Heuristic analysis

All the evaluation functions in this analysis consider the combination of the player's distance from the center of the board (or its inverse) and the inverse of the distance between the players.

To better describe it I divided the returned score values into two parts ***a*** and ***b***.

custom_score function

In the ***custom_score*** function the first part ***a*** is the inverse of the player's distance from the center of the board multiplied by the number of blank spaces left at the current time of the game, this latter part ***b*** is the inverse of the distance between the two players multiplied by the difference between the player and the opponent's legal moves.

In a 7x7 board the inverse of the player's distance from the centre in the ***a*** term is in the range [0.08, 2], the larger values correspond to the positions near the center and the smaller to those near the edges. Multiplying those values by the number of blank spaces left corresponds to having at the beginning of the game (when there are 49 blank spaces) values in the range [3.92, 98] and in the range [0.08, 2] when there is only a blank space left.

In the same kind of board the inverse of the distance between the two players in the ***b*** term will assume values in the range [0.0139, 1], the smaller values when the two players are in opposite diagonal positions and 1 when they are adjacent on the same row or column. The difference between the player and the opponent's legal moves could vary in the range [-7, 7] and this could shift *the absolute values* of the ***b*** term in the range [0.0973, 7].

At the beginning of the game the ***a*** term prevails over the term ***b***, occurring in higher values of heuristic in positions that are near the center. Towards the end of the game the ***b*** term starts

to be more relevant. The heuristic increase near the area of the opponent mostly where there are larger differences between the player and opponent's legal moves.

This allow the agent to consume positions near the centre at the beginning of the game, where there are more available moves, trying to reduce the space of the opponent at the end of the game.

custom_score_2 function

In the ***custom_score_2*** function the ***a*** term considers the player's distance from the centre of the board multiplied by the blank spaces left at the current time of the game and the player's legal moves.
Similarly as in the ***custom_score*** function the player's distance from the center is in the range [0.5, 12.5] that is multiplied by the blank space left at the current time of the game. The range of this product is [24.5, 612.5] at the start of the game (when there are 49 blank spaces) and and in the range [0.5, 12.5] when there is only a blank space left. Multiplying further by the legal moves (from 1 to 8) the ***a*** term might assume higher values (from 24.5 to 4,900 at the start of the game and from 0.5 to 102.4 when there is only a blank space).

The ***b*** term is the same of the ***custom_score*** function and in this case affects the agent's behavior later.

Therefore the agent moves towards the edges at the beginning of the game and towards the opponent's zone at the end.

custom_score_3 function

In the ***custom_score_3*** function the ***a*** term is the same as ***custom_score*** and the ***b*** term is that of ***custom_score*** multiplied by the two dimensions of the board. This provides an enhancement of the ***b*** term of ***custom_score*** anticipating its effect during the game.

These three types of custom score improve of about ten

percent the performance of the agent compared to that of the AB_Improved agent.

In the following 7 tables it is possible to show these improvements.

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                      **************************
                           Playing Matches
                      **************************

Match #   Opponent    AB_Improved   AB_Custom   AB_Custom_2  AB_Custom_3
                       Won | Lost   Won | Lost   Won | Lost   Won | Lost
   1       Random       9  |   1     9  |   1     9  |   1     9  |   1
   2       MM_Open      7  |   3     6  |   4     7  |   3     6  |   4
   3      MM_Center     9  |   1     9  |   1    10  |   0     9  |   1
   4     MM_Improved    6  |   4     9  |   1    10  |   0     5  |   5
   5       AB_Open      4  |   6     7  |   3     5  |   5     4  |   6
   6      AB_Center     4  |   6     5  |   5     4  |   6     5  |   5
   7     AB_Improved    6  |   4     4  |   6     3  |   7     8  |   2
-----------------------------------------------------------------------------
          Win Rate:      64.3%        70.0%        68.6%        65.7%
```

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                      **************************
                           Playing Matches
                      **************************

Match #   Opponent    AB_Improved   AB_Custom   AB_Custom_2  AB_Custom_3
                       Won | Lost   Won | Lost   Won | Lost   Won | Lost
   1       Random       8  |   2     9  |   1    10  |   0     9  |   1
   2       MM_Open      7  |   3     8  |   2     5  |   5     6  |   4
   3      MM_Center     9  |   1     8  |   2     6  |   4     7  |   3
   4     MM_Improved    5  |   5     7  |   3     5  |   5     9  |   1
   5       AB_Open      8  |   2     8  |   2     5  |   5     4  |   6
   6      AB_Center     4  |   6     8  |   2     6  |   4     6  |   4
   7     AB_Improved    5  |   5     5  |   5     6  |   4     7  |   3
-----------------------------------------------------------------------------
          Win Rate:      65.7%        75.7%        61.4%        68.6%
```

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                      **************************
                           Playing Matches
                      **************************

Match #   Opponent    AB_Improved   AB_Custom   AB_Custom_2  AB_Custom_3
                       Won | Lost   Won | Lost   Won | Lost   Won | Lost
   1       Random       8  |   2     8  |   2    10  |   0     8  |   2
   2       MM_Open      5  |   5     7  |   3     5  |   5     7  |   3
   3      MM_Center     4  |   6     9  |   1     8  |   2     7  |   3
   4     MM_Improved    5  |   5     4  |   6     8  |   2     8  |   2
   5       AB_Open      6  |   4     3  |   7     6  |   4     5  |   5
   6      AB_Center     3  |   7     5  |   5     6  |   4     5  |   5
   7     AB_Improved    6  |   4     6  |   4     5  |   5     6  |   4
-----------------------------------------------------------------------------
          Win Rate:      52.9%        60.0%        68.6%        65.7%
```

```
                      **************************
                           Playing Matches
                      **************************

Match #   Opponent    AB_Improved   AB_Custom   AB_Custom_2  AB_Custom_3
                       Won | Lost   Won | Lost   Won | Lost   Won | Lost
   1       Random      10  |   0     9  |   1    10  |   0     8  |   2
   2       MM_Open      5  |   5     8  |   2     8  |   2     7  |   3
   3      MM_Center     7  |   3     7  |   3     8  |   2     9  |   1
   4     MM_Improved    5  |   5     7  |   3     7  |   3     6  |   4
```

```
    5       AB_Open      4  |   6     5  |   5     6  |   4     6  |   4
    6      AB_Center     6  |   4     8  |   2     6  |   4     6  |   4
    7     AB_Improved    6  |   4     7  |   3     5  |   5     6  |   4
-------------------------------------------------------------------------
          Win Rate:      61.4%        72.9%        71.4%        68.6%


                      *************************
                           Playing Matches
                      *************************

 Match #   Opponent    AB_Improved   AB_Custom   AB_Custom_2  AB_Custom_3
                        Won | Lost   Won | Lost   Won | Lost   Won | Lost
    1       Random       6  |   4    10  |   0     8  |   2     9  |   1
    2       MM_Open      6  |   4     5  |   5     7  |   3     7  |   3
    3      MM_Center     5  |   5     8  |   2     8  |   2     9  |   1
    4     MM_Improved    6  |   4     7  |   3     9  |   1     6  |   4
    5       AB_Open      5  |   5     6  |   4     6  |   4     5  |   5
    6      AB_Center     4  |   6     5  |   5     5  |   5     6  |   4
    7     AB_Improved    3  |   7     5  |   5     4  |   6     6  |   4
-------------------------------------------------------------------------
          Win Rate:      50.0%        65.7%        67.1%        68.6%


                      *************************
                           Playing Matches
                      *************************

 Match #   Opponent    AB_Improved   AB_Custom   AB_Custom_2  AB_Custom_3
                        Won | Lost   Won | Lost   Won | Lost   Won | Lost
    1       Random      10  |   0     9  |   1     8  |   2    10  |   0
    2       MM_Open      4  |   6     9  |   1     6  |   4     7  |   3
    3      MM_Center     6  |   4     9  |   1     9  |   1     5  |   5
    4     MM_Improved    5  |   5     6  |   4     3  |   7     8  |   2
    5       AB_Open      6  |   4     6  |   4     7  |   3     5  |   5
    6      AB_Center     4  |   6     4  |   6     6  |   4     6  |   4
    7     AB_Improved    5  |   5     5  |   5     6  |   4     4  |   6
-------------------------------------------------------------------------
          Win Rate:      57.1%        68.6%        64.3%        64.3%


                      *************************
                           Playing Matches
                      *************************

 Match #   Opponent    AB_Improved   AB_Custom   AB_Custom_2  AB_Custom_3
                        Won | Lost   Won | Lost   Won | Lost   Won | Lost
    1       Random       9  |   1    10  |   0    10  |   0    10  |   0
    2       MM_Open      3  |   7     7  |   3     8  |   2     9  |   1
    3      MM_Center     8  |   2    10  |   0     8  |   2     7  |   3
    4     MM_Improved    5  |   5     4  |   6     7  |   3     7  |   3
    5       AB_Open      6  |   4     8  |   2     5  |   5     6  |   4
    6      AB_Center     5  |   5     5  |   5     5  |   5     3  |   7
    7     AB_Improved    6  |   4     5  |   5     6  |   4     4  |   6
-------------------------------------------------------------------------
          Win Rate:      60.0%        70.0%        70.0%        65.7%
```

By doing 7 simulation (490 games per agent) and finding the mean value, the results are that the percentage of wins are:

AB_Improved : 58.8 %
AB_Custom : 69 %
AB_Custom_2 : 67.4 %

AB_Custom_3: 66.7 %

In particular, considering the games against the MM_Center agent the percentage of wins are:

AB_Improved : 68.6 %
AB_Custom : 85.7 %
AB_Custom_2 : 81.4 %
AB_Custom_3: 75.7 %

considering the games against the MM_Improved agent the percentage of wins are:

AB_Improved : 51.4 %
AB_Custom : 62.9 %
AB_Custom_2 : 70 %
AB_Custom_3: 70 %

considering the games against the AB (open, center and improved) agents the percentage of wins are:

AB_Improved : 57 %
AB_Custom : 57 %
AB_Custom_2 : 54 %
AB_Custom_3: 54 %

Therefore according to the results, ***custom_score*** heuristic gives better results because improve the chances of winning of about the 10%, compared to that used by the AB_Improved agent.
In particular good results are an improvement of 17.1% compared to the same agent against the MM_Center and 11.5% more against the MM_Improved agent.
The performance against the AB (open, center and improved) agents is the same to that of the AB_Improved agent. The AB_Custom agent has a performance that is slightly better than the AB_Custom_2 and AB_Custom_3 agents.