Communication systems

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ELECTRONICS AND COMMUNICATIONS SYSTEMS

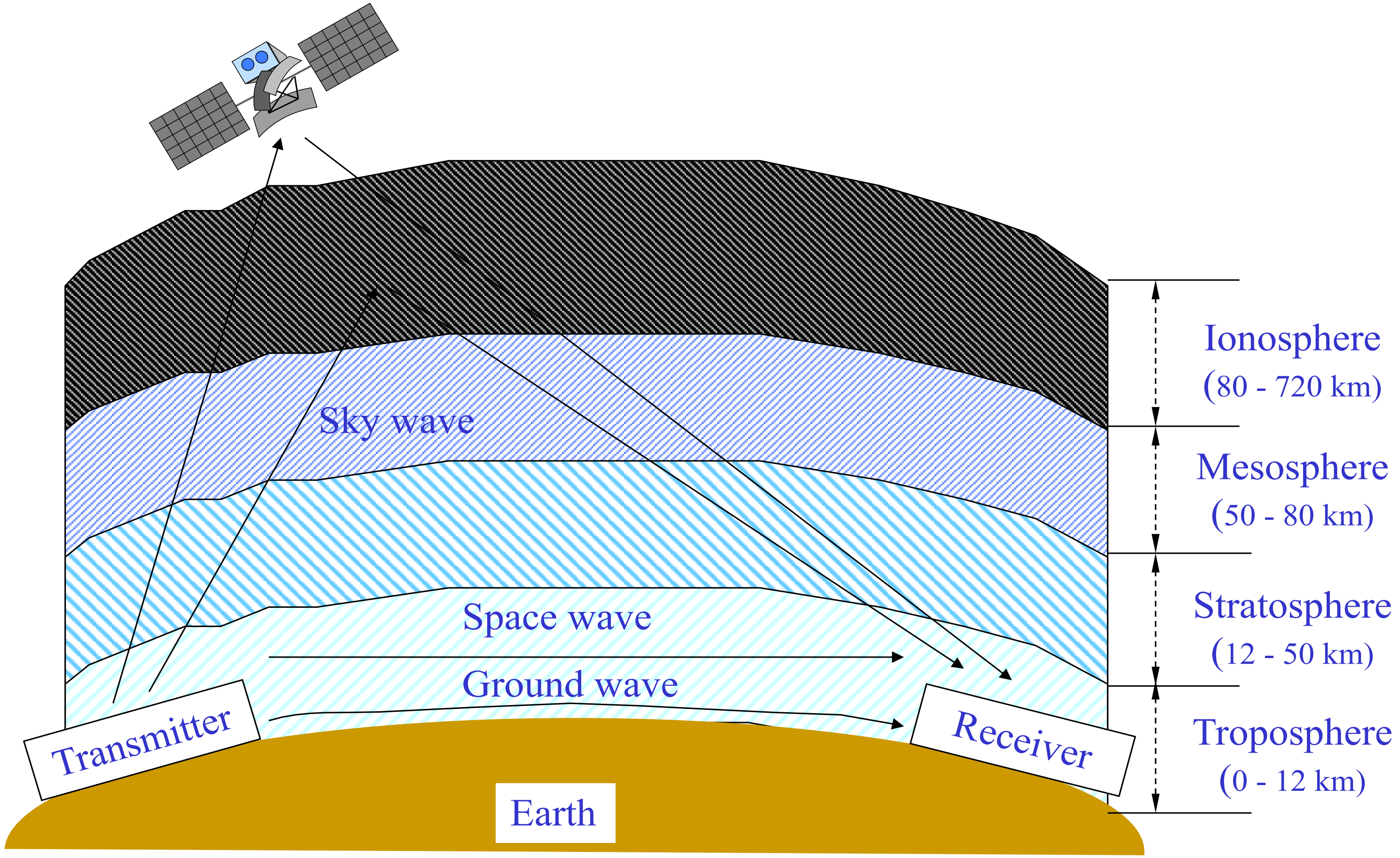
COMPUTER ENGINEERING

2. The wireless propagation channel

- Long-term fading: path-loss e shadowing
- Short-term fading: multipath fading

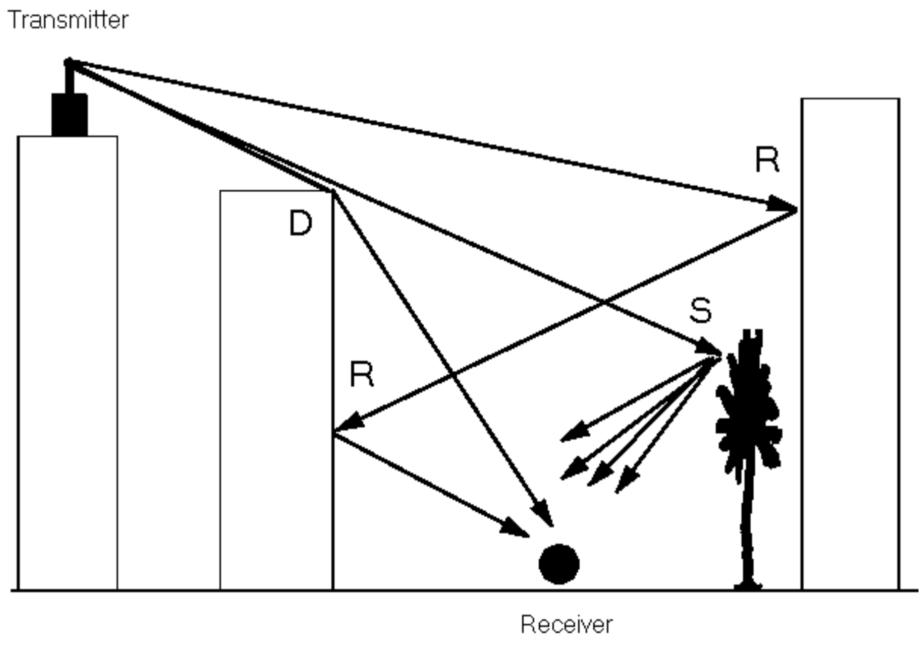
Signal propagation in the air

Classification Band	Initials	Frequency Range	Characteristics
Extremely low	ELF	< 300 Hz	Ground wave
Infra low	ILF	300 Hz - 3 kHz	
Very low	VLF	3 kHz - 30 kHz	
Low	LF	30 kHz - 300 kHz	
Medium	MF	300 kHz - 3 MHz	Ground/Sky wave
High	HF	3 MHz - 30 MHz	Sky wave
Very high	VHF	30 MHz - 300 MHz	Space wave
Ultra high	UHF	300 MHz - 3 GHz	
Super high	SHF	3 GHz - 30 GHz	



The wireless propagation channel (space wave)

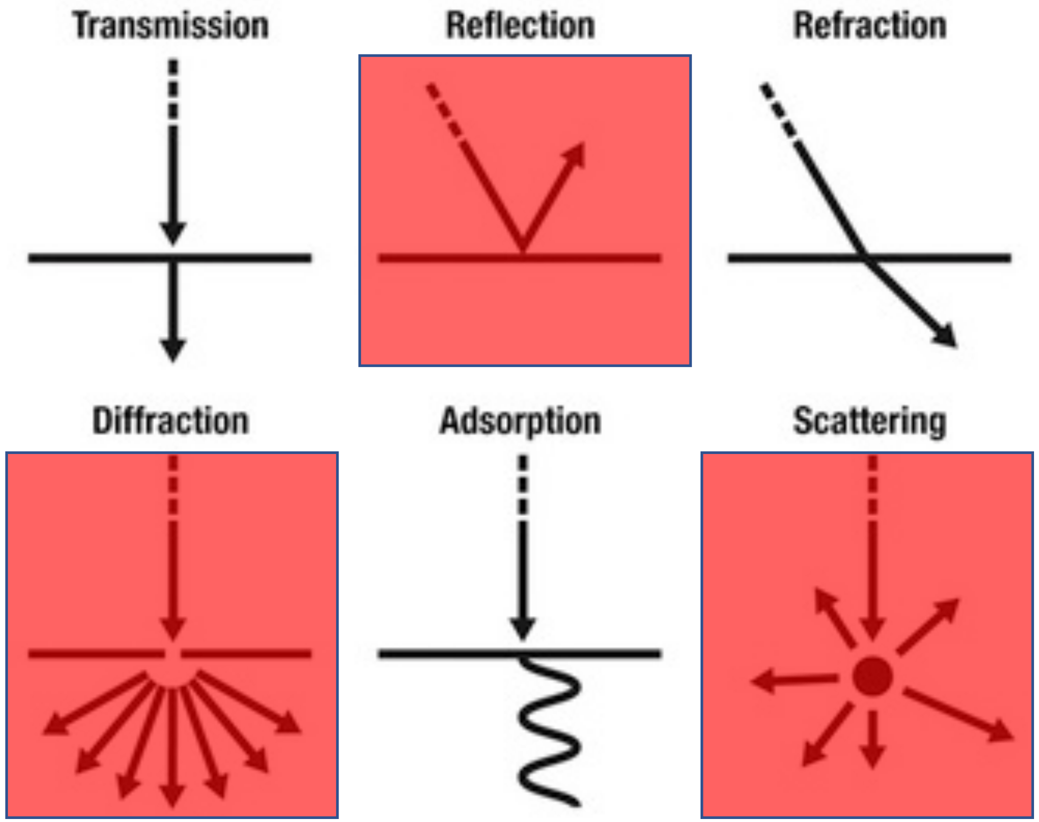
- Because, mobile services are mostly in the bandwidth 30MHz-30 GHz, *spacewave* is the most important wave propagation mechanism we need to consider.
- Most wireless radio systems operate in urban areas: No direct line-of-sight (los) between transmitter and receiver.
- The main physical phenomena are: reflection, diffraction, scattering.
- Can be categorized into two types:
    - Large-scale propagation models
    - Small-scale propagation models



Reflection (R), diffraction (D) and scattering (S).

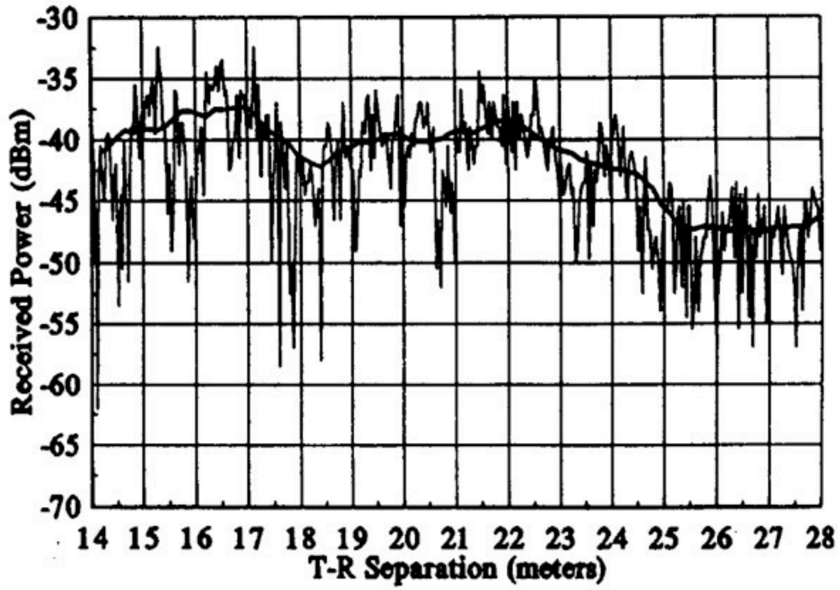
Propagation phenomena

- Three major propagation mechanisms:
  - **Reflection**
    Signal impinges on very large (w.r.t. to signal wavelength) objects. When a wave meets a boundary, it can be either reflected or transmitted.
  - **Diffraction**
    Signal is obstructed by objects that have sharp irregularities. Diffraction depends on the size of the object relative to the wavelength of the wave.
  - **Scattering**
    Propagation medium populated by small (wrt to signal wavelength) objects or rough surfaces (e.g. foliage, street signs).



Large-scale fading

- Large-scale fading: propagation models that characterize signal strengths over Tx-Rx separation distance.
- Accounts for average received power, changes over distances > 1 m.
- Large-scale fading can be modelled as the combination of *path-loss* and *shadowing*.



Large-scale fading: path-loss

- Path-loss models simplify Maxwell's equations.
- Models vary in complexity and accuracy but, in general, mean power falloff w.r.t. the tx-rx distance d is proportional to d^2 in free space and to d^n in other environments.
- Considering only path-loss, the average received signal power is

$$P_{Rx} \propto P_{Tx}\Gamma(f_0, d_0)\left(\frac{d_0}{d}\right)^n \quad d > d_0$$

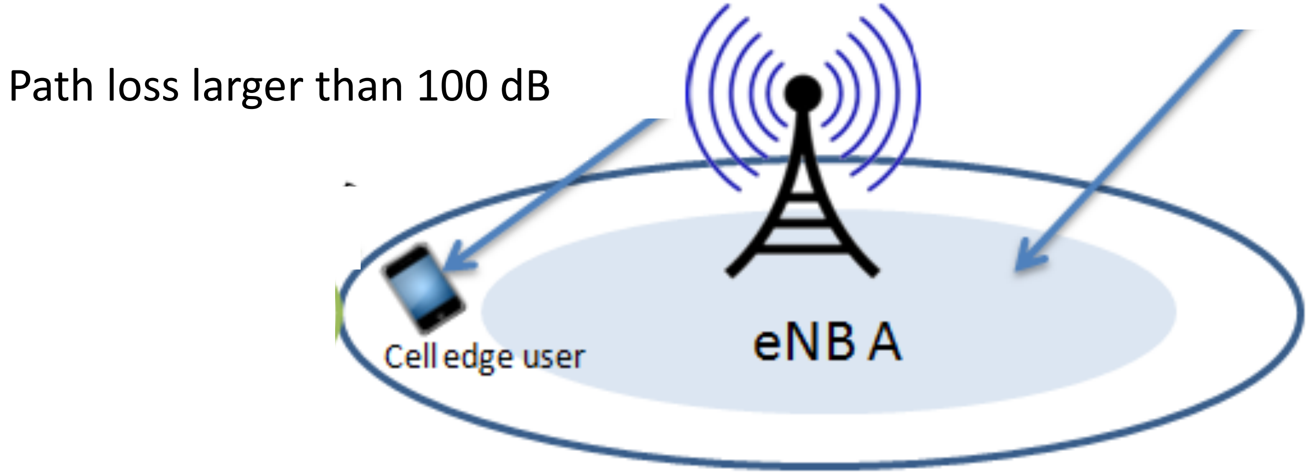
- *Near field term* $\Gamma(f_0, d_0) \approx \left(\frac{\lambda}{4\pi d_0}\right)^2$
- Path-loss A_{PL} is

$$A_{PL} = \frac{P_{Tx}}{P_{Rx}} = \Gamma(f_0, d_0)\left(\frac{d_0}{d}\right)^n$$

Environment	Path Loss Exponent, n
Free space	2
Urban area cellular radio	2.7 – 3.5
Urban area cellular (obstructed)	3 – 5
In-building line-of-sight	1.6 – 1.8
Obstructed in-building	4 – 6
Obstructed in-factories	2 – 3

Path-loss in cellular systems

- Path-loss: Signal attenuation defined as the ratio between the transmitted power and the average received power.
- The path-loss in a cellular system can be up to 100 dB for cell-edge users and represents the major impairment in any wireless cellular system



Large-scale fading: shadowing

- Two points with the same distance from the transmitter have theoretically the same path-loss, nevertheless their average attenuation may still greatly differ.
- *Shadowing* accounts for the random variations of the average channel attenuation.
- Shadowing fading A_S is a random variable log-normally distributed with parameters $\mu = 0$ and σ_S expressed in dB.
- The pdf in dB of A_S is

$$p(A_S) = \frac{1}{\sqrt{2\pi}\sigma_S} e^{-\frac{A_S^2}{2\sigma_S^2}}$$

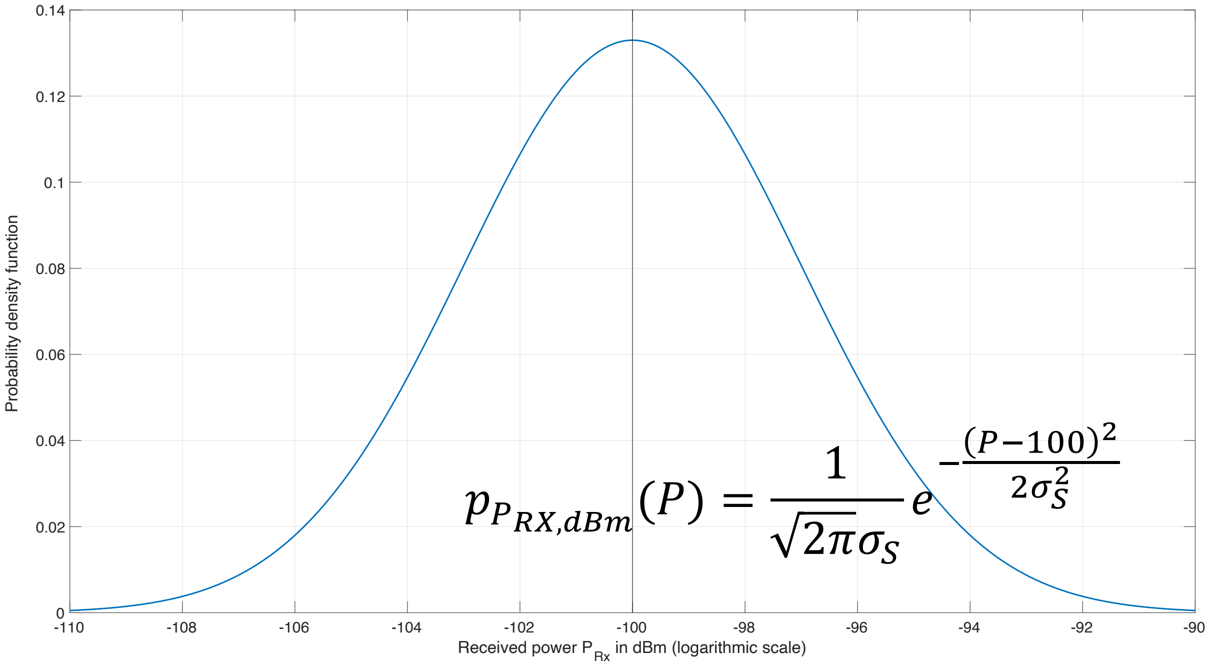
where σ_S is the standard deviation in decibels (typical values 0-9 dB)

Large-scale fading: shadowing

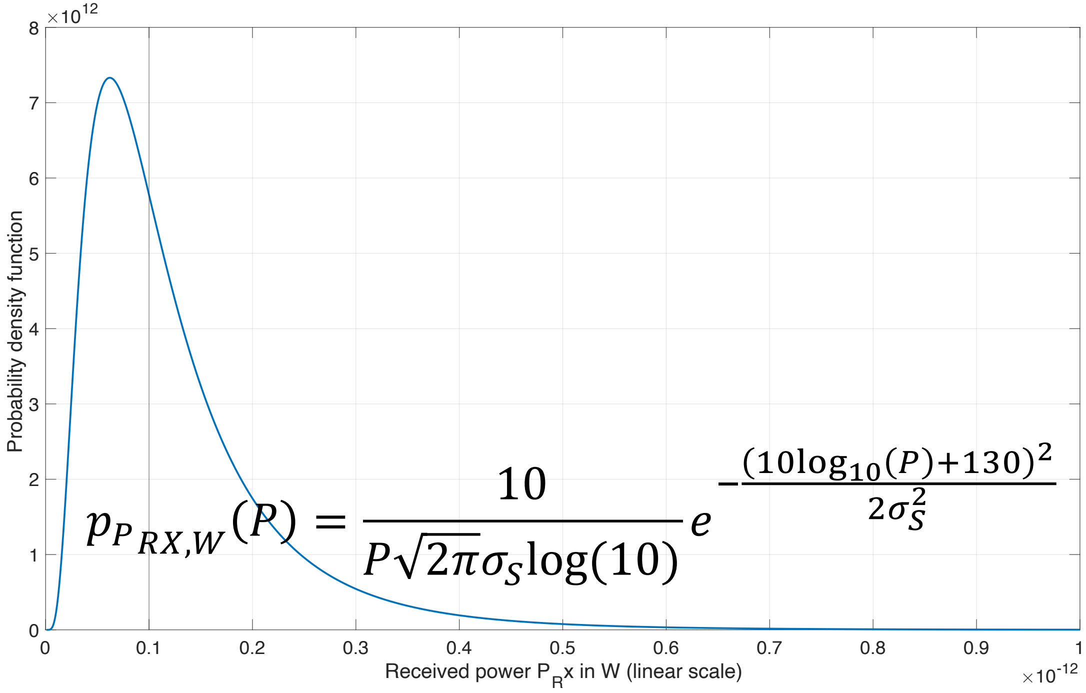
- Let's consider a channel with path-loss and shadowing ($\sigma_S = 3$ dB) only. The received power P_{RX} is

$$P_{RX} = P_{TX} A_{PL} A_S$$

- Assume that $P_{TX} A_{PL} = -100 \text{ dBm} = -130 \text{ dBW} = 10^{-13} \text{ W}$.
- Because of shadowing, P_{RX} is a random variable and its distribution in dBm and linear scale is



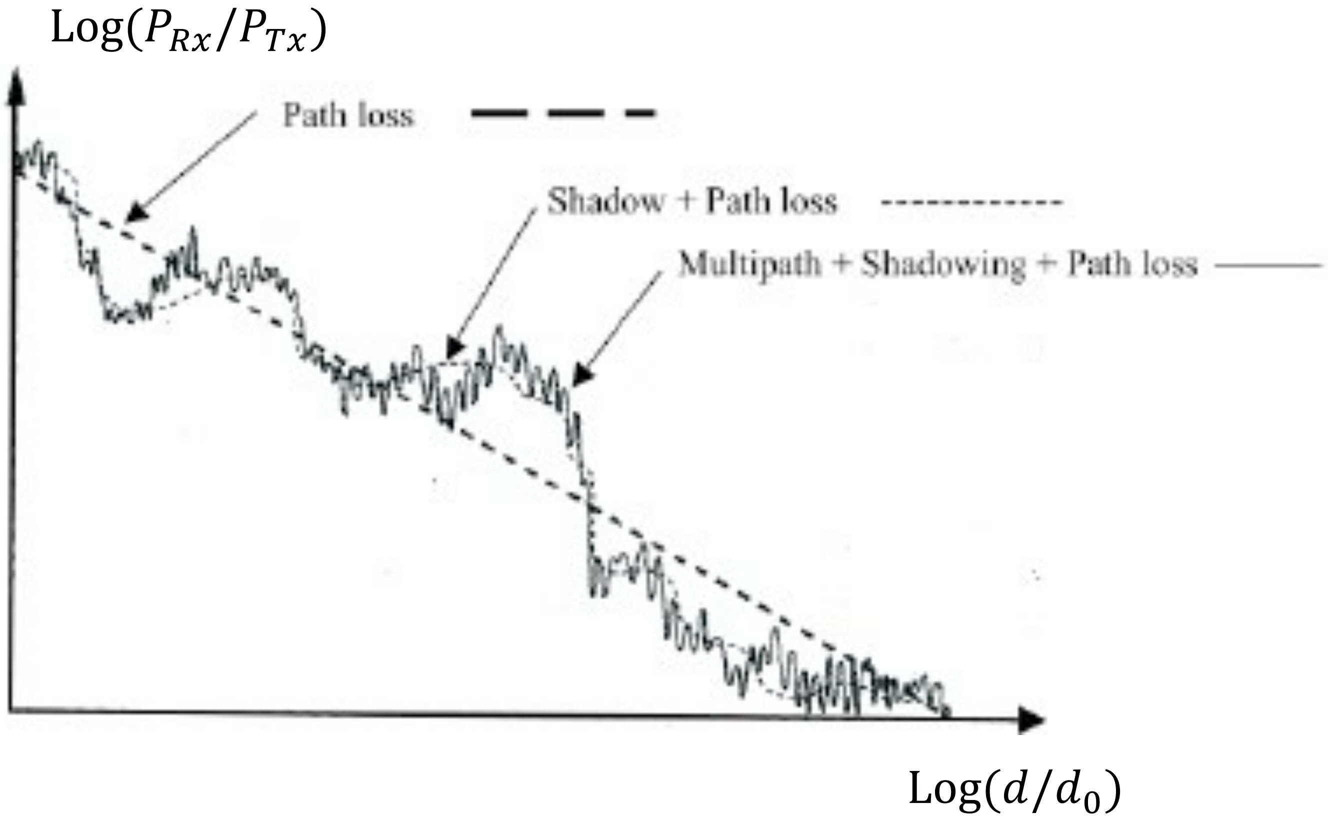
$$p_{P_{RX,dBm}}(P) = \frac{1}{\sqrt{2\pi}\sigma_S} e^{-\frac{(P-100)^2}{2\sigma_S^2}}$$

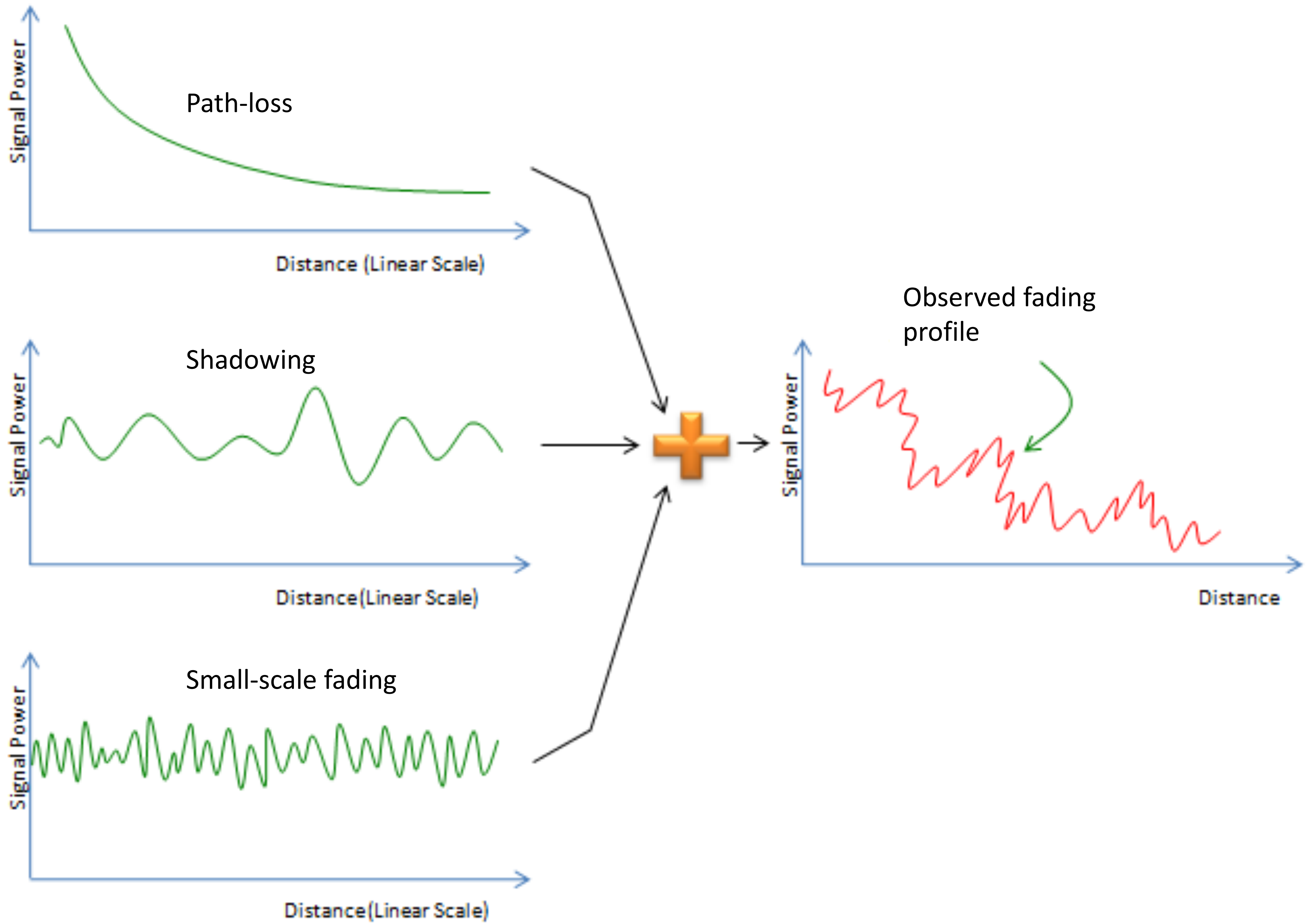


$$p_{P_{RX,W}}(P) = \frac{10}{P\sqrt{2\pi}\sigma_S \log(10)} e^{-\frac{(10\log_{10}(P)+130)^2}{2\sigma_S^2}}$$

Large-scale fading

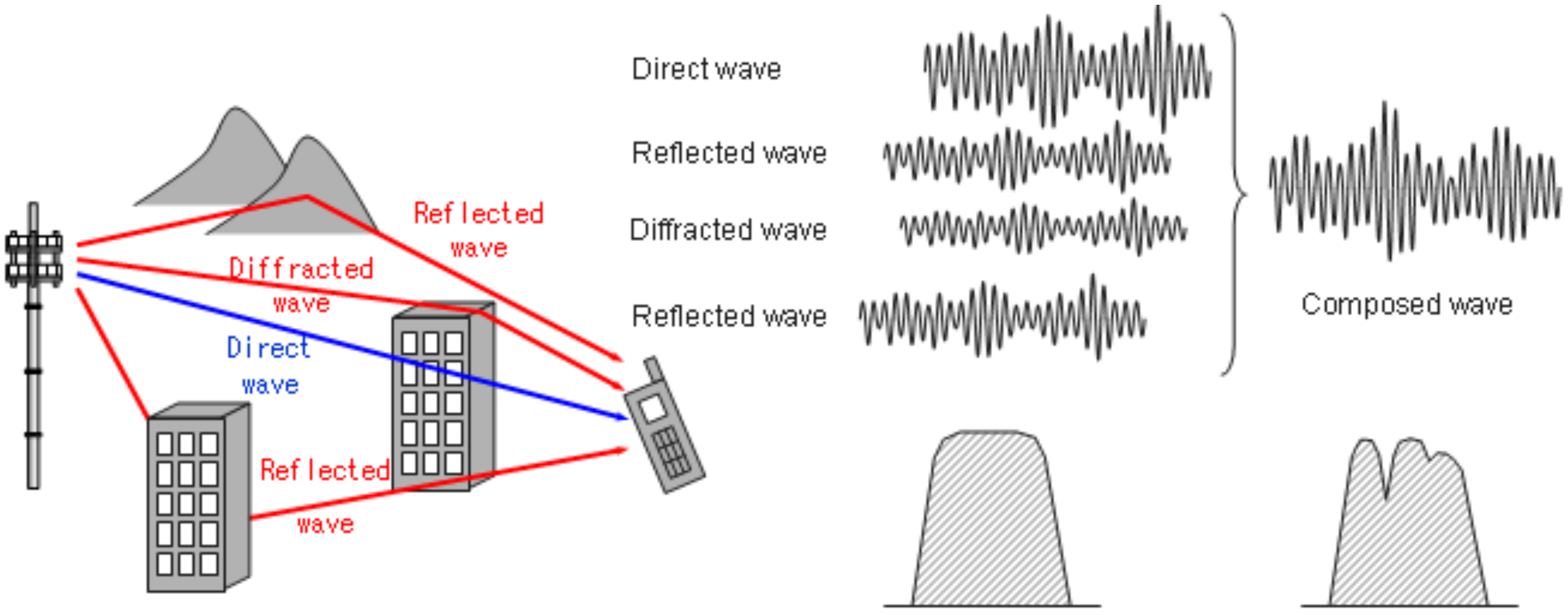
- The received power in dB is computed as
  $$P_{Rx}[dBm] = P_{Tx}[dBm] + \overbrace{A_{PL}\,[dB] + A_S[dB]}^{A_{LS}} + A_{SS}[dB]$$
- A_{PL} deterministic depends on the distance d.
- A_S random, log-normally distributed.
- A_{SS} is the attenuation due to small scale fading, which fluctuates rapidly with the distance




Signal Power
Path-loss
Distance (Linear Scale)
Signal Power
Shadowing
Distance(Linear Scale)
Signal Power
Small-scale fading
Distance(Linear Scale)
Observed fading profile
Signal Power
Distance

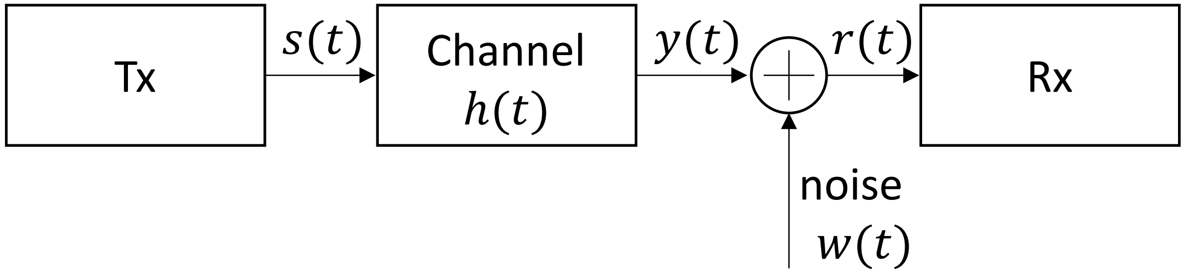
Small-scale fading

- Small-scale fading: accounts for the random variations of the instantaneous received power over distances of the order of a wavelength.
- Because of the various propagation phenomena, a large number of waves, each carrying a replica of the transmitted signal, arrives at the receiver.



Propagation channel

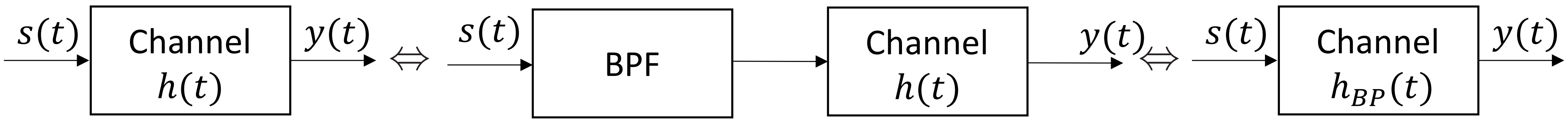
- The propagation channel can be modeled as a LTI filter.



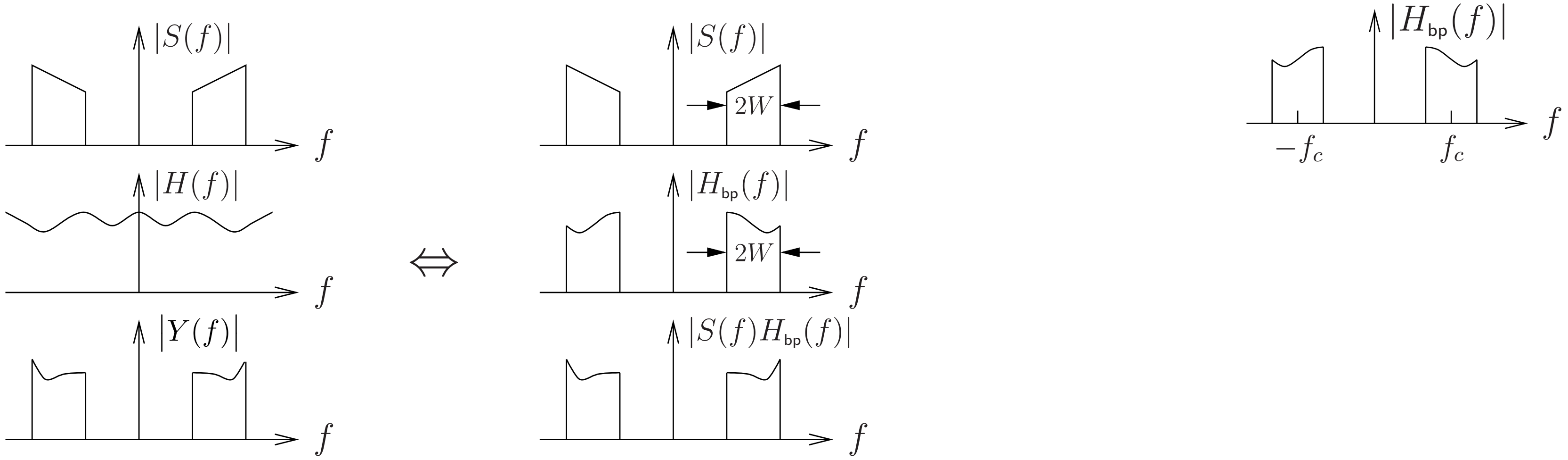
- The filter impulse response $h(t)$ depends on the small-scale fading characteristics.

Channel complex envelope

- Given the passband signal $s(t)$, with spectrum centered in f_c and bandwidth $B = 2W$, these three systems are equivalent

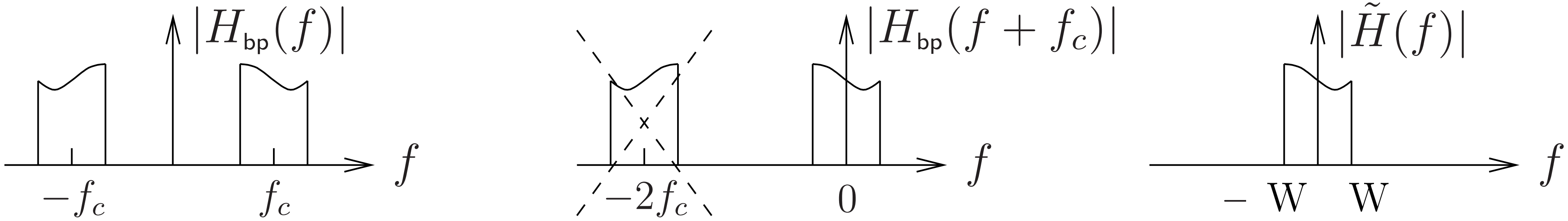


...because filtering $s(t)$ with $h(t)$ is equivalent to filtering $s(t)$ with $h_{\text{bp}}(t)$:



Channel impulse response

- The complex envelope of the channel is $\tilde{h}(t) \rightleftharpoons \widetilde{H}(f) = H_{\mathrm{bp}}(f + f_c)$.



- The corresponding passband channel is

$$h(t) = \mathcal{R}e\left\{\tilde{h}(t)e^{j2\pi f_c}\right\}$$

- The complex envelope of the received signal is

$$\tilde{y}(t) = \tilde{s}(t) \otimes \tilde{h}(t)$$

Channel's complex envelope

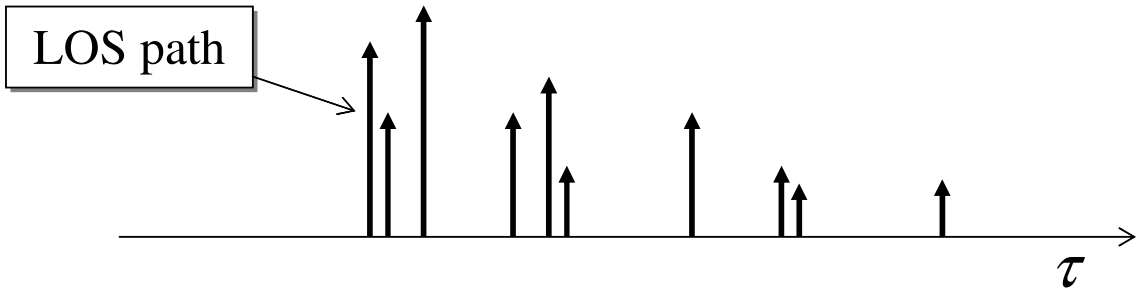
- The complex envelope (baseband) of the channel is

Large-scale fading, path-loss and shadowing

$$\tilde{h}(t) = A_{LS} \sum_{w=0}^{N_w - 1} \alpha_w e^{j\phi_w} \delta(t - \tau_w)$$

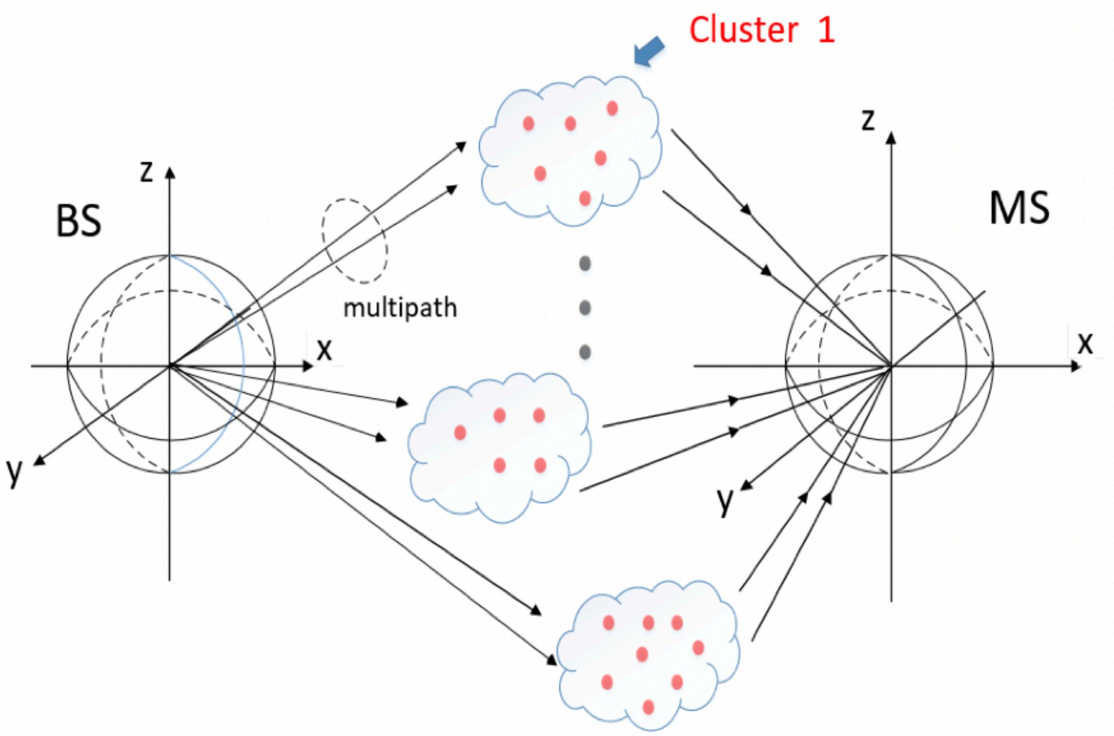
Small-scale fading, multipath

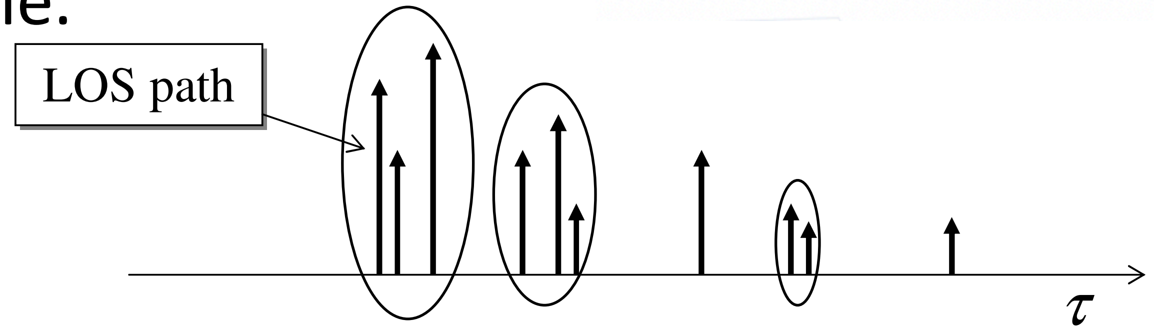
where A_{LS} is large-scale fading attenuation, N_w is the number of waves arriving at the receiver, α_w, ϕ_w and τ_w are attenuation, phase and delay of the w-th wave.



Small-scale fading

- Propagating from the transmitter to the receiver, signals may encounter several major obstacles.
- Depending on their delay, *the various paths can be grouped in different clusters*, where each cluster roughly corresponds to a specific obstacle.





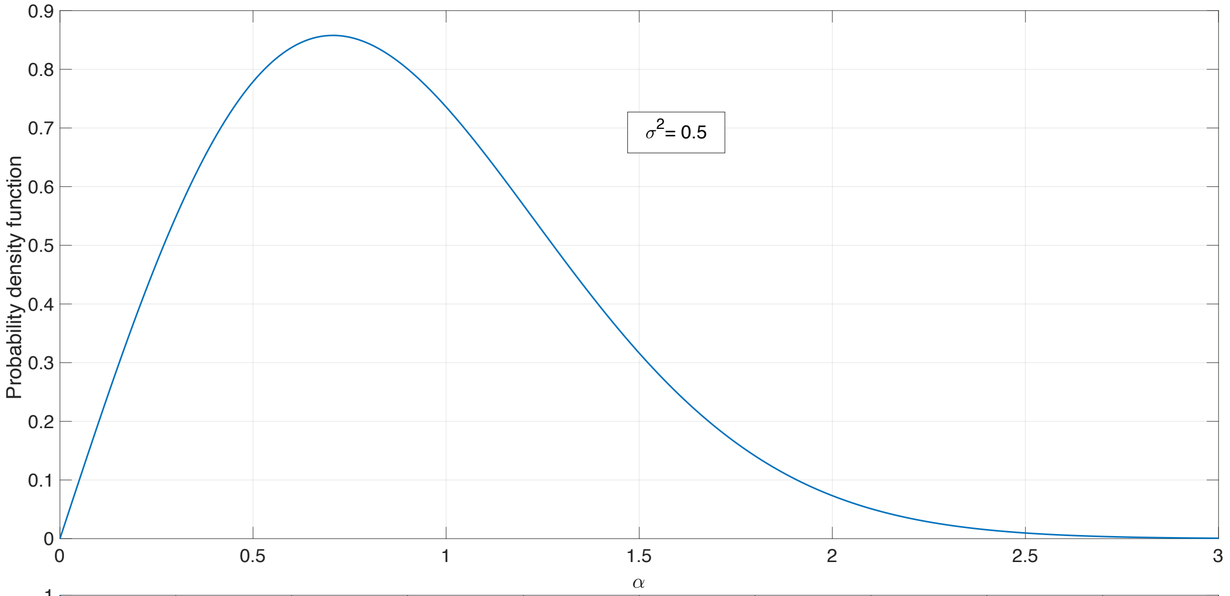
Small-scale fading: Rayleigh distribution

- If there is a sufficiently large scatter, for each cluster at the receiver we will have the sum of many different replicas of the signal, each with approximately the same delay and different complex gains.
- Because of the *Central limit theorem*, the complex gain of each cluster can be modelled as a *complex Gaussian* variable irrespective of the distribution of the individual components.
  - Phase ϕ is uniformly distributed in $[0,2\pi]$.
  - Amplitude α is *Rayleigh* distributed, if there is no los, or *Rician* distributed if there is los.
- Central limit theorem: when a sufficiently large number of random variables are added, their sum tends to be normally distributed regardless of the original distribution of the random variables.

Channel gain characterization

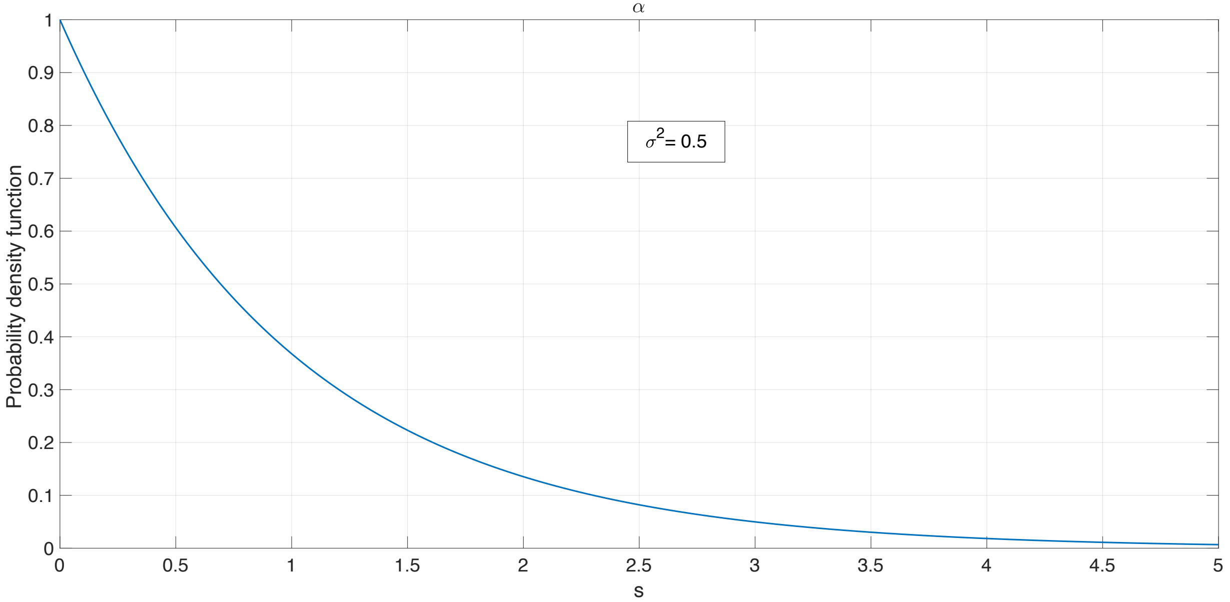
- The distribution for channel amplitude α is *Rayleigh*

$$p(\alpha) = \begin{cases} \frac{\alpha}{\sigma^2} e^{-\frac{\alpha^2}{2\sigma^2}} & \alpha \geq 0 \\ 0 & \alpha < 0 \end{cases}$$



- The distribution for channel power $s = \alpha^2$ is *exponential*

$$p(s) = \begin{cases} \frac{1}{2\sigma^2} e^{-\frac{s}{2\sigma^2}} & s \geq 0 \\ 0 & s < 0 \end{cases}$$



Small-scale fading

- The channel impulse response (CIR) can be modelled as

$$h(t) = A_{LS} \sum_{\ell=0}^{N_{C-1}} \alpha_\ell e^{j\phi_\ell} \delta(t - \tau_\ell)$$

where $\alpha_\ell e^{j\phi_\ell}$, the complex gain of the ℓ-th cluster is the sum of the complex gains of all the paths belonging to the cluster.

- Let $s(t)$ be the transmitted signal, neglecting the noise, the complex envelope of the signal at the receiver is

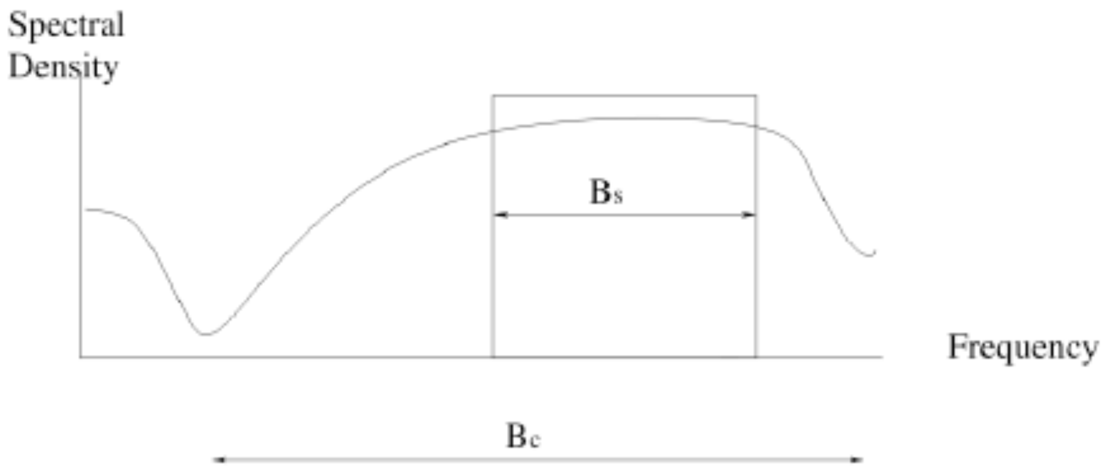
$$y(t) = s(t) \otimes h(t) = A_{LS} \sum_{\ell=0}^{N_{C-1}} \alpha_\ell e^{j\phi_\ell} s(t - \tau_\ell)$$

Multipath fading and ISI

- The received signal is modelled as the sum of a series of attenuated, time-delayed phase shifted replicas of the transmitted signal, *one different path* for each cluster.
- Depending on the symbol duration, the propagation channel might be composed by one or several *resolvable* paths, where each resolvable path roughly corresponds to a given cluster.
  - If for the signal of interest the channel can be approximated with one single path, the channel is *flat* fading
  - If there is more than one resolvable path, the channel is *multipath* and we have inter-symbol interference (ISI).
- The *coherence bandwidth* of the channel is the bandwidth over which the channel has approximately a constant gain and a linear phase response.

Flat fading channel

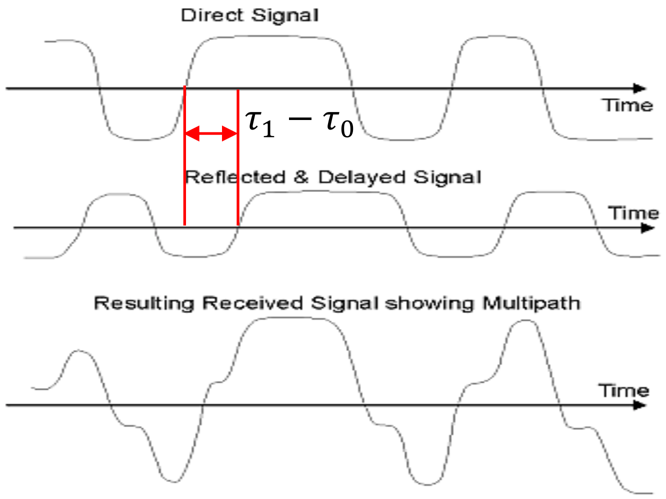
- When the channel can be modelled with only one path, its coherence bandwidth B_c is larger than the bandwidth B_s of the transmitted signal.



  - The spectral characteristics of the transmitted signal are preserved at the receiver.
  - The channel does not cause any non-linear distortion due to time dispersion.

Multipath fading

- If the resolvable paths are more than one, the received signal includes multiple versions of the same symbol, each one attenuated (faded), rotated in phase and delayed.
- The received signal is distorted and is affected by ISI
- The channel is said to be subject to *frequency selective fading.*



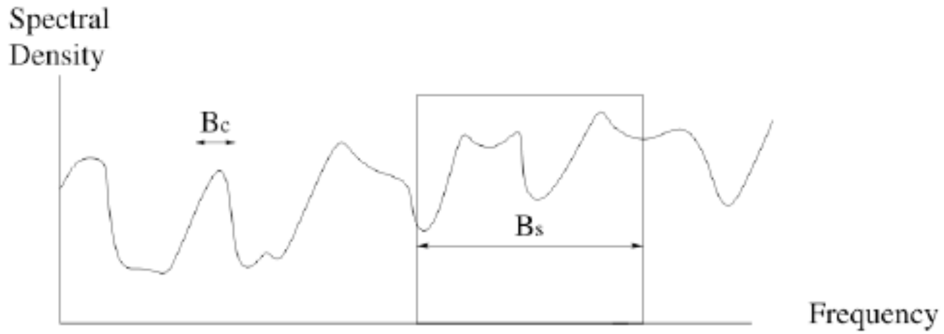
$\ell = 0$

$\ell = 1$

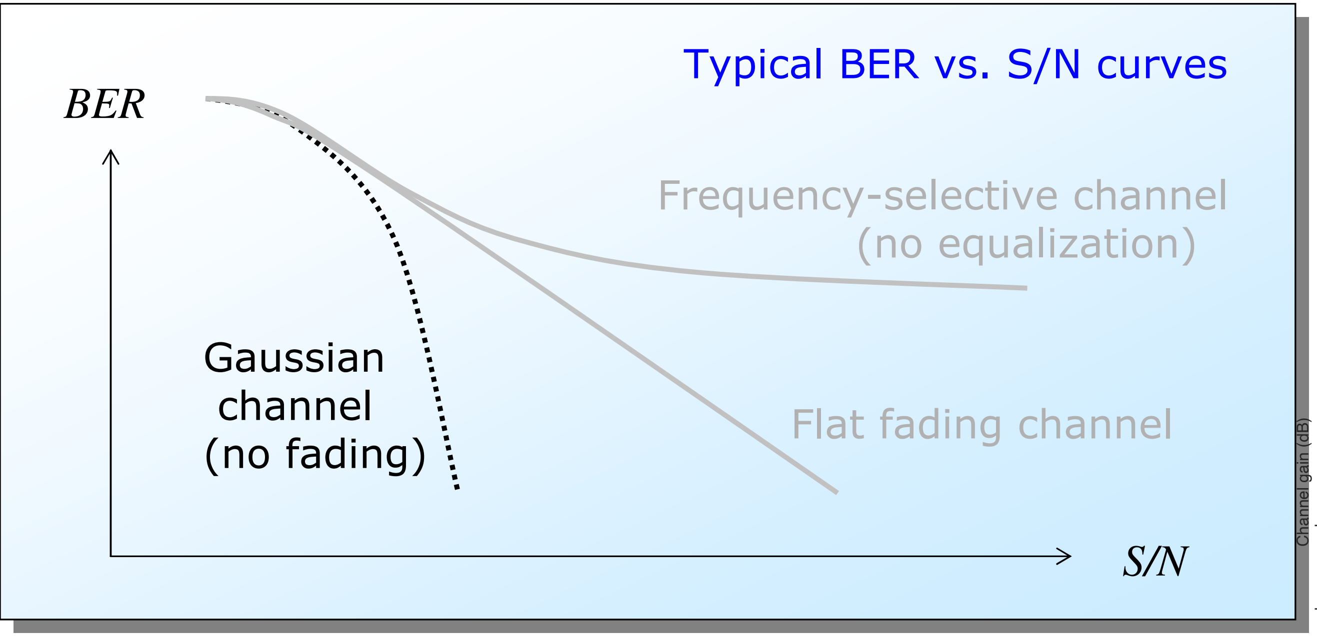
$$y(t) = \alpha_0 e^{j\phi_0} s(t - \tau_0) + \alpha_1 e^{j\phi_1} s(t - \tau_1)$$

Frequency selective fading

- Frequency selective fading
  - The coherence bandwidth B_C of the channel is smaller than the bandwidth B_s of the transmitted signal.
  - The spectral characteristics of the transmitted signal are not preserved at the receiver: certain frequency components have larger gains than others
  - The channel is *selective* in frequency.

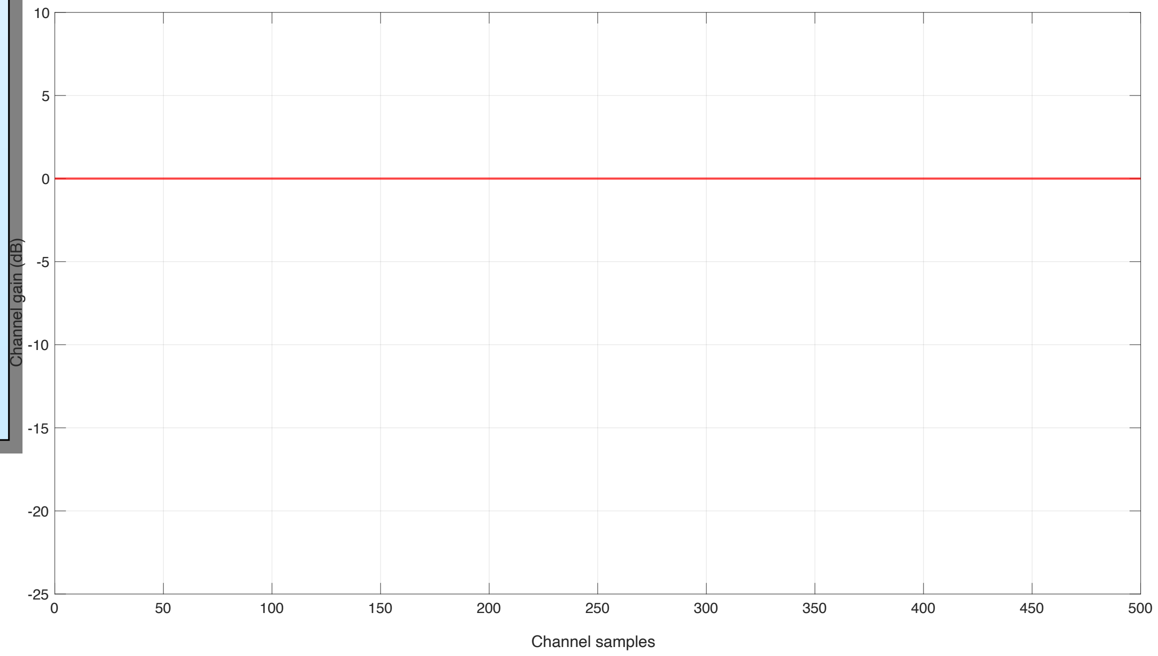


Flat fading channel: BER on AWGN

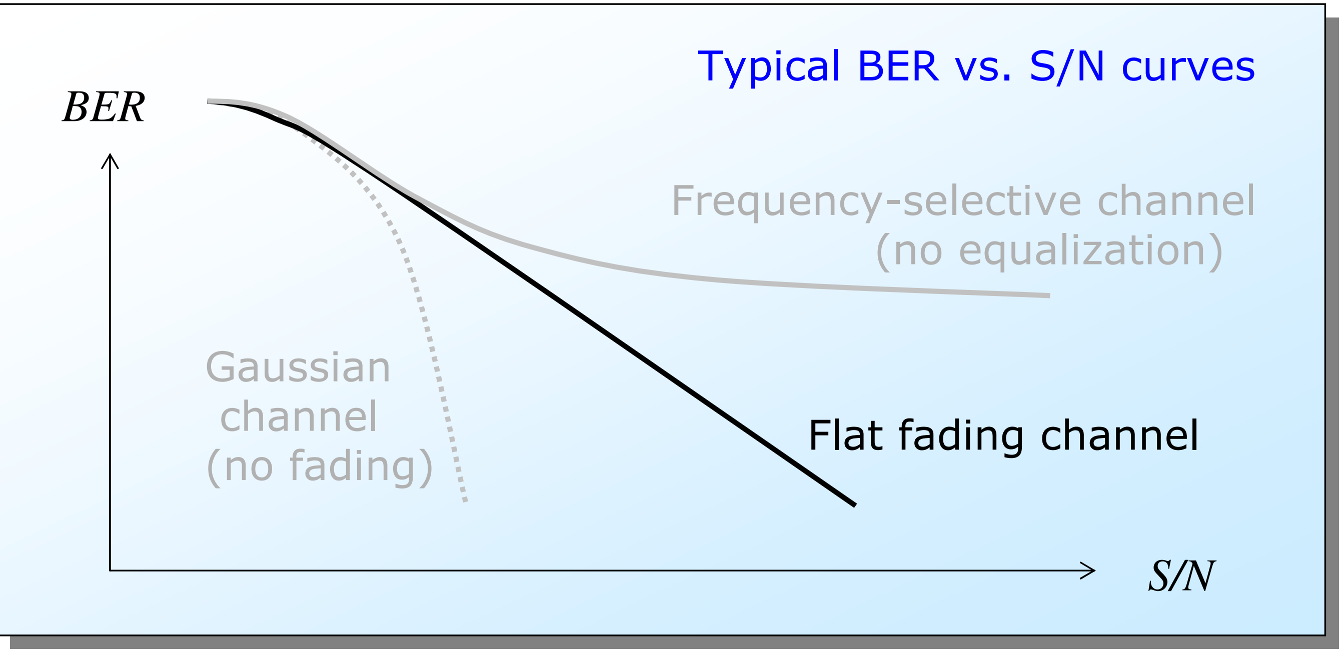


- With an AWGN channel, the decision variable is

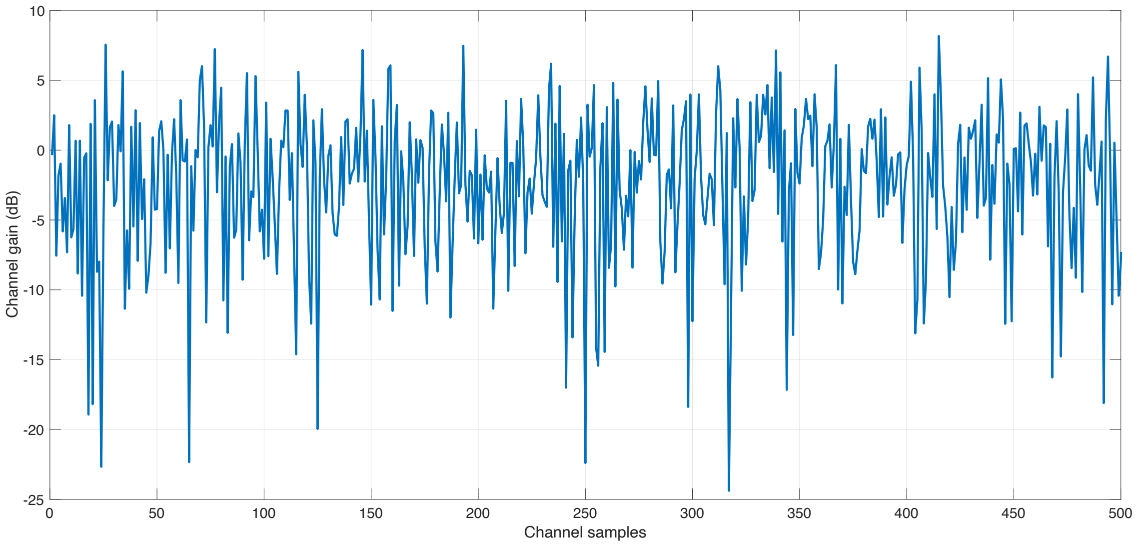
$$x(m) = c_m + n(m)$$



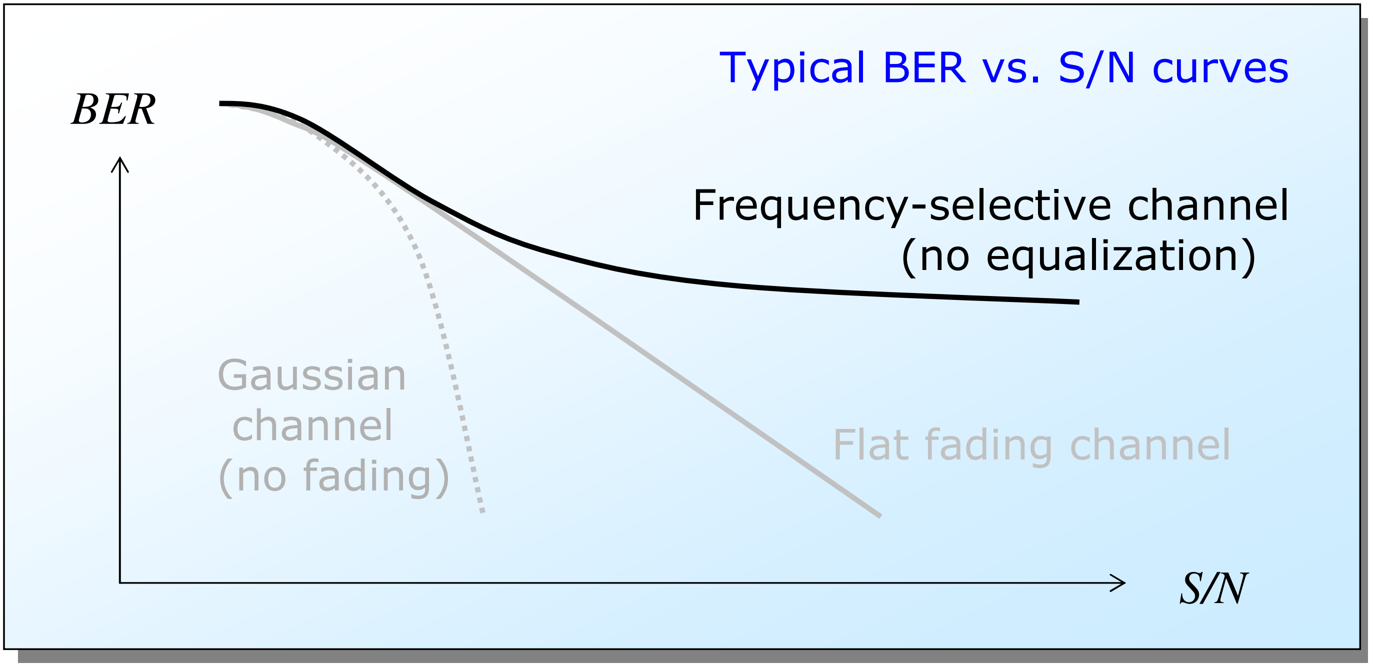
BER on flat Rayleigh fading channel



- With a flat fading channel, the decision variable is
$$x(m) = \alpha c_m + n(m)$$
- The mean error probability is obtained by averaging it over the channel
$$P_e = \int_0^{+\infty} P(e|\alpha)p(\alpha)d\alpha$$



BER on multipath Rayleigh fading channel



- With a frequency selective channel, the decision variable is

$$x(m) = \alpha_0 c_m + \underbrace{\sum_{\ell,\ell\neq 0} \alpha_\ell e^{j\phi_\ell} c_{m-\ell}}_{\text{ISI}} + n(m)$$

- If no countermeasures are taken, the error probability has an irreducible error-floor.

Channel’s delay spread

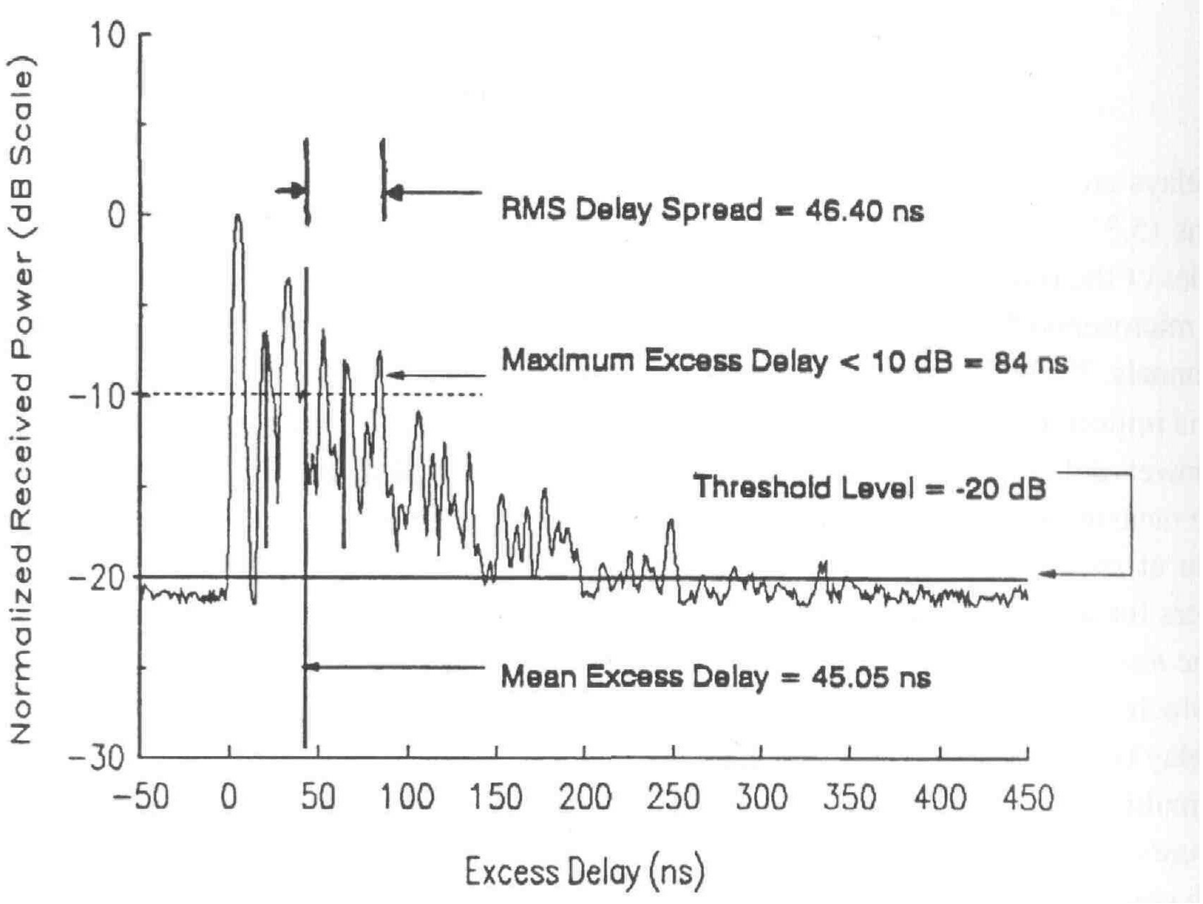
- One practical way to assess if the channel is multipath fading is to compare the symbol timing with the root mean square (RMS) delay spread.
- To compute the delay statistics one should integrate over the density function of the delays.
- Too difficult, instead the delay of each path is weighted by the coefficient $0 < \alpha_\ell^2 \backslash \sum_{\ell=0}^{L-1} \alpha_\ell^2 < 1$, which are equivalent to empirical mass probabilities.
- *Mean excess delay*

$$\bar{\tau} = \frac{\sum_{\ell=0}^{L-1} \alpha_\ell^2 \tau_\ell}{\sum_{\ell=0}^{L-1} \alpha_\ell^2}$$

- *RMS delay spread*

$$\sigma_\tau = \sqrt{\overline{\tau^2} - \bar{\tau}^2} \text{ with } \overline{\tau^2} = \frac{\sum_{\ell=0}^{L-1} \alpha_\ell^2 \tau_\ell^2}{\sum_{\ell=0}^{L-1} \alpha_\ell^2}$$

RMS delay spread



Delay spread and coherence bandwidth

- The selectivity of the channel can be evaluated both in the time and frequency domain
- *Delay spread*
  - If the delay spread is smaller than the symbol time, $\sigma_\tau \ll T$, there is only one resolvable path and the channel is *flat* fading.
  - If $\sigma_\tau > T$, there are more than one resolvable path and the channel is *multipath*.
- *Coherence bandwidth*. The channel coherence bandwidth B_C is approximately computed as the inverse of the delay spread
$$B_C \approx 1/\sigma_\tau$$
  - If the signal bandwidth is smaller than the coherence bandwidth, $B < B_c$ the channel is *flat.*
  - If $B > B_C$, the channel is *frequency selective*.

Typical values of RMS delay spread

- Measurements depend on signal frequency and environment.
- Typical values of delay spread are 0.2μs (rural area), 0.5μs (suburban area), 3-8μs (urban area), <2 μs (urban microcell) and 50-300ns (indoor picocell)

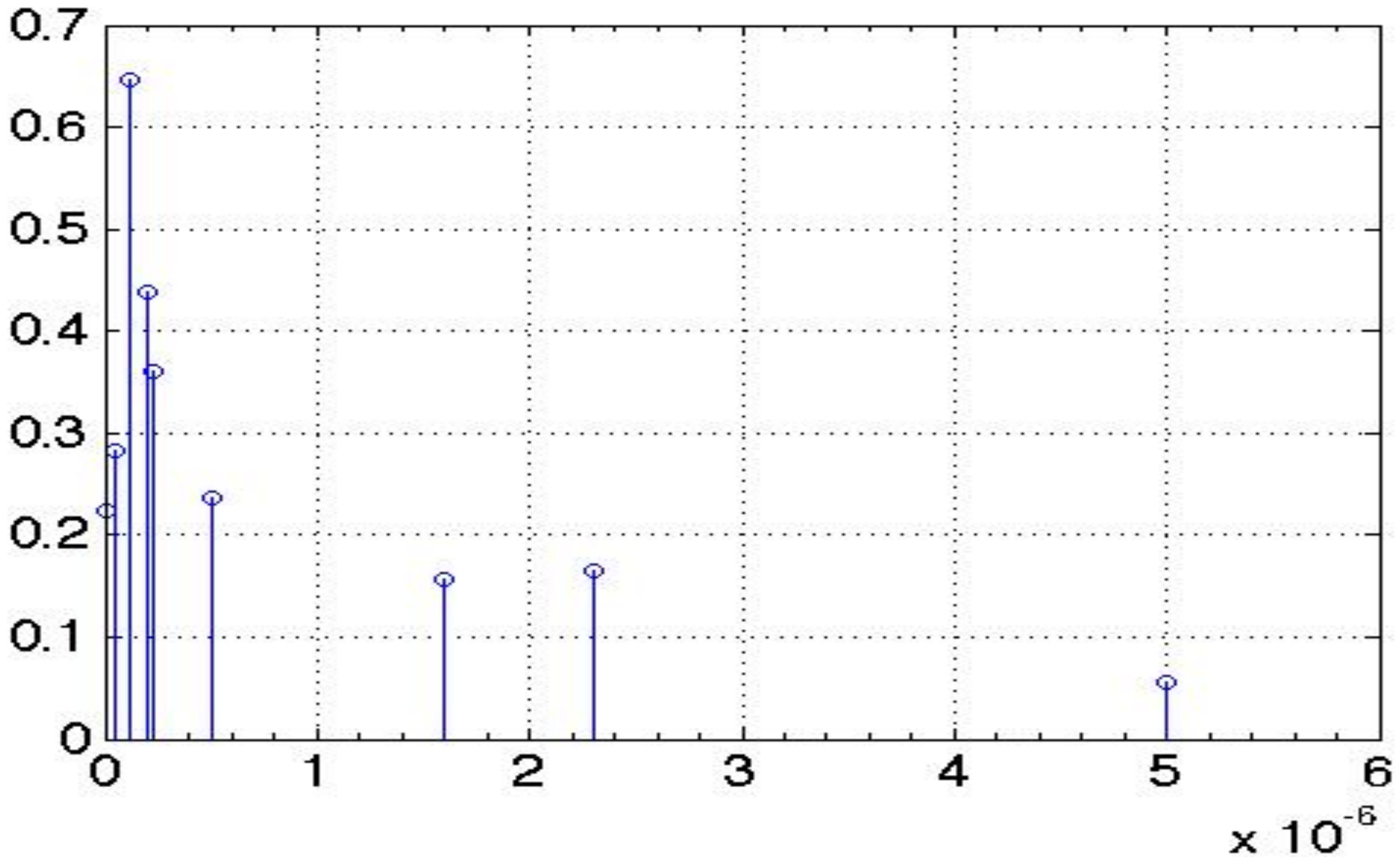
Environment	RMS delay spread (σ_τ)	Notes
Urban	1300 ns (3500 ns max)	NYC
LTE ETU	Up to 5 μs	Averaged typical case
Suburban	1960-2110 ns	Averaged extreme case
Indoor	10-50 ns	Office building
Indoor	70-94 ns (1470 ns max)	Office building

LTE channel models

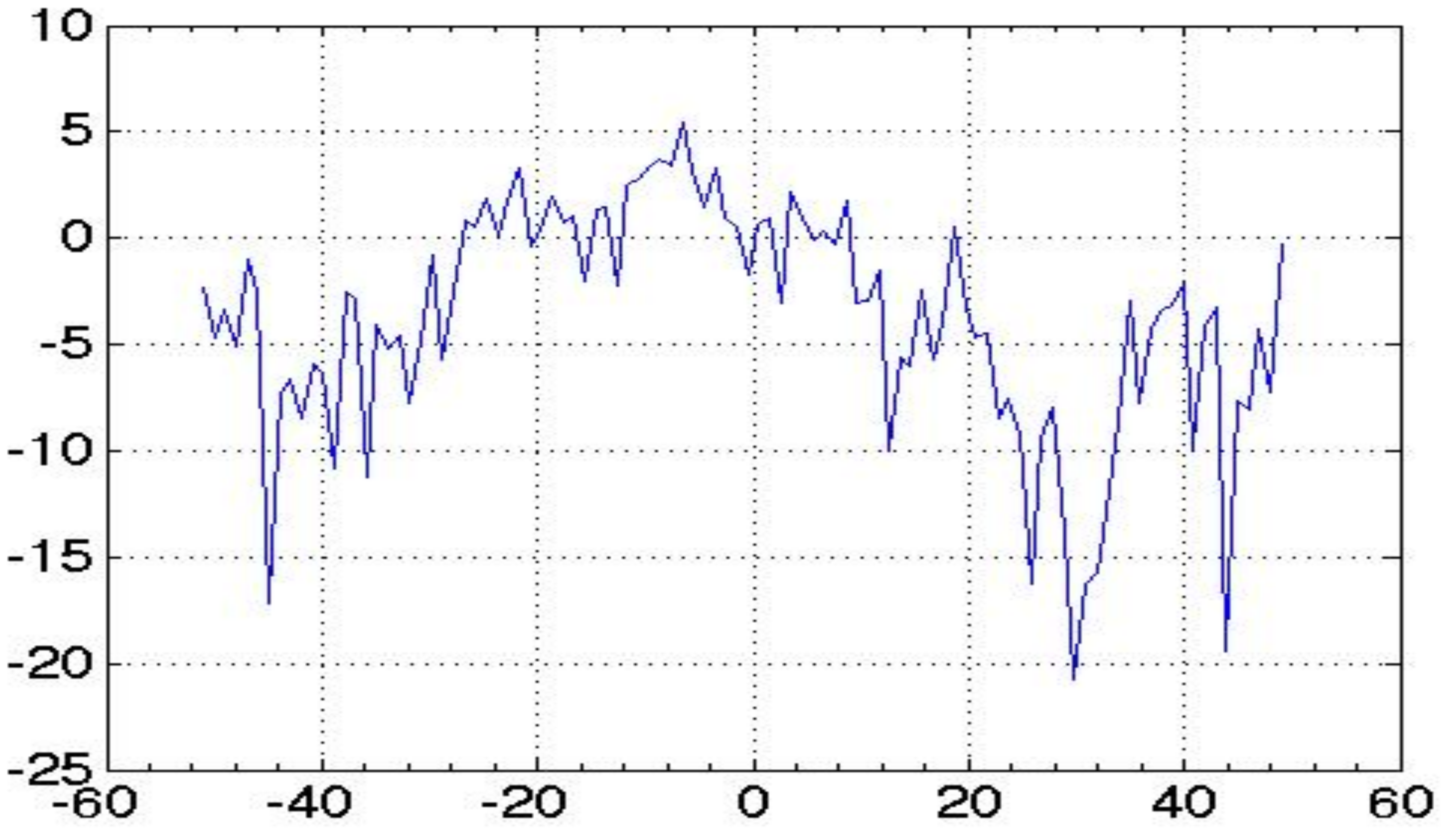
- Extended Typical Urban model (ETU)

Excess tap delay (ns)	Relative power (dB)
0	–1.0
50	–1.0
120	–1.0
200	0.0
230	0.0
500	0.0
1600	–3.0
2300	–5.0
5000	–7.0

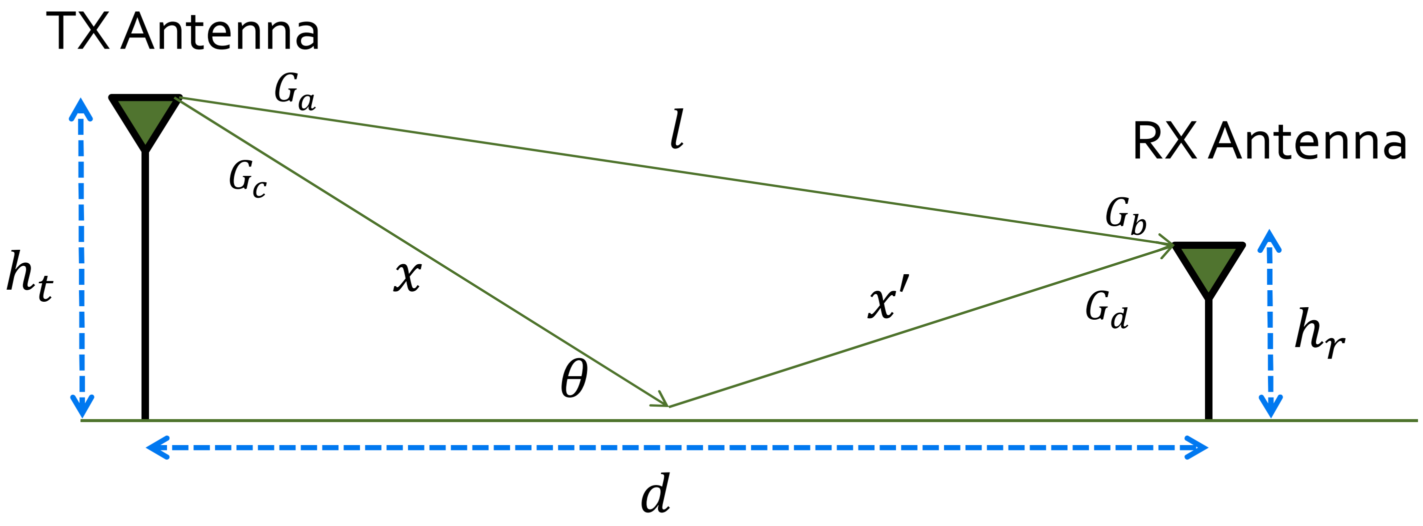
CIR snapshot



Frequency channel snapshot



Example: the two-ray channel model



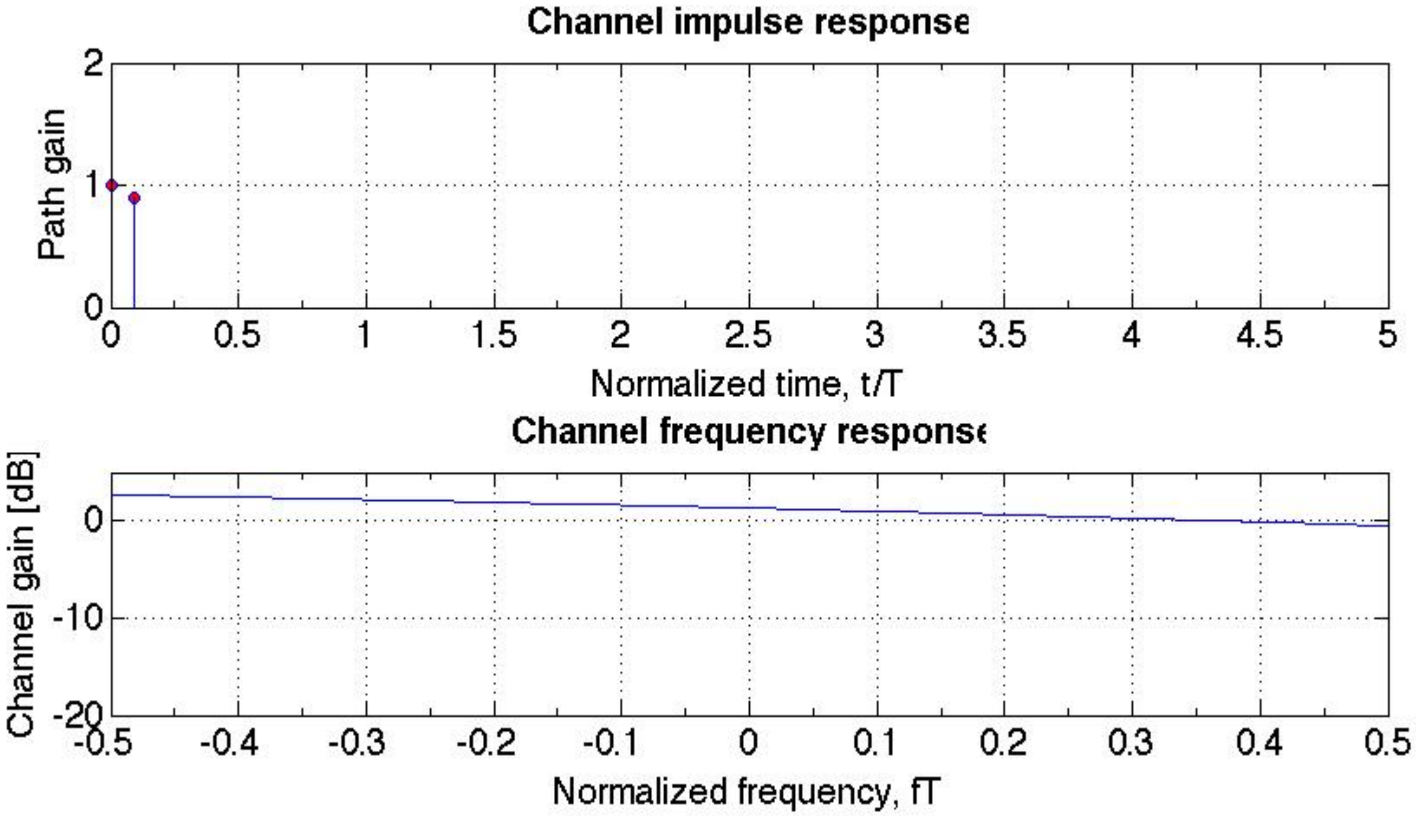
Delayed since x+x' is longer. $\tau = (x + x' - l)/c$

$$r_{2-ray}(t) = Re\left\{\frac{\lambda}{4\pi}\left[\frac{\sqrt{G_a G_b}\tilde{g}(t)\exp\left(-\frac{j2\pi l}{\lambda}\right)}{l} + \frac{R\sqrt{G_c G_d}\tilde{g}(t-\tau)\exp\left(-\frac{j2\pi(x+x')}{\lambda}\right)}{x+x'}\right]\exp(j2\pi f_c t)\right\}$$

R: ground reflection coefficient (phase and amplitude change)

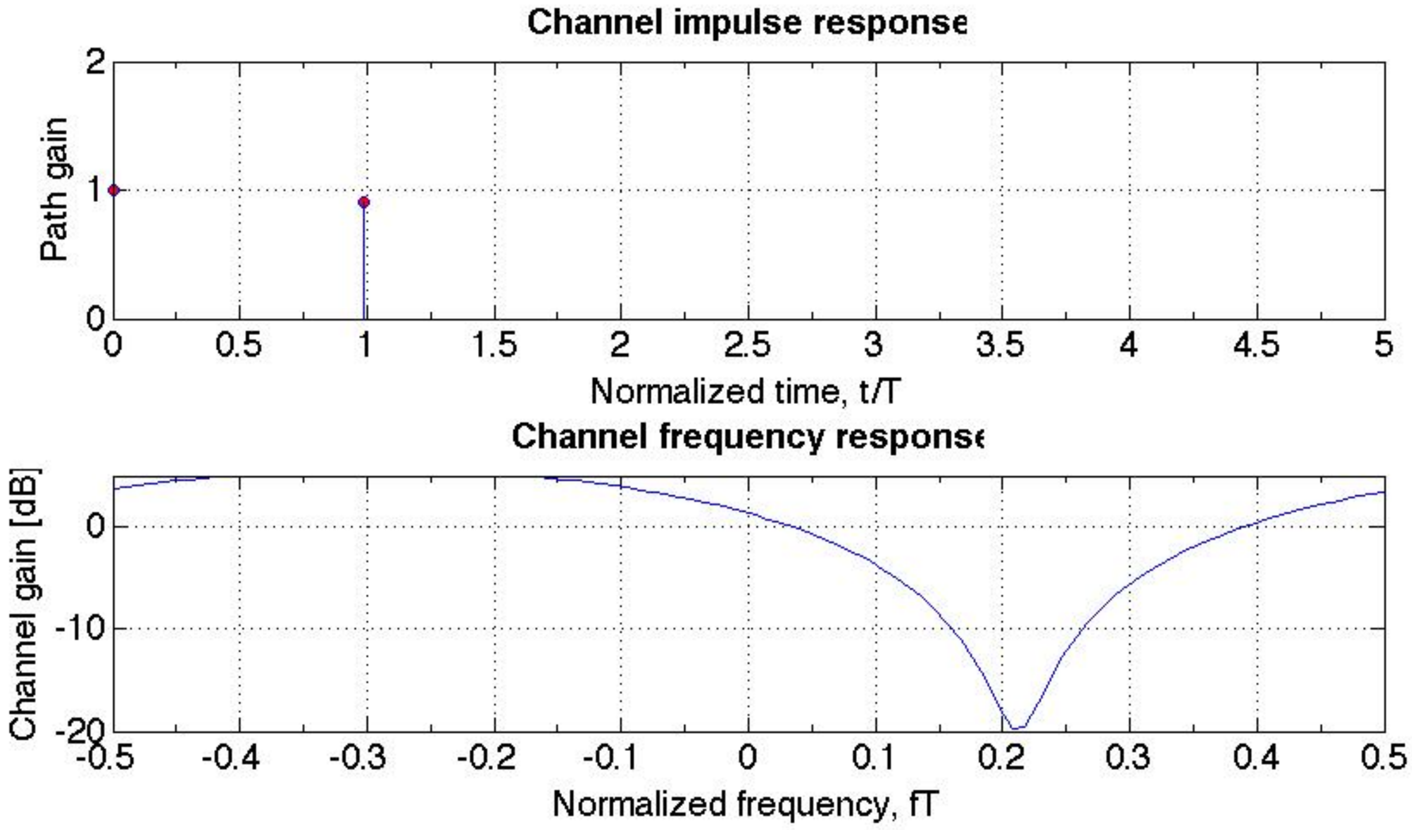
Two-path channel, $h(t) = \alpha_1 \delta(t) + \alpha_2 \delta(t - \tau)$

Channel parameters
$\alpha_1 = 1, \alpha_2 = 0.9$
$\tau = 0.1T$



Two-path channel, $h(t) = \alpha_1\delta(t) + \alpha_2\delta(t-\tau)$

Channel parameters
$\alpha_1 = 1, \alpha_2 = 0.9$
$\tau = T$

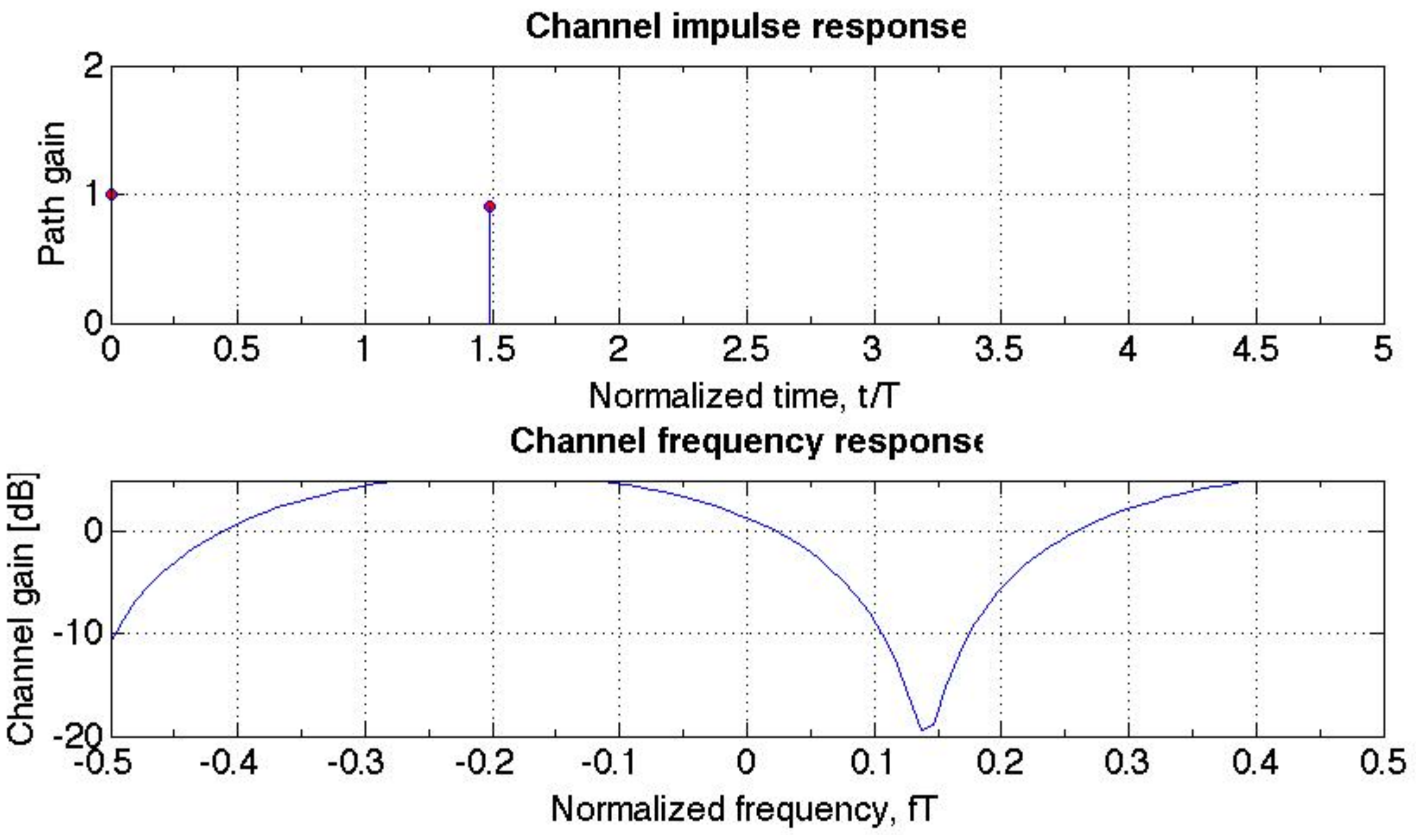


Two-path channel, $h(t) = \alpha_1 \delta(t) + \alpha_2 \delta(t - \tau)$

Channel paramete

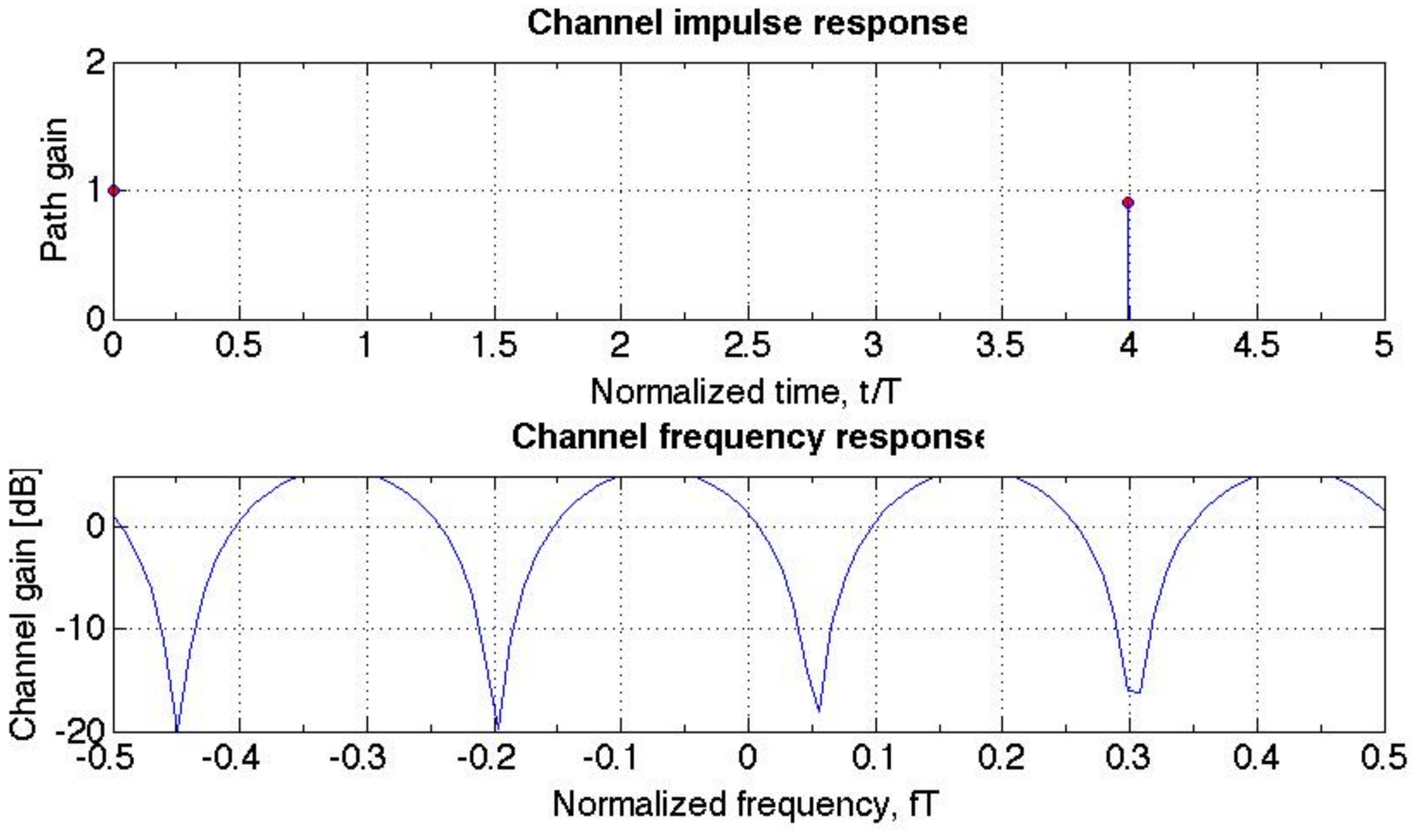
$\alpha_1 = 1, \alpha_2 = 0.9$

$\tau = 1.5T$



Two-path channel, $h(t) = \alpha_1\delta(t) + \alpha_2\delta(t-\tau)$

Channel parameters
$\alpha_1 = 1, \alpha_2 = 0.9$
$\tau = 4\,T$



Time-varying channel

- If the mobile receiver is in movement, the gains and the phase of the various paths of the channel vary in time

$$h(t,\tau) = A_{LS} \sum_{\ell=0}^{N_{C-1}} \alpha_\ell(t) e^{j\phi_\ell(t)} \delta(\tau - \tau_\ell)$$

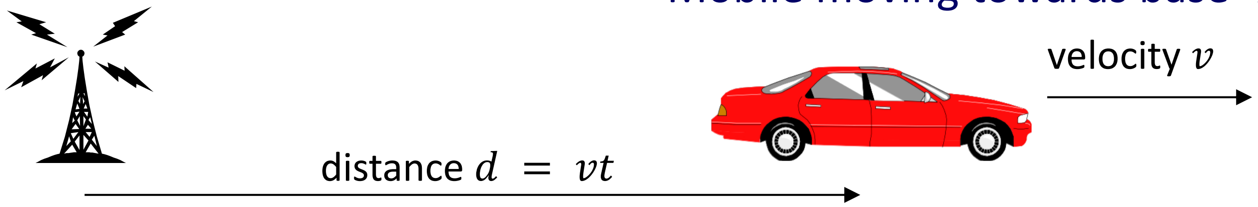
- The received signal is

$$y(t) = A_{LS} \sum_{\ell=0}^{N_{C-1}} \alpha_\ell(t) e^{j\phi_\ell(t)} s(t - \tau_\ell)$$

- The channel gains and phases change much faster than the large scale fading A_{LS} and the delays τ_ℓ.

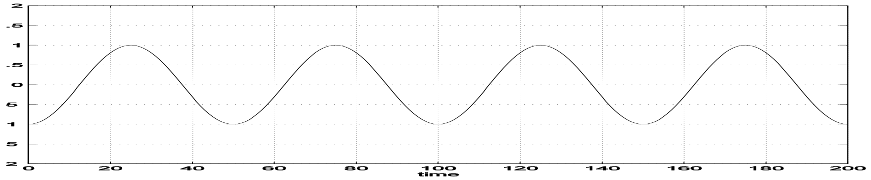
Doppler shift

Mobile moving away from base ➔ $v > 0$, Doppler shift < 0
Mobile moving towards base ➔ $v < 0$, Doppler shift > 0



Propagation delay = distance d/speed of light c, $\tau = vt/c$

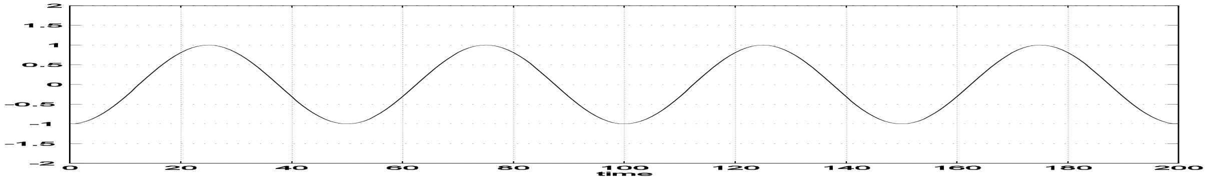
Transmitted signal $s(t)$



delay increases

Received signal $y(t)$

propagation delay



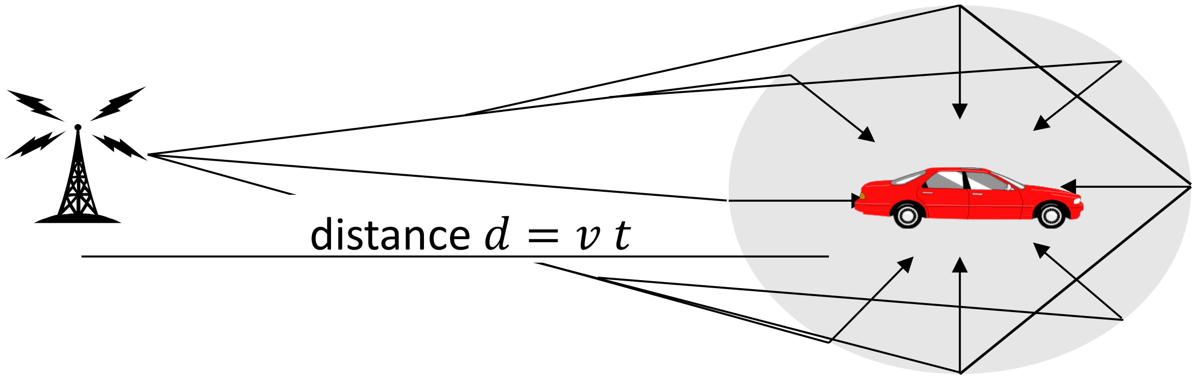
Received signal $y(t) = \sin 2\pi f_c\,(t - vt/c) = \sin 2\pi(f_c - f_c v/c)\,t$

received frequency

Doppler shift $f_d = -f_c v/c$

Scattering: Doppler Spectrum

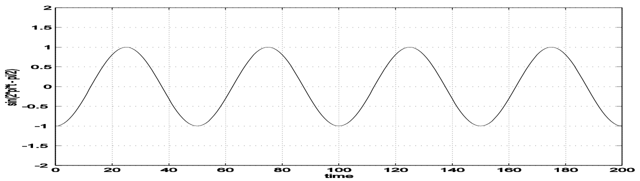
- In fading channels many signal replicas arrive at the receiver with different angles. The effect is a *Doppler spread* rather than a single shift.



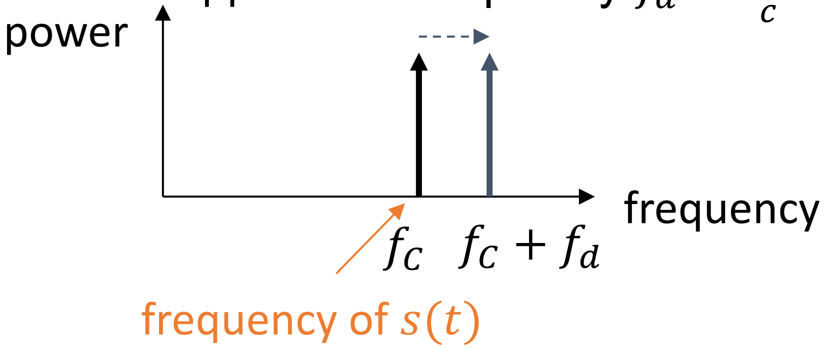
- Received signal is the sum of all scattered waves and is not anymore deterministic but is modeled as a *stochastic process*.
  - Doppler shift for each path depends on angle θ, each path has a shift $f\,\frac{v cos\theta}{c}$.
  - Typically assume that the received energy is the same from all directions (uniform scattering).
  - The received signal is described by its autocorrelation and power spectral density.

Jakes' Doppler spectrum for a sinusoidal tone

Transmitted signal $s(t)$

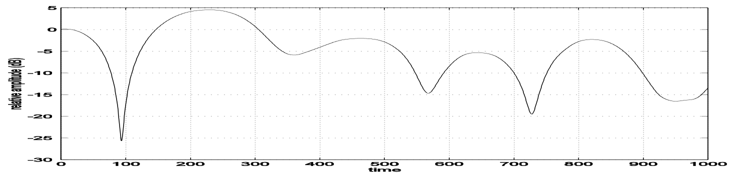


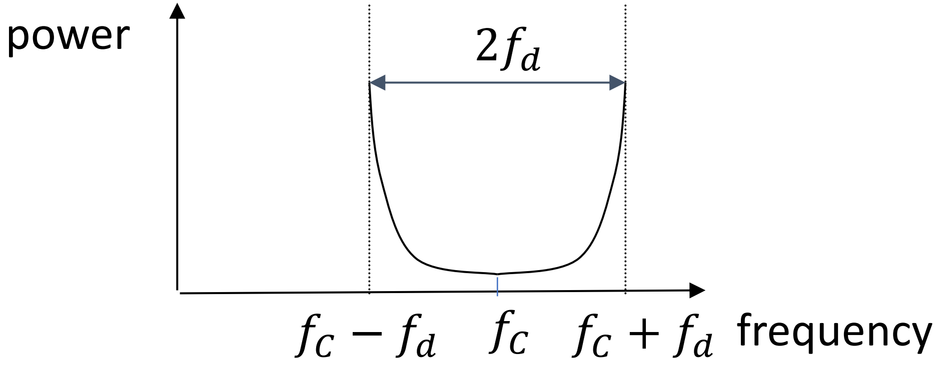
Doppler shift frequency $f_d = \frac{f_c v}{c} = \frac{v}{\lambda}$



$$S(f) = \frac{1}{\pi f_d \sqrt{1 - \left(\frac{f}{f_d}\right)^2}}$$

Received signal $y(t)$



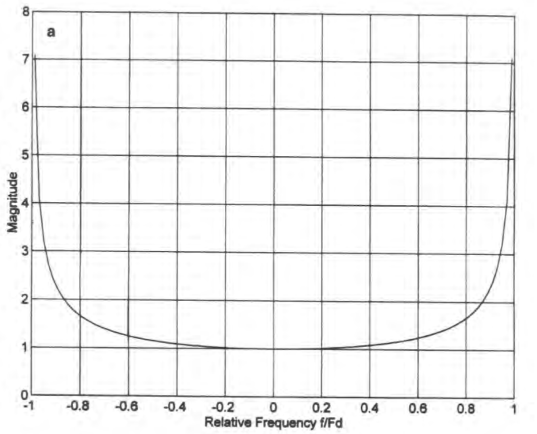


The spectral broadening caused by the receiver movement is called *Doppler spread.*

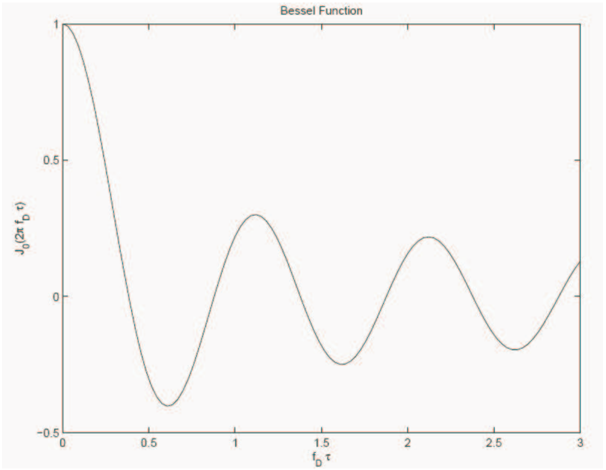
Time varying channel

- The Doppler spectrum $S(f)$ is the power spectral density of a sinusoid on a time varying channel.
- In the time domain $\rho(t) = J_0(2\pi f_d t) \leftrightharpoons S(f)$ is the autocorrelation function.
- $J_0(2\pi x) \approx 0$ for $x = \frac{1}{2} \Longrightarrow$ The channel can be assumed uncorrelated for $f_d T_c = \frac{1}{2}$.
- Channel coherence time is

$$T_c = \frac{1}{2 f_d}$$



$\leftrightharpoons$



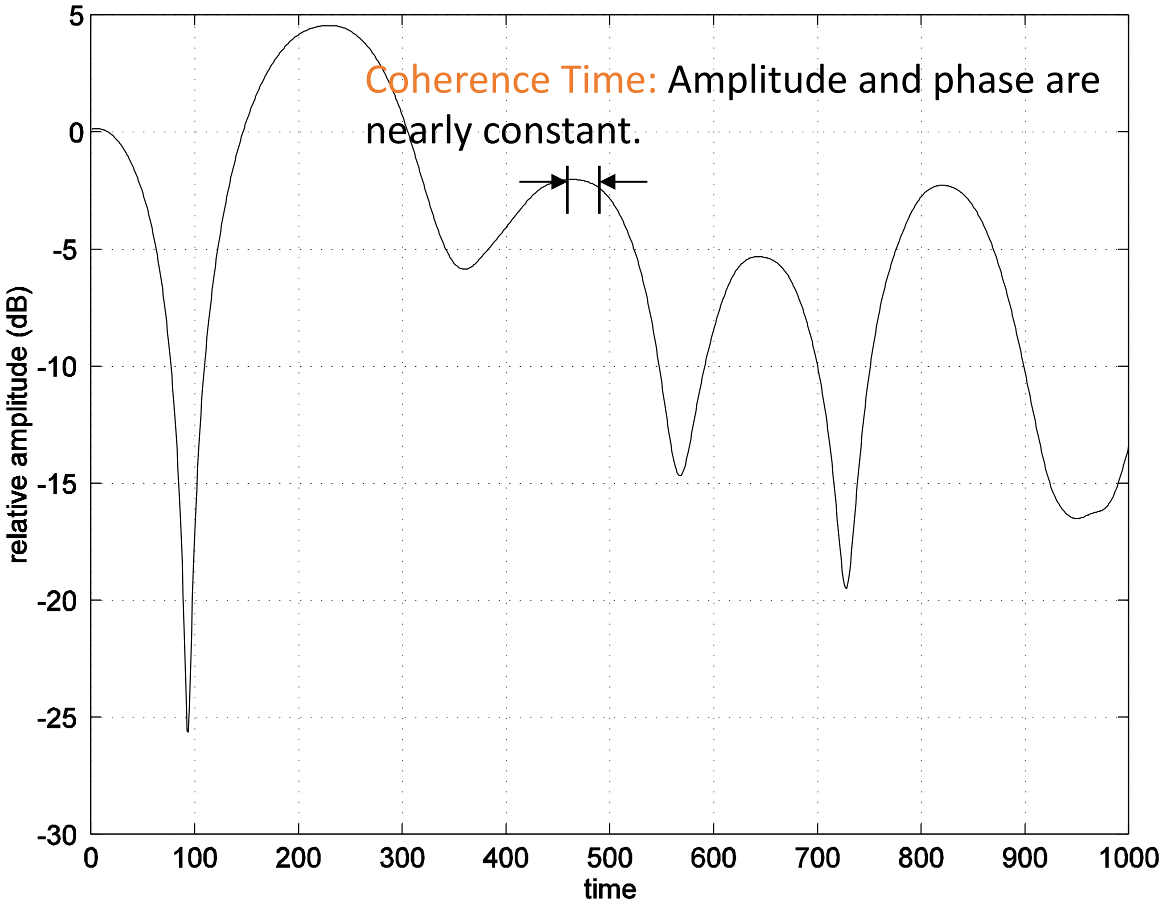
Channel Coherence Time

- The *channel coherence time* T_c is defined as the time interval over which the channel can be approximated as constant.

$$T_c = \frac{1}{2f_d}$$

- In terms of distance, it is

$$d_c = vT_c = \frac{1}{2}v\frac{c}{f_c v} = \frac{\lambda}{2}$$



Doppler spectrum

- Doppler spread is a measure of the *spectral broadening* caused by motion.
  - If the baseband signal bandwidth $B_s \gg f_d$ then the effect of Doppler spread is negligible at the receiver and the channel is *slow fading*.
  - If $B_s < f_d$ then the channel is *fast fading* and the Doppler spread severely distorts the received signal, which often results in an irreducible BER and synchronization problems.
- Similar considerations can be made in terms of symbol duration
  - A channel is *slow fading* if $T_c > T$.
  - A channel is said to be *fast fading* if $T_c < T$.

Fading channel example

- Consider a transmission at $f_c = 2.1$ GHz in a suburban aerea (delay spread $\sigma_\tau = 2\ \mu s$) to a user moving at a speed 90 km/h $\Longrightarrow v = 25$ m/s. The signal bandwidth is $B_S = 2$ MHz $\Longrightarrow$ the symbol time can be approximated as $T \sim \frac{1}{B_S} = 500$ ns.
- The Doppler spread is

$$f_d = \frac{f_c v}{c} = \frac{2.1 \cdot 10^9 \cdot 25}{3 \cdot 10^8} = 175 \text{ Hz} \Longrightarrow T_c = \frac{1}{2 f_d} \sim 3 \text{ ms.}$$

- The channel coherence bandwidth is $B_c = \frac{1}{\sigma_\tau}$ =500 kHz.
- The channel is *slow* ($B_s \gg f_d$ or $T \ll T_c$) and *frequency-selective* ($B_s > B_c$ or $T < \sigma_\tau$).

Small-scale fading recap

Small-Scale Fading
(Based on multipath time delay spread)

Flat Fading
1. BW of signal < BW of channel
2. Delay spread < Symbol period

Frequency Selective Fading
1. BW of signal > BW of channel
2. Delay spread > Symbol period

Small-Scale Fading
(Based on Doppler spread)

Fast Fading
1. High Doppler spread
2. Coherence time < Symbol period
3. Channel variations faster than base-band signal variations

Slow Fading
1. Low Doppler spread
2. Coherence time > Symbol period
3. Channel variations slower than baseband signal variations

Small-scale fading recap