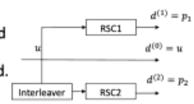
Turbo codes and LDPC

- In the channel capacity theorem, Shannon uses infinitely long codes, i.e. the code rate $R = k/n$ is fixed but $k \to \infty$ and $n \to \infty$.
- In practical system, the length of the code is limited because of the complexity of decoding: the performance of physical systems are far from Shannon theoretical limits.
- Around the turning of the century there have been two major breakthroughs:
  - Turbo codes (1993) OLUTION
  - Low-density parity check codes (LDPC, 1999).

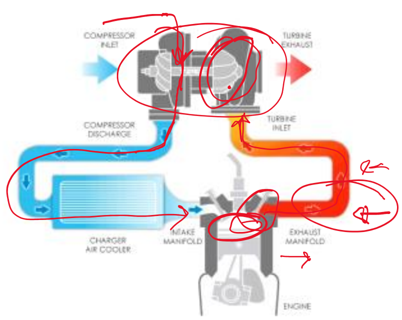
from Trellis and Turbo Coding, Schlegel and Perez, IEEE Press, 2004

Turbo codes

- The combination of coding and interleaving can be exploited to boost the performance of convolutional codes in general.
- *Turbo codes* are a particular example of concatenated codes, where two encoders are in parallel and the input data of the second encoder are first interleaved.
- Thanks to the interleaver, the decoder will have two independent replicas of the same data and can use both streams to decode the information sequence.
- Increase the redundancy of the transmitted information but also the *diversity* of the system.

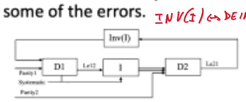


Turbo.....

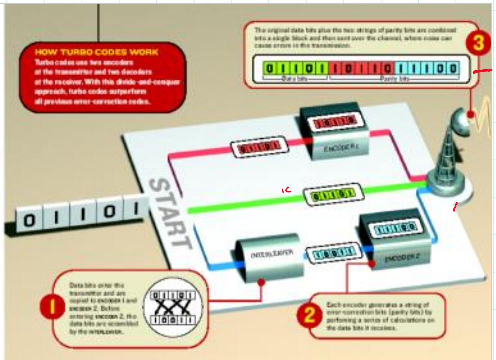


Turbo decoding

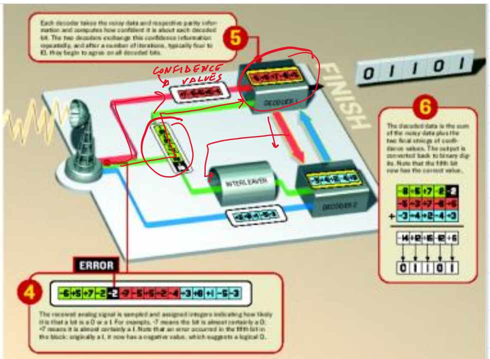
- The turbo decoder employs the cascade of two decoders, with the output of one decoder used as prior information to the next.
- Feedback in decoding circuit allows for multiple iterations and improves bit error performance.
- The process is iterative and at each step the combination of the two decoders corrects some of the errors. INV(I) ⟺ DEINTERLEAVE



Turbo codes (© IEEE spectrum)



Turbo codes (© IEEE spectrum)



Turbo codes: latency

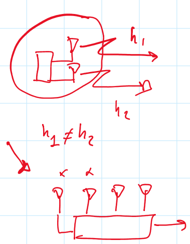
- Convolutional codes in general and turbo codes in particular have a problem with *latency* due to the presence of the interleaver and the iterative decoding process.
- At any given SNR there is a tradeoff between latency due to interleaver and QOS
  - Small block sizes (~300 bits) can be used for real time voice (medium-high BER can be tolerated).
  - Mid range block sizes (~4000 bits) used for video play back (low BER).
  - Large block sizes (~16000 bits) large latency, very low BER, useful for file transfer (very low BER).

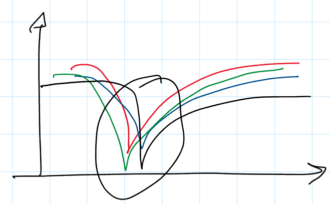
1st Source of delay is the interleaver at TRx
2nd Source of delay is " " " Rx
3rd Source " " is the iterative process

Spatial diversity

Receive diversity

- Among the many ways to obtain diversity:
  - *Frequency* and *time* diversity require expensive resources (bandwidth or time) and do not provide array gain.
  - *Space* diversity by means of multiple antennas does not sacrifice bandwidth or time and may provide array gain.
- *Array gain*: is the power *gain* achieved by using multiple antennas with respect to the single antenna case. The more correlated is the spatial channel the higher the potential array gain.
- *Diversity gain*: is the power gain due to the exploitation of the diversity of the spatial channel. It is maximum when the spatial channel is uncorrelated.



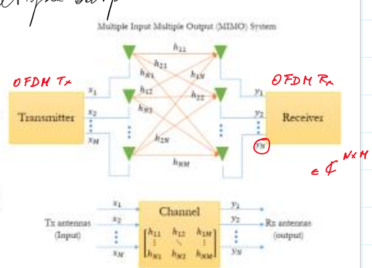


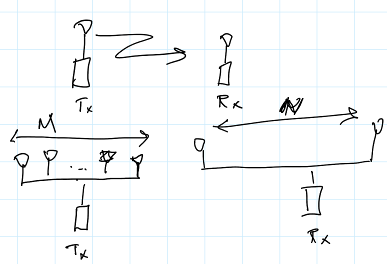
Array gain means the average SNR ↑
Diversity gain changes the distributions of the gain in a more favorable way

MIMO channel

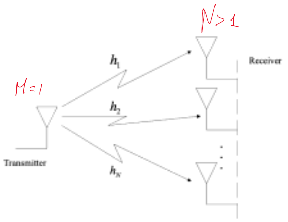
Multiple-input
Multiple output

- Multiple-input multiple-output systems are systems where where both the transmitter and the receiver are equipped with several antennas.
- *Narrowband* assumption: the channel linking the n-th receive antenna with the m-th transmit antenna is the scalar $h_{n,m}$





SIMO channel: receive diversity



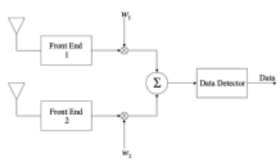
- $N > 1$ antennas at the receiver, $M = 1$ at the transmitter.
- The decision variable at the i-th receive antenna is
  $$x_i(m) = h_i c_m + n_i(m)$$
- The signals received at the N antennas are combined together and the decision variable is
  $$z(m) = w_1 x_1(m) + \cdots + w_N x_N(m)$$

w combining weights

$$h = \begin{bmatrix} h_1 \\ h_2 \\ | \\ h_N \end{bmatrix}$$

Maximal ratio combining (MRC)

- The decision variable is
  $$z(m) = \sum_{i=1}^{N} w_i h_i c_m + \sum_{i=1}^{N} w_i n_i(m)$$
- For the i-th antenna the optimal weight is
  $$w_i = h_i^*$$
- In the 1x2 case, the signal-to-noise ratio is
  $$SNR = (|h_1|^2 + |h_2|^2)\frac{A}{\sigma^2}$$



selection combining
Set to 1 the coefficient for which the channel gain is maximum & all the other coeff.

1x2 channel

$$z(m) = w_1 h_1 c_m + w_2 h_2 c_m + w_1 n_1(m) + w_2 n_2(m)$$
$$\overset{[opt.]}{=} (h_1^* h_1 + h_2^* h_2) c_m + h_1^* n_1(m) + h_2^* n_2(m)$$
$$= \underbrace{(|h_1|^2 + |h_2|^2) c_m} + \underline{h_1^* n_1(m)} + \underline{h_2^* n_2(m)}$$

$$SNR_{1\times 2} = \frac{(|h_1|^2 + |h_2|^2)^2 A}{\cancel{(|h_1|^2 + |h_2|^2)}\sigma^2} = (|h_1|^2 + |h_2|^2)\frac{A}{\sigma^2}$$

$$SNR_{1\times 1} = |h_1|^2 \frac{A}{\sigma^2}$$