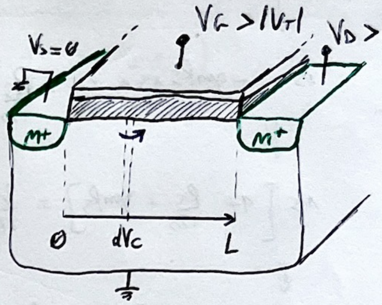
MOSFET FIGURES OF MERIT

Let's consider a MOSFET n-type with source and bulk shorted to ground while the gate is biased at $V_G > V_T$. The drain is biased with $V_D > 0$. In the channel we have

$$dV_C = I_{DS}\cdot dR = I_{DS}\cdot \frac{dx}{q n \mu_n W \Delta} = I_{DS}\cdot \frac{dx}{Q'_n(x)\,\mu_n W}$$

where $\quad Q'_n(x) = C'_{ox}(V_G - V_C - V_T)$



$$\int_0^{V_{DS}} W\mu_n C'_{ox}(V_G - V_C - V_T)\,dV_C = \int_0^{L} I_{DS}\,dx$$

$$\downarrow$$

$$I_{DS} = \mu_n C'_{ox}\frac{W}{L}\left[(V_G - V_T)V_{DS} - \frac{V_{DS}^2}{2}\right]$$

$$I_{DS}^{SAT} = \frac{1}{2}\mu_n C'_{ox}\frac{W}{L}(V_{GS} - V_T)^2$$

For V_{DS} values $> V_{DS}^{SAT} = (V_{GS} - V_T)$ the previous relation is no more valid. At steady state the current leaving the source is equal to the one reaching the drain side, so the drift velocity (and the electric field) must increase as we move towards the drain, since $n \downarrow$ and $n\cdot v \simeq$ CONSTANT. In the point called PINCH-OFF we have $Q'_n \simeq 0$ (negligible) and it is expected to move backwards as V_{DS} increases above V_{DS}^{SAT}, while in the area behind the pinch-off point the previous relation is still valid, so

$$I_{DS}^{SAT'} = K'\frac{W}{L'}(V_G - V_T)^2 \qquad \left[K' = \frac{1}{2}\mu_n C'_{ox}\right]$$

where $L' < L$, so $I_{DS}^{SAT'} > I_{DS}^{SAT}$. Assuming that $(L - L')$ remains small, as suggested by the experimental I-V curves, we get that

$$I_{DS} = I_{DS}^{SAT} + \left(\frac{\partial I_{DS}}{\partial V_{DS}}\right)\Bigg|_{V_{DS} = V_{DS}^{SAT}}\cdot\left(V_{DS} - V_{DS}^{SAT}\right)$$

where

$$\frac{\partial I_{DS}}{\partial V_{DS}} = \frac{\partial I_{DS}^{SAT}}{\partial L}\,\frac{\partial L}{\partial V_{DS}^{SAT}} = -\frac{I_{DS}^{SAT}}{L}\cdot\frac{\partial L'}{\partial V_{DS}}\Bigg|_{V_{DS} = V_{DS}^{SAT}}$$

so

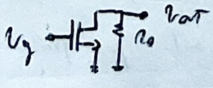
$$I_{DS} = I_{DS}^{SAT}\left[1 + \lambda\left(V_{DS} - V_{DS}^{SAT}\right)\right] \qquad \left(\lambda = -\frac{1}{L}\frac{\partial L'}{\partial V_{DS}}\Bigg|_{V_{DS} = V_{DS}^{SAT}}\right)$$

We define $\frac{1}{\lambda} = V_A$ EARLY VOLTAGE.

Does not depend on current!

$\longrightarrow$ GAIN

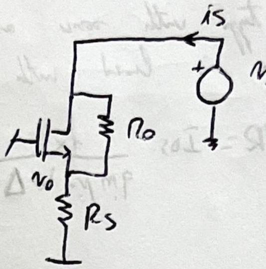
The maximum gain of a MOSFET is $g_m r_0 = \mu = \frac{2I}{V_{ov}}\cdot\frac{V_A}{I} = \frac{2V_A}{V_{ov}}$



②

→ **RESISTANCES**

• (R_D)



$v_o = i_s \cdot R_S$

$i_s = -g_m v_o + \frac{v_s - v_o}{r_o}$

↓

$i_s = -g_m R_S i_s + \frac{v_s}{r_o} - \frac{R_S}{r_o} i_s$

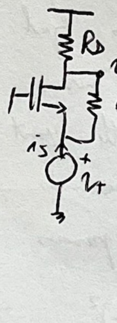
↓

$i_s \left[1 + \frac{R_S}{r_o} + g_m R_S \right] = \frac{v_s}{r_o}$

⇓

$\frac{v_s}{i_s} = r_o + R_S + g_m R_S r_o = r_o + R_S \left[1 + g_m r_o \right]$

• (R_S)



$v_o = i_s \cdot R_D$

$i_s = g_m v_T + \frac{v_T - v_o}{r_o}$

↓

$i_s = -\frac{R_D}{r_o} i_s + \frac{v_T}{r_o} + g_m v_T$

↓

$v_T \left[g_m + \frac{1}{r_o} \right] = i_s \left[1 + \frac{R_D}{r_o} \right]$

↓

$\frac{v_T}{i_s} = \frac{1 + \frac{R_D}{r_o}}{\frac{1}{r_o} + g_m} = \frac{r_o + R_D}{1 + g_m r_o}$

→ **WEAK INVERSION**

In this region electrons are not dominating the electrostatics in the channel, so the bands are almost flat. The concentration of electrons at the source side is computed by using the M-B statistics

$$n(0) = N_S \cdot e^{-\frac{q\phi_B}{kT}}$$

being $\phi_B = V_{BI} - \psi_S$. The current is due to diffusion!

$$J_n = q D_n \frac{dn}{dx} = q D_n \frac{n(0)}{L} = q D_n \frac{n_i^2}{L N_A} e^{\frac{q\psi_S}{kT}}$$

It can be shown that

$$I_{DS} = 4m \left(\frac{1}{2} \mu_n C'_{ox} \frac{W}{L} V_{TH}^2 \right) e^{\frac{q(V_{GS} - V_T)}{mkT}} \qquad \left(m = 1 + \frac{C_{dep}}{C_{ox}} \simeq 1,5 \right)$$

so $\frac{\partial I_{DS}}{\partial V_{GS}} = g_m = \frac{I_{DS}}{m V_{TH}}$ and $\mu = g_m r_o = \frac{V_A}{m V_{TH}}$.

→ MODERATE INVERSION

EKV model

$$IC = \frac{I_D}{4m\left[\frac{1}{2}\mu_n C'_{ox}\frac{W}{L}(V_{TH})^2\right]}$$

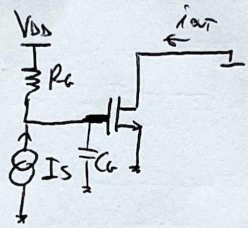
$$g_m = \frac{2}{1+\sqrt{1+4\cdot IC}}\ \frac{I_D}{mV_{TH}}$$

TAXONOMY	IC	BIAS RANGE
WEAK INV.	$IC \leq 0,1$	$V_{GS} \leq \lvert V_T \rvert - 0,1V$
MODERATE INV.	$0,1 \leq IC \leq 10$	$\lvert V_T \rvert - 0,1V \leq V_{GS} \leq \lvert V_T \rvert + 0,2V$
STRONG INV.	$IC \geq 10$	$V_{GS} \geq \lvert V_T \rvert + 0,2V$

$$\left(m = 1 + \frac{C'_{dep}}{C'_{ox}} = 1,5\right)$$

→ CUT-OFF FREQUENCY

Frequency for a unity current gain



$$\frac{i_{out}}{i_S} = g_m R_G$$

$$f_P = \frac{1}{2\pi}\ \frac{1}{R_G \cdot C_G}$$

$$\Downarrow$$

$$f_T = g_m \cancel{R_G} \cdot \frac{1}{2\pi\, C_G \cdot \cancel{R_G}} = \frac{g_m}{2\pi\, C_G}$$

Assuming that $C_G = C_{gs} \simeq C'_{ox} \cdot WL$ then

$$f_T \simeq \frac{\cancel{2}\cdot\frac{1}{\cancel{2}}\mu_n \cancel{C'_{ox}}\frac{W}{L}V_{ov}}{2\pi\, \cancel{C'_{ox}}\, WL} = \frac{1}{2\pi}\ \frac{\mu_n F}{L} = \frac{1}{2\pi}\ \frac{v}{L} = \frac{1}{2\pi}\ \frac{1}{\tau_{DRIFT}}$$

In weak inversion instead, we have that

$$Q' = \frac{q}{2}\cdot n(0)\cdot L \qquad \text{[charge per unit area]}$$

$$J_n = q\, D_n\, \frac{n(0)}{L} \qquad \text{[current density]}$$

$$\tau_{DIFF} = \frac{Q'}{J_n} = \frac{L^2}{2D_n}$$

and $$f_T = \frac{1}{2\pi\, \tau_{DIFF}}$$

$$\tau_{DRIFT} < \tau_{DIFF} \implies \frac{L^2}{\mu_n V_{ov}} < \frac{L^2}{2D_n} \implies V_{ov} > 2\frac{\mu_n}{\mu_n}\frac{kT}{q} = 2V_{TH}$$