and $f^{-1} : R \to R$.

ii. g has no inverse since it is not onto. For example 1/2 is the image of 1 under g but it is not an element of the co-domain, Z.

iii. the function SUM from part (a) has no inverse since, as shown earlier, it is neither one to one nor onto.

Question 5

a. The majority of candidates calculated the first four terms of the sequence correctly by substituting 2, 3, 4 and 5 for n, in the formula. Thus:

i. $u_2 = 3u1 - 2u_0 : u_2 = 3 \times 2 - 2 \times 1 = 4$ $u_3 = 3u_2 - 2u_1 : u_3 = 3 \times 4 - 2 \times 2 = 8$ $u_4 = 3u_3 - 2u_2 : u_4 = 3 \times 8 - 2 \times 4 = 16$ $u_5 = 3u_4 - 2u_3 : u_5 = 3 \times 16 - 2 \times 8 = 32$.

ii. The non-recursive formula is $u_n = 2^n$. The solution $u_{n+1} = 2u_n$ given by some is a recursive formula.

b. i. Was given by $s_k+1 = s_k + 5(k + 1) - 4$, which is s_k + the (k + 1)th term.

ii. Candidates were required to perform a proof by induction on the sum of the terms of this sequence. There is a standard procedure involved in such a proof. This is broken down into several stages, which candidates should know and have practiced beforehand, so that although the individual proofs differ, their structure is the same.

Here BASE CASE: substitute $r = 1$ into $5r - 4$ to give $5 \times 1 - 4 = 1$ for the left hand side of the statement.

The right hand side is given by substituting $n = 1$ into $\frac{n(5n-3)}{2} = 1$, so the statement is true for $n = 1$.

Now INDUCTION HYPOTHESIS: Suppose the statement is true for some $n = k$ then), $k_r = 1\ (5r - 4) = \frac{k(5k - 3)}{2}$ true, where k ≥ 1.

Then INDUCTION STEP: Show it is then also true for $n = k + 1$, that is show r.h.s of the statement is equal to

$$\frac{(k+1)\ 5\ (k+1) - 3}{2} = \frac{(k+1)\ (5k+2)}{2}$$

We know from part (i) and the induction hypotheses that $\sum_{r=1}^{k+1}(5r-4) = \frac{k(5k-3)}{2} + 5(k+1) - 4$ This equals

$$\frac{5k^2 - 3k + 10k + 2}{2} = \frac{5k^2 + 7k + 2}{2} = \frac{(k+1)(5k+2)}{2}.$$

This is what we wanted to prove from the induction step, so the final comment is needed to complete the proof: So the statement is also true for n = k + 1 and by induction it is true for all k ≥ 1.

Question 6

a. The vertices of the digraph represent each of the players. There is an arc from one vertex to another if the first player has beaten the other, or alternatively the first player has lost to the other. There are n vertices in the digraph D and $\frac{n(n-1)}{2}$ arcs, since it a complete graph and each player plays every other player once only.

b. The matrix is:

$$\begin{pmatrix} 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

  i. The rows and columns of the matrix are indexed by the players, A, B, C, D, E and for any vertex u there is a 1 in the column v if there is an edge or arc uv, that is if player u beat player v. This matrix is acceptable if transposed and defined with the arc occurring when player u lost to player v

  ii. The matrix is not symmetric about the leading diagonal.

c. i. The graph G must be drawn.

  ii. Then a non-isomorphic graph with the same number of vertices and the same degree sequence, H, must be drawn such as: u : v, y v : u, w, x w : v x : v, y y : u, x, z z : y. This is not isomorphic to G as the degree 3 vertices in H are not adjacent, whereas in G they are adjacent.

Question 7

a. The smallest value of a = 3 since –40 is divisible by 5. Although a and b are greater than or equal to zero, a – b can be ≤ 0.

b. The set of numbers required is {5n : n ∈ N} i.e. all positive multiples of 5.

c. The set is {5n + 3 : n ∈ N}.

d. i. The relation is reflexive since for all a ∈ N we have a – a = 0 which is divisible by 5.

  i. The relation is symmetric since for all a, b ∈ N, if a – b is divisible by 5 then b – a = –(a – b) is also divisible by 5.

  ii. It is also transitive since for all a, b, c ∈ N, if a – b = 5k, k ∈ N and b – c = 5l, l ∈ N, then (a – b) + (b – c) = 5(k + l). So a – c = 5(k + l) which means a – c is divisble by 5.

  iii. It is an equivalence relation as it is reflexive, symmetric and transitive.

The formal definitions of reflexivity, symmetry and transitivity should be revised before the examination, and candidates should be able to reproduce them, with appropriate counter examples, where required in the examination.

Question 8

a. i. Candidates require a Venn diagram with three sets interconnecting in the most general way. The numbers in each region are given by the following table.

P	C	J	$No.\ in\ region$
0	0	0	20
0	0	1	21
0	1	0	28
0	1	1	8
1	0	0	29
1	0	1	3
1	1	0	6
1	1	1	5

ii. There are 20 students who do not study any language in the Venn diagram, so the probability is 20/120 =1/6.

b. i. We have P (A) = 10/20 = 1/2.

ii. P (B) = 8/20 = 2/5.

iii. P (C) = 11/20.

iv. P (A $\cap$ C) = 5/20 = 1/4.

v. P (B $\cap$ C) = 4/20 = 1/5.

vi. P (A $\cup$ C) = 16/20 = 4/5.

Some candidates included 1 as a prime number whereas 2 is the first prime.

Question 9

a. There are a few basic definitions in each chapter of the subject guide which are key to understanding the concepts in that section. These should be noted and learnt as part of the revision process. These include knowing what properties a graph must have if it is simple; what is meant by the degree of a vertex of a graph; what properties a graph must have in order for it to be a tree. The concept of iso- morphism is fundamental to graph theory and regularly included in the exam so candidates should ensure they have a good understanding of it and can construct and identify graphs which are either isomorphic or non-isomorphic, and give clear, specific reasons why. To say graphs are non-isomorphic as there is no one to correspondence function between them is not sufficient as a justification.

i. Many candidates answered part (a) i) correctly by saying that a graph is a tree if it is a connected graph with no cycles. Some only said it is connected with no loops which is not sufficient.

ii. The three non-isomorphic trees on 5 vertices are in the subject guide, Vol II page 32, the last three trees in Example 3.1.

iii. A path graph has only vertices of degree 1 and 2, in fact two vertices of degree 1 and the rest of degree 2. So only one of the graphs drawn in (ii) is a path graph.

    iv. There are $n - 1$ edges in a path graph with n vertices so in this case there are 99.

b. i. The first three levels of the tree showing the records stored at each node is required.

    ii. If the floor function method is used the first record stored at the root is 25, the records at the next level are 12 and 38 and the records at the third level are 6, 18, 31, 44.

    The height of this tree is 6 since $1 + 2 + 4 + 18 + 16 + 32 = 63 \geq 50$.

Question 10

a. i. The first part of this question required candidates to multiply two matrices together, and candidates should ensure they can do this correctly.

    ii. We have MA exists and its size is 3×3. M + B exists and its size is 3×2. BM is not possible since the number of columns of B does not match the number of rows of M.

b. Was a standard question involving Gaussian elimination.

    i. First, candidates were asked to copy down the augmented matrix and circle the pivot. Since the first row is already in the required form, the pivot is the 1 in the second row.

    ii. The next operation is $r3 - 2 \times r2$.

    iii. In the Gaussian elimination marks were given for method where the working was clear. Many candidates lost a mark as they did not fully reduce the matrix to one with ones on the leading diagonal, but left other numbers in the third row. Row-echelon form is needed. It is helpful if the row operations employed are clearly labelled and the order of transformations is shown, so any errors can be worked through and credit given. The final solution is $(4, -1, 2)$.