THIS PAPER IS NOT TO BE REMOVED FROM THE EXAMINATION HALLS

UNIVERSITY OF LONDON **CO1102 ZA**

BSc, CertHE and Diploma Examination

COMPUTING AND INFORMATION SYSTEMS, CREATIVE COMPUTING AND COMBINED DEGREE SCHEME

Mathematics for Computing

Date and Time: Tuesday 9 May 2017: 10.00 - 13.00

Duration: 3 hours

There are TEN questions in this paper. Candidates should answer all **TEN** questions. All questions carry equal marks and full marks can be obtained for complete answers to **TEN** questions. Each question carries equal marks. The marks for each part of a question are indicated at the end of the part in [.] brackets.

There are 100 marks available on this paper.

A hand held calculator may be used when answering questions on this paper but it must not be pre-programmed or able to display graphics text or algebraic equations. The make and type of machine must be stated clearly on the front cover of the answer book.

Question 1

(a) Working in base 2 and showing all carries, compute the following:

$$(101011)_2 + (11010)_2$$

[2]

(b) i. Define what is meant by a rational number.

ii. Give an example of a rational number a, where $1 < a < 2$.

iii. Give an example of an irrational number b, where $1 < b < 2$.

[3]

(c) Express the binary number 101.011 as a decimal and say whether or not it is rational. [2]

(d) Showing all working convert the repeating decimal number

$$3.424242...$$

to a fraction in its lowest terms. [3]

Question 2

(a) Let A, B and C be subsets of a universal set $\mathcal{U}$.

i. Draw a labelled Venn diagram showing these three sets dividing $\mathcal{U}$ into 8 disjoint regions.

ii. On your diagram shade the region corresponding to X as given in the following membership table. Be sure to include a key to your shaded area.

A	B	C	X
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

iii. Construct a membership table which shows the region Y, where

$$Y = (B \cup C) - A.$$

iv. By comparing Y to X or otherwise state a simple relation between these two sets using set notation. [7]

(b) i. Give the set $\{5n - 2 : n \in \mathbb{Z}, -2 \leq n \leq 3\}$ by the listing method.

ii. Give the set $P = \{1, 3, 9, 27, ..., 6561\}$ by the rules of inclusion method.

[3]

Question 3

Let $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and let p, q be the following propositions concerning an integer $n \in S$.

$$p \quad : \quad n \text{ is a multiple of three;}$$
$$q \quad : \quad n \text{ is an odd number.}$$

(a) For each of the following compound statements find the set of values $n \in S$ for which it is true:

$$p \wedge q; \quad p \vee q; \quad \neg p \wedge q.$$

[3]

(b) Express the following statement using p, q and logic symbols:

If n is even then it is not a multiple of 3.

[1]

(c) Use truth tables to prove that:

$$q \rightarrow p \equiv \neg p \rightarrow \neg q.$$

[2]

(d) List the elements of S which are in the truth set for the statement $q \rightarrow p$. [2]

(e) Write the contrapositive of the following statement concerning an integer n.

If the last digit of n is 4, then n is not odd.

[2]

Question 4

The exponential function f where $f : \mathbb{R} \to \mathbb{R}$, is defined by the rule $f(x) = 2^x$.

(a) Find $f(0), f(3), f(-1)$ and $f(\frac{1}{2})$. [2]

(b) Showing your working, find x such that $f(x) = 100$. [2]

(c) Say, with reason, whether f is

i. one-to-one;

ii. onto.

[2]

(d) Say what properties a function must have for it to have an inverse. [2]

(e) The invertible function $g : \mathbb{R} \to \mathbb{R}^+$, is given by the rule $g(x) = 10^x$. Give the domain, range and function rule for the function g^{-1}. [2]

Question 5

(a) A sequence is defined by the formula:

$$u_n = 3n + 5 \text{ for } n \geq 1.$$

i. Calculate u_1, u_2, u_3 and u_4, showing your working.

ii. Define this same sequence in terms of a recurrence relation and initial term. [4]

(b) Prove by induction that

$$\sum_{r=1}^{n}(3r+5) = \frac{3n^2 + 13n}{2}$$

for all $n \geq 1$.

[4]

(c) Use the formula from (b) to evaluate the following sum:

$$11 + 14 + 17 + 21 + ... + 2018.$$

[2]

Question 6

(a) State the two properties a graph must have in order for it to be **simple**. [2]

(b) A graph is called k-regular if each of its vertices has degree k.
Draw an example of each of the following graphs.

i. A 3-regular graph on 6 vertices.

ii. A 4-regular graph on 5 vertices.

[2]

(c) i. Draw a simple graph G with degree sequence $3, 3, 3, 3, 2, 2$.

ii. Draw a simple graph, H, which has the same number of vertices and edges as G but is not isomorphic to G. Also say why the graph you have drawn is not isomorphic to G.

iii. Draw another simple graph with the same degree sequence as G which is non-isomorphic to both G and H.

[4]

(d) Given a 7-regular graph on $2n$ vertices where $n \geq 4$ find how many edges there are in this graph.

[2]

Question 7

(a) Given the graph M with adjacency matrix

$$\mathbf{A}(M) = \begin{pmatrix} 0 & 2 & 0 & 1 \\ 2 & 0 & 2 & 1 \\ 0 & 2 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}.$$

i. Draw the graph M.

ii. Draw two spanning trees of M which are non-isomorphic to one another and say why they are non-isomorphic. Label these trees T_1 and T_2.

iii. Draw another spanning tree of M, called T_3, and determine whether or not it is isomorphic to either of the trees you drew in (ii).

[5]

(b) Given two graphs A and B, a relation R is defined such that graph A is related to graph B if it is non-isomorphic to graph B. Say whether or not this relation is

i. reflexive;

ii. symmetric;

iii. transitive.

Justify your answer in each case. In the cases where the given property does not hold provide a counter example to justify this. [5]

Question 8

In a tournament two teams, A and B, play a series of just two matches. The outcome of each match may be an outright win or lose for each team, or a draw. The winner of the tournament is the team that wins the most matches. If both teams win the same number of matches the result is a draw.

(a) Draw a tree to model the possible outcomes for the tournament. [4]

(b) If the probability that A wins any match is $\frac{1}{3}$ and the probability that B wins any match is $\frac{1}{2}$ find

i. the probability that A wins the tournament;

ii. the probability that B wins the tournament;

iii. the probability that the outcome of the tournament is a draw.

[6]

Question 9

(a) A ternary tree is a rooted tree in which each internal node has exactly 3 children. Let Q be a ternary tree of height h in which all the external nodes lie on level h.

i. Suppose that $h > 6$, give the number of nodes on level 6 of the tree Q.

ii. Say why the number of **internal** nodes in Q is $\sum\limits_{r=0}^{h-1} 3^r$.

iii. What is the smallest possible height of the tree Q if it is known to have at least 1000 internal nodes?

[5]

(b) Let $X = \{x_1, x_2, x_3, ..., x_8\}$ be a set of eight elements.

i. Say how to assign a unique 8-bit binary string to each subset of X.

ii. Give the 8-bit binary string for the subset $S = \{x_1, x_3, x_5\}$ and find the subset of X coded by the binary string 01010101.

iii. Find the total number of subsets of X. [5]

Question 10

(a) Given two matrices **A** and **B** where **A** $= \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, **B** $= \begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix}$ and $x, y \in \mathbb{R}$.

i. Find **AB** and **BA**.

ii. Given **AB** $=$ **BA**, state a relation between x and y.

[4]

(b) i. Write down the augmented matrix for the following system of equations.

$$\begin{aligned} 2x + 2z &= 8 \\ x + y + 3z &= 12 \\ x - y + 2z &= 5. \end{aligned}$$

ii. Use Gaussian elimination to solve the system.

[6]

END OF PAPER