Continuous Optimization

Chapter 3: Constrained Optimization

1 Stationarity

In this chapter we will consider constrained optimization problems with the following shape

$$\begin{aligned} \min \quad & f(x) \\ \text{s.t.} \quad & x \in C \end{aligned} \tag{1}$$

Definition 1.1 (Convex Set)**.** *A set C is said to be convex if given $x_1, x_2 \in C$ and $\lambda \in [0, 1]$, then $\lambda x_1 + (1 - \lambda)x_2 \in C$.*

Definition 1.2 (Convex Function)**.** *A function $f : C \to \mathbb{R}$ defined on a convex set C is said to be convex if given $x_1, x_2 \in C$ and $\lambda \in [0, 1]$, then*

$$f(\lambda x_1 + (1 - \lambda)x_2) \leq \lambda f(x_1) + (1 - \lambda)f(x_2).$$

Definition 1.3 (Strictly Convex Function)**.** *A function $f : C \to \mathbb{R}$ defined on a convex set C is said to be strictly convex if given $x_1, x_2 \in C$ and $\lambda \in [0, 1]$, then*

$$f(\lambda x_1 + (1 - \lambda)x_2) < \lambda f(x_1) + (1 - \lambda)f(x_2).$$

Also, a function is called concave if $-f$ is convex and strictly concave if $-f$ is strictly convex.

References