Communication systems

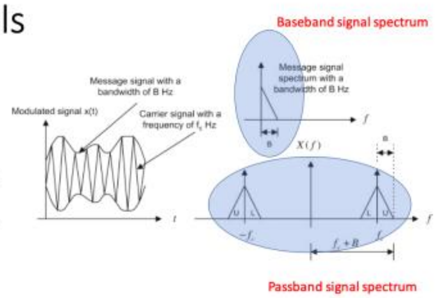
Prof. Marco Moretti
marco.moretti@unipi.it
ELECTRONICS AND COMMUNICATIONS SYSTEMS
COMPUTER ENGINEERING

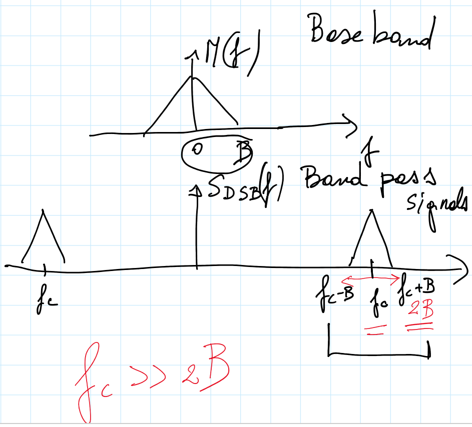
2nd lesson

Monday, 28 September 2020 16:27

Passband signals

- The vast majority of communication systems are passband systems.
- The transmitted signal $s(t)$ has its energy concentrated in a bandwidth $2B$ centered around some nominal carrier frequency f_c and above and relatively far away from dc.
- For a *passband* signal it is
  $$f_c \gg 2B$$





Complex envelope of a passband signal

- The passband modulator-demodulator can be drawn in a more compact form by using complex notation.
- Any passband signal $s(t)$ can be represented as

$$s(t) = \mathrm{Re}\{\tilde{s}(t)e^{j2\pi f_c t}\} = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t)$$

  where $\tilde{s}(t) = s_I(t) + js_Q(t)$ is the *complex envelope* of the signal with $s_I(t)$ and $s_Q(t)$ the in-phase and quadrature components.
- Complex envelope for known modulated signals
  - $\tilde{s}_{DSB}(t) = A_c m(t); s_I(t) = A_c m(t), s_Q(t) = 0.$
  - $\tilde{s}_{QAM}(t) = A_c m_1(t) + jA_c m_2(t); s_I(t) = A_c m_1(t),\ s_Q(t) = A_c m_2(t).$

$\tilde{s}(t)$ complex envelope

$s(t) = \mathrm{Re}\{\tilde{s}(t)e^{j2\pi f_c t}\}$

$$s(t) = \Re\left\{\tilde{s}(t)\, e^{j2\pi f_c t}\right\}$$

$$\tilde{s}(t) = \tilde{s}_R(t) + j\tilde{s}_I(t)$$ is a complex signal

$$s(t) = \Re\left\{\left(\tilde{s}_R(t) + j\tilde{s}_I(t)\right)\left(\cos(2\pi f_c t) + j\sin(2\pi f_c t)\right)\right\}$$

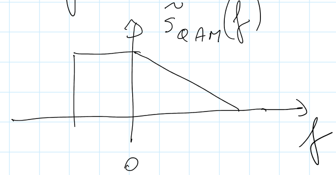
$$s(t) = \tilde{s}_R(t)\cos 2\pi f_c t - \tilde{s}_I(t)\sin(2\pi f_c t)$$

Exemple : A QAM signal

$$s_{QAM}(t) = A_c m_1(t)\cos(2\pi f_c t) - A_c m_2(t)\sin(2\pi f_c t)$$

$$\Downarrow$$

$$\tilde{s}_{QAM}(t) = A_c m_1(t) + jA_c m_2(t)$$



In-phase $\Downarrow$ Quadrature $\Downarrow$

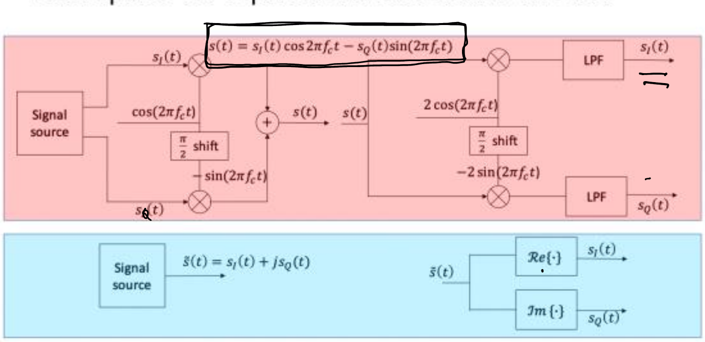
$$\tilde{s}(t) = \tilde{s}_R(t) + j\tilde{s}_I(t) = s_I(t) + js_Q(t)$$

$\Uparrow$ Real $\Uparrow$ immaginary

Complex envelope of a passband signal

- The complex envelope is an equivalent baseband representation of a passband signal.
- Employing the baseband equivalent has several benefits:
  - A baseband model is simpler to study, since it removes the effects of the carrier frequency from the signal model.
  - A baseband model can be numerically simulated with much lower computation than a passband model because the bandwidth and, as a consequence, the sampling rate is much lower.
  - A baseband model is often the basis for a digital implementation of a bandpass communications system.

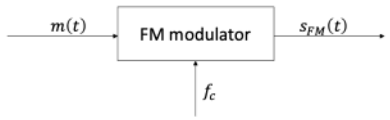
Bandpass vs. equivalent baseband model



Analog communications: frequency modulation (FM)

- In the FM modulation, the message is embedded in the signal phase

$$s_{FM}(t) = A_c \cos\left(2\pi f_c t + \underbrace{2\pi k_f \int_{-\infty}^{t} m(\tau)\, d\tau}_{\varphi(t)}\right)$$

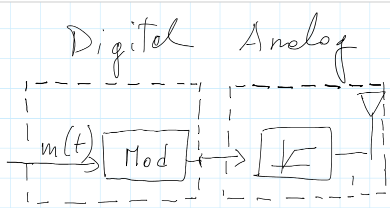


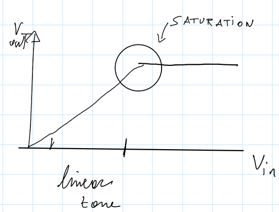
$$\tilde{s}_{FM}(t) = A_c e^{j2\pi k_f \int_{-\infty}^{t} m(t)\, d\tau}$$

$$\mathrm{Re}\left\{ \tilde{s}_{FM}(t) e^{j2\pi f_c t} \right\} = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)\, d\tau\right)$$

FM radio

- Advantages:
  - Constant envelope modulation: greatly simplifies amplifier design.
  - By properly adjusting FM parameters, it is possible to trade spectral efficiency with energy efficiency.
  - Commercial FM transmits an audio signal with bandwidth $B = 15$ kHz over a bandwidth of approx 200 kHz.





Amplifier gain curve

FM radio

- The complex envelope of a FM signal is

Phase $\phi(t)$ of the complex envelope

$$\tilde{s}_{FM}(t) = A_c e^{j2\pi k_f \int_{-\infty}^{t} m(\tau) d\tau}$$

- Frequency deviation of an FM signal

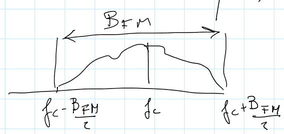
$$f_d(t) = \frac{1}{2\pi} \frac{d}{dt} \phi(t)$$

- Maximum frequency deviation $\Delta f = \max\{|f_d(t)|\} = k_f \max\{|m(t)|\}$
- Modulation index $m_f = \frac{\Delta f}{B_m}$ bandwidth of the modulating signal

$$\frac{d}{dt}\phi(t) = 2\pi k_f \underline{m(t)} \Rightarrow \frac{1}{2\pi} \frac{d}{d\phi(t)} \phi(t) = k_f m(t)$$

$$\frac{d}{dt}\varphi(t) = 2\pi k_f \, \underline{m(t)} \quad \Rightarrow \quad \frac{1}{2\pi}\frac{d}{d\,\phi(t)}\varphi(t) = k_f\, m(t)$$

$$\boxed{\Delta F \neq B_{FM}}$$



FM signal with a modulating sinusoid

- Let $m(t)$ be a sinusoid

$$m(t) = V_m \cos(2\pi f_m t)$$

- The FM signal is

$$s_{FM}(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} V_m \cos(2\pi f_m \tau)\, d\tau\right)$$

$$= A_c \cos\left(2\pi f_c t + \cancel{2\pi} k_f V_m \frac{\sin(2\pi f_m t)}{\cancel{2\pi} f_m}\right) = A_c \cos\left(2\pi f_c t + \frac{K_f V_m}{f_m} \sin(2\pi f_m t)\right)$$

$$= A_c \cos(2\pi f_c t + m_f \sin(2\pi f_m t))$$

Modulation index

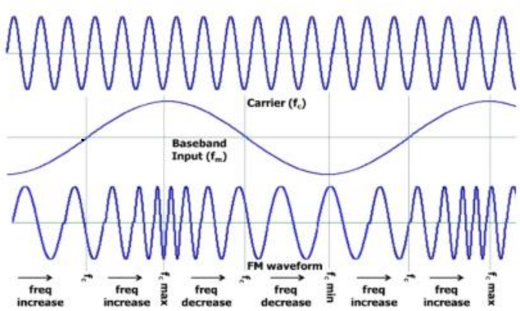
B_m

- Complex envelope is

$$\tilde{s}_{FM}(t) = A_c e^{j m_f \sin(2\pi f_m t)}$$

$$\max |V_m \cos(2\pi f_m t)| = V_m \qquad B = f_m \quad \Rightarrow \quad \frac{K_f V_m}{B} = m_f$$

Frequency modulation



$\Leftarrow$ B_{FM}?

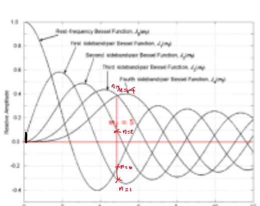
$$\tilde{s}_{FM}(t) = A_c e^{j m_f \sin(2\pi f_m t)}$$ periodic of period $T_m = 1/f_m$

FM signal spectrum

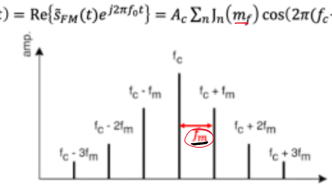
- Expoliting the periodicity of $\tilde{s}_{FM}(t) = A_c e^{j m_f \sin(2\pi f_m t)}$, the complex envelope can be written as a sum of Fourier coefficients

$$\tilde{s}_{FM}(t) = A_c \sum_n J_n(m_f) e^{j2\pi n f_m t}$$

Bessel function of the first type of order n



$$s_{FM}(t) = \mathrm{Re}\{\tilde{s}_{FM}(t) e^{j2\pi f_0 t}\} = A_c \sum_n J_n(\underline{m_f}) \cos(2\pi(f_c + n f_m)t)$$



FM signal spectrum

- It is is impossible to calculate a closed form expression for FM spectrum
- A good approximation is the Carson bandwidth rule

$$B_{FM} \approx 2(m_f + 1)B = 2(\Delta f + B)$$

- Any frequency modulated signal has an *infinite* number of sidebands and hence an infinite bandwidth but most of the energy (98% or more) is concentrated within the bandwidth defined by Carson's rule.
- In commercial mono FM we have $B_{FM} \approx 180$ kHz
  - $B = 15$ kHz (high quality audio)
  - $\Delta f = 75$ kHz
  - $m_f = 5$

$m_f = \frac{\Delta F}{B} = \frac{75}{15} = 5$

$B = 2(1+m_f)\cdot B =$

$\underline{180 \text{ kHz}}$

88 MHz → 108 MHz

$\underbrace{\text{88 MHz → 108 MHz}}_{}$ 20 MHz bandwidth reserved for commercial FM

Guard interval between adjacent channels

Each channel is $(180 + 20) = 200$ KHz

$B = 15$KHz; $\Delta F = 75$ KHz

$m_f = \frac{75}{15} = 5 \Rightarrow B_{FM} = 2(5+1)\cdot 15 \text{ KHz} = 2(75+15) = 180 \text{ KHz}$

FM receiver

- Neglecting the effect of noise and channel, the complex envelope of the received signal is

$$\tilde{v}(t) = A_c e^{j2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau}$$

- The modulating signal can be recovered by differentiating the phase of $v(t)$

$$\hat{m}(t) = \frac{1}{2\pi k_f}\frac{d}{dt}\angle\tilde{v}(t)$$

Conceptual FM baseband receiver