

Instituto Superior Técnico, Universidade de Lisboa

Master Degree in Naval and Oceanic Engineering

Modelling of Sea Waves

**Author:**

Leonardo Lombardi

**IST Number:**

1101888

**Professor:**

Shan Wang

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**1. Introduction**

This report documents the procedures, calculations, and analysis of the results of proposed exercises for Modelling of Sea Waves course Project 2, which consists of the study of wave climate and wave energy conversion considering different wave energy converts.

**2. Part One - Wave energy conversion**

For the first part, data were made available in a text file (named "NOAAwave\_1979010103to2007123121.txt") containing, in each line, measurements of significant height (Hs, in meters), peak period (Tp, in seconds) and wave direction (Dp, in degrees) for a given year, month (1 representing January, 2 February, and so on), day and hour. Data were measured at the position determined by latitude 39N longitude 9.5W, at 52 meters of water depth, near the coast of Ericeira, in Portugal.

**2.1 Exercise 1 – Scatter Tables**

A scatter table is a representation of the frequency distribution of variables at defined intervals. For this exercise, it was asked to obtain the scatter table relating significant height (Hs) with peak periods (Tp), significant height with wave direction (Dp), and peak periods with wave direction. To facilitate the construction of these tables, the data from the text file was imported into Excel. Then the bin sizes were determined considering the minimum and maximum values ​​of the variables to be scattered (Hs, Tp and Dp) and aiming to compromise a significant amount of data in each interval.

With the data available in Excel, the table cell value of a given scatter table was determined, using the excel COUNTIFS function, counting the number of measurements within the specified intervals for that cell. From this procedure, the following tables were obtained:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | **Scatter Table of Hs vs Tp** | | | | | | | | | | | | | | | | | | | | | |
| **Tp (s)** | | | | | | | | | | | | | | | | | | | | | |
| 0-1 | 1-2 | 2-3 | 3-4 | 4-5 | 5-6 | 6-7 | 7-8 | 8-9 | 9-10 | 10-11 | 11-12 | 12-13 | 13-14 | 14-15 | 15-16 | 16-17 | 17-18 | 18-19 | 19-20 | 20-21 | 21-22 |
| **Hs (m)** | 0-0,5 | 0 | 0 | 3 | 2 | 6 | 0 | 6 | 68 | 53 | 41 | 50 | 37 | 16 | 17 | 10 | 4 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0,5-1 | 0 | 0 | 0 | 33 | 106 | 153 | 376 | 1031 | 2167 | 1783 | 1306 | 759 | 385 | 208 | 118 | 56 | 27 | 16 | 4 | 1 | 0 | 1 |
| 1-1,5 | 0 | 0 | 0 | 9 | 121 | 422 | 856 | 1698 | 3611 | 3627 | 3271 | 2298 | 1173 | 635 | 305 | 121 | 62 | 22 | 13 | 2 | 1 | 1 |
| 1,5-2 | 0 | 0 | 0 | 0 | 7 | 234 | 1050 | 1518 | 2368 | 3282 | 3781 | 2977 | 1797 | 973 | 478 | 216 | 118 | 51 | 15 | 7 | 0 | 0 |
| 2-2,5 | 0 | 0 | 0 | 0 | 0 | 13 | 284 | 923 | 1016 | 1425 | 2284 | 2630 | 2126 | 1152 | 572 | 282 | 127 | 39 | 16 | 4 | 1 | 0 |
| 2,5-3 | 0 | 0 | 0 | 0 | 0 | 0 | 25 | 346 | 535 | 516 | 1162 | 1777 | 1807 | 1331 | 687 | 288 | 124 | 37 | 13 | 4 | 3 | 0 |
| 3-3,5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 52 | 208 | 245 | 451 | 924 | 1217 | 1103 | 722 | 288 | 87 | 31 | 9 | 2 | 0 | 0 |
| 3,5-4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 6 | 46 | 127 | 232 | 441 | 703 | 770 | 593 | 242 | 95 | 32 | 9 | 2 | 0 | 0 |
| 4-4,5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 11 | 53 | 114 | 208 | 379 | 506 | 459 | 320 | 99 | 26 | 6 | 0 | 1 | 0 |
| 4,5-5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 20 | 61 | 98 | 214 | 252 | 306 | 247 | 122 | 42 | 11 | 5 | 0 | 0 |
| 5-5,5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4 | 21 | 50 | 74 | 94 | 161 | 156 | 103 | 29 | 10 | 1 | 0 | 0 |
| 5,5-6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 6 | 22 | 44 | 72 | 103 | 115 | 83 | 30 | 8 | 1 | 0 | 0 |
| 6-6,5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 8 | 18 | 48 | 61 | 72 | 67 | 26 | 6 | 3 | 0 | 0 |
| 6,5-7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 11 | 14 | 40 | 42 | 22 | 9 | 2 | 0 | 0 | 0 |
| 7-7,5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7 | 8 | 23 | 22 | 22 | 7 | 0 | 0 | 0 | 0 |
| 7,5-8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 3 | 14 | 22 | 17 | 9 | 0 | 0 | 0 | 0 |
| 8-8,5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 4 | 15 | 7 | 2 | 2 | 0 | 0 | 0 |
| 8,5-9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 3 | 6 | 8 | 1 | 1 | 0 | 0 |
| 9-9,5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 1 | 3 | 1 | 1 | 0 | 0 |
| 9,5-10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 0 | 0 | 0 | 0 | 0 |
| 10-10,5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 10,5-11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 11-11,5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Table 1 - Scatter Table of Hs vs Tp

The scatter table relating significant wave height and peak period is analyzed in more detail later in report, but it can be deduced (calculating the sum of row and column values) that range of period in which more waves occur is between 10 and 11 seconds and the range of significant heights in which more waves occur is between 1,5 and 2.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | **Scatter Table of Hs vs Dp** | | | | | | | | | | | | | | | | | |
| **Dp (°)** | | | | | | | | | | | | | | | | | |
| 0-20 | 20-40 | 40-60 | 60-80 | 80-100 | 100-120 | 120-140 | 140-160 | 160-180 | 180-200 | 200-220 | 220-240 | 240-260 | 260-280 | 280-300 | 300-320 | 320-340 | 340-360 |
| **Hs (m)** | 0-0,5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7 | 1 | 0 | 46 | 146 | 68 | 32 | 13 |
| 0,5-1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 6 | 45 | 19 | 159 | 1307 | 3127 | 2251 | 1064 | 550 |
| 1-1,5 | 0 | 0 | 0 | 2 | 6 | 6 | 1 | 0 | 9 | 34 | 72 | 74 | 232 | 2525 | 6049 | 5103 | 3012 | 1123 |
| 1,5-2 | 0 | 0 | 0 | 1 | 2 | 3 | 0 | 0 | 8 | 30 | 63 | 93 | 210 | 2563 | 5793 | 5252 | 3664 | 1190 |
| 2-2,5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 12 | 62 | 95 | 194 | 1899 | 4648 | 3572 | 1854 | 556 |
| 2,5-3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 9 | 43 | 76 | 138 | 1355 | 3638 | 2398 | 748 | 250 |
| 3-3,5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 28 | 55 | 116 | 761 | 2505 | 1518 | 308 | 43 |
| 3,5-4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 5 | 31 | 61 | 564 | 1619 | 889 | 120 | 8 |
| 4-4,5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4 | 16 | 30 | 371 | 1081 | 618 | 62 | 0 |
| 4,5-5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 9 | 18 | 260 | 663 | 398 | 30 | 0 |
| 5-5,5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3 | 15 | 141 | 349 | 193 | 2 | 0 |
| 5,5-6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4 | 95 | 260 | 125 | 0 | 0 |
| 6-6,5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 59 | 183 | 68 | 0 | 0 |
| 6,5-7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 30 | 88 | 24 | 0 | 0 |
| 7-7,5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 19 | 52 | 18 | 0 | 0 |
| 7,5-8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 15 | 29 | 22 | 0 | 0 |
| 8-8,5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 13 | 11 | 8 | 0 | 0 |
| 8,5-9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 9 | 11 | 4 | 0 | 0 |
| 9-9,5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4 | 6 | 0 | 0 | 0 |
| 9,5-10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 0 | 0 | 0 |
| 10-10,5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 10,5-11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 11-11,5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

Table 2 - Scatter Table of Hs vs Dp

For the scatter table relating significant wave height and wave directions, we get that the range of wave directions in which more waves is occur is 280 and 300 degrees, and that the range of significant heights in which more waves occur is between, as expected, again between 1,5 and 2 meters.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | **Scatter Table of Tp vs Dp** | | | | | | | | | | | | | | | | | |
| **Dp (°)** | | | | | | | | | | | | | | | | | |
| 0-20 | 20-40 | 40-60 | 60-80 | 80-100 | 100-120 | 120-140 | 140-160 | 160-180 | 180-200 | 200-220 | 220-240 | 240-260 | 260-280 | 280-300 | 300-320 | 320-340 | 340-360 |
| **Tp (s)** | 0-1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1-2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2-3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 2 | 0 | 0 | 0 |
| 3-4 | 0 | 0 | 0 | 0 | 5 | 2 | 1 | 0 | 0 | 2 | 0 | 1 | 0 | 1 | 2 | 1 | 8 | 21 |
| 4-5 | 0 | 0 | 0 | 3 | 3 | 7 | 0 | 0 | 3 | 6 | 2 | 6 | 6 | 2 | 9 | 12 | 61 | 120 |
| 5-6 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 10 | 6 | 19 | 22 | 21 | 39 | 19 | 56 | 342 | 286 |
| 6-7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 26 | 64 | 45 | 136 | 118 | 108 | 166 | 1049 | 880 |
| 7-8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 18 | 89 | 107 | 159 | 305 | 616 | 973 | 2169 | 1205 |
| 8-9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 31 | 79 | 123 | 265 | 542 | 2112 | 2908 | 3023 | 932 |
| 9-10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8 | 55 | 94 | 214 | 647 | 3649 | 3960 | 2212 | 285 |
| 10-11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 14 | 57 | 178 | 1360 | 5494 | 4380 | 1254 | 4 |
| 11-12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 12 | 130 | 2560 | 6046 | 3173 | 310 | 0 |
| 12-13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 48 | 2618 | 5054 | 2104 | 146 | 0 |
| 13-14 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 6 | 0 | 9 | 1796 | 3390 | 1882 | 105 | 0 |
| 14-15 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 9 | 1165 | 2032 | 1379 | 77 | 0 |
| 15-16 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 525 | 1015 | 917 | 54 | 0 |
| 16-17 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 240 | 465 | 438 | 48 | 0 |
| 17-18 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 83 | 173 | 137 | 24 | 0 |
| 18-19 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 27 | 49 | 38 | 12 | 0 |
| 19-20 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8 | 20 | 4 | 2 | 0 |
| 20-21 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 5 | 0 | 0 | 0 |
| 21-22 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |

Table 3 - Scatter Table of Tp vs Dp

From the scatter table relating peak periods and wave directions, we can see that the majority of peak periods occur for big angles (between 240 and 340 degrees). So we can summarize as this: the most predominant peak period is between 10 and 11 seconds, the most predominant significant height between 1,5 and 2 meters and the most predominant wave direction is between 260 and 340 degrees. This is coherent with the shape of the coast as show in the image in project specifications, where the wave direction is perpendicular to the coast.

**2.2 Exercise 2 – Wave Power Analysis**

Write

### **2.2.1 – Average Wave Power per unit crest length**

From the project statement, we calculate the wave power per unit crest with the following formula:

Where p is the seawater density (defined as 1025 kilograms per cubic meter) , g is the gravity acceleration at the earth’s surface (9.8 meters per second squared) and Te is the energetic period, being 0.9 times the peak period for a typical JONSWAP peak enhancement of 𝛾 = 3.3. To obtain the average power per unit crest, we add the wave power for each significant height and peak period, and then divide the summation by the number of data, that is:

With the significant height in meters and the peak period in seconds, we get the result in watts per meter. For the whole time series data, we obtain that the average power is 31,053 kilo watts per meter.

### **2.2.2 – Average Wave Power per unit crest length per Season**

Using the same formulas from above and considering the months of each season as winter (December, January, February), spring (March, April, May), summer (June, July, August), autumn (September, October, November) and using MATLAB, we get the following table of results:

|  |  |  |  |
| --- | --- | --- | --- |
| **Average Wave Power per unit crest** | | | |
| kW/m | | | |
| **Winter** | **Summer** | **Spring** | **Autumn** |
| 56,48 | 10,47 | 2,85 | 28,95 |

Table 4 - Average Wave Power per unit crest per season

From the table, we can see that the biggest average wave power occurs in the winter – almost 20 times more than in the lowest average wave power season, the spring. We know that the average wave power increases when the waves are higher – this happens in winter, probably because in this season wind speed, that contributes to wave generation, is greater because of the increased temperature difference between the poles and equator.

### **2.2.3 – Normalized Occurrence**

To develop the normalized occurrence table of Hs vs Te, we just divide every table cell value from the Hs vs Tp scatter table to the sum of all Tp values (this works because Te is just Tp times 0,9). Tp. This table give us the probability of finding a particular sea state based on the given data. With values as percentages and considering the same bin sizes defined for Hs vs Tp (converted to Te), the obtained normalized occurrence table is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | **Normalized Occurrence Table of Hs vs Te** | | | | | | | | | | | | | | | | | | | | | |
| **Te (s)** | | | | | | | | | | | | | | | | | | | | | |
|  | | 0-0,9 | 0,9-1,8 | 1,8-2,7 | 2,7-3,6 | 3,6-4,5 | 4,5-5,4 | 5,4-6,3 | 6,3-7,2 | 7,2-8,1 | 8,1-9 | 9-9,9 | 9,9-10,8 | 10,8-11,7 | 11,7-12,6 | 12,6-13,5 | 13,5-14,4 | 14,4-15,3 | 15,3-16,2 | 16,2-17,1 | 17,1-18 | 18-18,9 | 18,9-19,8 |
| **Hs (m)** | 0-0,5 | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,1% | 0,1% | 0,1% | 0,1% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% |
| 0,5-1 | 0,0% | 0,0% | 0,0% | 0,0% | 0,1% | 0,2% | 0,5% | 1,3% | 2,7% | 2,2% | 1,6% | 0,9% | 0,5% | 0,3% | 0,1% | 0,1% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% |
| 1-1,5 | 0,0% | 0,0% | 0,0% | 0,0% | 0,1% | 0,5% | 1,0% | 2,1% | 4,4% | 4,4% | 4,0% | 2,8% | 1,4% | 0,8% | 0,4% | 0,1% | 0,1% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% |
| 1,5-2 | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,3% | 1,3% | 1,9% | 2,9% | 4,0% | 4,6% | 3,6% | 2,2% | 1,2% | 0,6% | 0,3% | 0,1% | 0,1% | 0,0% | 0,0% | 0,0% | 0,0% |
| 2-2,5 | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,3% | 1,1% | 1,2% | 1,7% | 2,8% | 3,2% | 2,6% | 1,4% | 0,7% | 0,3% | 0,2% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% |
| 2,5-3 | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,4% | 0,7% | 0,6% | 1,4% | 2,2% | 2,2% | 1,6% | 0,8% | 0,4% | 0,2% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% |
| 3-3,5 | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,1% | 0,3% | 0,3% | 0,6% | 1,1% | 1,5% | 1,4% | 0,9% | 0,4% | 0,1% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% |
| 3,5-4 | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,1% | 0,2% | 0,3% | 0,5% | 0,9% | 0,9% | 0,7% | 0,3% | 0,1% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% |
| 4-4,5 | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,1% | 0,1% | 0,3% | 0,5% | 0,6% | 0,6% | 0,4% | 0,1% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% |
| 4,5-5 | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,1% | 0,1% | 0,3% | 0,3% | 0,4% | 0,3% | 0,1% | 0,1% | 0,0% | 0,0% | 0,0% | 0,0% |
| 5-5,5 | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,1% | 0,1% | 0,1% | 0,2% | 0,2% | 0,1% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% |
| 5,5-6 | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,1% | 0,1% | 0,1% | 0,1% | 0,1% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% |
| 6-6,5 | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,1% | 0,1% | 0,1% | 0,1% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% |
| 6,5-7 | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,1% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% |
| 7-7,5 | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% |
| 7,5-8 | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% |
| 8-8,5 | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% |
| 8,5-9 | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% |
| 9-9,5 | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% |
| 9,5-10 | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% |
| 10-10,5 | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% |
| 10,5-11 | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% |
| 11-11,5 | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% |

Table 5 - Normalized Occurrence Table of Hs vs Te

Multiplying the occurrence by the wave power (for each bin peak period and significant height, considering the maximum value for that bin), we can obtain the final matrix of contribution to the wave power in each sea state as:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | **Power Table** | | | | | | | | | | | | | | | | | | | | | |
| **Te (s)** | | | | | | | | | | | | | | | | | | | | | |
| **Using Tp bin size** | | 0-0,9 | 0,9-1,8 | 1,8-2,7 | 2,7-3,6 | 3,6-4,5 | 4,5-5,4 | 5,4-6,3 | 6,3-7,2 | 7,2-8,1 | 8,1-9 | 9-9,9 | 9,9-10,8 | 10,8-11,7 | 11,7-12,6 | 12,6-13,5 | 13,5-14,4 | 14,4-15,3 | 15,3-16,2 | 16,2-17,1 | 17,1-18 | 18-18,9 | 18,9-19,8 |
| **Hs (m)** | 0-0,5 | 0,00 | 0,00 | 0,01 | 0,01 | 0,04 | 0,00 | 0,06 | 0,74 | 0,65 | 0,55 | 0,74 | 0,60 | 0,28 | 0,32 | 0,20 | 0,09 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| 0,5-1 | 0,00 | 0,00 | 0,00 | 0,71 | 2,87 | 4,97 | 14,25 | 44,64 | 105,56 | 96,51 | 77,76 | 49,30 | 27,09 | 15,76 | 9,58 | 4,85 | 2,48 | 1,56 | 0,41 | 0,11 | 0,00 | 0,12 |
| 1-1,5 | 0,00 | 0,00 | 0,00 | 0,44 | 7,37 | 30,84 | 72,97 | 165,44 | 395,79 | 441,72 | 438,20 | 335,84 | 185,71 | 108,27 | 55,72 | 23,58 | 12,84 | 4,82 | 3,01 | 0,49 | 0,26 | 0,27 |
| 1,5-2 | 0,00 | 0,00 | 0,00 | 0,00 | 0,76 | 30,40 | 159,13 | 262,93 | 461,43 | 710,58 | 900,49 | 773,46 | 505,79 | 294,93 | 155,24 | 74,83 | 43,43 | 19,88 | 6,17 | 3,03 | 0,00 | 0,00 |
| 2-2,5 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 2,64 | 67,25 | 249,80 | 309,34 | 482,07 | 849,94 | 1067,66 | 934,98 | 545,60 | 290,26 | 152,64 | 73,04 | 23,75 | 10,28 | 2,71 | 0,71 | 0,00 |
| 2,5-3 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 8,53 | 134,84 | 234,56 | 251,37 | 622,67 | 1038,79 | 1144,36 | 907,75 | 502,00 | 224,48 | 102,69 | 32,44 | 12,03 | 3,90 | 3,07 | 0,00 |
| 3-3,5 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 27,58 | 124,13 | 162,45 | 328,94 | 735,20 | 1049,03 | 1023,90 | 718,10 | 305,54 | 98,07 | 37,00 | 11,34 | 2,65 | 0,00 | 0,00 |
| 3,5-4 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 4,16 | 35,85 | 109,99 | 221,01 | 458,31 | 791,47 | 933,59 | 770,34 | 335,33 | 139,87 | 49,88 | 14,81 | 3,46 | 0,00 | 0,00 |
| 4-4,5 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 10,85 | 58,09 | 137,45 | 273,58 | 540,04 | 776,46 | 754,65 | 561,19 | 184,47 | 51,30 | 12,50 | 0,00 | 2,30 | 0,00 |
| 4,5-5 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 27,06 | 90,80 | 159,13 | 376,46 | 477,40 | 621,11 | 534,78 | 280,65 | 102,30 | 28,28 | 13,53 | 0,00 | 0,00 |
| 5-5,5 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 6,55 | 37,82 | 98,24 | 157,51 | 215,48 | 395,42 | 408,68 | 286,70 | 85,47 | 31,11 | 3,27 | 0,00 | 0,00 |
| 5,5-6 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 12,86 | 51,44 | 111,46 | 196,42 | 301,06 | 358,54 | 274,95 | 105,22 | 29,62 | 3,90 | 0,00 | 0,00 |
| 6-6,5 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 2,29 | 5,03 | 21,95 | 53,51 | 153,68 | 209,25 | 263,45 | 260,48 | 107,03 | 26,07 | 13,72 | 0,00 | 0,00 |
| 6,5-7 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 6,37 | 37,93 | 51,98 | 159,13 | 178,23 | 99,19 | 42,97 | 10,08 | 0,00 | 0,00 | 0,00 |
| 7-7,5 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 27,71 | 34,10 | 105,04 | 107,17 | 113,87 | 38,36 | 0,00 | 0,00 | 0,00 | 0,00 |
| 7,5-8 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 4,50 | 14,55 | 72,75 | 121,94 | 100,11 | 56,12 | 0,00 | 0,00 | 0,00 | 0,00 |
| 8-8,5 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 10,95 | 23,46 | 93,86 | 46,54 | 14,08 | 14,86 | 0,00 | 0,00 | 0,00 |
| 8,5-9 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 32,88 | 21,04 | 44,72 | 63,13 | 8,33 | 8,77 | 0,00 | 0,00 |
| 9-9,5 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 14,66 | 15,63 | 8,30 | 26,38 | 9,28 | 9,77 | 0,00 | 0,00 |
| 9,5-10 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 17,32 | 18,40 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| 10-10,5 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| 10,5-11 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| 11-11,5 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |

Table 6 - Power Table

From where we can see that most occurrences are in peak periods (and corresponding energetic periods) between 7 and 14 seconds and significant heights between 0,5 and 3,5 meters (this range accounts for 78,3% of the occurrences). As for the wave contribution, we know that the average wave power is proportional to the square of the significant height and to the peak period, so the range that provides the most energy is shifted a bit to the right (because of higher peak periods) and even more to the bottom (because of higher significant heights). In this case, the range of peak periods between 7 and 14 seconds and significant wave heights between 0,5 and 3,5 meters accounts for 48,7% of the energy contribution – which is considerably less.

**2.3 Exercise 3 – Wave Energy Converters**

Three wave energy converters were available with corresponding power matrices – relating the power provided by the instrument in kW for associated significant height and energetic period. To estimate the electricity production of a wave converter, one must associate the converter power matrix with the wave activity of the specific site (which, in the present case, is given by the developed normalized occurrence table relating significant wave height with energetic period) considering the following equation:

Where is the number of period and is the number of wave height in the table, is the probability of occurrence of the sea state corresponding to the bin defined by the line i and the column j and is the electric power corresponding to the same sea state or energy bin for the wave energy converter considered. We then use Excel to calculate this value for each wave converter, by considering the power matrices of each wave converter given as a function of significant height and energetic period. Carefully ensuring that the power matrix values ​​have been multiplied by the correct values ​​from the Normalized Occurance scatter table of Hs vs Te, we get the results in table below:

|  |  |  |
| --- | --- | --- |
| **Average Eletric Power for WEC** | | |
| MW | | |
| **Aquabuoy** | **Wave Dragon** | **Pelamis** |
| 0,689 | 7,759 | 2,469 |

Table 7 - Average Eletric Power for WEC

From which it appears that, for the conditions analyzed, the wave dragon is the most efficient wave energy converter - generating an average electrical power more than 11 times greater than that of Aquabuoy and more than 3 times greater than that of Pelamis.

**3. Part Two - Wave forces and mooring analysis of a buoy**

To solve the next exercises, we consider the following mooring system:

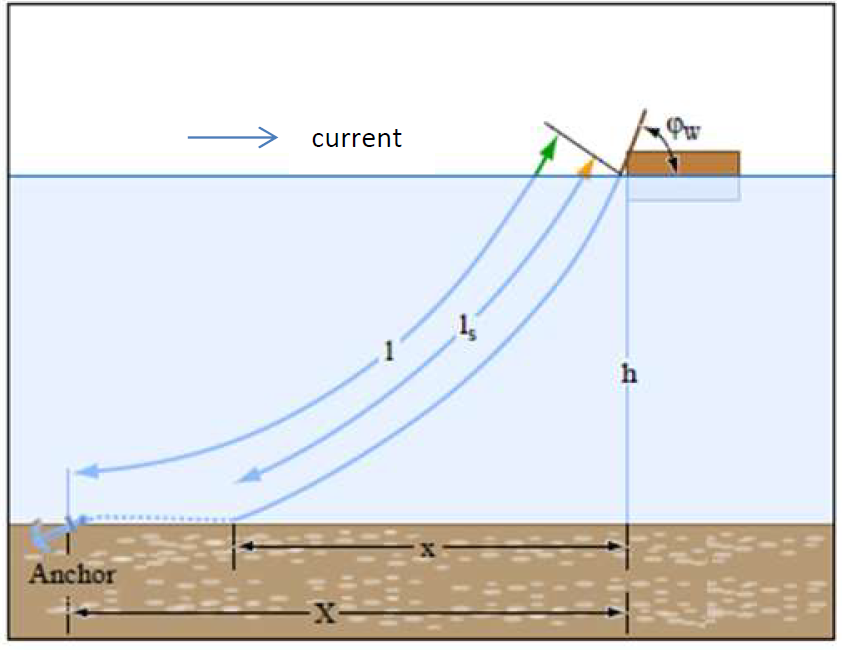


Figure 1- Mooring System of Part Two

And were given the following parameters:

|  |  |  |  |
| --- | --- | --- | --- |
| **Parameter** | **Value** | **Abbreviation** | **Unity** |
| Buoy Diameter | 2,5 | Dbuoy | m |
| Chain Weight | 800,8 | wchain | N/m |
| Density of seawater | 1025 | p | kg/m3 |
| Drag Coefficient | 0,9 | Cd | - |
| Current Speed | 3 | Uc | m/s |
| Buoy Draft | 10 | Tbuoy | m/s |

Table 8 - Mooring System problem given parameters

We then use MATLAB to solve all following exercises.

**a) Horizontal Wave Force per Unit Length**

We can calculate the horizontal wave force per unit length as:

We obtain

**b) Total Horizontal Wave Force**

We can calculate the total horizontal wave force as:

We obtain

**c) Minimum Chain Length**

Considering a fairlead angle of 45 degrees, the minimum chain length (s), the horizontal distance from touch down to fairlead point (x) and the thension at the buoy (Th) are obtained using:

We obtain:

**d) Horizontal Distance with axial stiffness**

Considering the stiffness as S, the new horizontal distance from touch down to fairlead will be:

The result is:

**e) Total Buoy Mass**

The total buoy mass can be obtained with:

Which yields: