5th lesson

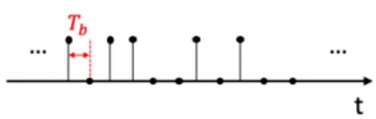
Monday, 12 October 2020 10:39

Digital communications

How can we transmit a sequence of bits?

- What happens if we want to transmit a *sequence of bits* in the place of an analog signal?

$d_k = \cdots 1,0,1,1,0,0,1,0,1,0,1,0,0, \ldots$



$$\sum_i d_k \delta(t - kT_b)$$

- A train of delta occupies infinite bandwidth, before transmission the bits need to be passed through a low pass filter.
- For reasons already discussed, any signal transmitted in the air needs to be translated in frequency.

How can we transmit a sequence of bits?

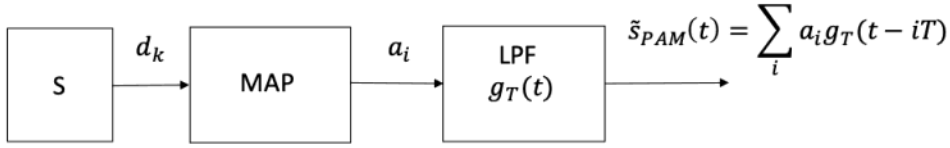
- Each bit of the sequence can be modelled as an equiprobable random variable

$$P\{d_k = 0\} = P\{d_k = 1\} = \frac{1}{2}$$

  so that $E\{d_k\} = \frac{1}{2}$.
- In general, to save energy it is better to transmit 0-mean information.
- Bits d_k are mapped to 0-mean *information symbols*: $a_i = 2d_i - 1$.
- One information symbol can be used to map more than just one bit.

Pulse amplitude modulation

- Pulse amplitude modulation (PAM) is the modulation obtained by
  1. Mapping the bits d_k to the information symbols a_i
  2. Filtering the symbols with a low pass filter with impulse response $g_T(t)$



- Since the mapper can map a sequence of m bits on just one information symbol, the bit duration T_b and the symbol duration T may be different.

Pulse amplitude modulation

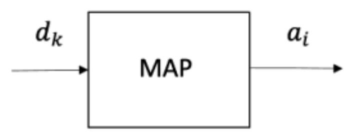
- The signal $\tilde{s}_{PAM}(t)$ is a *real* baseband signal that can be modulated at any frequency f_c

$$s_{PAM}(t) = \sum_i a_i g_T(t - iT)\cos(2\pi f_c t)$$

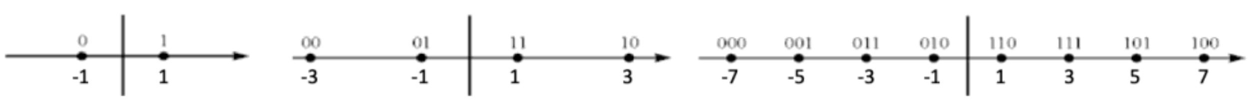
- The PAM signal is equivalent to an analog DSB where the modulating (and complex envelope) signal $m(t)$ is

$$m(t) = \sum_i a_i g_T(t - iT)$$

PAM: symbol mapping

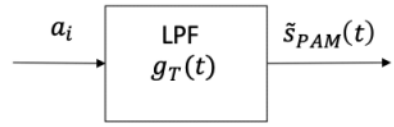


- The mapper block associates a sequence of m bits to a symbol.
- The symbol constellation contains $M = 2^m$ bits, $m = \log_2 M$.
- If the source generates bits with a rate $R_b = \frac{1}{T_b}$, the mapper outputs symbols with a rate m times slower, i.e. $R = \frac{R_b}{m} = \frac{R_b}{\log_2 M}$ or, in terms of bit and symbol timing, $T = T_b \log_2 M$
- Usually bit-to-symbol mapping is performed so that $E\{a_i\} = 0$.

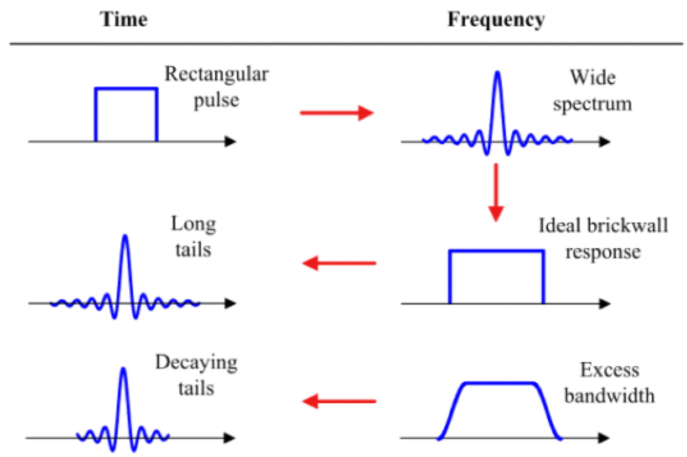


$M = 2$, $m = 1$ $\quad$ $M = 4$, $m = 2$ $\quad$ $M = 8$, $m = 3$

PAM: pulse shaping



- Intuitively, the choice of the impulse response of the low-pass pulse shaping filter determines the bandwidth of the PAM signal.
- If the pulse shape has duration longer than 1 symbol time T, the spectrum is more compact but the energy of one symbols is spread over several intervals.



PAM: power spectral density

- A PAM signal is modelled as a *stochastic process* because the symbols a_i are samples of a discrete-time random process.
- The bandwidth occupied by a stochastic process is measured by its *power spectral density* (Fourier transform of its autocorrelation function).
- The PSD of the PAM signal $\tilde{s}(t)$ is

$$S_{\tilde{s}}(f) = \frac{1}{T} S_a(f) |G_T(f)|^2$$

where $S_a(f)$ is the PSD of a_i and $G_T(f)$ is the frequency response of the transmit filter $g_T(t)$.

From now on, we omit the tilde for ease of notation.

PAM: power spectral density

- $S_a(f)$ is computed as the Fourier transform of the autocorrelation function $R_a(m)$ of the stationary, discrete, independent process a_i.

$$R_a(m) = E\{a_i a_{i+m}\} = \begin{cases} E\{a_i^2\} = A & m = 0 \\ (E\{a_i\})^2 & m \neq 0 \end{cases}$$

when symbols are zero-mean, it is

$$R_a(m) = A\delta(m)$$

and

$$S_a(f) = A$$

$$R_e(m) = E\{a_i a_{i+m}\} = \begin{cases} E\{a_i^2\} = A & m = \phi \\ E\{a_i\} E\{a_{i+m}\} & m \neq 0 \end{cases} \Rightarrow R_e(m) = A\delta(m)$$

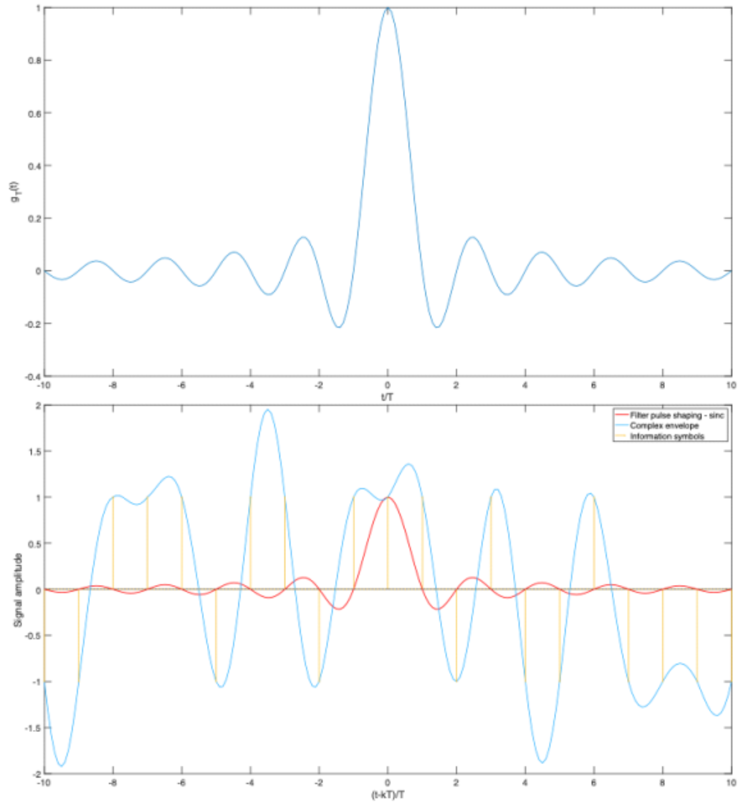
$E\{a_i^2\} = A$ mean square value

$E\{a_i\} = E\{a_{i+m}\}$ stationarity

$E\{a_i\} = \phi$ symbols are ϕ mean

PAM: pulse shaping

- The most compact spectrum is obtained when $G_T(f) = \mathrm{rect}(fT)$, which in the time domain corresponds to a *sinc*.
- The pulse shape of a *sinc* spans an interval of several symbols.
- One single symbol 'mixes' its information with several adjacent symbols.
- This type of interference is denominated *inter-symbol interference* (ISI).

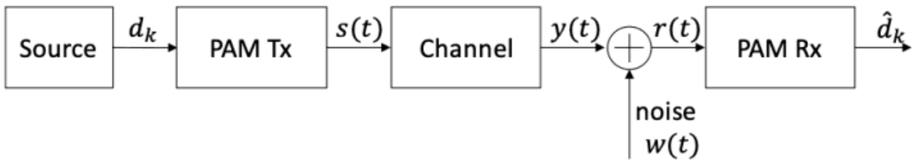


PAM: occupied bandwidth

- Because of the espression of the PSD, the bandwidth occupied by the PAM signal depends on $G_T(f)$, the frequency response of the transmit filter.
- There is a trade-off to make:
    - compact spectrum → large amount of interference in the time domain (Extreme choice: a *rect* in the frequency domain and a *sinc* in time).
    - wide spectrum → most of the symbol energy is contained within one symbol interval (Extreme choice: a *rect* in the time domain and a *sinc* in frequency).

PAM: receiver architecture

- PAM system block diagram



  - The propagation channel is in general modelled as a LTI filter with impuls response $h(t)$. When the channel is ideal, it is $h(t) = \delta(t)$.
  - The noise term is a white, zero-mean, Gaussian stationary process with PSD $S_w(f) = N_0/2$ ($S_w(f) = 2N_0$ for its complex envelope).
  - The receiver's task is to reconstruct the sequence of transmitted bits from the received signal $r(t)$.