4th lesson

Tuesday, 6 October 2020 09:53

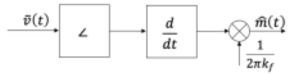
FM receiver

- Neglecting the effect of noise and channel, the complex envelope of the received signal is
$$\tilde{v}(t) = A_c e^{j2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau}$$
- The modulating signal can be recovered by differentiating the phase of $\tilde{v}(t)$
$$\hat{m}(t) = \frac{1}{2\pi k_f}\frac{d}{dt}\angle\tilde{v}(t)$$

$\measuredangle \tilde{v}(t) = 2\pi K_f \int_{-\infty}^{t} m(\tau)d\tau$

$\frac{d}{dt}\measuredangle\tilde{v}(t) = 2\pi K_f m(t)$

Conceptual FM baseband receiver



$\frac{d}{dt}\int_{-\infty}^{t} m(\tau)d\tau = m(t)$

$\tilde{s}_{FM}(t) = A_c e^{j2\pi K_f \int_{-\infty}^{t} m(\tau)d\tau}$

At the transmitter

$\tilde{v}(t) = A_c \underbrace{e^{j2\pi K_f \int_{-\infty}^{t} m(\tau)d\tau}}$

At the receiver

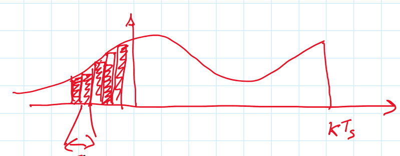
In ideal conditions (no noise and perfect channel) $\tilde{s}_{FM}(t) = \tilde{v}(t)$

FM receiver – practical implementation

- At the SDR output there is the signal complex envelope sampled at frequency $f_s = \frac{1}{T_s}$
$$\tilde{v}(k) = \tilde{v}(t)|_{t=kT_s} = A_c e^{j2\pi k_f \int_{-\infty}^{kT_s} m(\tau)d\tau} \approx A_c e^{j2\pi k_f \sum_{\ell=-\infty}^{k} m(\ell)T_s}$$
- The product of two consecutive baseband samples yields
$$\tilde{v}(k)\tilde{v}^*(k-1) \approx A_c e^{j2\pi k_f \sum_{\ell=-\infty}^{k} m(\ell)T_s} A_c e^{-j2\pi k_f \sum_{\ell=-\infty}^{k-1} m(\ell)T_s}$$
$$= A_c^2 e^{j2\pi k_f m(k)T_s}$$
- An estimate of $m(k)$, the k-th sample of $m(t)$, is
$$\hat{m}(k) = \frac{1}{T_s}\frac{1}{\Delta f}\angle\tilde{v}(k)\tilde{v}^*(k-1)$$

$\Rightarrow \tilde{v}(k) = v_I(k) + j v_Q(k)$

$$\begin{cases} v_I(k) = A_c \cos 2\pi K_f \int_{-\infty}^{kT_s} m(\tau)d\tau \\ v_Q(k) = A_c \sin\left(2\pi K_f \int_{-\infty}^{kT_s} m(\tau)d\tau\right) \end{cases}$$

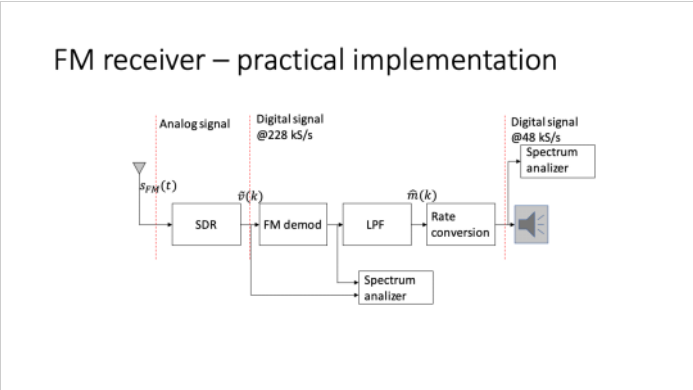


1. $\tilde{v}(k) = A_c e^{j2\pi K_f \int_{-\infty}^{kT_s} m(\tau)d\tau}$

2. $\tilde{v}(k) \approx A_c e^{j2\pi K_f \sum_{\ell=-\infty}^{k} m(\ell)T_s}$ $\qquad \int_{-\infty}^{kT_s} m(\tau)d\tau \approx \sum_{\ell=\infty}^{k} m(k)T_s$

3. $\tilde{v}(k)\,\tilde{v}^*(k-1) \approx \underbrace{A_c e^{j2\pi K_f \sum_{-\infty}^{k} m(\ell)T_s}}_{\tilde{v}(k)} \cdot \underbrace{A_c e^{-j2\pi K_f \sum_{\ell=-\infty}^{(k-1)} m(\ell)T_s}}_{\tilde{v}^*(k-1)}$
$= A_c^2 e^{j2\pi K_f m(k)}$

4. $\hat{m}(k) = \frac{1}{2\pi K_f} \measuredangle \tilde{v}(k)\tilde{v}^*(k-1)$



$$\measuredangle\, \tilde{v}(k)\, \tilde{v}^*(k-1)$$

$$= \tan^{-1} \frac{\mathrm{Im}\{\tilde{v}(k)\, \tilde{v}^*(k-1)\}}{\mathrm{Re}\{\tilde{v}(k)\, \tilde{v}^*(k-1)\}} = 2\pi K_f\, m(k)\, T_s$$

$$\text{rxData}: \underbrace{\text{rtlsdr\_fs}}_{=\frac{1}{T_s}} \cdot \frac{1}{2\pi \cdot \text{freqDev}}\, \measuredangle\, \tilde{v}(k)\, \tilde{v}^*(k-1) = \frac{1}{\cancel{T_s}} \cdot \frac{1}{\cancel{2\pi}\, \Delta_f} \cdot \cancel{2\pi} K_f\, m(k)\, \cancel{T_s}$$

$$= \frac{1}{K_f \max\{|m(k)|\}} \cdot K_f\, m(k) = \frac{m(k)}{\max\{|m(k)|}$$