

Lógica para Computação

Exercícios de Dedução Natural em Lógica de Predicados

Saída da API

1 Informações:

MODELO BASE = gpt 4.1

Modelo Treinado

2 Questões:

1. $\forall x P(x) \vdash \neg \exists x \neg P(x)$

RESPOSTA CHAT-GPT

Prova:""

1. $\forall x P(x)$ pre
2. { $\exists x \neg P(x)$ hip
3. { $a \neg P(a)$ hip
4. $P(a)$ Ae 1
5. @ $\neg e$ 3, 4
- }
6. @ Ee 2, 3-5
- }
7. $\neg \exists x \neg P(x)$ $\sim i$ 2-6

""

2. $\neg \exists x \neg P(x) \vdash \forall x P(x)$

RESPOSTA CHAT-GPT

Prova:""

1. $\neg \exists x \neg P(x)$ pre
2. { a
3. { $\neg P(a)$ hip
4. $\exists x \neg P(x)$ Ei 3
5. @ $\neg e$ 4,1
- }
6. $P(a)$ raa 3-5
- }
7. $\forall x P(x)$ Ai 2-6

""

3. $\exists x P(x) \vdash \neg \forall x \neg P(x)$

RESPOSTA CHAT-GPT

Prova:""

1. $\exists x P(x)$ pre
2. { $\forall x \neg P(x)$ hip
3. { $a P(a)$ hip
4. $\neg P(a)$ Ae 2
5. @ $\neg e$ 3,4
- }
6. @ Ee 1, 3-5
- }
7. $\neg \forall x \neg P(x)$ $\sim i$ 2-6

""

4. $\neg \forall x \neg P(x) \vdash \exists x P(x)$

RESPOSTA CHAT-GPT

Prova:""

1. $\neg \forall x \neg P(x)$ pre
2. { $\neg \exists x P(x)$ hip
3. { a
4. { $P(a)$ hip

```

5.    ExP(x)  Ei 4
6.    @      ~e 2,5
    }
7.    ~P(a)   ~i 4-6
    }
8.    Ax~P(x)  Ai 3-7
9.    @      ~e 1,8
    }
10. Ex P(x)    raa 2-9

""

```

5. $\text{Ax } (P(x) \& Q(x)) \vdash \text{Ax } P(x) \& \text{Ax } Q(x)$
 RESPOSTA CHAT-GPT

```

Prova: ""

1. Ax(P(x) & Q(x)) pre
2. { a
3.   P(a) & Q(a) Ae 1
4.   P(a) &e 3
}
5. Ax P(x) Ai 2-4
6. { b
7.   P(b) & Q(b) Ae 1
8.   Q(b) &e 7
}
9. Ax Q(x) Ai 6-8
10. Ax P(x) & Ax Q(x) &i 5, 9

""

```

6. $\text{Ax Ay } P(x,y) \vdash \text{Ay Ax } P(x,y)$
 RESPOSTA CHAT-GPT

```

Prova: ""

1. AxAyP(x,y) pre
2. { b
3.   { a
4.     AyP(a,y) Ae 1
5.     P(a,b) Ae 4
}
6.   AxP(x,b) Ai 3-5
}
7. AyAxP(x,y) Ai 2-6

""

```

7. $\text{Ax } (P \rightarrow Q(x)) \vdash P \rightarrow \text{Ax } Q(x)$
 RESPOSTA CHAT-GPT

```

Prova: ""

1. Ax(P->Q(x)) pre
2. { P hip
3. { a
4.   P -> Q(a) Ae 1
5.   Q(a) ->e 2,4
}
6. Ax Q(x) Ai 3-5
}
7. P -> Ax Q(x) ->i 2-6

""

```

8. $\text{Ex } (P(x) | Q(x)) \vdash \text{Ex } P(x) | \text{Ex } Q(x)$
 RESPOSTA CHAT-GPT

```

Prova: ""

1. Ex(P(x) | Q(x)) pre
2. { a P(a) | Q(a) hip

```

```

3. { P(a) hip
4. Ex P(x) Ei 3
5. Ex P(x) | Ex Q(x) |i 4
}
6. { Q(a) hip
7. Ex Q(x) Ei 6
8. Ex P(x) | Ex Q(x) |i 7
}
9. Ex P(x) | Ex Q(x) |e 2, 3-5, 6-8
}
10. Ex P(x) | Ex Q(x) Ee 1, 2-9

```

""

9. $\sim Ax P(x) \vdash Ex \sim P(x)$
 RESPOSTA CHAT-GPT

Prova: ""

```

1. {  $\sim Ax P(x)$  hip
2.   {  $\sim Ex \sim P(x)$  hip
3.     { a
4.       {  $\sim P(a)$  hip
5.         Ex  $\sim P(x)$  Ei 4
6.         @  $\sim e$  5,2
       }
7.       P(a) raa 4-6
     }
8.     Ax P(x) Ai 3-7
9.     @  $\sim e$  8,1
   }
10.   Ex  $\sim P(x)$  raa 2-9
}
11.  $\sim Ax P(x) \rightarrow Ex \sim P(x) \rightarrow i$  1-10

```

""

10. $Ex \sim P(x) \vdash \sim Ax P(x)$
 RESPOSTA CHAT-GPT

Prova: ""

```

1. Ex  $\sim P(x)$  pre
2. { Ax P(x) hip
3. { a  $\sim P(a)$  hip
4.   P(a) Ae 2
5.   @  $\sim e$  3,4
}
6. @ Ee 1, 3-5
}
7.  $\sim Ax P(x) \sim i$  2-6

```

""

11. $\sim Ex P(x) \vdash Ax \sim P(x)$
 RESPOSTA CHAT-GPT

Prova: ""

```

1.  $\sim Ex P(x)$  pre
2. { a
3.   { P(a) hip
4.     Ex P(x) Ei 3
5.     @  $\sim e$  4,1
}
6.  $\sim P(a) \sim i$  3-5
}
7. Ax  $\sim P(x)$  Ai 2-6

```

""

12. $Ax \sim P(x) \vdash \sim Ex P(x)$
 RESPOSTA CHAT-GPT

Prova:""

```
1.  $\forall x \neg P(x)$  pre
2. {  $\exists x P(x)$  hip
3.   {  $a P(a)$  hip
4.      $\neg P(a)$  Ae 1
5.       @  $\neg e$  3, 4
6.   }
7. @  $Ee$  2, 3-5
8. }
9.  $\neg \exists x P(x)$   $\sim i$  2-6
```

""

13. $\exists x (P(x) \& Q) \vdash \exists x P(x) \& Q$
RESPOSTA CHAT-GPT

Prova:""

```
1.  $\exists x (P(x) \& Q)$  pre
2. {  $a P(a) \& Q$  hip
3.    $P(a) \& e$  2
4.      $Q \& e$  2
5.      $\exists x P(x)$   $Ei$  3
6. }
7.  $\exists x P(x) \& Q$   $\& i$  5, 4
8.  $\exists x P(x) \& Q$   $Ee$  1, 2-6
```

""

14. $\exists x P(x) \& Q \vdash \exists x (P(x) \& Q)$
RESPOSTA CHAT-GPT

Prova:""

```
1.  $\exists x P(x) \& Q$  pre
2.  $\exists x P(x) \& e$  1
3.  $Q \& e$  1
4. {  $a P(a)$  hip
5.    $P(a) \& Q$   $\& i$  4,3
6.    $\exists x (P(x) \& Q)$   $Ei$  5
7. }
8.  $\exists x (P(x) \& Q)$   $Ee$  2, 4-6
```

""

15. $\forall x (P(x) \mid Q) \vdash \forall x P(x) \mid Q$
RESPOSTA CHAT-GPT

Prova:""

```
1.  $\forall x (P(x) \mid Q)$  pre
2. {  $\neg Q$  hip
3.   {  $a$ 
4.      $P(a) \mid Q$  Ae 1
5.     {  $P(a)$  hip
6.        $P(a) \& i$  5,5
7.     }
8.     {  $Q$  hip
9.       @  $\neg e$  2,7
10.       $P(a) \& e$  8
11.    }
12.     $P(a) \mid e$  4, 5-6, 7-9
13.  }
14.  $\forall x P(x)$   $Ai$  3-10
15. }
16.  $\forall x P(x) \mid Q$   $\mid i$  11
17. }
18. {  $Q$  hip
19.    $\forall x P(x) \mid Q$   $\mid i$  13
20. }
21.  $\forall x P(x) \mid Q$   $\mid e$  2-12, 13-14
```

""

16. $\forall x (P(x) \mid Q) \vdash \forall x (P(x) \mid Q)$
RESPOSTA CHAT-GPT

Prova:""

1. $\forall x (P(x) \mid Q)$ pre
2. { $\forall x (P(x)$ hip
3. { a
4. $P(a)$ Ae 2
5. $P(a) \mid Q$ |i 4
}
6. $\forall x (P(x) \mid Q)$ Ai 3-5
}
7. { Q hip
8. { a
9. $P(a) \mid Q$ |i 7
}
10. $\forall x (P(x) \mid Q)$ Ai 8-9
}
11. $\forall x (P(x) \mid Q)$ |e 1, 2-6, 7-10

" "

17. $\exists x (P(x) \rightarrow Q) \vdash \forall x (P(x) \rightarrow Q)$
RESPOSTA CHAT-GPT

Prova:""

1. $\exists x (P(x) \rightarrow Q)$ pre
2. { $\forall x (P(x)$ hip
3. { a $P(a) \rightarrow Q$ hip
4. $P(a)$ Ae 2
5. $Q \rightarrow e$ 4,3
}
6. Q Ee 1, 3-5
}
7. $\forall x (P(x) \rightarrow Q) \rightarrow i$ 2-6

" "

18. $\forall x (P(x) \rightarrow Q) \vdash \exists x (P(x) \rightarrow Q)$
RESPOSTA CHAT-GPT

Prova:""

1. $\forall x (P(x) \rightarrow Q)$ pre
2. { $\neg(\exists x (P(x) \rightarrow Q))$ hip
3. { a
4. { $P(a)$ hip
5. $\forall x (P(x) \rightarrow Q)$ Ai 3-4
6. $Q \rightarrow e$ 1,5
7. $P(a) \rightarrow Q \rightarrow i$ 4-6
8. $\exists x (P(x) \rightarrow Q)$ Ei 7
9. @ $\neg e$ 2,8
}
10. $\neg P(a) \rightarrow i$ 4-9
}
11. $\exists x \neg P(x)$ Ei 10
12. $Q \rightarrow e$ 1,11
13. $P(x) \rightarrow Q$ Ei 12
14. $\exists x (P(x) \rightarrow Q)$ Ei 13
15. @ $\neg e$ 2,14
}
16. $\exists x (P(x) \rightarrow Q)$ raa 2-15

" "

19. $\exists x (P \rightarrow Q(x)) \vdash P \rightarrow \exists x Q(x)$
RESPOSTA CHAT-GPT

Prova:""

1. $\exists x (P \rightarrow Q(x))$ pre
2. { P hip
3. { a $P \rightarrow Q(a)$ hip
4. $Q(a) \rightarrow e$ 2, 3

```

5.    Ex Q(x) Ei 4
}
6.    Ex Q(x) Ee 1, 3-5
}
7. P -> Ex Q(x) ->i 2-6

""

```

20. $P \rightarrow \text{Ex } Q(x) \mid - \text{Ex } (P \rightarrow Q(x))$
 RESPOSTA CHAT-GPT

```

Prova:""

1. P -> Ex Q(x) pre
2. { a
3. { P hip
4.    Ex Q(x) ->e 1,3
5.    { b Q(b) hip
6.      Q(a) Ai 2-5
}
7.    Ex Q(x) Ae 6
}
8. P -> Q(a) ->i 3-7
9. Ex (P -> Q(x)) Ei 8
}
10. Ex (P -> Q(x)) Ai 2-9

""

```

21. $\mid - (A \mid (A \& B)) \rightarrow A$
 RESPOSTA CHAT-GPT

```

Prova:""

1. { A | (A & B) hip
2. { A hip
3. A copie 2
}
4. { A & B hip
5. A &e 4
}
6. A | (A & B) ->e 1, 2-3, 4-5
}
7. (A | (A & B)) -> A ->i 1-6

""

```

22. $\mid - (A \& (A \mid B)) \rightarrow A$
 RESPOSTA CHAT-GPT

```

Prova:""

1.{ A & (A | B) hip
2.A &e 1
}
3.(A & (A | B)) -> A ->i 1-2

""

```

23. $\mid - (A \rightarrow (B \rightarrow C)) \rightarrow (B \rightarrow (A \rightarrow C))$
 RESPOSTA CHAT-GPT

```

Prova:""

1. { A -> (B -> C) hip
2. { B hip
3.    { A hip
4.      B -> C ->e 3,1
5.      C ->e 2,4
}
6.    A -> C ->i 3-5
}
7. B -> (A -> C) ->i 2-6

```

```

}
8. (A -> (B -> C)) -> (B -> (A -> C)) ->i 1-7

"""

```

24. $\vdash (A \rightarrow (A \rightarrow B)) \rightarrow (A \rightarrow B)$
 RESPOSTA CHAT-GPT

```

Prova: ""

1. { A -> (A -> B) hip
2. { A hip
3. A -> B ->e 2,1
4. B ->e 2,3
}
5. A -> B ->i 2-4
}
6. (A -> (A -> B)) -> (A -> B) ->i 1-5

"""

```

25. $\vdash (\neg A \rightarrow B) \rightarrow ((\neg A \rightarrow \neg B) \rightarrow A)$
 RESPOSTA CHAT-GPT

```

Prova: ""

1. {  ~A -> B                hip
2. {  ~A -> ~B              hip
3. {    ~A                  hip
4. ~B                      ->e 3,2
5. B                      ->e 3,1
6. @                      ~e 4,5
}
7. A                      raa 3-6
}
8. (~A -> ~B) -> A          ->i 2-7
}
9. (~A -> B) -> ((~A -> ~B) -> A) ->i 1-8

"""

```

26. $\vdash A \mid \neg A$
 RESPOSTA CHAT-GPT

```

Prova: ""

1. { ~(A | ~A) hip
2. { A hip
3. A | ~A |i 2
4. @ ~e 3,1
}
5. ~A ~i 2-4
6. A | ~A |i 5
7. @ ~e 6,1
}
8. A | ~A raa 1-7

"""

```

27. $\vdash (A \rightarrow B) \mid (B \rightarrow A)$
 RESPOSTA CHAT-GPT

```

Prova: ""

1. { ~(A -> B) & ~(B -> A)  hip
2. { A hip
3. { B hip
4. B copie 3
}
5. A -> B ->i 3-4
6. @ ~e 1,5
}
7. ~B ~i 2-6

```

```

8. { B hip
9. @ ~e 7,8
10. A copie 2
}
11. B -> A ->i 8-10
12. @ ~e 1,11
}
13. A & ~B hip
14. { A hip
15. { ~B hip
16. A & ~B &i 14,15
17. ~(A & ~B) pre
18. @ ~e 16,17
}
19. ~~B ~i 15-18
20. B copie 13
21. @ ~e 19,20
}
22. ~A ~i 14-21
23. ~A & ~B &i 22, 7
24. (A & ~B) | (~A & ~B) |i 23
25. (A & ~B) -> B pre
26. B ->e 24,25
27. @ ~e 7,26
}
28. A | B |i 13
29. { A hip
30. B | A |i 29
}
31. { B hip
32. B | A |i 31
}
33. B | A |e 28,29-30,31-32
34. { B | A hip
35. ~ (B | A) pre
36. @ ~e 33,35
}
37. EEA ~i 34-36
38. (B | A) |i 37
}
39. @ ~e 1,38
}
40. (A -> B) | (B -> A) raa 1-39

""

```

28. |- A->A
 RESPOSTA CHAT-GPT

Prova:""

```

1. { A hip
2.   A copie 1
}
3. A -> A ->i 1-2

""

```

29. |- (A->B)->((C->A)->(C->A))
 RESPOSTA CHAT-GPT

Prova:""

```

1. { A -> B hip
2. { C -> A hip
3.   { C hip
4.     A ->e 3,2
5.     B ->e 4,1
   }
6.   C -> B ->i 3-5
}
7. (C -> A) -> (C -> B) ->i 2-6
}
8. (A -> B) -> ((C -> A) -> (C -> B)) ->i 1-7

""

```

30. (A&B)->C |- B->(A->C)
 RESPOSTA CHAT-GPT


```

Prova:""

1. A & B -> C pre
2. { B hip
3. { A hip
4. A & B &i 3,2
5. C ->e 4,1
}
6. A -> C ->i 3-5
}
7. B -> (A -> C) ->i 2-6

""

```

31. $B \rightarrow (A \rightarrow C) \mid - (A \& B) \rightarrow C$
 RESPOSTA CHAT-GPT

```

Prova:""

1. B -> (A -> C) pre
2. { A & B hip
3. A &e 2
4. B &e 2
5. A -> C ->e 4,1
6. C ->e 3,5
}
7. (A & B) -> C ->i 2-6

""

```

32. $\mid - (A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow C))$
 RESPOSTA CHAT-GPT

```

Prova:""

1. { A -> (B -> C) hip
2. { A -> B hip
3. { A hip
4. B -> C ->e 1,3
5. B ->e 2,3
6. C ->e 4,5
}
7. A -> C ->i 3-6
}
8. (A -> B) -> (A -> C) ->i 2-7
}
9. (A -> (B -> C)) -> ((A -> B) -> (A -> C)) ->i 1-8

""

```

33. $\mid - A \rightarrow (B \rightarrow A)$
 RESPOSTA CHAT-GPT

```

Prova:""

1. { A hip

2. { B hip

3. A copie 1

}

4. B -> A ->i 2-3

}

5. A -> (B -> A) ->i 1-4

""

```

34. $\mid - ((A \rightarrow B) \rightarrow A) \rightarrow A$
 RESPOSTA CHAT-GPT

```

Prova:""

1.{ (A -> B) -> A hip
2.{ A hip
3.B copie 2
}
4.A -> B ->i 2-3
5.A ->e 4,1
}
6.((A -> B) -> A) -> A ->i 1-5

""

```

35. $A \rightarrow C, B \rightarrow C, A|B \vdash C$
 RESPOSTA CHAT-GPT

```

Prova:""

1. A -> C           pre
2. B -> C           pre
3. A | B           pre
4. { A             hip
5. C               ->e 4,1
}
6. { B             hip
7. C               ->e 6,2
}
8. C               |e 3,4-5,6-7

""

```

36. $A \vdash \sim\sim A$
 RESPOSTA CHAT-GPT

```

Prova:""

1. A pre
2. { ~A hip
3. @ ~e 1, 2

}

4. ~~A ~i 2-3

""

```

37. $\sim\sim A \vdash A$
 RESPOSTA CHAT-GPT

```

Prova:""

1.~ ~ A hip
2.{ ~A hip
3.@ ~e 1,2
4.A @e 3
}
5.A raa 2-4

""

```

38. $A \rightarrow B, \sim B \vdash \sim A$
 RESPOSTA CHAT-GPT

```

Prova:""

1. A -> B pre
2. ~B pre
3. {A hip
4. B ->e 3,1
5. @ ~e 4,2

```

```
}
6. ~A ~i 3-5
```

39. $\sim B \rightarrow \sim A \mid - A \rightarrow B$

RESPOSTA CHAT-GPT

Prova:""

```
1. ~B -> ~A pre
2. { A hip
3. { ~B hip
4. ~A ->e 1,3
5. @ ~e 2,4
}
6. B raa 3-5
}
7. A -> B ->i 2-6
```

""

40. $A \rightarrow B \mid - \sim B \rightarrow \sim A$

RESPOSTA CHAT-GPT

Prova:""

```
1. A -> B pre
2. { ~B hip
3. { A hip
4. B ->e 3,1
5. @ ~e 4,2
}
6. ~A ~i 3-5
}
7. ~B -> ~A ->i 2-6
```

""

41. $\sim(A \mid B) \mid - \sim A \& \sim B$

RESPOSTA CHAT-GPT

Prova:""

```
1. ~ (A | B) pre
2. { A hip
3. A | B |i 2
4. @ ~e 3, 1
}
5. ~A ~i 2-4
6. { B hip
7. A | B |i 6
8. @ ~e 7, 1
}
9. ~B ~i 6-8
10. ~A & ~B &i 5, 9
```

""

42. $\sim A \& \sim B \mid - \sim(A \mid B)$

RESPOSTA CHAT-GPT

Prova:""

```
1. ~A & ~B pre
2. A | B Ei 1 (Eliminação da conjunção &e)

3. { A hip
4. ~A ~e 1
5. @ ~e 3, 4
}
6. { B hip
7. ~B ~e 1
8. @ ~e 6, 7
}
```

9. @ |e 2, 3-5, 6-8

10. $\sim(A \mid B) \sim i$ 2-9

""

43. $\sim(A \& B) \mid - \sim A \mid \sim B$

RESPOSTA CHAT-GPT

Prova: ""

```
1. {  $\sim(A \& B)$  hip
2.   {  $\sim A$  hip
3.      $\sim A \mid \sim B \mid i$  2
   }
4.   {  $\sim B$  hip
5.      $\sim A \mid \sim B \mid i$  4
   }
6.    $\sim A \mid \sim B \mid e$  1, 2-3, 4-5
   }
7.  $(A \& B) \rightarrow (\sim A \mid \sim B) \rightarrow i$  1-6
```

""

44. $\sim A \mid \sim B \mid - \sim(A \& B)$

RESPOSTA CHAT-GPT

Prova: ""

```
1.  $\sim A \mid \sim B$  pre
2. {  $A \& B$  hip
3.    $A \& e$  2
4.    $B \& e$  2
5. {  $\sim A$  hip
6.   @  $\sim e$  3,5
   }
7. {  $\sim B$  hip
8.   @  $\sim e$  4,7
   }
9.   @ |e 1,5-6,7-8
   }
10.  $\sim(A \& B) \sim i$  2-9
```

""

45. $A \mid (B \& C) \mid - (A \mid B) \& (A \mid C)$

RESPOSTA CHAT-GPT

Prova: ""

```
1.  $A \mid (B \& C)$  pre
2. {  $A$  hip
3.  $A \mid B \mid i$  2
   }
4. {  $B \& C$  hip
5.  $B \& e$  4
6.  $A \mid B \mid i$  5
   }
7.  $A \mid B \mid e$  1, 2-3, 4-6
8. {  $A$  hip
9.  $A \mid C \mid i$  8
   }
10. {  $B \& C$  hip
11.  $C \& e$  10
12.  $A \mid C \mid i$  11
   }
13.  $A \mid C \mid e$  1, 8-9, 10-12
14.  $(A \mid B) \& (A \mid C) \& i$  7, 13
```

""

46. $(A \mid B) \& (A \mid C) \mid - A \mid (B \& C)$

RESPOSTA CHAT-GPT

Prova:""

```
1. (A | B) & (A | C) pre
2. A | B &e 1
3. A | C &e 1
4. { A hip
5. A | (B & C) |i 4
}
6. { B hip
7. { A hip
8. A | (B & C) |i 7
}
9. { C hip
10. B & C &i 6,9
11. A | (B & C) |i 10
}
12. A | (B & C) |e 3, 7-8, 9-11
}
13. { C hip
14. { A hip
15. A | (B & C) |i 14
}
16. { B hip
17. B & C &i 16,13
18. A | (B & C) |i 17
}
19. A | (B & C) |e 2, 14-15, 16-18
}
20. A | (B & C) |e 2, 4-5, 6-12
""
```

47. $A \& (B|C) \vdash (A \& B) | (A \& C)$
RESPOSTA CHAT-GPT

Prova:""

```
1. A & (B | C) pre
2. A &e 1
3. B | C &e 1
4. { B hip
5. A & B &i 2, 4
6. (A & B) | (A & C) |i 5
}
7. { C hip
8. A & C &i 2, 7
9. (A & B) | (A & C) |i 8
}
10. (A & B) | (A & C) |e 3, 4-6, 7-9
""
```

48. $(A \& B) | (A \& C) \vdash A \& (B|C)$
RESPOSTA CHAT-GPT

Prova:""

```
1. (A & B) | (A & C) pre
2. { A & B hip
3. A &e 2
4. B &e 2
5. B | C |i 4
6. A & (B | C) &i 3,5
}
7. { A & C hip
8. A &e 7
9. C &e 7
10. B | C |i 9
11. A & (B | C) &i 8,10
}
12. A & (B | C) |e 1, 2-6, 7-11
""
```

49. $A|B, \sim B \vdash A$
RESPOSTA CHAT-GPT

```

Prova:""

1. A | B pre
2. ~B pre
3. { A hip
4. A copie 3
}
5. { B hip
6. @ ~e 2,5
7. A @e 6
}
8. A |e 1,3-4,5-7

""

```

50. $A|B \vdash \sim A \rightarrow B$
 RESPOSTA CHAT-GPT

```

Prova:""

1. A | B pre
2. { ~A hip
3. { A hip
4. @ ~e 2,3
5. B @e 4
}
6. { B hip
7. B copie 6
}
8. B |e 1,3-5,6-7
}
9. ~A -> B ->i 2-8

""

```

51. $\sim A \rightarrow B \vdash A|B$
 RESPOSTA CHAT-GPT

```

Prova:""

1. ~A -> B pre
2. { ~(A | B) hip
3. { A hip
4. A | B |i 3
5. @ ~e 4,2
}
6. ~A ~i 3-5
7. B ->e 6,1
8. A | B |i 7
9. @ ~e 8,2
}
10. A | B raa 2-9

""

```

52. $A \& B \vdash \sim (A \rightarrow \sim B)$
 RESPOSTA CHAT-GPT

```

Prova:""

1. A & B pre

2. { A -> ~B hip

3. A &e 1

4. ~B ->e 3, 2

5. B &e 1

6. @ ~e 4, 5

}

7. ~(A -> ~B) ~i 2-6

""

```

53. $\sim(A \rightarrow \sim B) \vdash A \& B$
RESPOSTA CHAT-GPT

Prova:""

1. $\sim(A \rightarrow \sim B)$ pre
2. { $\sim(A \& B)$ hip
3. { A hip
4. { B hip
5. A & B &i 3,4
6. @ $\sim e$ 5,2
}
7. $\sim B$ $\sim i$ 4-6
}
8. $A \rightarrow \sim B \rightarrow i$ 3-7
9. @ $\sim e$ 8,1
}
10. A & B raa 2-9

" "

54. $A|B \vdash \sim(\sim A \& \sim B)$
RESPOSTA CHAT-GPT

Prova:""

1. A | B pre
2. { $\sim A \& \sim B$ hip
3. $\sim A$ &e 2
4. { A hip
5. @ $\sim e$ 4,3
}
6. $\sim A \rightarrow @ \rightarrow i$ 4-5
7. { B hip
8. @ $\sim e$ 7,3
}
9. B $\rightarrow @ \rightarrow i$ 7-8
10. @ |e 1,4-6,7-9
}
11. $\sim(\sim A \& \sim B)$ $\sim i$ 2-10

" "

55. $\sim(\sim A \& \sim B) \vdash A|B$
RESPOSTA CHAT-GPT

Prova:""

1. $\sim(\sim A \& \sim B)$ pre
2. { $\sim(A|B)$ hip
3. { A hip
4. A | B |i 3
5. @ $\sim e$ 4,2
}
6. $\sim A$ $\sim i$ 3-5
7. { B hip
8. A | B |i 7
9. @ $\sim e$ 8,2
}
10. $\sim B$ $\sim i$ 7-9
11. $\sim A \& \sim B$ &i 6,10
12. @ $\sim e$ 1,11
}
13. A | B raa 2-12

" "

56. $A \rightarrow B \vdash \sim(A \& \sim B)$
RESPOSTA CHAT-GPT

Prova:""

```

1. A -> B pre
2. { A & ~B hip
3. A &e 2
4. ~B &e 2
5. B ->e 3,1
6. @ ~e 5,4
}
7. ~(A & ~B) ~i 2-6

""

```

57. $\sim(A \& \sim B) \vdash A \rightarrow B$
 RESPOSTA CHAT-GPT

```

Prova: ""

1. ~(A & ~B) pre
2. { A hip
3. { ~B hip
4.   A & ~B &i 2,3
5.   @ ~e 1,4
}
6.   B raa 3-5
}
7. A -> B ->i 2-6

""

```

58. $A \& B \vdash \sim(\sim A \mid \sim B)$
 RESPOSTA CHAT-GPT

```

Prova: ""

1. A & B pre
2. { ~A | ~B hip
3. { ~A hip
4. A &e 1
5. @ ~e 4,3
}
6. { ~B hip
7. B &e 1
8. @ ~e 7,6
}
9. @ |e 2, 3-5, 6-8
}
10. ~(~A | ~B) ~i 2-9

""

```

59. $\sim(\sim A \mid \sim B) \vdash A \& B$
 RESPOSTA CHAT-GPT

```

Prova: ""

1. ~(~A | ~B) pre
2. { ~ (A & B) hip
3. { A hip
4. { B hip
5. A & B &i 3,4
6. @ ~e 5,2
}
7. ~B ~i 4-6
8. ~A | ~B |i 7
9. @ ~e 8,1
}
10. ~A ~i 3-9
11. ~A | ~B |i 10
12. @ ~e 11,1
}
13. A & B raa 2-12

""

```

60. $A \rightarrow B \vdash \sim A \mid B$
 RESPOSTA CHAT-GPT

Prova:""

```
1. A -> B pre
2. { ~ (~A | B) hip
3.   { A hip
4.     B ->e 3,1
5.     ~A | B |i 4
6.     @ ~e 2,5
7.   ~A ~i 3-6
8.   ~A | B |i 7
9.   @ ~e 2,8
10.  }
11. ~A | B raa 2-9
```

""

61. $\sim A|B \vdash A \rightarrow B$

RESPOSTA CHAT-GPT

Prova:""

```
1. ~A | B pre
2. { A hip
3. { ~A hip
4. @ ~e 2,3
5. B @e 4
6. }
7. { B hip
8. B copie 6
9. }
10. B |e 1, 3-5, 6-7
11. }
12. A -> B ->i 2-8
```

""