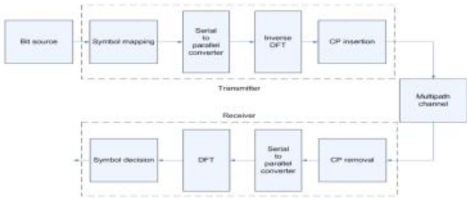
16th lesson

Tuesday, 17 November 2020 11:32

OFDM baseband transceiver



OFDM baseband transceiver

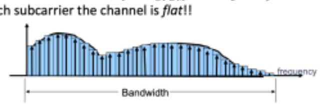
1. In the serial-to-parallel block, a block of N consecutive data symbols are collected in the vector $\mathbf{S} = [S(0), S(1), \dots, S(N-1)]$.
2. The IDFT block converts $\mathbf{S}$ into a 'time-domain' vector
$$\mathbf{s} = \mathbf{F}^H\mathbf{S}$$
3. A N_{CP}–long cyclic prefix is inserted to create the new time-domain vector of length $N + N_{CP}$
$$\bar{\mathbf{s}} = [s(N - N_{CP} - 1), \dots, s(N-1), s(0), \dots, s(N-1)]$$

OFDM baseband transceiver

4. The signal propagates through the wireless channel with impulse response $\mathbf{h} = [h(0), h(1), \dots, h(L-1)]$
$$y(k) = \sum_{\ell=0}^{L-1} h(\ell)\bar{s}(k-\ell)$$
5. At the receiver the samples corresponding to the CP, which do not carry any information, are discarded and the remaining samples are frequency converted $\mathbf{Y} = \mathbf{F}\mathbf{y}$, yielding
$$\mathbf{Y} = \mathbf{F}\mathcal{H}\mathbf{s} = \mathbf{F}(\mathbf{F}^H\mathbf{H}\mathbf{F})\mathbf{s}$$

OFDM on multipath channel

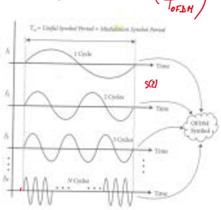
- The overall signal bandwidth is B.
- The sampling duration is $T = 1/B$.
- The OFDM block duration is $T_{OFDM} = T(N + N_{CP})$
- The bandwidth for each subcarrier is $\Delta f = B/N$
- By accurately choosing N we have
$$T < \sigma_\tau \ll T_{OFDM},\ B > B_c \gg \Delta f$$
- On each subcarrier the channel is *flat*!!



OFDM interpretation

$e^{j2\pi\frac{Kn}{N}}$ $rect\left(\frac{t}{T_{OFDM}}\right)$

- Each frequency symbol $S(n)$ is multiplied by a complex exponential for a duration of N samples (plus the CP).
- The k-th sample corresponding to the n-th subcarrier is
$$S(n)e^{j2\pi n\Delta f t}\Big|_{t=kT} = S(n)e^{j2\pi n\Delta f kT} = S(n)e^{j2\pi n\frac{B}{N}kT} = S(n)e^{\frac{j2\pi nk}{N}}$$
- The waveform corresponding to subcarrier n is
$$S(n)e^{\frac{j2\pi nk}{N}}, k = 0, \dots, N-1$$



$$\mathbf{s} = \sum_{n=0}^{N-1} S(n)e^{j2\pi\frac{Kn}{N}} \quad K = 0, 1, \_ N-1$$

$$\mathbf{s} = [s(0), s(1), \_, s(N-1)]$$

$\frac{n}{N}$

$K\ \underbrace{0 \_\_ N-1}_{N}$

$$s(k) = \sum_{n=0}^{N-1} S(n)e^{-j2\pi\frac{Kn}{N}}$$

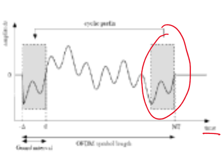
subcarrier index

time index

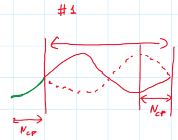
message transmitted on subcarrier n

OFDM interpretation

- All the N time-waveforms are periodic of period N.
- The CP insertion exploits this periodicity to render the channel *flat* for each subcarrier.
- In facts, during an OFDM block the received signal on each subcarrier depends only on the the transmitted symbol on that subcarrier.



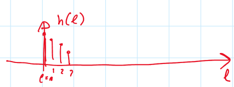
#1



$S(1) = 1$ —

$S(1) = -1$ - -

$\circledast\ h(t)$



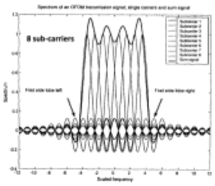
subcarrier #1

$s(k)$ $S(1)e^{j2\pi\frac{K\cdot 1}{N}}$

symbol on that subcarrier.

OFDM frequency orthogonality

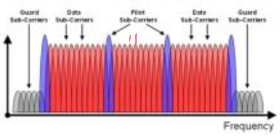
- The symbol transmitted on a subcarrier is fixed for the duration of an OFDM block.
- This is equivalent to multiply the complex exponential by a 'rect' function for a duration of NT seconds.
- The power spectral density of the OFDM signal is the sum of N 'sinc' functions, one for each subcarrier.
- All the sinc functions are orthogonal by construction and they do not interfere with each other.



$\mathrm{rect}(t) \rightleftharpoons \mathrm{sinc}(f)$

OFDM example: WiFi – IEEE 802.11a/g/n/ac

- A WiFi transmission occupies a bandwidth $B = 20$ MHz, which is divided in $N = 64$ sub-carriers spaced $\Delta f = 312.5$ kHz.
  - 802.11a/g use 48 subcarriers for data, 4 for pilot, and 12 as null subcarriers.
  - 802.11n/ac use 52 subcarriers for data, 4 for pilot, and 8 as null.



OFDM example: WiFi – IEEE 802.11a/g/n/ac

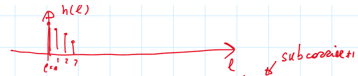
- The OFDM block is composed by $N = 64$ and $N_{CP} = 16$ samples.
- The duration of each sample is $T = \frac{1}{B} = \frac{1}{20\cdot 10^6} = 50$ ns and the duration of a block is $T_{OFDM} = (64 + 16)\cdot 50 = 4\ \mu$s.
- In general, the delay spread of an indoor channel is $\sigma_\tau < 500$ ns, so that the channel is indeed *flat*
$$T_{OFDM} \gg \sigma_\tau$$
- Assuming that the maximum indoor mobility is $v = 3$ m/s, the Maximum Doppler shift is $f_d = \frac{5\cdot 10^{\wedge}9\cdot 3}{3\cdot 10^{\wedge}8} = 50$ Hz $\Longrightarrow T_c = \frac{1}{2\cdot 50} = 0.01$ s and the channel is *slow*
$$T_{OFDM} \ll T_c$$

OFDM example: WiFi – IEEE 802.11a/g/n/ac

- Each subcarrier carries a new symbol every $T_{OFDM} = 4$ ms.
- The symbol rate per subcarrier is $\frac{1}{T_{OFDM}} = 0.25 \cdot 10^6$ sym/s.
- There are 48 subcarriers dedicated to data transmissions and the overall symbol rate is $48 \cdot 0.25 \cdot 10^6 = 12 \cdot 10^6$ sym/s.
- Loss of (spectral and energy) efficiency due to the CP insertion is
$$\eta_{CP} = \frac{N_{CP}}{N} = \frac{16}{80} = 20\%$$
- Additional loss of spectral efficiency due to guard subcarriers
$$\eta_{GS} = \frac{16}{64} = 25\%$$

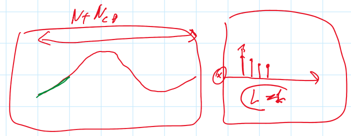
Error rate for OFDM systems

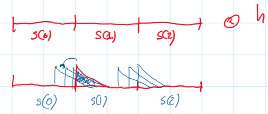
- Considering the presence of noise, the output of the FFT is
$$R(n) = Y(n) + N(n) = H(n)S(n) + N(n)$$
where $N(n) = \mathbf{F}\mathbf{n}$ and the vector $\mathbf{n}$ collects the received noise samples in time, $\mathbf{n} = [n(0), n(1), \ldots, n(N-1)]$.
- Due to the properties of the unitary matrix $\mathbf{F}$, the statistics of $N(n)$ are equal to the statistics of the noise samples $n(k)$
$$n(k) \in \mathcal{N}(0, \sigma^2) \Leftrightarrow N(n) \in \mathcal{N}(0, \sigma^2)$$
- The decision variable is
$$X(n) = \frac{R(n)}{H(n)} = S(n) + \frac{N(n)}{H(n)}$$

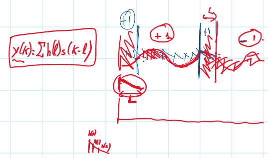


$s_1(k) = S(1)\, e^{j2\pi \frac{k \cdot 1}{N}}$

$y_1(k) = s_1(k) \otimes h(k)$







$B = \frac{1}{T} \Rightarrow T = \frac{1}{B} \Rightarrow NT = \frac{N}{B} = \frac{1}{\Delta f}$

$y_1(t) = S(1)\, e^{j2\pi \frac{t \cdot 1}{NT}} \cdot \mathrm{rect}\left(\frac{t}{NT}\right) \otimes h(t) = S(1)\, e^{j2\pi \Delta f t}\, \mathrm{rect}\left(\frac{t}{NT}\right) \otimes h(t)$

$e^{j2\pi \Delta f t}\, \mathrm{rect}(t/NT) \rightleftharpoons TN \mathrm{sinc}(fNT) \otimes \delta(f - \Delta f) = NT \mathrm{sinc}\left(\frac{f}{\Delta f} - 1\right)$

$Y(1) = S(1)\, NT\, \mathrm{sinc}(f/\Delta f - 1)\, H(f)$

12 Msym/s $\quad$ 16-QAM $\Rightarrow$ Each symbol carries 4 bits

$\Rightarrow \sim 50$ Mb/s