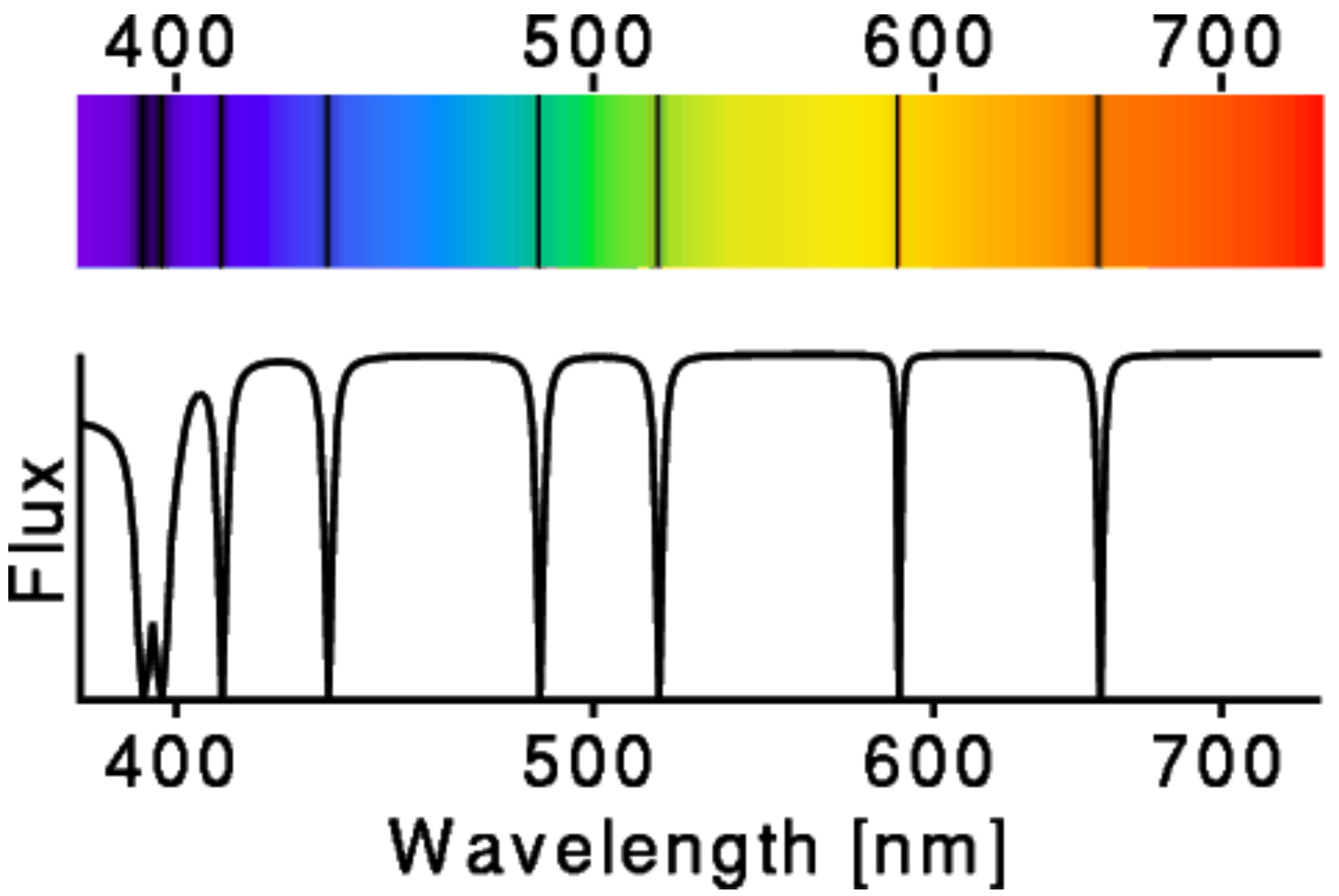
Introduction to

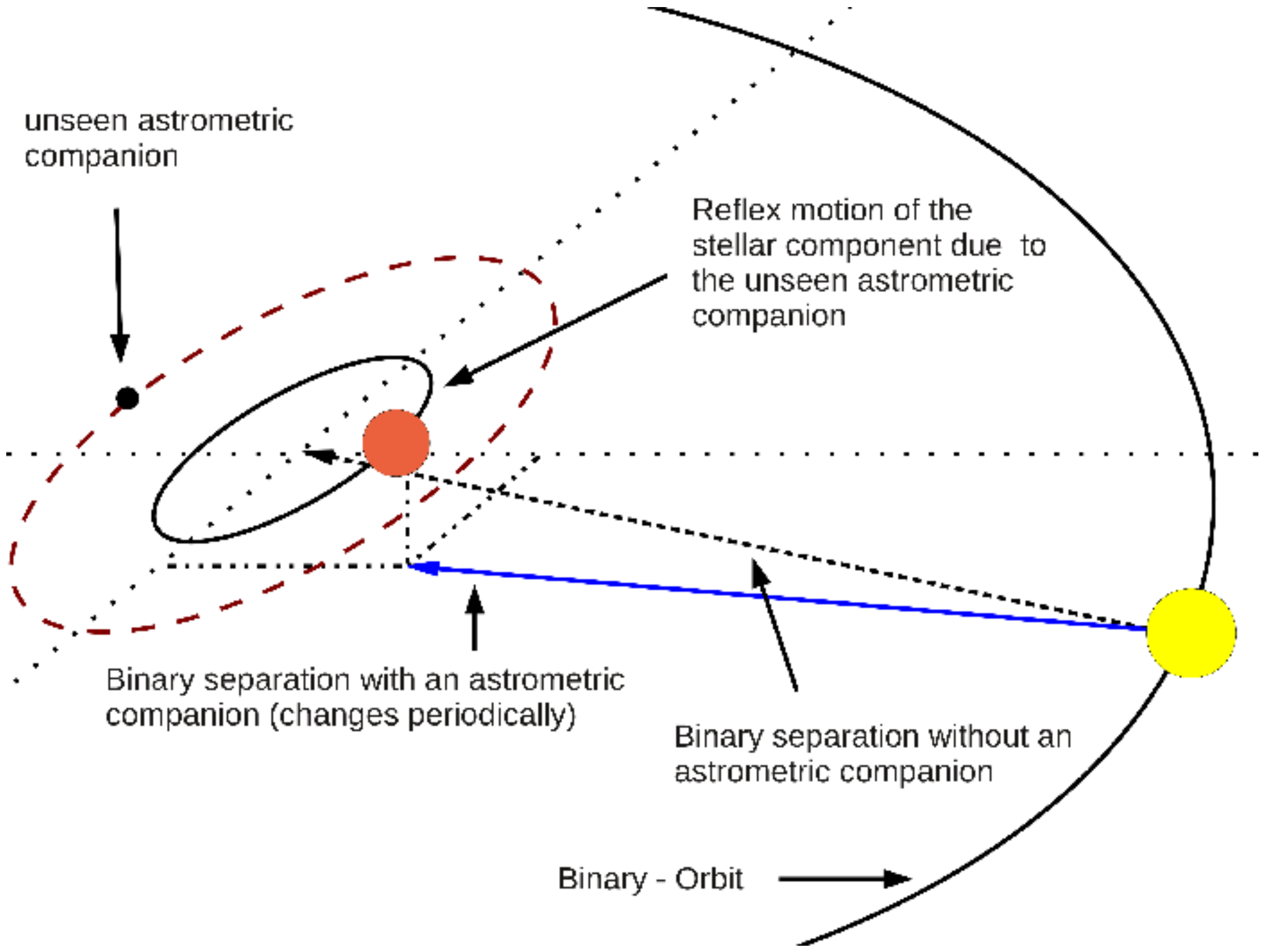
Astrometry

Leonardo Tiditada Pedersen

What is Astrometry?

Astrometry is a branch of astronomy that involves precise measurements of the positions and movements of stars and other celestial bodies





Topics that we will cover

Angular Size

Parallax

Doppler effect

Proper motion

Space velocity

Angles in astronomy

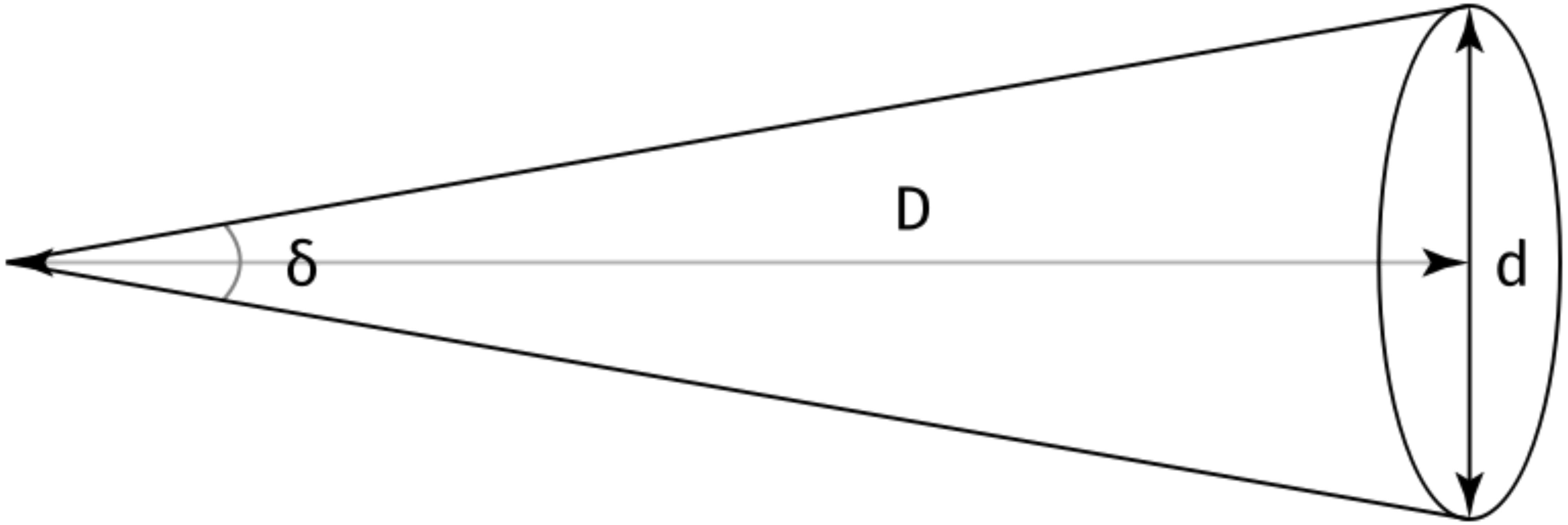
definition of arcminutes and arcsecond

1 degree = 60 arcminutes (60’)

1 arcminutes = 60 arcseconds (60”)

Hence 1 degree = 3600 arcminutes (3600”)

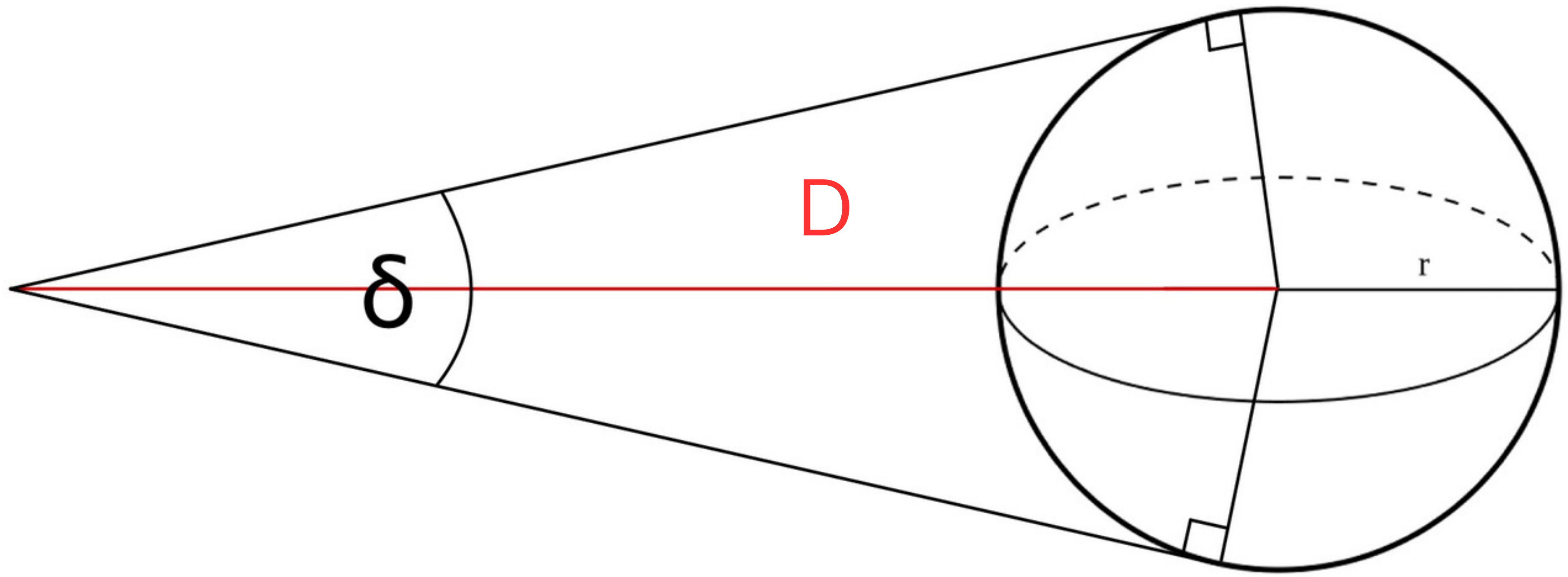
Angular size for circular object



$$\delta = 2\arctan\left(\frac{d}{2D}\right),$$ **Approximately** $$\delta \approx \frac{d}{D}$$

$$\arcsin x \approx \arctan x \approx x$$

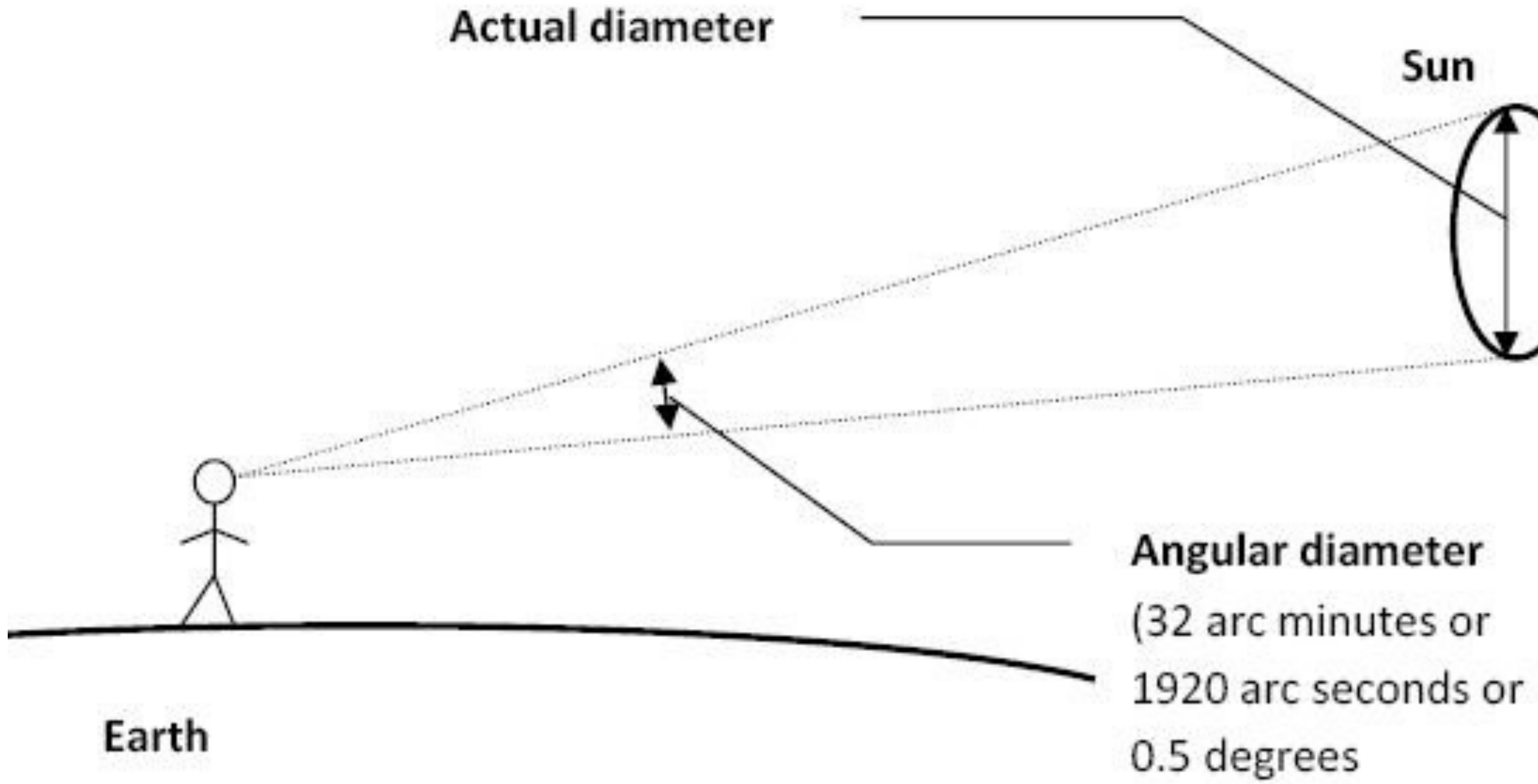
Angular size for Spherical object



$$\delta = 2\arcsin\left(\frac{r}{D}\right)$$ **Approximately** $$\delta \approx \frac{2r}{D} = \frac{d}{D}$$

$\arcsin x \approx \arctan x \approx x$

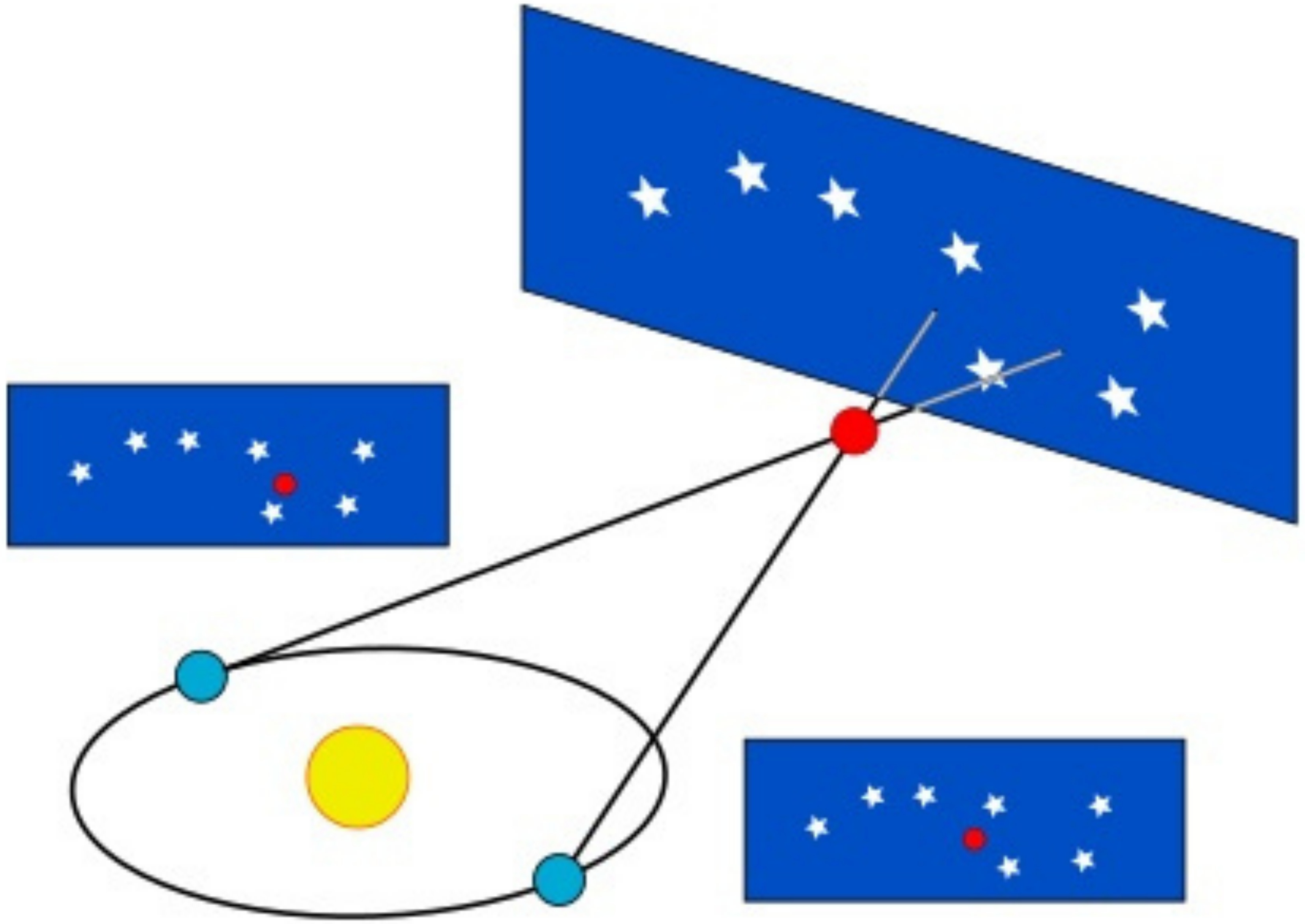
Conclusion

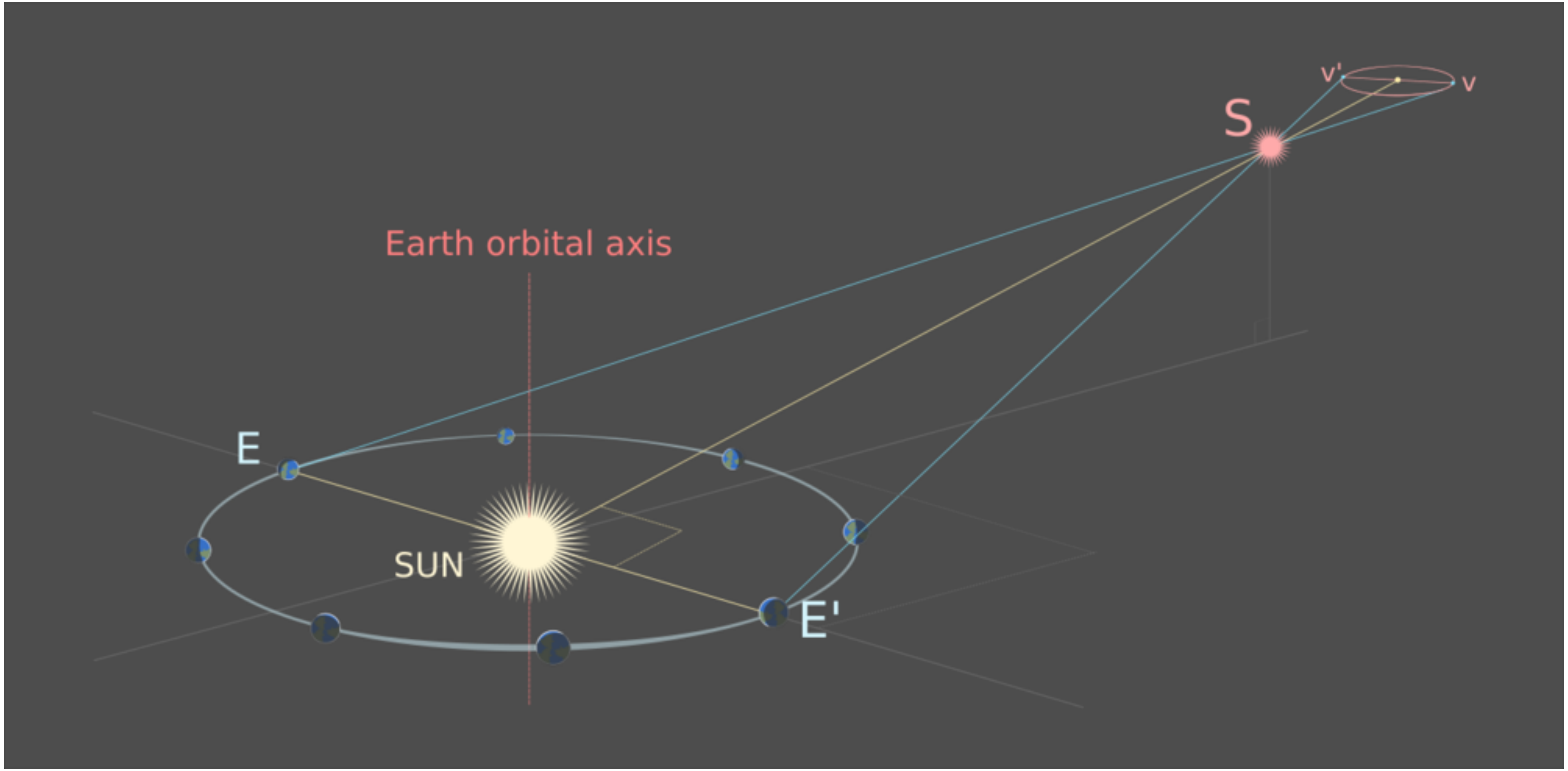


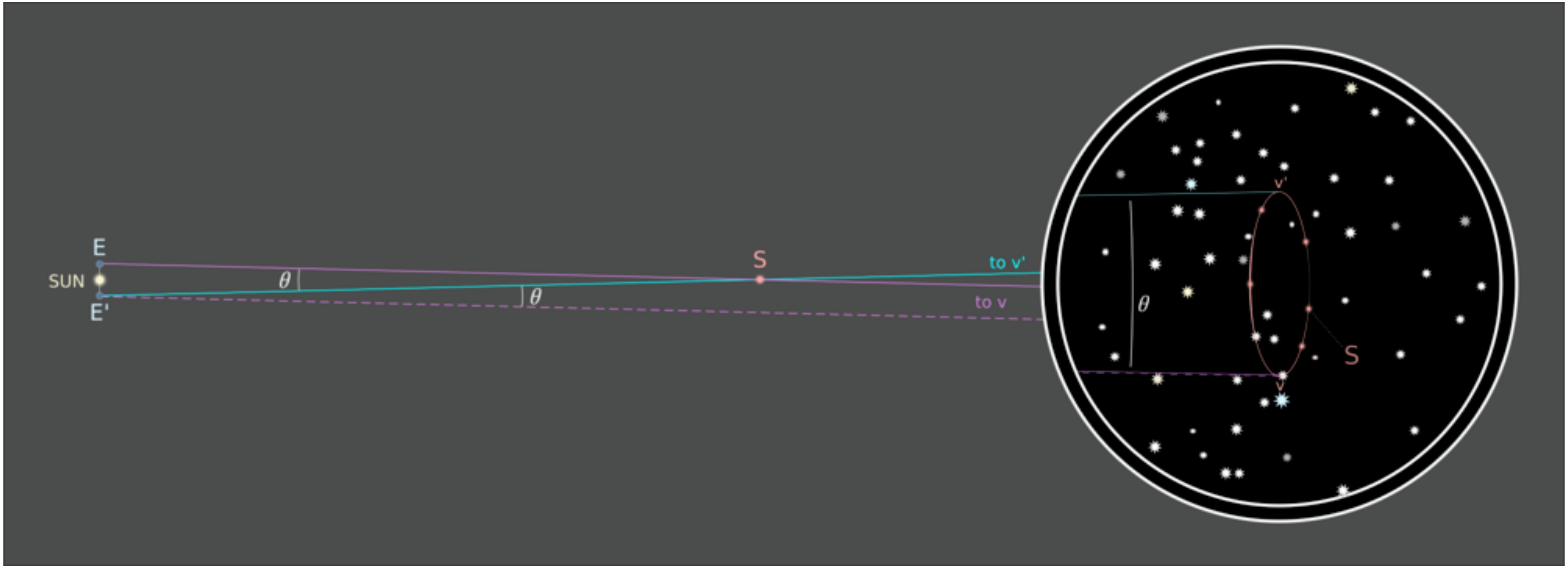
Distance spherical object can be approximatedas a circle

Parallax

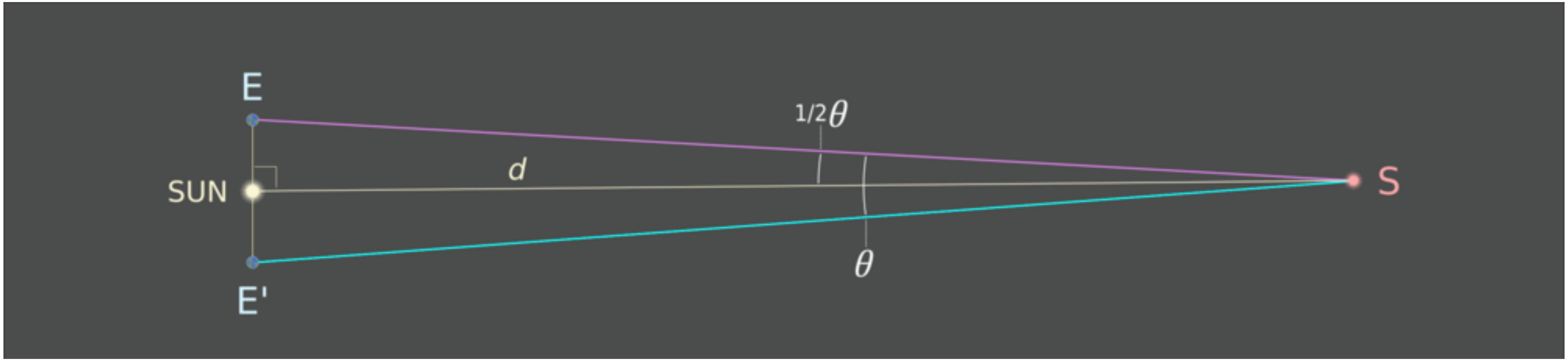
Parallax is a displacement or difference in the apparent position of an object viewed along two different lines of sight and is measured by the angle or half-angle of inclination between those two lines.




v'
v
S
Earth orbital axis
E
SUN
E'



E
SUN
E'
θ
θ
S
to v'
to v
v'
θ
S
v



$$\tan(\tfrac{1}{2}\theta) = (1AU) / d$$

d = 1/p derivation

For a right triangle,

$$\tan p = \frac{1\,\text{au}}{d},$$

where p is the parallax, 1 au (149,600,000 km) is approximately the average distance from the Sun to Earth, and d is the distance to the star. Using small-angle approximations (valid when the angle is small compared to 1 radian),

$$\tan x \approx x \text{ radians} = x \cdot \frac{180}{\pi} \text{ degrees} = x \cdot 180 \cdot \frac{3600}{\pi} \text{ arcseconds},$$

so the parallax, measured in arcseconds, is

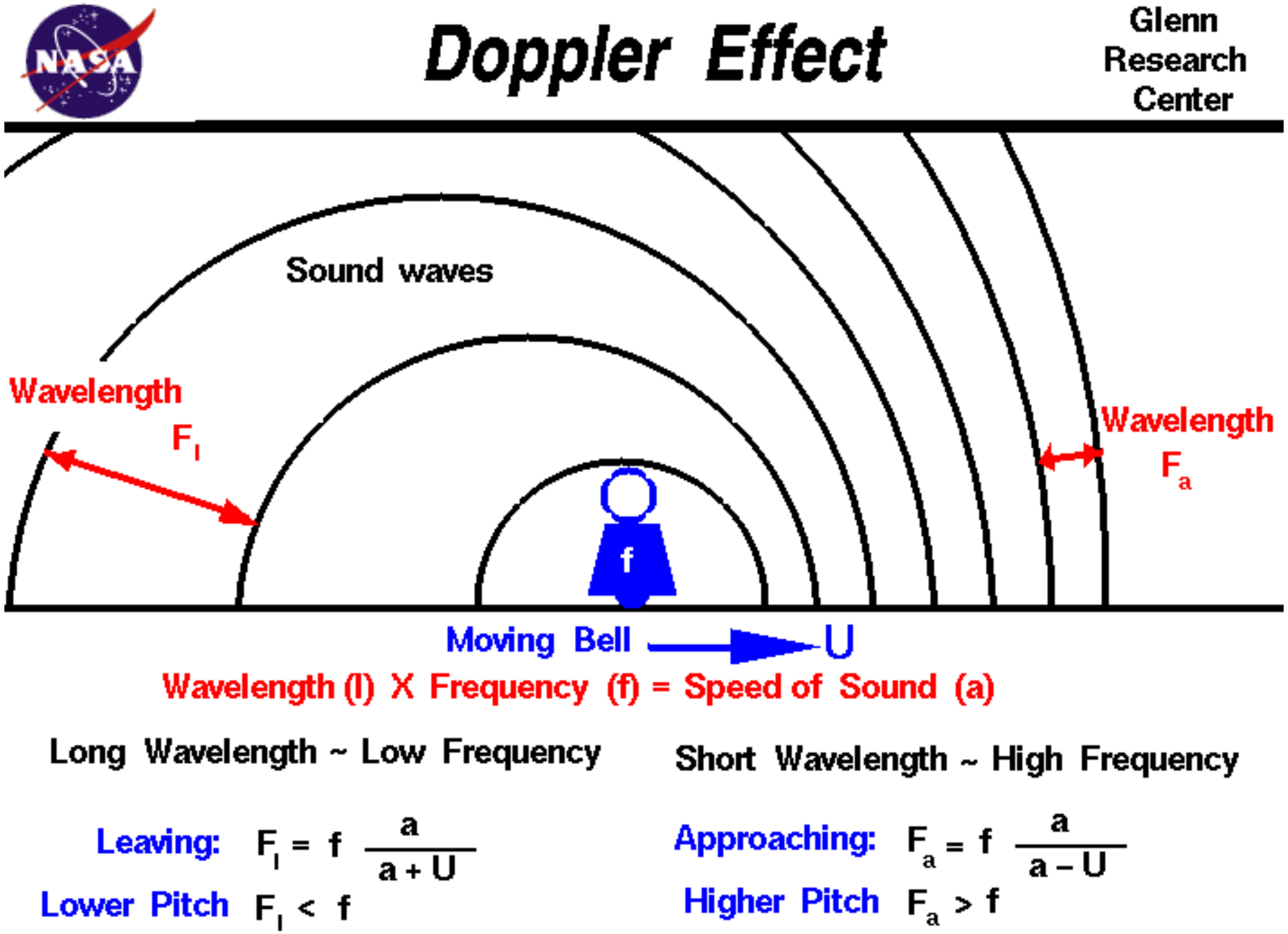
$$p'' \approx \frac{1 \text{ au}}{d} \cdot 180 \cdot \frac{3600}{\pi}.$$

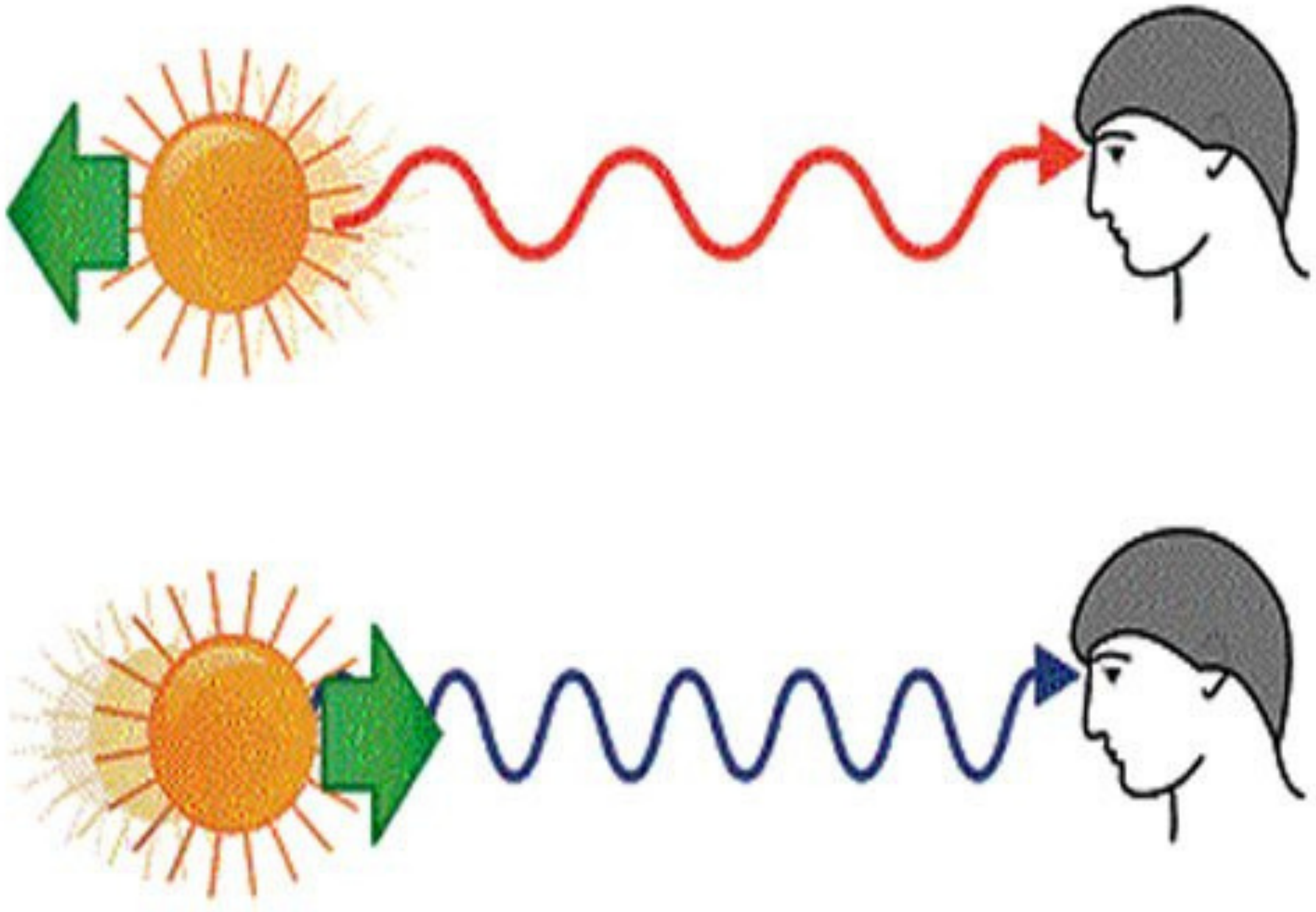
If the parallax is 1", then the distance is

$$d = 1 \text{ au} \cdot 180 \cdot \frac{3600}{\pi} \approx 206{,}265 \text{ au} \approx 3.2616 \text{ ly} \equiv 1 \text{ parsec}.$$

This *defines* the parsec, a convenient unit for measuring distance using parallax. Therefore, the distance, measured in parsecs, is simply $d = 1/p$, when the parallax is given in arcseconds.[2]

What is Doppler effect?





Note that the velocity are radial velocity

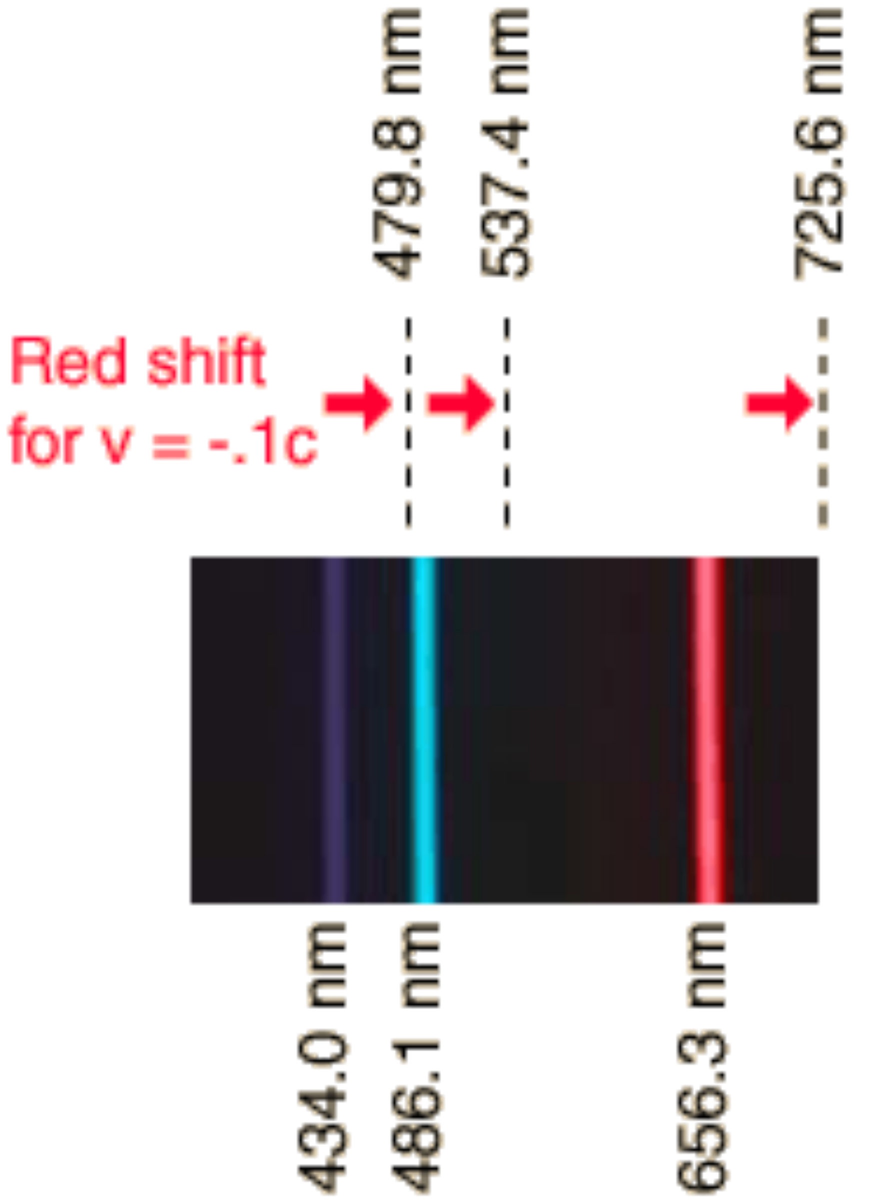
$$z \equiv \frac{\Delta\lambda}{\lambda} \equiv \boxed{\left(\sqrt{\frac{c+v}{c-v}}\right) - 1}$$

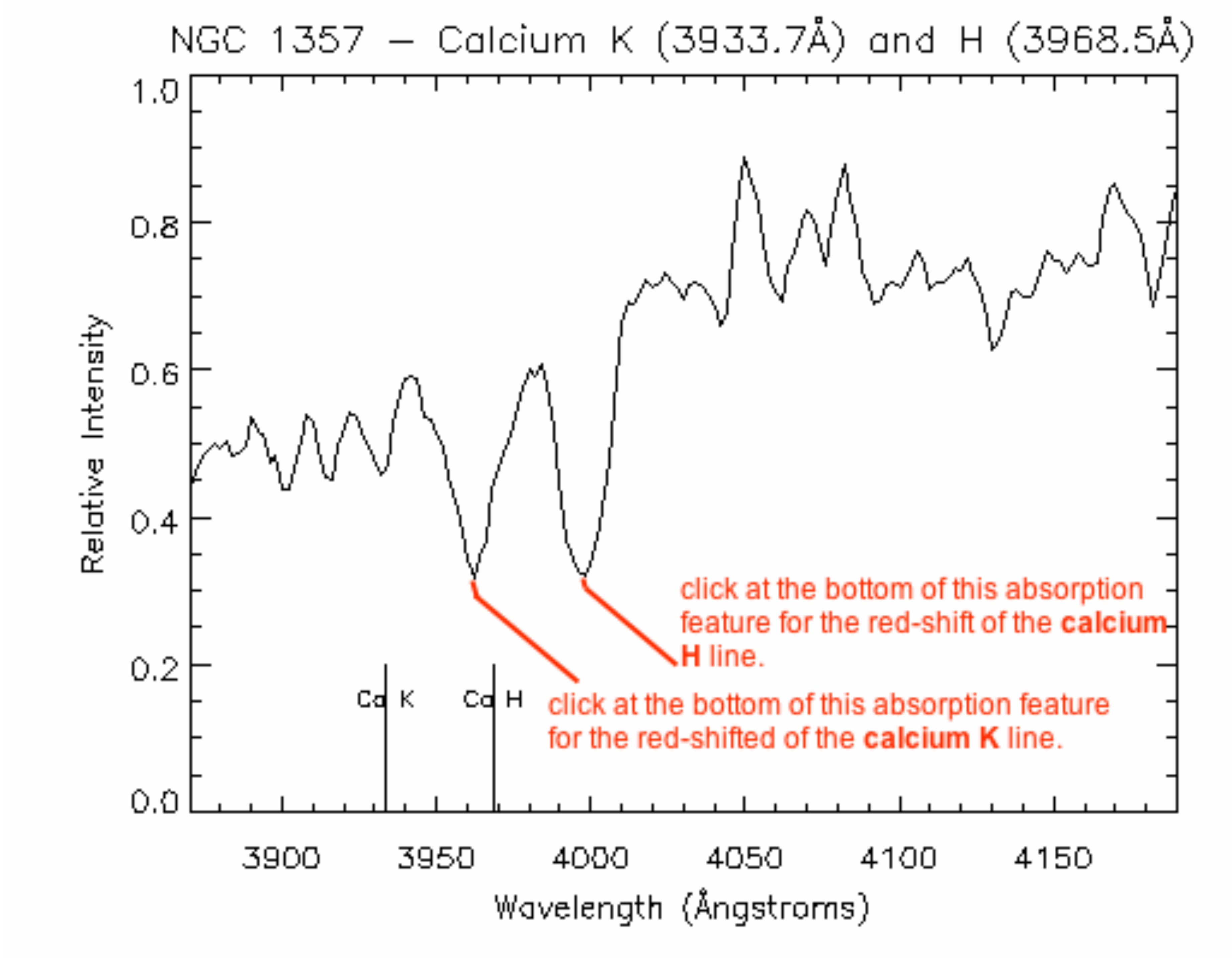
Relativistic case

normal Case

$$\frac{V}{C} = \frac{\Delta\lambda}{\lambda}$$

Example of redshift





In a distant galaxy, an astronomer identifies a spectral line as being CaII (singly ionized Calcium), which has a rest wavelength of 393.3 nm. If in this galaxy, the wavelength is observed to by 410.0nm, then what would the equivalent recessional velocity be in km/sec, and what is the galaxy's redshift? Using a Hubble constant of 75 km/sec/Mpc, what is the distance to this galaxy?

The wavelength of the line hasn't changed too much, so just use nonrelativistic expressions:

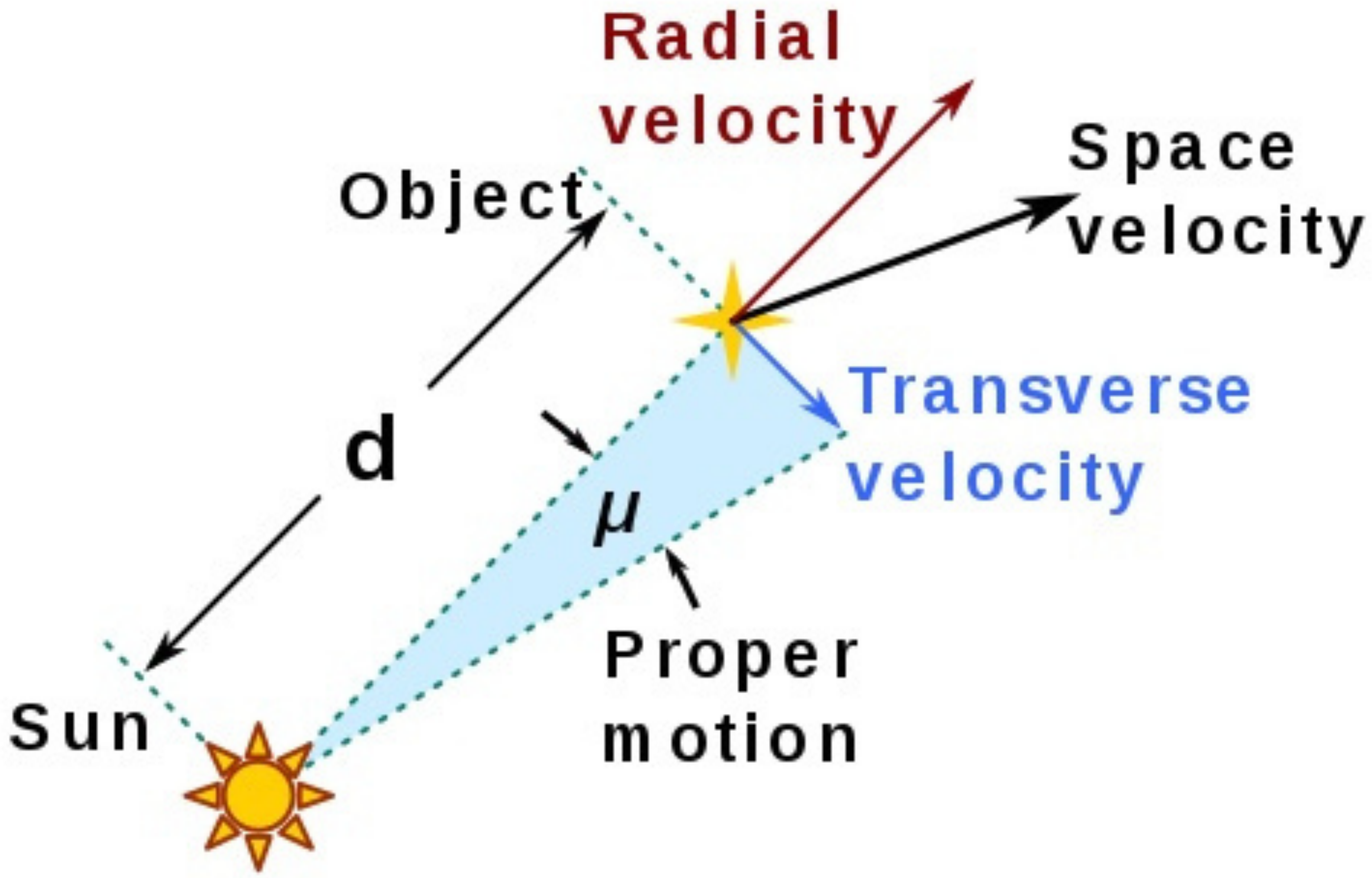
$$z = \frac{\Delta\lambda}{\lambda} = \frac{16.7\,\text{nm}}{410.0\,\text{nm}} = 0.0407$$

$$v = z\,c = 0.0407 \times (3 \times 10^5\,\text{km/s}) = 12200\,\text{km/s}$$

Special case: **If instead the observed wavelength is 460nm**

$$Z = \sqrt{\frac{C+V}{C-V}} - 1 = \frac{\Delta\lambda}{\lambda} \Rightarrow \frac{66.7}{410} = \sqrt{\frac{3\times10^5 + V}{3\times10^5 - V}} - 1 \Rightarrow V = 44879.7\ \text{km/s}$$

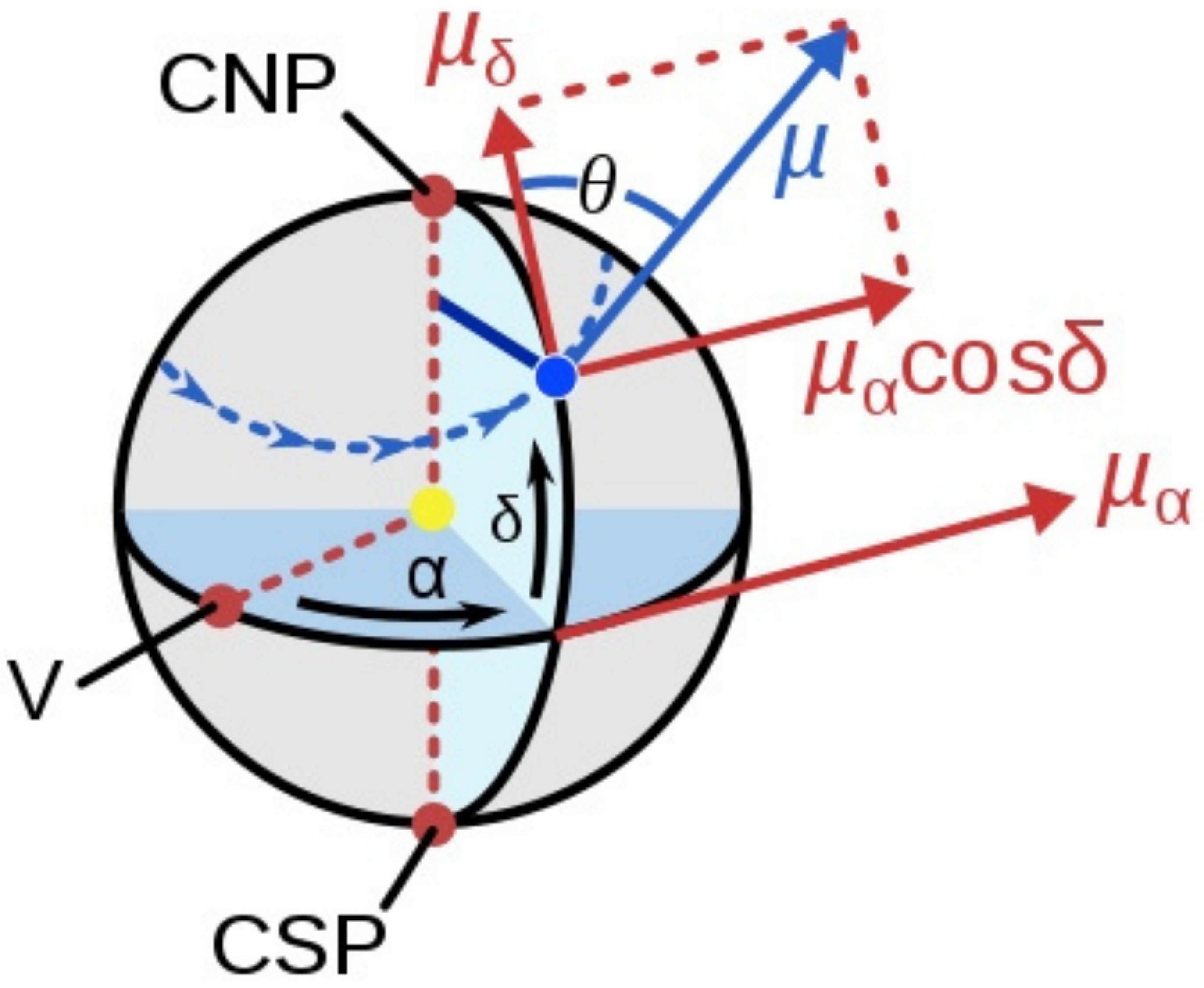
Proper motion



$$V_T = \mu d$$

(The unit of μ is arcsecond/year

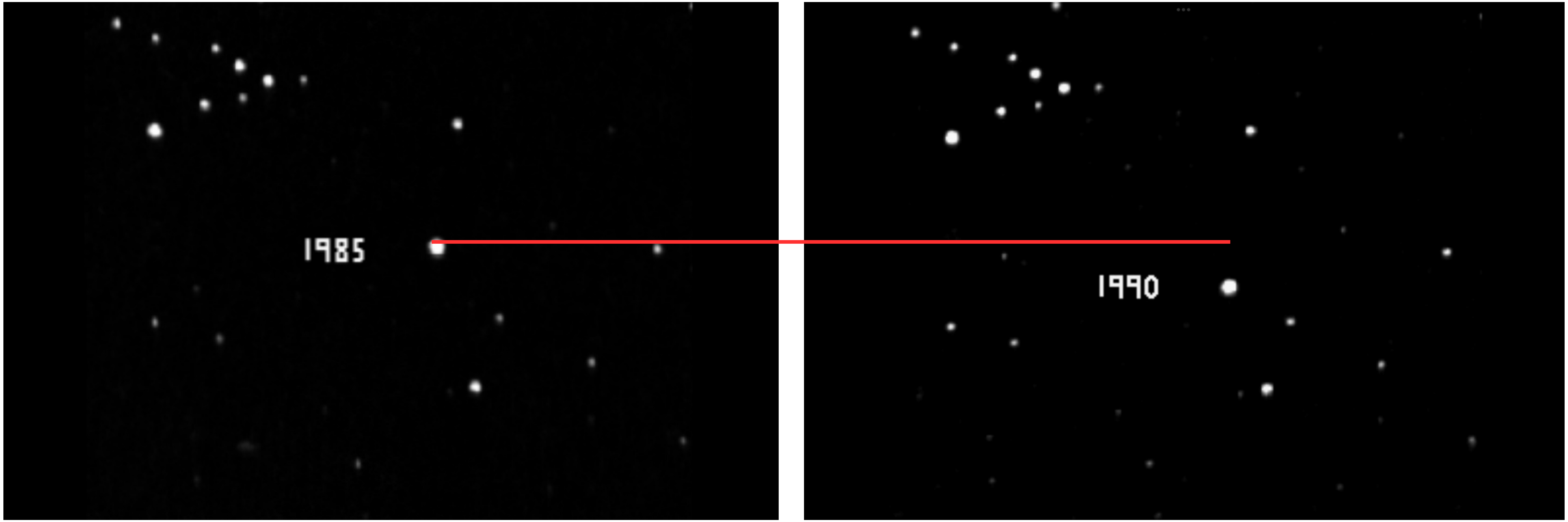
Where V_T is the transverse velocity and μ is the proper motion, d is the distance.

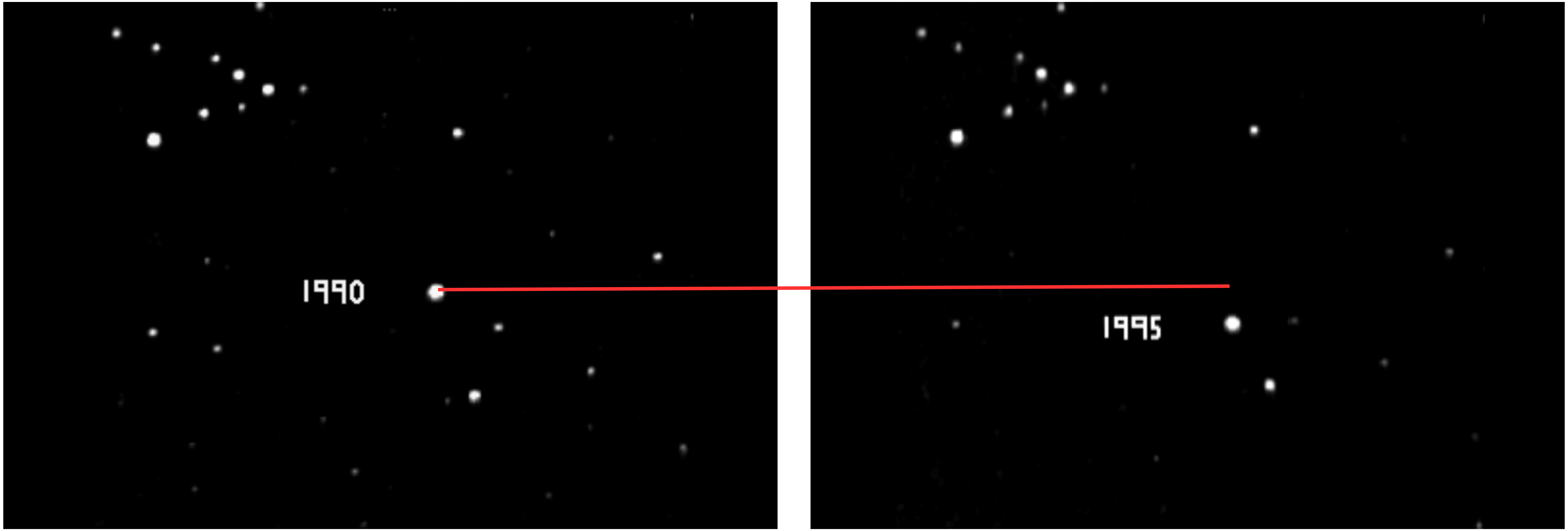


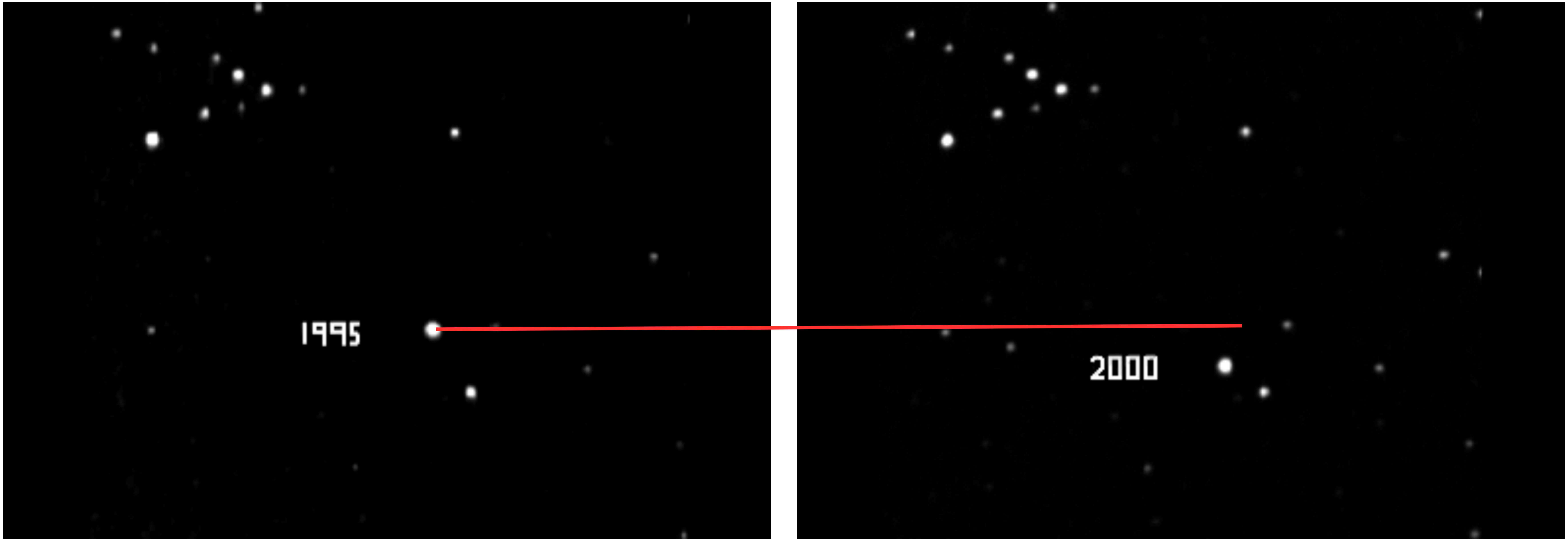
$$\mu^2 = {\mu_\delta}^2 + {\mu_\alpha}^2 \cdot \cos^2 \delta \,,$$

$$\mu_\alpha = \frac{\alpha_2 - \alpha_1}{\Delta t}$$

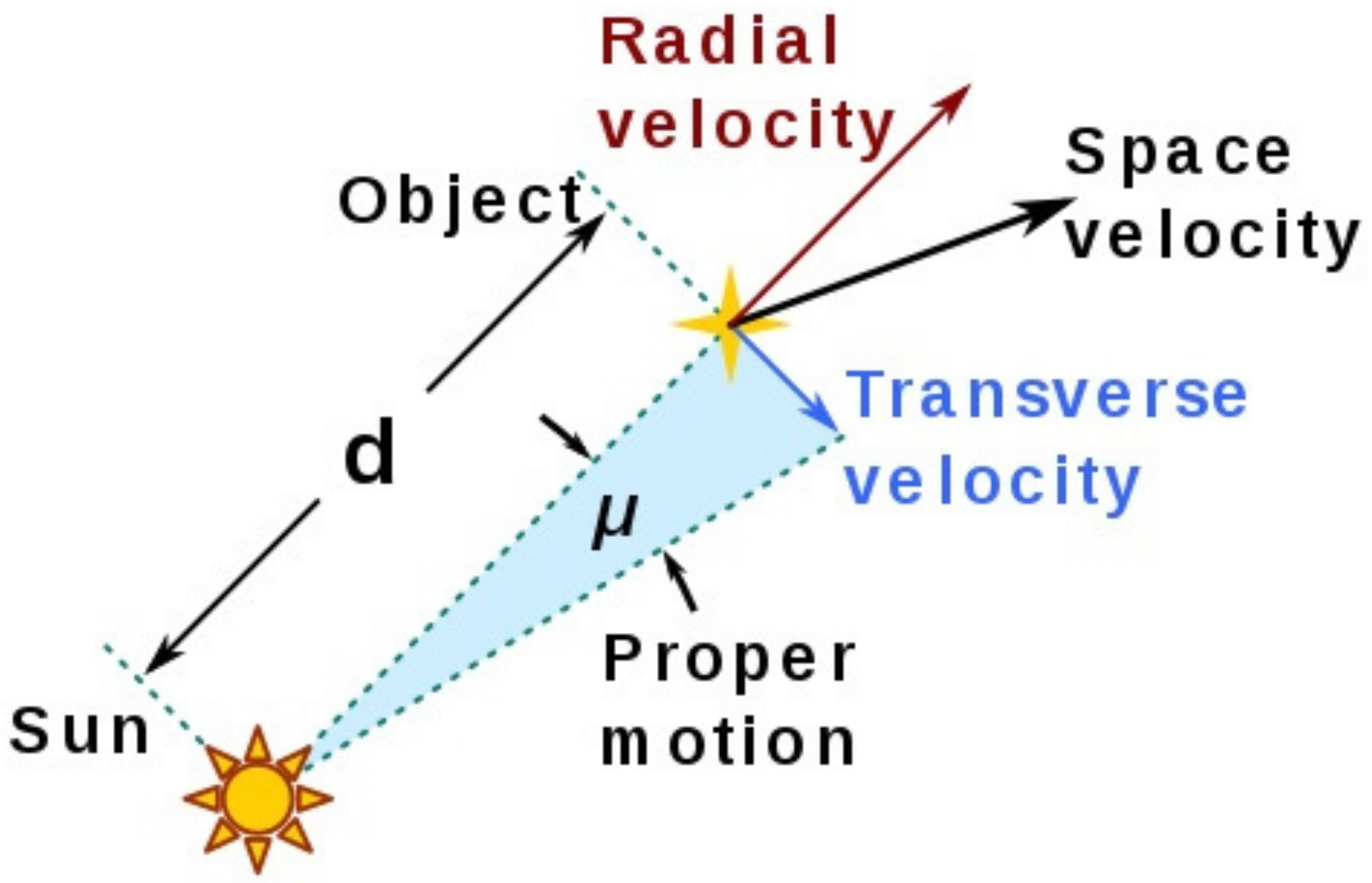
$$\mu_\delta = \frac{\delta_2 - \delta_1}{\Delta t}$$


1985
1990


1990
1995


1995
2000

Space velocity



From this picture the space velocity can be expressed as

$$V^2 = V_T^2 + V_R^2$$