17th lesson

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Communication systems

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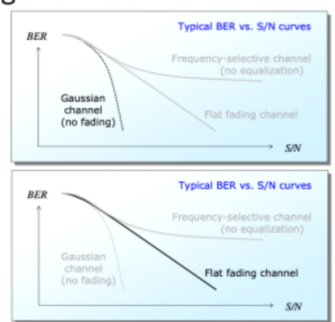
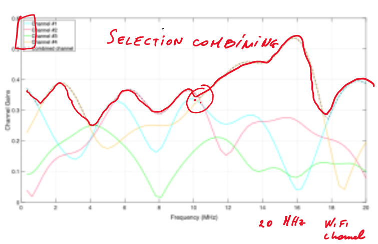
Transmissions over fading channels

- Signal fading is the main problem in wireless communications.
- OFDM is a technique designed to combat the destructive effects of *multipath* fading.
- *Slow flat* Rayleigh fading is still a big problem.
- One of the most effective resources against the effects of channel fading is *diversity*.



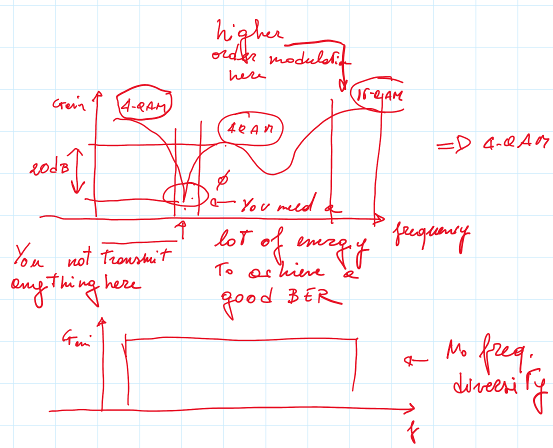
Diversity in wireless communications

- *Diversity* refers to the possibility of improving the reliability of a message by transmitting it over two or more communication channels with different characteristics.
- Diversity is a common technique for combatting fading and co-channel interference and avoiding error bursts.



Diversity in wireless communication systems

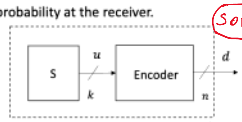
- *Time diversity*: relates to the *coherence time*
  - Transmission over multiple time slots by channel coding plus interleaving
  - The amount of diversity is small over very slow fading channels
- *Frequency diversity*: relates to the *coherence bandwidth*
  - Transmission over multiple frequency bands
  - The amount of diversity is small over very flat fading channels
- *Spatial diversity*: relates to the *coherence distance*
  - Transmission and reception employing multiple antennas.



Time diversity: interleaving and coding

- The main idea was formalized by Claude Shannon in 1948.
- Channel coding introduces some redundancy in the transmitted bits to either
  - Detect errors at the receiver
  - Improve the bit error probability at the receiver.

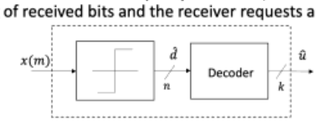
Sovrce



- The redundancy is measured by the code rate $R = \frac{k}{n} < 1$, the ratio between k, the number of bits at the input of the encoder and n, the number of bits at the output of the encoder.

Error detection coding

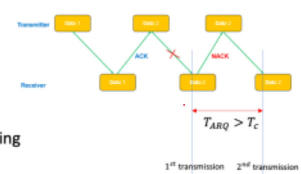
- Very simple technique: one or more bits of parity are added at the end of a word.
- The receiver computes the parity check applying the same algorithm implemented at the receiver:
  - If the result computed at the receiver matches the parity check bits, the parity bits are discarded and the received bits are considered error-free.
  - If the result does not match the parity check bits, there is one or more errors in the string of received bits and the receiver requests a re-transmission.



Data retransmission

- The receiver feeds back an ACK for a correct reception and a NACK for a faulty reception.
- After receiving a NACK, the transmitter resends the data packet
- ARQ exploits the *time diversity* of the channel by retransmitting the data after T_{ARQ}, a time interval, longer than the channel coherence time T_c.
- The new transmission will experience a different and hopefully better channel
- Advanced receivers are capable of combining the two received messages to further improve the chances of a successful reception.

Automatic Repeat Request (ARQ)



Error correction coding and channel capacity

- Channel codes can be employed to *correct* the errors introduced by the channel.
- Given a communication channel of bandwidth B, Shannon proved that the channel capacity can be computed as

$$C = B\log_2(1 + SNR)\ \text{b/s}$$

  - For any transmission with rate $R < C$ and an arbitrarily small ϵ, it is possible to find an error correction code such that the error probability is $P_e < \epsilon$.
  - On the contrary, if $R > C$ it is not possible to find any code that can make the probability of error of the transmission over the channel arbitrarily small.

Error correction codes

$\boxed{\text{bits } 0,1}$

- Linear algebric codes are the most used type of codes:
  - Block codes
  - Convolutional codes
- All algebraic operations are performed in the GF(2), the Galois field of two elements {0,1} .

+	0	1
0	0	1
1	1	0

x	0	1
0	0	0
1	0	1

- They have been initially studied for AWGN transmissions but can be employed over fading channels.

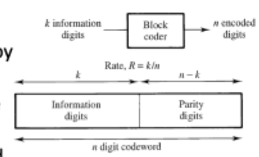
Block codes: encoder

- Block codes are most of the times in *systematic* form: the coded word is formed by k information bits and $n-k$ parity bits.
- The encoder can be represented as the code *generator matrix* $\boldsymbol{G}$ the encoder.
- The word $\boldsymbol{u}$ of k bits is encoded in the coded word $\boldsymbol{d}$ of n bits

$$\boldsymbol{d} = \boldsymbol{uG}$$

- The encoder add redundancy so that all coded words differ of as many bits as possible.

$G = [I_k | P]$



REPETITION CODES

$0 \to 000$
$1 \to 111$

0 $\boxed{00}$
1 $\boxed{11}$

$000 \to 0$
$\left.\begin{matrix}010\\001\\100\end{matrix}\right\} \to 0$
$\left.\begin{matrix}110\\101\\011\end{matrix}\right\} \to 1$
$111 \to 1$

MAJORITY DECODING

Assume we have received
$\hat{x} = [0\,1\,0]$

$d(\hat{x}, \underline{000}) = 1$
$d(\hat{x}, 111) = \underline{2}$

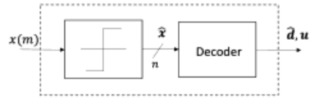
$1 \quad 11 + e = 010 \qquad \boxed{101}$
$1 \quad 11 + \boxed{101} = 010$

Block codes: decoder

- After having received the string of n bits $\hat{\boldsymbol{x}}$, the decoder selects the codeword $\hat{\boldsymbol{d}}$, as the one that has minimal distance from $\hat{\boldsymbol{x}}$

$$\hat{\boldsymbol{d}} = \arg\min d(\boldsymbol{d}, \hat{\boldsymbol{x}})$$ MAXIMUM LIKELIHOOD

- The distance is computed as the number of bits that are different in the two string of bits.
- An error event occurs when, due to the noise, the received vector $\hat{\boldsymbol{x}}$ is closer to a codeword different from the transmitted one.
- Codes where the distance between words is large are more robust against noise and fading than codes where the distance is small.



block code with $\boxed{K = 20}$ $n = 30$ $R = 2/3$
What is the number of different codewords

$K = 1$, $n = 3$ 000 111

In total I have $2^1 = 2$

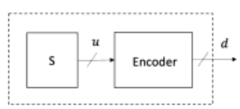
$2^{20} = 2^{10 \cdot 2} \approx 1$ Mio

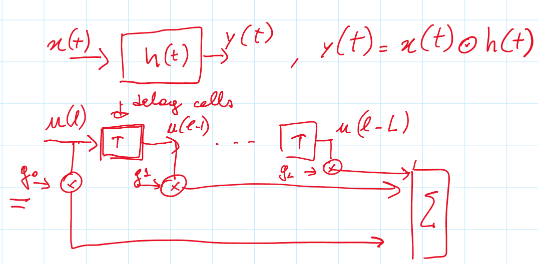
3 Mb/s $\Rightarrow$ 1 codword is 30 bits

100 000 of received codwords per second and for ear colword you need 1 Mio comparisons.

Convolutional codes: encoder

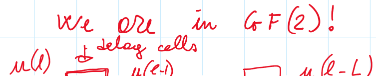
- The encoder of a k, n convolutional code works as $\underline{n \text{ linear filters}}$ in GF(2). Each of the n outputs of the encoder is a linear combination of the input
- The filter impulse response for the j-th output bit, the j-th code generator, is a series of 0 and 1.





$y(t) = x(t) \otimes h(t)$

We are in GF(2)!

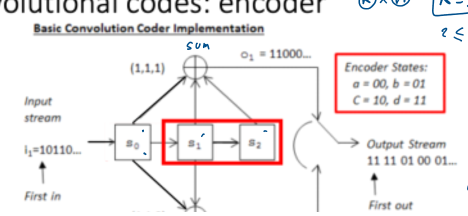


Convolutional codes: encoder

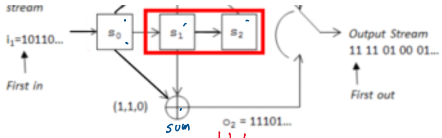
$K \times n$ $\boxed{K=1}$

$2 \leq n \leq 5$

$R = 1/2$



$K = 1$
$n = 2$
$d_n^{(1)} = u_n + u_{n-1} + u_{n-2}$
$d_n^{(2)} = u_n + u_{n-1}$



Coder Rate ½, and configuration (n,k,m) = (2,1,3) where:
n: is the number of output bits **k:** *is the number of input bits*
m: is the number of shift register stages

$d_n^{(1)} = u_n + u_{n-1} + u_{n-2}$

$d_n^{(2)} = u_n + u_{n-1}$

We are in GF(2)!