11th lesson

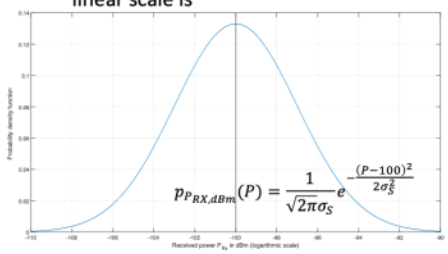
Monday, 2 November 2020 09:45

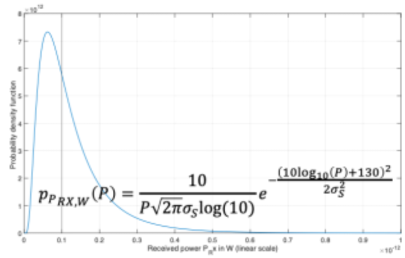
Large-scale fading: shadowing

- Let's consider a channel with path-loss and shadowing ($\sigma_S = 3$ dB) only. The received power P_{RX} is

$$P_{RX} = P_{TX} A_{PL} A_S$$

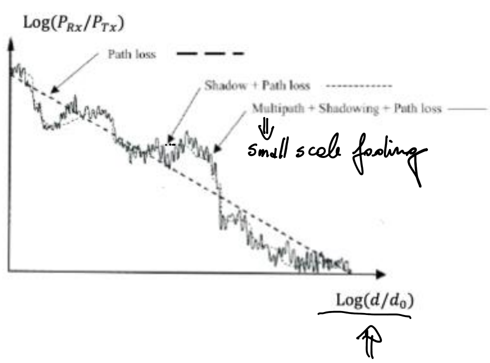
- Assume that $P_{TX} A_{PL} = -100 \text{ dBm} = -130 \text{ dBW} = 10^{-13}$ W .
- Because of shadowing, P_{RX} is a random variable and its distribution in dBm and linear scale is





Large-scale fading

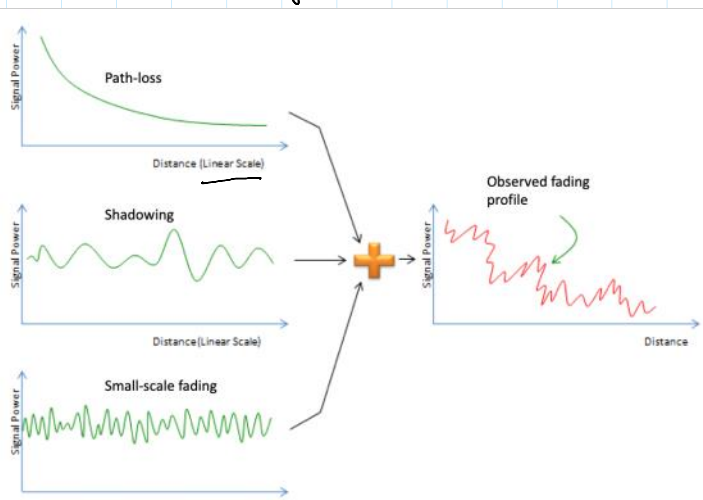
- The received power in dB is computed as
  $P_{RX}[dBm] = P_{TX}[dBm] + \underbrace{A_{PL}[dB] + A_S[dB]}_{A_{LS}} + A_{SS}[dB]$
- A_{PL} deterministic depends on the distance d.
- A_S random, log-normally distributed.
- A_{SS} is the attenuation due to small scale fading, which fluctuates rapidly with the distance



$$P_{Rx} = P_{Tx} \cdot A_{PL} \cdot A_S \cdot A_{SS}$$

$$[P_{Rx}]_{dBm} = P_{Tx_{dBm}} + A_{PL_{dB}} + A_{S_{dB}} + A_{SS_{dB}}$$

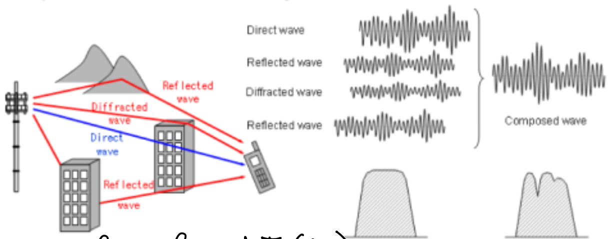
$$A_{PL} = \frac{P_{Rx}}{P_{Tx}} \propto \Gamma \cdot d^{-n} \Rightarrow \log(\Gamma d^{-n}) = \log\Gamma + \log d^{-n} = \log\Gamma - \overset{\text{slope of the curve}}{n} \log d$$



Small-scale fading

$\lambda = \frac{c}{f} = \frac{300.000\ Km/s}{2 \cdot 10^9\ s^{-1}} = \frac{3 \cdot 10^8}{2 \cdot 10^9}\ m = 15\ cm$

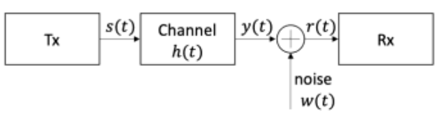
- Small-scale fading: accounts for the random variations of the instantaneous received power over distances of the order of a wavelength.
- Because of the various propagation phenomena, a large number of waves, each carrying a replica of the transmitted signal, arrives at the receiver.



Direct wave = line of sight (los)

Propagation channel

- The propagation channel can be modeled as a LTI filter.



- The filter impulse response $h(t)$ depends on the small-scale fading characteristics.

Ideal channel $h(t) = \delta(t)$

$\Updownarrow$

$H(f) = 1$

A LTI system is fully described by its impulse response $h(t) \rightleftharpoons H(f)$

$x(t) \to \boxed{h(t)} \to y(t)$

$h(t)$

$y(t) = x(t) \otimes h(t)$

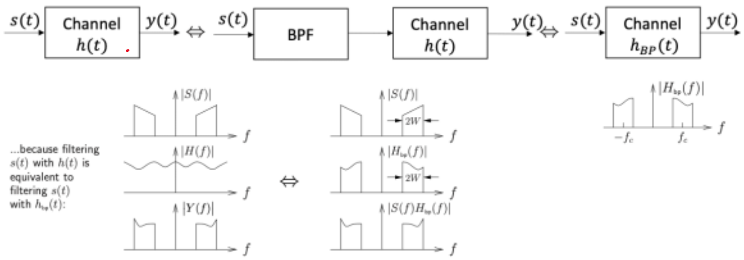
$\Updownarrow \quad \Updownarrow \quad \Updownarrow$

$Y(f) = X(f) \cdot H(f)$

ONLY true for LTI systems !!

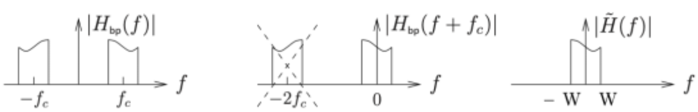
Channel complex envelope

- Given the passband signal $s(t)$, with spectrum centered in f_c and bandwidth $B = 2W$, these three systems are equivalent



Channel impulse response

- The complex envelope of the channel is $\tilde{h}(t) \rightleftharpoons \tilde{H}(f) = H_{\mathrm{bp}}(f + f_c)$.



- The corresponding passband channel is
$$h(t) = \mathcal{R}e\{\tilde{h}(t)e^{j2\pi f_c}\}$$
- The complex envelope of the received signal is
$$\tilde{y}(t) = \tilde{s}(t) \otimes \tilde{h}(t)$$

Channel's complex envelope

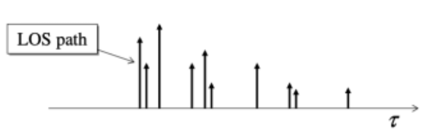
- The complex envelope (baseband) of the channel is

Large-scale fading, path-loss and shadowing

$$\tilde{h}(t) = A_{LS} \sum_{w=0}^{N_w - 1} \alpha_w e^{j\phi_w} \delta(t - \tau_w)$$

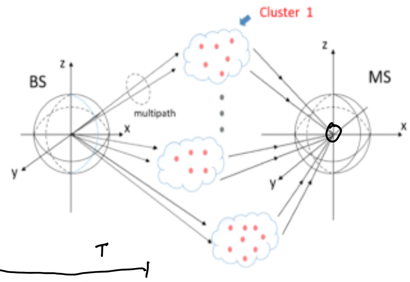
Small-scale fading, multipath

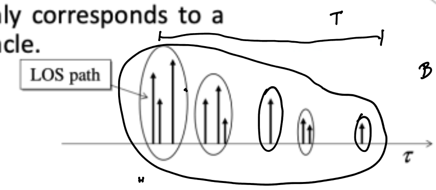
where A_{LS} is large-scale fading attenuation, N_w is the number of waves arriving at the receiver, α_w, ϕ_w and τ_w are attenuation, phase and delay of the w-th wave.



Small-scale fading

- Propagating from the transmitter to the receiver, signals may encounter several major obstacles.
- Depending on their delay, *the various paths can be grouped in different clusters*, where each cluster roughly corresponds to a specific obstacle.





$B \propto \frac{1}{T}$

Small-scale fading: Rayleigh distribution

- If there is a sufficiently large scatter, for each cluster at the receiver we will have the sum of many different replicas of the signal, each with approximately the same delay and different complex gains.
- Because of the *Central limit theorem*, the complex gain of each cluster can be modelled as a *complex Gaussian* variable irrespective of the distribution of the individual components.
  - Phase ϕ is uniformly distributed in $[0,2\pi]$.
  - Amplitude α is *Rayleigh* distributed, if there is no los, or *Rician* distributed if there is los.
- Central limit theorem: when a sufficiently large number of random variables are added, their sum tends to be normally distributed regardless of the original distribution of the random variables.

For a given cluster there are W_c replicas arriving at the receiver, with W_c a large number $\Rightarrow$ CLT applies

$a_c = \sum_w^{W_c} \alpha_w e^{j\phi_w}$ $\Rightarrow$ because of CLT a is a complex Gaussian distributed random variable.

$a_c \in \mathcal{N}(0, 2\sigma^2)$

$\Rightarrow \alpha_c = |a_c|$ is Rayleigh distributed

$\mu_{\alpha_c} = E\{\alpha_c\} = \int_{-\infty}^{+\infty} \alpha_c \, p(\alpha) \, d\alpha = \sigma\sqrt{\pi/2}$

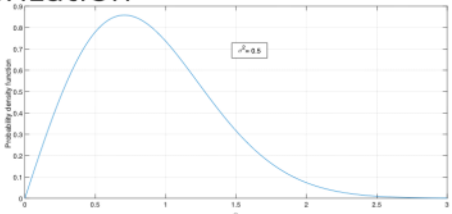
mean square value of α_c: $E\{\alpha_c^2\} = \int_{-\infty}^{+\infty} \alpha_c^2 \, p(\alpha) \, d\alpha = 2\sigma^2$

$2\sigma^2$ is the variance of the corresponding complex Gaussian variable a

Channel gain characterization

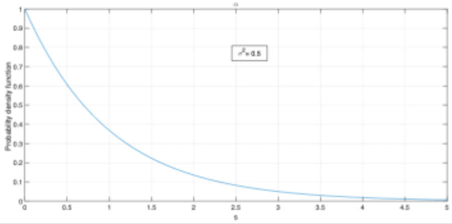
- The distribution for channel amplitude α is *Rayleigh*

$$p(\alpha) = \begin{cases} \frac{\alpha}{\sigma^2} e^{-\frac{\alpha^2}{2\sigma^2}} & \alpha \geq 0 \\ 0 & \alpha < 0 \end{cases}$$



- The distribution for channel power $s = \alpha^2$ is *exponential*

$$p(s) = \begin{cases} \frac{1}{2\sigma^2} e^{-\frac{s}{2\sigma^2}} & s \geq 0 \\ 0 & s < 0 \end{cases}$$



Small-scale fading

- The channel impulse response (CIR) can be modelled as

$$h(t) = A_{LS} \sum_{\ell=0}^{N_{C-1}} \alpha_\ell e^{j\phi_\ell} \delta(t - \tau_\ell)$$

where $\alpha_\ell e^{j\phi_\ell}$, the complex gain of the ℓ-th cluster is the sum of the complex gains of all the paths belonging to the cluster.

- Let $s(t)$ be the transmitted signal, neglecting the noise, the complex envelope of the signal at the receiver is

$$y(t) = s(t) \otimes h(t) = A_{LS} \sum_{\ell=0}^{N_{C-1}} \alpha_\ell e^{j\phi_\ell} s(t - \tau_\ell)$$