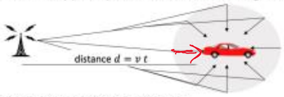
14th lesson

Monday, 9 November 2020 13:31

Scattering: Doppler Spectrum

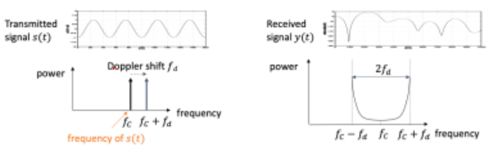
- In fading channels many signal replicas arrive at the receiver with different angles. The effect is a *Doppler spread* rather than a single shift.



- Received signal is the sum of all scattered waves.
- Doppler shift for each path depends on angle θ, each path has a shift $f\frac{v\cos\theta}{c}$.
- Typically assume that the received energy is the same from all directions (uniform scattering).

Jakes' Doppler spectrum

1. The effect of movement is modelled as a stochastic process
2. There are components at right and at the left



$$S(f) = \frac{1}{\pi f_d\sqrt{1-\left(\frac{f}{f_d}\right)^2}}$$

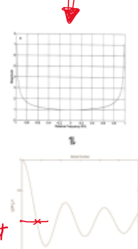
The fequency $f_d = \frac{f_c v}{c} = \frac{v}{\lambda}$ is the *Doppler spread.*

Power spectral density $\rightleftarrows$ autocorrelation

Time varying channel

- The Doppler spectrum $S(f)$ is the power spectral density of a sinusoid on a time varying channel.
- In the time domain $\rho(t) = J_0(2\pi f_d t) \leftrightarrows S(f)$ is the autocorrelation function.
- $J_0(2\pi x) \approx 0$ for $x = \frac{1}{2} \Longrightarrow$ The channel can be assumed uncorrelated for $f_d T_c = \frac{1}{2}$.
- Channel coherence time is

$$T_c = \frac{1}{2f_d}$$



$S(f)$

d = doppler shift

$f_d = \frac{f_c \cdot v_{max}}{c} = \frac{v_{max}}{\lambda_c}$

$\lambda_c = \frac{c}{f_c}$

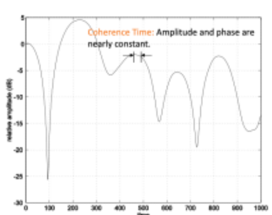
Channel Coherence Time

- The *channel coherence time* T_c is defined as the time interval over which the channel can be approximated as constant.

$$T_c = \frac{1}{2f_d}$$

- In terms of distance, it is

$$d_c = vT_c = \frac{1}{2}v\frac{c}{f_c v} = \frac{\lambda}{2}$$

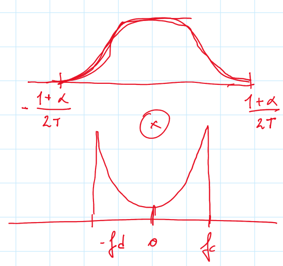


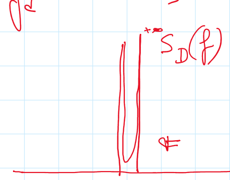
$S_s(f) \otimes S_D(f)$

$f_d << B_s$

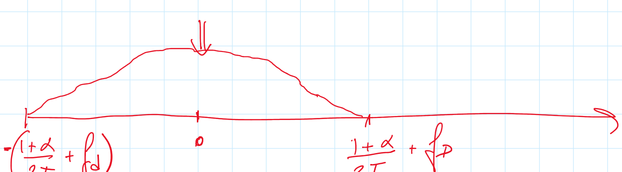
Doppler spectrum

- Doppler spread is a measure of the *spectral broadening* caused by motion.
  - If the baseband signal bandwidth $B_s \gg f_d$ then the effect of Doppler spread is negligible at the receiver and the channel is *slow fading.*
  - If $B_s < f_d$ then the channel is *fast fading* and the Doppler spread severely distorts the received signal, which often results in an irreducible BER and synchronization problems.
- Similar considerations can be made in terms of symbol duration
  - A channel is *slow fading* if $T_c > T$.
  - A channel is said to be *fast fading* if $T_c < T$.





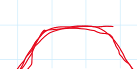
$\sim \delta(f - f_c)$



Fading channel example

- Consider a transmission at $f_c = 2.1$ GHz in a suburban aerea (delay spread $\sigma_\tau = 2\ \mu s$) to a user moving at a speed 90 km/h $\Longrightarrow v = 25$ m/s. The signal bandwidth is $B_S = 2$ MHz $\Longrightarrow$ the symbol time can be approximated as $T \sim \frac{1}{B_s} = 500$ ns.
- The Doppler spread is

$$f_d = \frac{f_c v}{c} = \frac{2.1 \cdot 10^9 \cdot 25}{3 \cdot 10^8} = 175 \text{ Hz} \Longrightarrow T_c = \frac{1}{2f_d} \sim 3 \text{ ms}.$$



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- The Doppler spread is
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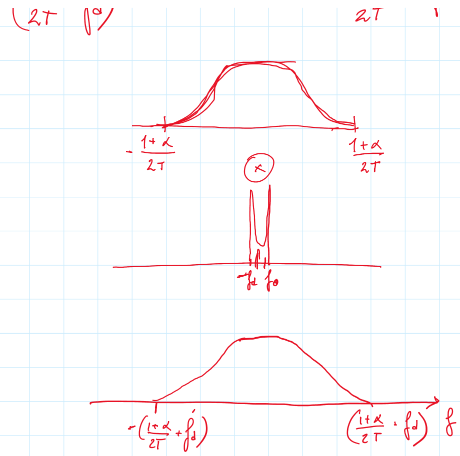
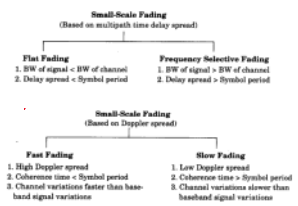
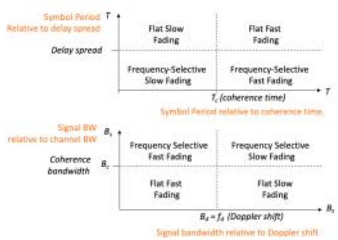
- The channel coherence bandwidth is $B_c = \frac{1}{\sigma_\tau} = 500$ kHz.
- The channel is *slow* ($B_s \gg f_d$ or $T \ll T_c$) and *frequency-selective* ($B_s > B_c$ or $T < \sigma_\tau$).



Small-scale fading recap



Small-scale fading recap



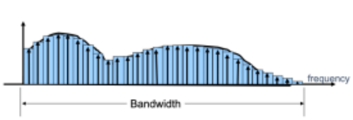
Multi-carrier signals

Multicarrier transmissions

- Main reasons for the success of multicarrier modulations:
  - Robustness versus multipath fading
    - As the data rates increase, multipath becomes a major problem for single carrier transmissions
  - Spectrally efficient
  - Low implementation complexity
    - DFT and IDFT can efficiently implemented with the FFT algorithm
  - Flexible resource allocation
    - OFDMA exploits channel frequency diversity by dynamically assigning the radio resources to the users.

OFDM technology

- The wideband multipath channel is divided into N narrowband sub-channels.
- Provided that the system is accurately dimensioned, each sub-channel can be approximated as flat fading.



Channel as a tapped delay line

- The channel impulse repsonse can be resampled at intervals multiple di T and the channel gains

Equivalent representations under the condition That I am using the channel to transmit a signal with symbol time T

$$h(t) = \sum_{m=0}^{N_c-1} \alpha_m e^{j\phi_m}\delta(t-\tau_m) = \sum_{\ell=0}^{L-1} h(\ell)\delta(t-\ell T)$$

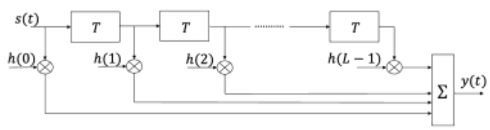
- Even if L might be different from N_c, the channel characteristics do not change.

OFDM signal model (1)

- The complex envelope of the signal received through the multipath channel is

$$y(t) = s(t) \otimes h(t) = \sum_{\ell=0}^{L-1} h(\ell)s(t-\ell T)$$

- The signal $y(t)$ is affected by ISI!



OFDM signal model (2)

finite length block of N samples

Let's consider a block $\boldsymbol{s} = [s(0), s(1), \dots, s(N-1)]$ of N samples. After passing through the channel, the received samples are

DISCRETE CONVOLUTION

$$\Rightarrow \quad y(k) = \sum_{\ell=0}^{L-1} h(\ell)s(k-\ell)$$
$$= h(0)s(k) + \dots + h(L-1)s(k-L+1)$$

Since the elements of $\boldsymbol{s}$ are not defined for negative indices, the values of the samples $s(-1), s(-2), \dots, s(L-1)$ is 0. Accordingly, the received signal is

$k=0$

$$\begin{cases} y(0) = h(0)s(0) \\ y(1) = h(0)s(1) + h(1)s(0) \\ \vdots \\ y(N-1) = \underbrace{h(0)s(N-1) + h(1)s(N-2) + \dots + h(L-1)s(N-L)}_{\text{last } L \text{ elements of } S.} \end{cases}$$

OFDM signal model (3): matrix notation

$y(0) = h(0)s(0) + 0\ 0$

$y(1) = h(1)s(0) + h(0)s(1)$

- In matrix notation the block of received samples $\boldsymbol{y}$ can be represented as $\boldsymbol{y} = \mathcal{H}\boldsymbol{s}$

$$\begin{bmatrix} y(0) \\ y(1) \\ \vdots \\ \vdots \\ y(N-1) \end{bmatrix} = \begin{bmatrix} h(0) & 0 & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ h(1) & h(0) & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & h(1) & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ h(L-1) & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & h(L-1) & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \ddots & 0 & h(L-1) & \ddots & \ddots & h(1) & h(0) \end{bmatrix} \begin{bmatrix} s(0) \\ s(1) \\ \vdots \\ \vdots \\ s(N-1) \end{bmatrix}$$

The elements along any diagonal of the $N \times N$ matrix $\mathcal{H}$ are all equal and $\mathcal{H}$ is called a *Toeplitz* matrix.