

- LEONARDO PACHECO, *IGL via  $\omega$ -rules*.

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IGL is an intuitionistic version of the provability logic GL using both box and diamond modalities. It was first studied by Das, van der Giessen, and Marin [2], who provided two ill-founded proof systems and two semantics for this logic. Later, Aguilera and Pacheco [1] defined a cyclic proof system  $\text{cm}\ell\text{IGL}$  for IGL; they also proved that the set of theorems of IGL is recursively enumerable.

We define a well-founded labeled proof system  $\omega\text{m}\ell\text{IGL}$  for IGL characterized by the following  $\omega$ -rule:

$$\frac{x : \Box^n \perp, \mathbf{R}, \Gamma \vdash \Delta \ (\forall n \in \omega)}{\mathbf{R}, \Gamma \vdash \Delta},$$

where  $x$  is a label variable. To prove that all valid formulas of IGL can be proved using the  $\omega$ -rule, we use a proof search game argument. To show the soundness of the  $\omega$ -rule are valid, we closely analyze the completeness of  $\text{cm}\ell\text{IGL}$ . Note that an  $\omega$ -rule for classical GL was studied by Tanaka [3].

[1] JUAN P. AGUILERA and LEONARDO PACHECO, *IGL without sharps*, submitted.

[2] ANUPAM DAS and IRIS VAN DER GIESSEN and SONIA MARIN, *Intuitionistic Gödel-Löb logic, à la Simpson: labelled systems and birelational semantics*, **EACSL Annual Conference on Computer Science Logic** vol. 288, Schloss Dagstuhl. Leibniz-Zent. Inform., Wadern, 2024, Art. No. 22, 18 pages.

[3] YOSHIHITO TANAKA, *A cut-free proof system for a predicate extension of the logic of provability*, **Reports on Mathematical Logic**, (2018), no. 53, pp. 97–109.