13th lesson

Monday, 9 November 2020 10:07

Channel's delay spread

- One practical way to assess if the channel is multipath fading is to compare the symbol timing with the root mean square (RMS) delay spread.
- To compute the delay statistics one should integrate over the density function of the delays.
- Too difficult, instead the delay of each path is weighted by the coefficient $0 < \alpha_\ell^2 \backslash \sum_{\ell=0}^{L-1} \alpha_\ell^2 < 1$, which are equivalent to empirical mass probabilities.
- *Mean excess delay*

$$\bar{\tau} = \frac{\sum_{\ell=0}^{L-1} \alpha_\ell^2 \tau_\ell}{\sum_{\ell=0}^{L-1} \alpha_\ell^2}$$

- *RMS delay spread*

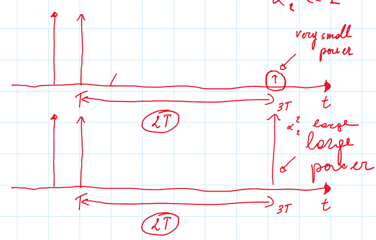
$$\sigma_\tau = \sqrt{\overline{\tau^2} - \bar{\tau}^2} \text{ with } \overline{\tau^2} = \frac{\sum_{\ell=0}^{L-1} \alpha_\ell^2 \tau_\ell^2}{\sum_{\ell=0}^{L-1} \alpha_\ell^2}$$

$$\bar{\tau} = \int_{-\infty}^{+\infty} \tau \, pdf(\tau) \, d\tau \qquad \bar{\tau} = \sum_{\ell=0}^{L-1} \tau_\ell P(\tau_\ell)$$

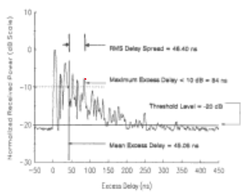
$$\Rightarrow$$

$$\sigma_\tau^2 = \int_{-\infty}^{+\infty} (\tau - \bar{\tau})^2 pdf(\tau) \, d\tau \qquad \sigma_\tau^2 = \sum_{\ell=0}^{L-1} (\tau_\ell - \bar{\tau}) P(\tau_\ell)$$

$$P(\tau_m) = \frac{\alpha_m^2}{\sum_{\ell=0}^{L-1} \alpha_\ell^2} \qquad \sum P(\tau_m) = 1$$



RMS delay spread



Delay spread and coherence bandwidth

- The selectivity of the channel can be evaluated both in the time and frequency domain
- *Delay spread*
  - If the delay spread is smaller than the symbol time, $\sigma_\tau \ll T$, there is only one resolvable path and the channel is *flat* fading.
  - If $\sigma_\tau > T$, there are more than one resolvable path and the channel is *multipath*.
- *Coherence bandwidth*. The channel coherence bandwidth B_C is approximately computed as the inverse of the delay spread

$$B_C \approx 1/\sigma_\tau$$

  - If the signal bandwidth is smaller than the coherence bandwidth, $B < B_c$ the channel is *flat*.
  - If $B > B_c$, the channel is *frequency selective*.

Typical values of RMS delay spread

- Measurements depend on signal frequency and environment.
- Typical values of delay spread are 0.2μs (rural area), 0.5μs (suburban area), 3-8μs (urban area), <2 μs (urban microcell) and 50-300ns (indoor picocell)

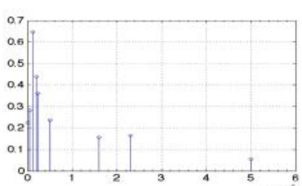
Environment	RMS delay spread (σ_τ)	Notes
Urban	1300 ns (3500 ns max)	NYC
LTE ETU	Up to 5 μs	Averaged typical case
Suburban	1960-2110 ns	Averaged extreme case
Indoor	10-50 ns	Office building
Indoor	70-94 ns (1470 ns max)	Office building

LTE channel models

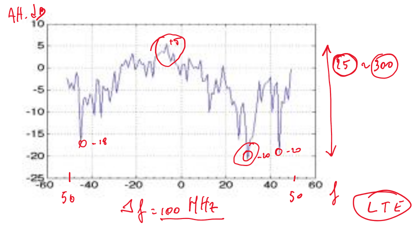
- Extended Typical Urban model (ETU)

Excess tap delay (ns)	Relative power (dB)
0	−1.0
50	−1.0
120	−1.0
200	0.0
230	0.0
500	0.0
1600	−3.0
2300	−5.0
5000	−7.0

CIR snapshot

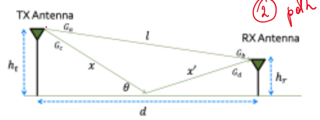


Frequency channel snapshot



Example: the two-ray channel model

② path



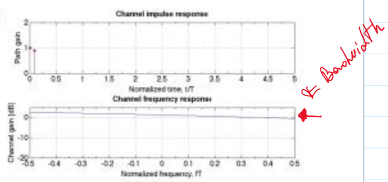
Delayed since x+x' is longer. $\tau = (x + x' - l)/c$

$$r_{2\text{-ray}}(t) = Re\left\{\frac{\lambda}{4\pi}\left[\frac{\sqrt{G_a G_b}\,\beta(t)\exp\left(-\frac{j2\pi l}{\lambda}\right)}{l} + \frac{R\sqrt{G_c G_d}\,\beta(t-\tau)\exp\left(-\frac{j2\pi(x+x')}{\lambda}\right)}{x+x'}\right]\exp(j2\pi f_c t)\right\}$$

R: ground reflection coefficient (phase and amplitude change)

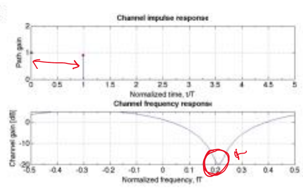
Two-path channel, $h(t) = \alpha_1\delta(t) + \alpha_2\delta(t-\tau)$

Channel parameters
$\alpha_1 = 1, \alpha_2 = 0.9$
$\tau = 0.1T$



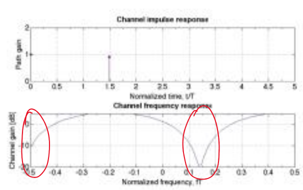
Two-path channel, $h(t) = \alpha_1\delta(t) + \alpha_2\delta(t-\tau)$

Channel parameters
$\alpha_1 = 1, \alpha_2 = 0.9$
$\tau = T$



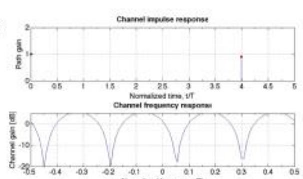
Two-path channel, $h(t) = \alpha_1\delta(t) + \alpha_2\delta(t-\tau)$

Channel paramete
$\alpha_1 = 1, \alpha_2 = 0.9$
$\tau = 1.5T$



Two-path channel, $h(t) = \alpha_1\delta(t) + \alpha_2\delta(t-\tau)$

Channel parameters
$\alpha_1 = 1, \alpha_2 = 0.9$
$\tau = 4T$



Time-varying channel

- If the mobile receiver is in movement, the gains and the phase of the various paths of the channel vary in time

$$h(t,\tau) = A_{LS} \sum_{\ell=0}^{N_{c-1}} \alpha_\ell(t) e^{j\phi_\ell(t)} \delta(\tau - \tau_\ell)$$

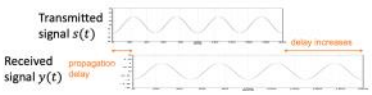
- The received signal is

$$y(t) = A_{LS} \sum_{\ell=0}^{N_{c-1}} \alpha_\ell(t) e^{j\phi_\ell(t)} s(t - \tau_\ell)$$

- The channel gains and phases change much faster than the large scale fading A_{LS} and the delays τ_ℓ.

Doppler shift

Propagation delay = distance d/speed of light c, $\tau = vt/c$



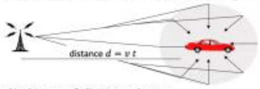
Received signal $y(t) = \sin 2\pi f_c (t - vt/c) = \sin 2\pi(f_c - f_c v/c)\, t$

received frequency

Doppler shift $f_d = -f_c v/c$

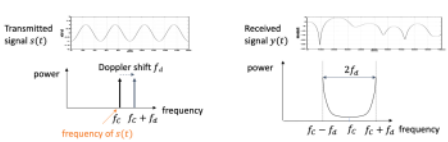
Scattering: Doppler Spectrum

- In fading channels many signal replicas arrive at the receiver with different angles. The effect is a *Doppler spread* rather than a single shift.



- Received signal is the sum of all scattered waves.
- Doppler shift for each path depends on angle θ, each path has a shift $f \frac{v\cos\theta}{c}$.
- Typically assume that the received energy is the same from all directions (uniform scattering).

Jakes' Doppler spectrum



$$S(f) = \frac{1}{\pi f_d \sqrt{1 - \left(\frac{f}{f_d}\right)^2}}$$

The fequency $f_d = \frac{f_c v}{c} = \frac{v}{\lambda}$ is the *Doppler spread.*