Applications of Continuous Distributions in Finance and Economics

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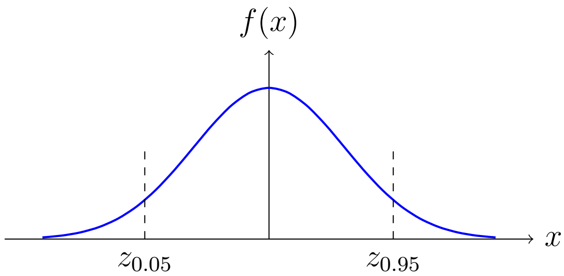
1. Introduction: Why Normality Matters

Financial and economic variables are uncertain. Prices, returns, GDP growth, or inflation are outcomes influenced by many small factors (interest rates, consumer behavior, macro shocks). The **Normal Distribution** is central because:

1. **Central Limit Theorem (CLT)**: Sums of many independent shocks approximate a normal distribution. This explains why returns or aggregate economic variables often appear bell-shaped.
2. **Mathematical tractability**: Known formulas for probabilities, means, variances, and quantiles allow precise risk and pricing calculations.

Visual intuition: Bell curve of returns:



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2. Applications in Finance

A. Modeling Asset Returns

Why not prices directly? Prices P_t can't be negative. Log-returns $r_t = \ln(P_t/P_{t-1})$ are better because:

- They can be positive or negative.

- Aggregation: The sum of log-returns over multiple periods gives the total log-return.
- The CLT applies: many small shocks in returns lead to approximately normal r_t.

Model: $r_t \sim N(\mu, \sigma^2)$ **Volatility**: σ measures the magnitude of fluctuations (risk). **Expected return**: μ is the mean trend over time.

B. Risk Management: Value at Risk (VaR)

VaR quantifies potential portfolio losses.

Logic: We want a threshold x such that losses worse than x occur with probability α (e.g., 5%). Using the CDF of a normal distribution:

$$P(X < x) = \alpha \implies x = \mu + z_\alpha \sigma$$

Example: 1-day 5% VaR

Portfolio returns: $X \sim N(\mu = 0.0005, \sigma = 0.012)$

$$P(X < x) = 0.05 \implies Z = \frac{x - \mu}{\sigma} = -1.645$$

$$x = 0.0005 + (-1.645)(0.012) = -0.01924 \approx -1.924\%$$

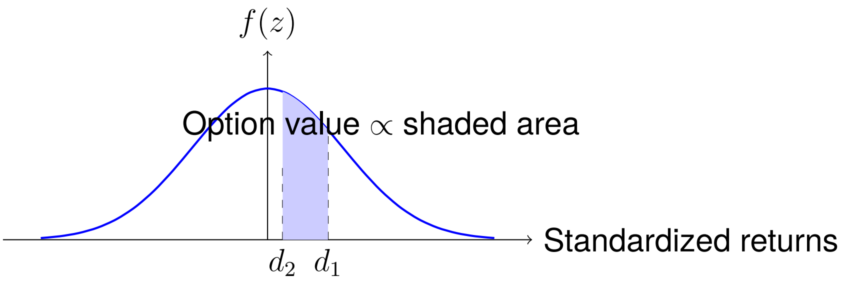
Interpretation: 95% confident the portfolio loses no more than 1.924% in one day.

C. Option Pricing: Black-Scholes

Core idea: Option prices depend on the probability-weighted outcomes of the underlying asset at expiry.

- Stock price modeled as **log-normal**: $S_T = S_0 e^{(\mu - \sigma^2/2)T + \sigma W_T}$.
- d_1 and d_2 arise from standardizing the log-normal variable relative to the strike price K:
$$d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma\sqrt{T}$$
- $N(d_1)$ = probability the option ends in-the-money in risk-neutral world, adjusted by S_0.
- $N(d_2)$ = discounted probability adjusted for strike price.

Visual:



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3. Applications in Economics

A. Economic Indicators

- GDP growth and inflation are aggregates of many small shocks $\Rightarrow$ approximately normal.
- Shocks (oil prices, disasters) modeled as $N(0, \sigma^2)$.

B. Regression Analysis and Errors

$$Y = \beta_0 + \beta_1 X + \varepsilon, \quad \varepsilon \sim N(0, \sigma^2)$$

- Assumption of normal errors allows exact t-tests and F-tests.
- If errors weren't normal, test statistics could be biased.

C. Forecasting and Confidence Intervals

Example: Forecast GDP = 2.5%, $\sigma = 0.3\%$, 95% CI:

$$CI = 2.5 \pm 1.96 \cdot 0.3 = [1.912\%, 3.088\%]$$

Logic: CI represents likely range of true GDP, based on normal error assumptions.

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4. Exercises (Consolidated)

Exercise 1: 1-day VaR $X \sim N(\mu = 0.001, \sigma = 0.02)$. Find 1% VaR. **Answer:** $z = -2.33$, $x = -0.0456$ (4.56% loss)

Exercise 2: Probability of Large Return $R \sim N(0.01, 0.03^2)$, find $P(R > 0.05)$. **Answer:** $Z = 1.333$, $P = 0.0918$

Exercise 3: Forecast CI $\hat{\mu} = 3\%$, $\sigma = 0.5\%$, 95% CI **Answer:** $CI = [2.02\%, 3.98\%]$

Exercise 4: Option Pricing d_1 $S_0 = 100$, $K = 105$, $\sigma = 0.2$, $r = 0.05$, $T = 0.5$ **Answer:** $d_1 = \frac{\ln(S_0/K)+(r+\sigma^2/2)T}{\sigma\sqrt{T}} \approx -0.051$

Exercise 5: PDF Probability $f(y) = 3y^2, 0 < y < 1$, find $P(Y > 0.5)$ **Answer:** $\int_{0.5}^{1} 3y^2 dy = 0.875$

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5. Optional: Extra Continuous Distributions

Uniform $U(a, b)$

PDF $f(x) = 1/(b-a)$, Mean $(a+b)/2$, Var $(b-a)^2/12$

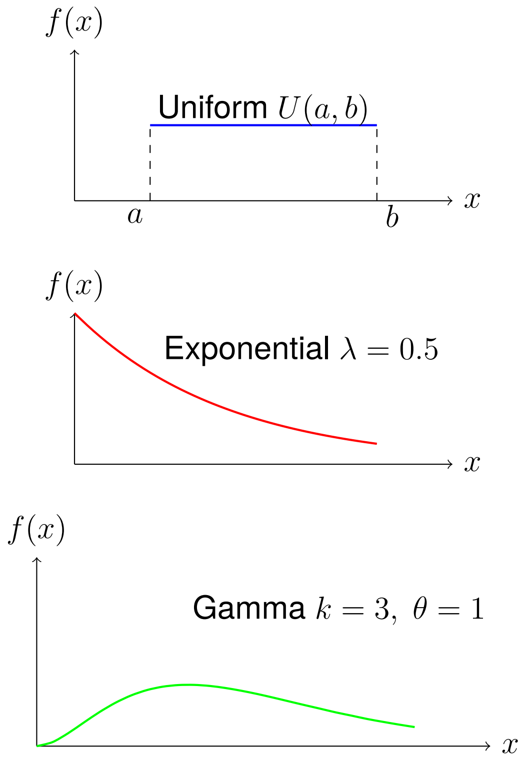
Exponential Exp(λ)

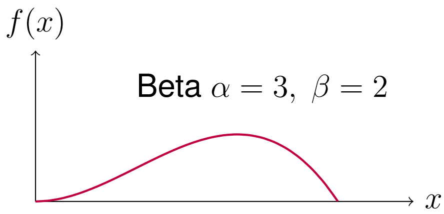
PDF $f(x) = \lambda e^{-\lambda x}$, Mean $1/\lambda$, Var $1/\lambda^2$

Gamma, Beta

Gamma: $f(x) \propto x^{k-1}e^{-x/\theta}$, Beta: $f(x) \propto x^{\alpha-1}(1-x)^{\beta-1}$, widely used in Bayesian econ/finance.

Visual Intuition: Common Continuous Distributions




$f(x)$
Beta $\alpha = 3,\ \beta = 2$
x