Towards Logical Time

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Outline

Steps to uncover
the inner structure
of distributed computations,
and to reason about them

- Models of distributed computations
- On the Notion of time
- Timestamping

Modelling Distributed Computations

Processes and Events

Our target system is made up of a (static) set of n processes $\{p_1, p_2, \ldots p_n\}$, each placed at a different node.
Nodes are connected by channels.
The execution of each process can be modeled by a sequence of **events**. Events can be:

- **Internal events** - significant actions, "state changes"
- **Communication events** - send/receive of a message (typically in an asynchronous way)

Process History

Each single process is **sequential**, i.e. its events are executed one after the other, i.e. they are **totally ordered**.
Usually, the k-th event executed by process p_i is indicated as e^k_i

The sequence of all the events executed by a process is named as its **history**: $\boldsymbol{h}_i = e^1_i, e^2_i, \ldots e^k_i \ldots$

A *prefix* of a process history corresponds to a progression point in the local execution.

Messages and Precedence

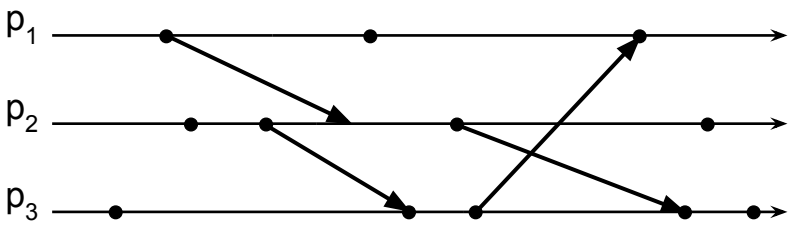
For any message m in the computation, handled in asynchronous communication, we can define an ordering on the related events:

$send(m)$ happens before $receive(m)$

With other types of communication, e.g. with “synchronous” communications (rendez-vous), these two operations are considered to logically happen at the same time, so they cannot be totally ordered.

Space-Time Diagrams

A distributed execution can be graphically depicted with *space-time diagrams*:



Consistent Cuts (I)

A set of prefixes for all process histories is named **cut**.
The set with the last event for each prefix is the cut *frontier*.
A **consistent cut** is a concept to formalize the notion of "possible progression point" of a whole distributed execution.

A cut is *consistent* if it satisfies the following property:
for any message m such that $receive(m)$ is in the cut, $send(m)$ belong to the cut as well.

Ordering over (Consistent) Cuts

An ordering among cuts is plainly induced by the set inclusion relation!

Given two cuts C_1, C_2 $\quad C_1 \rightarrow C_2 \ iff \ C_1 \subset C_2$

Notably, some cuts could not be ordered...

Notion of Time

No Global Time/Clock

In a distributed system it is not possible to have clocks at all the nodes *in perfect synch*. Such a synchronization can be achieved within a certain *tolerance*.

As a result, we cannot rely on a reference global time.

Without a global time, we need to understand **how to relate events executed at different progression points.**

Towards Logical Time

The flow of control of a distributed execution is described by the **precedence relations** among its event.

Such precedence relations can substitute the notion of global time for the purpose of analyzing the structure of the overall execution.

Any actual precedence could be captured by a unique relation to express the notion of *potential causality*.

Happens Before (I)

One single relation can be defined to account for all precedences throughout the execution: $\xrightarrow{HB}$

It can be constructed according to the following rules:

HB0 (transitivity) - for three events $e, f,$ and g, if $e \xrightarrow{HB} f$ and $f \xrightarrow{HB} g$ then also $e \xrightarrow{HB} g$ holds

...

Happens Before (II)

...

HB1 (in-process ordering) - for events e, f in the same process p_i, if $e \xrightarrow{i} f$ then also $e \xrightarrow{HB} f$ holds

HB2 (asynch. comm.) - for any event $e = send(m)$ (non-blocking) and the corresponding event $f = receive(m)$ for the same m, $e \xrightarrow{HB} f$ holds

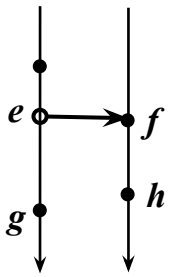
HB and Concurrency

The definition of the “Happens Before” relation let us also formalize the notion of concurrency:

Two events e, f are said *concurrent*, denoted as $e \,||\, f$, iff

$$\neg\left(e \xrightarrow{HB} f\right) \wedge \neg\left(f \xrightarrow{HB} e\right)$$

Happens Before with Rendez-vous



In case synchronous communication is present as well, another rule can be added to the definition:

HB3 (synch. comm.) - for any event $e = ssend(m)$ (blocking) and the corresponding event $f = receive(m)$ for the same m,
for any event g such that $e \xrightarrow{HB} g$ we have $f \xrightarrow{HB} g$ and
for any event h such that $f \xrightarrow{HB} h$ we have $e \xrightarrow{HB} h$

HB Adding Shared Variables

With shared variables as well, HB can be extended in different ways, depending on the ordering assumed for the read/write operations on the same variable V.

Let $\xrightarrow{ob(V)}$ be the total ordering of reads/writes on sh. variable V.

HB4-strong - for two different events e, f for operations on V, if at least one of them is a write event, then

if $e \xrightarrow{ob(V)} f$ then $e \xrightarrow{HB} f$

Weak HB with Shared Variables

Another extension considers only the precedence of a read operation w.r.t. the previous write.

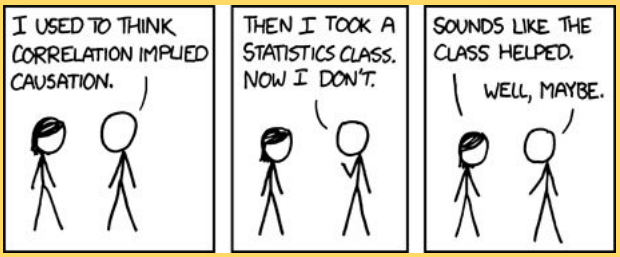
The version number of a variable V is incremented *at any write operation*.

$v(V,e)$ indicates the version number of V immediately after e.

HB4-weak - for a write event e and a read event f on the same V, if $v(V,e) = v(V,f)$ then $e \xrightarrow{HB} f$

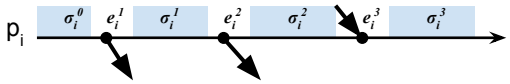
Pause for Thought

Happens Before vs Causality



Events and Local States

A *local* computation corresponds to a sequence of *local states*, with transitions triggered by events.

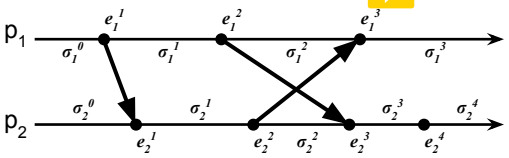


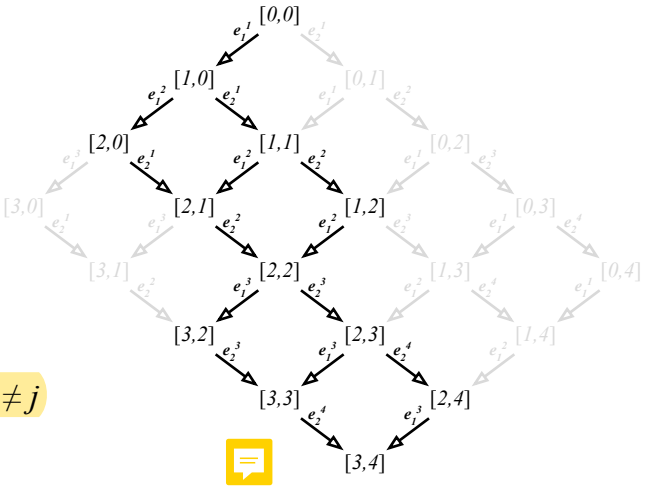
Let define $a \xrightarrow{HB'} b$ as HB + reflexivity, i.e. $\forall\, a,\ a \xrightarrow{HB'} a$

We can state $\sigma_i{}^a \xrightarrow{\sigma} \sigma_j{}^b \equiv e_i{}^{a+1} \xrightarrow{HB'} e_j{}^b$

From this definition, a distributed execution can be modeled as a partial order on the set of all states S: $(S, \xrightarrow{\sigma})$

Global States and Reachability Graph



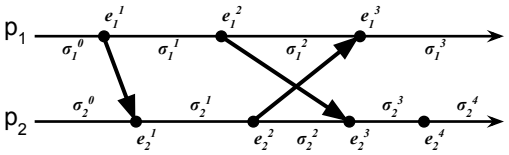


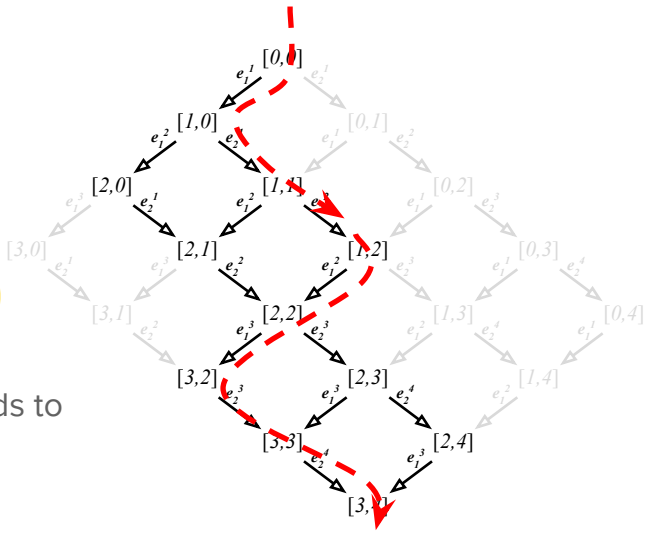
A "Global State" is a collection of one local state per process:

$\Sigma = [\sigma_i, \sigma_i, \ldots \sigma_n]$

A GS Σ is *consistent* if $\sigma_i \parallel \sigma_j \ \forall \ i, j, i \neq j$

Sequential Observations





A **sequential observation** is a *sequence* of all events/states, that respects their partial order.

A sequential observation corresponds to a path in the reachability graph.

Clocks for Logical Time

HB and Logical Clocks

If we need to deal with the relative orderings of event occurrences, we can substitute the “global time” with an index that relates to the ordering of events.

A **logical clock** $C(\cdot)$ is a means to map events onto a partial order, so that $e \xrightarrow{HB} f$ implies $C(e) < C(f)$ (*clock consistency property*)

The simplest co-domain for $C(\cdot)$: sequence of increasing integers. In this case, the corresponding time can be said *linear time*.

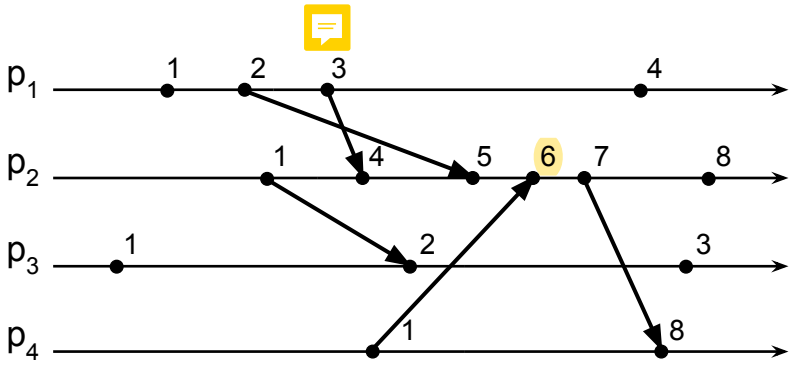
Lamport Timestamps

A classical logical clock T_{Lp}: an event e is associated with the length of the longest HB chain to reach e. How to assign such timestamps?

Each process p_i keeps a local counter $clock_i$;
It is initialized to 0, and updated according to the following rules:

1. for all events but *receive*s, $clock_i = clock_i + 1$ and then $T_{Lp}(e) \leftarrow clock_i$
2. On *send* events, its value is piggybacked in msg m - call it $ts(m)$
3. With *receive*s, $clock_i = max(clock_i, ts(m)) + 1$ and then $T_{Lp}(e) \leftarrow clock_i$

Lamport Timestamps - Example



Lamport Timestamps - Properties

First, the clock consistency: $e \xrightarrow{HB} f$ implies $T_{Lp}(e) < T_{Lp}(f)$ by construction

By contraposition, $T_{Lp}(e) \leq T_{Lp}(f)$ implies $\neg\left(f \xrightarrow{HB} e\right)$

I.e., if $T_{Lp}(e) \leq T_{Lp}(f)$, e happened before f or $e \,||\, f$.

Moreover, $T_{Lp}(e) = T_{Lp}(f)$ implies $e \,||\, f$.

Problem: it is possible to have $(T_{Lp}(e) < T_{Lp}(f)) \wedge \neg\left(e \xrightarrow{HB} f\right)$!!!

This means that T_{Lp} cannot be used to check precedence/causality!

Lamport Timestamps - Extensions

The algorithm for Lamport timestamps can be easily extended to deal with synchronous communication and use of shared variables.

Totally Ordered Lamport Timestamps

The linear time T_{Lp} can be used to obtain a total order T_{Lt} over all the events, such that it would be *consistent* with the HB relation.

For process p_i, T_{Lt} can be defined as the pair (T_{Lp}, i) and the total order states that $(a, b)<(c, d) \;\; iff \;\; (a<c) \vee ((a=c) \wedge (b<d))$

In practice, T_{Lt} can be an integer *conveniently* obtained as $T_{Lp} \ll B + i$, with $B=\lceil log_2 n \rceil$ and n number of processes ($\ll B$: shift left of B bits).

Towards Strong Clock Consistency

We'd like to have a clock able to indicate causal dependence,

i.e.: $C(e)<C(f) \;\Rightarrow\; e \xrightarrow{HB} f$

Possible solution: keep a *vector* counter V_i of n integers at each process p_i, so that $V_i[i]$ is the counter of events within p_i, and $V_i[j]$ corresponds to the most recent value of $V_j[j]$ as detected by p_i.

Whenever p_i sends a message, the most recent value of V_i is piggybacked in the message.

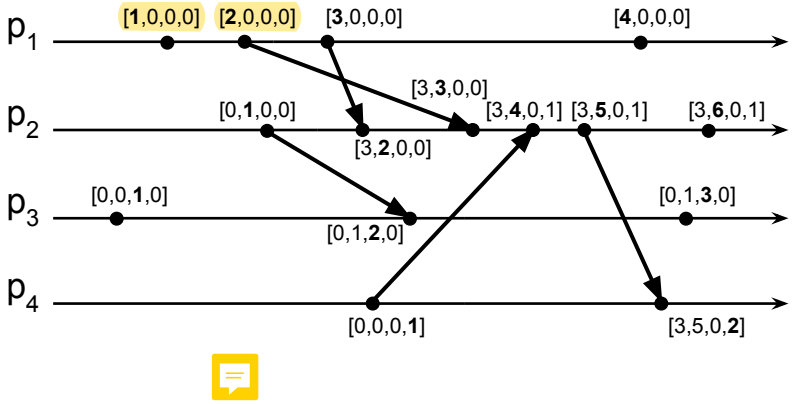
Algorithm for Vector Timestamps

At each event occurrence, the local vector counter V_i is updated, and its value is assigned to the event as its *vector* timestamp $T_V(e)$.

How to update local vector counters?

1. For all events, in the first place $V_i[i]=V_i[i]+1$
2. On *send* events, $T_V(e)$ is piggybacked in msg m - call it $ts(m)$
3. On *receive* events, $V_i=max_{compwise}(V_i, ts(m))$
4. Finally, $T_V(e) \leftarrow V_i$

Vector Timestamps - Example



How to Compare Vector Timestamps

Given two vector timestamps T_1 and T_2, we define:

$T_1 \leq T_2$ *iff* $T_1[k] \leq T_2[k] \quad \forall k \in [1, \ldots n]$

$T_1 < T_2$ *iff* $T_1 \leq T_2 \wedge T_1 \neq T_2$

$T_1 \parallel T_2$ *iff* $T_1 \not< T_2 \wedge T_2 \not< T_1$

The last two definitions relate to HB precedence and concurrency, as it will be shown.

Vector Timestamps - Properties

Clock consistency: $e \xrightarrow{HB} f$ implies $T_V(e) < T_V(f)$ by construction

Strong C.C. : $T_V(e) < T_V(f)$ implies $e \xrightarrow{HB} f$

Proof: by contraposition, let's show $\neg\left(e \xrightarrow{HB} f\right)$ implies $T_V(e) \not< T_V(f)$

$T_V(e) \not< T_V(f)$ means that, for at least one position i, $T_V(e)[i] > T_V(f)[i]$

Say e is in p_i, and f in a different p_j.

Just before e, p_i increases its $V_i[i]$, say to value t, thus $T_V(e)[i] = t$.

As $\neg\left(e \xrightarrow{HB} f\right)$, there is no way for value t to propagate to p_j,

thus $T_V(f)[i] < t$

Cons. Cuts & Vector Timestamps

Vector Timestamps let us check the consistency of a cut.

Let $F(C) = \{f_1, ...f_n\}$ be the set of events in the frontier of a cut C.

Let the "cut timestamp" be $\boldsymbol{T}_V(C) = max_{compwise}(\boldsymbol{T}_V(f_1), \ ... \ \boldsymbol{T}_V(f_n))$

C is consistent if $\forall i$, $\boldsymbol{T}_V(C)[i] = \boldsymbol{T}_V(f_i)[i]$

In a nutshell, the maximum i-th value must always be on the i-th process/position: **no other process in the cut knows my future!**

Cuts & Vector Timestamps - Check!