12th lesson

Monday, 2 November 2020 19:23

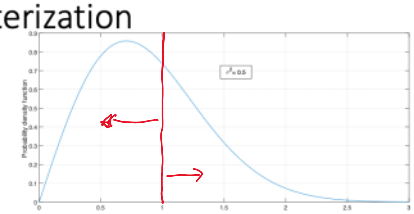
Small-scale fading: Rayleigh distribution

- If there is a sufficiently large scatter, for each cluster at the receiver we will have the sum of many different replicas of the signal, each with approximately the same delay and different complex gains.
- Because of the *Central limit theorem*, the complex gain of each cluster can be modelled as a *complex Gaussian* variable irrespective of the distribution of the individual components.
  - Phase ϕ is uniformly distributed in $[0,2\pi]$.
  - Amplitude α is *Rayleigh* distributed, if there is no los, or *Rician* distributed if there is los.
- Central limit theorem: when a sufficiently large number of random variables are added, their sum tends to be normally distributed regardless of the original distribution of the random variables.

Channel gain characterization

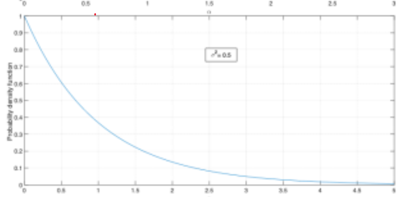
- The distribution for channel amplitude α is *Rayleigh*

$$p(\alpha) = \begin{cases} \dfrac{\alpha}{\sigma^2} e^{-\frac{\alpha^2}{2\sigma^2}} & \alpha \geq 0 \\ 0 & \alpha < 0 \end{cases}$$



- The distribution for channel power $s = \alpha^2$ is *exponential*

$$p(s) = \begin{cases} \dfrac{1}{2\sigma^2} e^{-\frac{s}{2\sigma^2}} & s \geq 0 \\ 0 & s < 0 \end{cases}$$



Small-scale fading

- The channel impulse response (CIR) can be modelled as

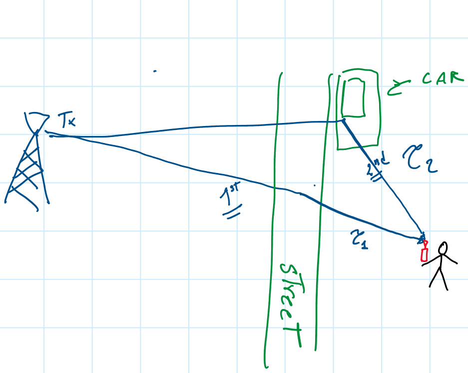
$$h(t) = A_{LS} \sum_{\ell=0}^{N_{C-1}} \alpha_\ell e^{j\phi_\ell} \delta(t - \tau_\ell) \Leftarrow \text{complex envelope of the channel}$$

  where $\alpha_\ell e^{j\phi_\ell}$, the complex gain of the ℓ-th cluster is the sum of the complex gains of all the paths belonging to the cluster.
- Let $s(t)$ be the transmitted signal, neglecting the noise, the complex envelope of the signal at the receiver is

$$y(t) = s(t) \otimes h(t) = A_{LS} \sum_{\ell=0}^{N_{C-1}} \alpha_\ell e^{j\phi_\ell} s(t - \tau_\ell)$$

Multipath fading and ISI

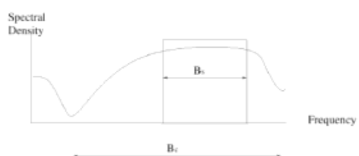
- The received signal is modelled as the sum of a series of attenuated, time-delayed phase shifted replicas of the transmitted signal, *one different path* for each cluster.
- Depending on the symbol duration, the propagation channel might be composed by one or several *resolvable* paths, where each resolvable path roughly corresponds to a given cluster.
  - If for the signal of interest the channel can be approximated with one single path, the channel is *flat* fading
  - If there is more than one resolvable path, the channel is *multipath* and we have inter-symbol interference (ISI).
- The *coherence bandwidth* of the channel is the bandwidth over which the channel has approximately a constant gain and a linear phase response.



What is the reason to decide if there is one or two RESOLVABLE paths?

Flat fading channel

- When the channel can be modelled with only one path, its coherence bandwidth B_c is larger than the bandwidth B_s of the transmitted signal.



  - The spectral characteristics of the transmitted signal are preserved at the receiver.
  - The channel does not cause any non-linear distortion due to time dispersion.

Flat fading channel — means only 1 replica!

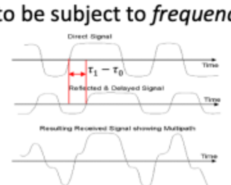
$h(t) \approx A_{LS} \cdot \alpha e^{j\phi} \cdot \delta(t-\tau_0)$ — uniform distrib. (ϕ); Rayleigh distributed (α)

$H(f) = A_{LS} \cdot \alpha\, e^{j\phi}\, e^{-j2\pi f \tau_0}$

$|H(f)| = A_{LS}\, \alpha$ flat in frequency.

Multipath fading

- If the resolvable paths are more than one, the received signal includes multiple versions of the same symbol, each one attenuated (faded), rotated in phase and delayed.
- The received signal is distorted and is affected by ISI
- The channel is said to be subject to *frequency selective fading.*



$\ell = 0$

$\ell = 1$

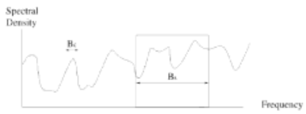
$A_{LS} = 1$

$y(t) = \alpha_0 e^{j\phi_0} s(t-\tau_0) + \alpha_1 e^{j\phi_1} s(t-\tau_1)$

Multipath fading, frequency selective } Equivalent definitions.

Frequency selective fading

- Frequency selective fading
  - The coherence bandwidth B_C of the channel is smaller than the bandwidth B_s of the transmitted signal.
  - The spectral characteristics of the transmitted signal are not preserved at the receiver: certain frequency components have larger gains than others
  - The channel is *selective* in frequency.



Multipath fading is frequency selective
Example with two clusters

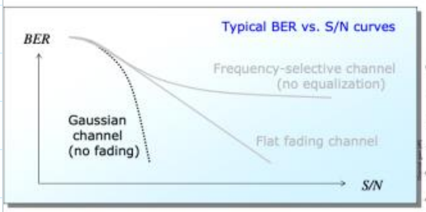
$$h(t) = \alpha_0 e^{j\phi_0}\delta(t-\tau_0) + \alpha_1 e^{j\phi_1}\delta(t-\tau_1)$$

$$\Downarrow\Uparrow$$

$$H(f) = \alpha_0 e^{j\phi_0} e^{-j2\pi f\tau_0} + \alpha_1 e^{j\phi_1} e^{-j2\pi f\tau_1}$$

Additive White Gaussian Noise

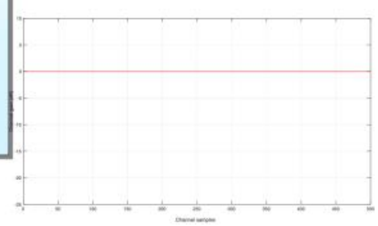
Flat fading channel: BER on AWGN



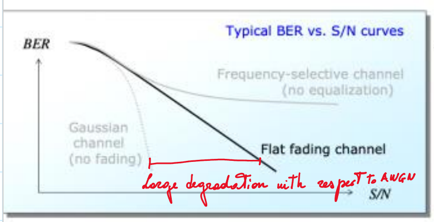
$$h(t) = \delta(t)$$

- With an AWGN channel, the decision variable is

$$x(m) = c_m + n(m)$$



BER on flat Rayleigh fading channel

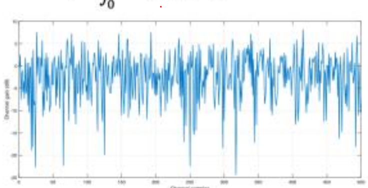


- With a flat fading channel, the decision variable is

$$x(m) = \alpha c_m + n(m)$$

- The mean error probability is obtained by averaging it over the channel

$$P_e = \int_0^{+\infty} P(e|\alpha)p(\alpha)d\alpha$$



No error floor
$\Rightarrow \lim_{E_s \to +\infty} P_e = \phi$!

Flat fading channel

$$h(t) = \alpha e^{j\phi}\delta(t-\tau_0)$$

$$\boxed{T_x} \xrightarrow{s(t)} \boxed{h(t)} \xrightarrow{y(t)} \oplus \longrightarrow \boxed{R_x} \longrightarrow \hat{a}_m \qquad \uparrow w(t)$$

PAM

$$r(t) = y(t) + w(t)$$

$$x(t) = r(t) \otimes g_R(t) = y(t) \otimes g_R(t) + \underbrace{w(t) \otimes g_R(t)}_{n(t)}$$

$$y(t) \otimes g_R(t) = \underbrace{s(t) \otimes h(t)}_{y(t)} \otimes g_R(t)$$

$$s(t) = \sum_i a_i g_T(t-iT)$$

$$y(t) \otimes g_R(t) = \sum_i a_i\, g_T(t-iT) \otimes \alpha e^{j\phi}\delta(t-\tau_0) \otimes g_R(t)$$

$$g_T(t) \otimes g_R(t) = g_N(t)$$

$$g_T(t) \otimes g_R(t) \otimes \delta(t-\tau_0) = g_N(t-\tau_0)$$

$$x(t) = \alpha e^{j\phi} \sum_i a_i\, g_N(t-iT-\tau_0) + n(t)$$

after synch $t = mT + \tau_0$

$$\longrightarrow \boxed{g_R(t)} \longrightarrow \diagup \longrightarrow \boxed{\mp} \longrightarrow$$

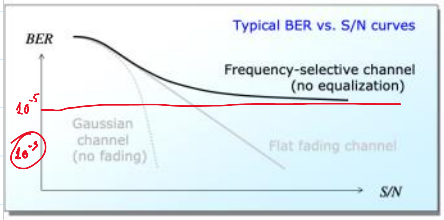
synchronization
$t = mT$

< I can estimate τ_0, ϕ

G

$x(m) = \alpha e^{j\phi} a_m + n(m)$ $\qquad x'(m) = x(m) e^{-j\phi} = \alpha a_m + n'(m)$

BER on multipath Rayleigh fading channel



- With a frequency selective channel, the decision variable is

$$x(m) = \alpha_0 c_m + \underbrace{\sum_{\ell,\ell\neq 0} \alpha_\ell e^{j\phi_\ell} c_{m-\ell}}_{\text{ISI}} + n(m)$$

- If no countermeasures are taken, the error probability has an irreducible error-floor.

$$\lim_{E_s \to +\infty} P_e = K, \quad K \text{ error floor}$$

S/N

$$SNIR = \frac{S}{N + I} = \frac{S}{S} \cdot \frac{1}{\frac{N}{S} + \frac{I}{S}} = \frac{1}{\cancel{\frac{N}{S}} + \frac{I}{S}} = \frac{1}{\frac{I}{S}} = \frac{S}{I}$$