Introduction to Cosmology

How Theory and Observation Revealed Our Cosmos

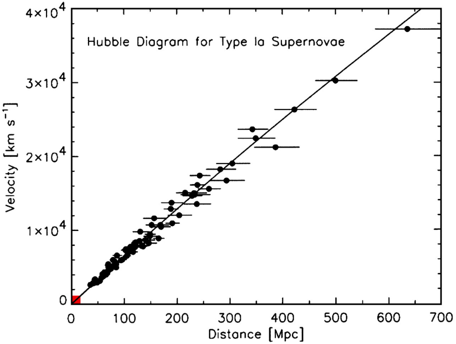
Leonardo Tiditada Pedersen

October 2024

The First Clue: The Universe is Expanding

Theory: Hubble's Law

- In the 1920s, Edwin Hubble discovered that nearly all galaxies are moving away from us.
- He found a linear relationship between a galaxy's distance (d) and its recession velocity (v).
- This is Hubble's Law: $v = H_0 d$, where H_0 is the Hubble Constant.
- This is the foundational evidence that our universe is not static; it's expanding.



A modern Hubble Diagram. The linear trend is clear.

Takeaway: The universe is growing, which implies it was smaller and

Describing Expansion: The Geometry of Spacetime

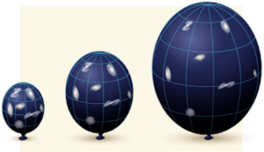
Theory: The FLRW Metric

- This is the solution to Einstein's equations for a homogeneous and isotropic universe.
- It introduces the single most important variable in cosmology: the **scale factor**, $a(t)$. This describes the relative size of the universe over time.

The metric gives the distance (ds) between two points in spacetime:

$$ds^2 = -c^2 dt^2 + a(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right]$$

where k is the curvature parameter $(+1, 0, -1)$.



The balloon analogy: as space expands ($a(t)$ grows), galaxies move apart.

The Engine of Expansion: The Friedmann Equations

Theory: The Rules of Expansion

- Applying General Relativity to the FLRW metric gives us the Friedmann Equations. They govern the dynamics of the scale factor $a(t)$.
- The first equation is the most important; it's the energy equation for the universe.

The First Friedmann Equation

$$(\text{Expansion Rate})^2 = (\text{Energy Density}) - (\text{Curvature})$$

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

- The expansion rate (H) is a battle between the density of stuff (ρ) pushing expansion, and the curvature of spacetime (k) holding it back.

The Cosmic Ingredients (The "Energies")

The density ρ is composed of different ingredients that dilute differently as space expands (as a increases):

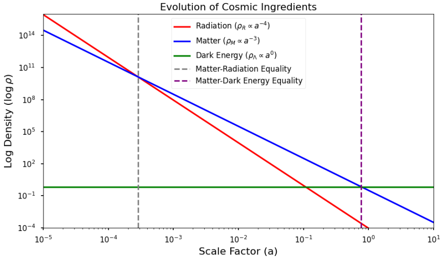
- **Matter** (ρ_M): Dilutes with volume.

$$\rho_M \propto a^{-3}$$

- **Radiation** (ρ_R): Dilutes with volume and loses energy (redshifts).

$$\rho_R \propto a^{-4}$$

- **Dark Energy** (ρ_Λ): Has a constant density.

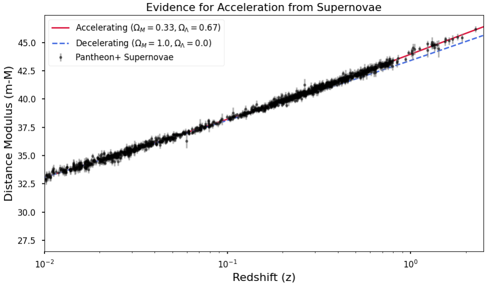


Evolution of the dominant energy component of the universe.

The Plot Twist: Supernovae Reveal Acceleration

Theory & Data

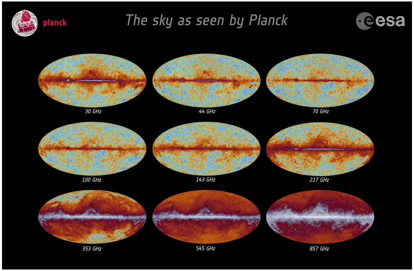
- To solve the Friedmann equation, we need to measure the expansion history.
- We use Type Ia Supernovae as "standard candles" to measure cosmic distances over billions of years.
- The data points are the measurements; the curves are theoretical models from the Friedmann eq.



The data (points) favor a model with Dark Energy, indicating acceleration.

Theory: The Cosmic Microwave Background

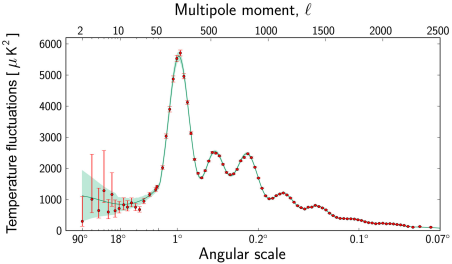
- The CMB is the "afterglow" of the Big Bang, a snapshot of the universe when it was just 380,000 years old.
- It is the oldest light we can possibly see.
- These tiny temperature fluctuations are the seeds of all galaxies.



The CMB temperature fluctuations as seen by the Planck satellite.

Theory & Data

- The characteristic size of the hot/cold spots in the CMB map tells us the geometry of the universe.
- The data from the Planck satellite matches the prediction of our theoretical (ΛCDM) model with incredible precision.



The CMB Power Spectrum. The first peak confirms the universe is flat ($k = 0$).

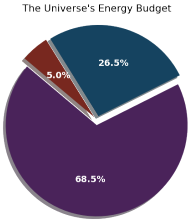
Our Standard Model: The Universe's Recipe

The ΛCDM Model

- SNe demand acceleration (Λ).
- CMB demands a flat universe with Cold Dark Matter (CDM).
- Combining all evidence gives us the Standard Model of Cosmology.

The recipe for our universe today:

- **68%** Dark Energy
- **27%** Dark Matter
- **5%** Ordinary Matter



The energy-density budget of the universe.

Summary & The Great Unknowns

Our Cosmic Story

1. We saw the universe expanding (Hubble).
2. We described it with the geometry of GR (FLRW).
3. We wrote down the rules for its expansion (Friedmann).
4. We measured the expansion and found Dark Energy (SNe).
5. We confirmed the whole picture with the Big Bang's afterglow (CMB).

The Great Unknowns

Despite this success, we still don't know what 95% of our universe is!

- What *is* Dark Energy?
- What *is* Dark Matter?