Scuola di Scienze Matematiche, Fisiche e Naturali
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Tesi di Laurea

RIFLESSIONE DELLA LUCE NELLA COMPUTER GRAFICA BASATA SULLA FISICA

REFLECTION OF LIGHT IN PHYSICALLY BASED COMPUTER GRAPHICS

LEONARDO ZETTI

Relatore: *Stefano Berretti*

Correlatore: *Michele Ginolfi*

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INTRODUCTION

The aim of this thesis is to discuss a set of methods used in 3D computer graphics for the generation of photo-realistic images.

These techniques can be grouped together under the umbrella term *Physically Based Rendering*. There's a lot that goes into the creation of physically based images, and our discussion will only scratch the surface of the problem. More specifically, we'll only be concerned with capturing realistic *reflection* of light on a 3D material, as there's a lot to be said on this topic alone.

The final part of the thesis will propose a C++/OpenGL implementation of the Disney Principled BRDF, a reflection model presented by Brent Burley in his 2012 talk *Physically Based Shading at Disney* (Burley 2012). The talk was part of the 2012 SIGGRAPH course *Practical Physically Based Shading in Film and Game Production*.

1.1 COMPUTER GRAPHICS

Computers have the potential to be powerful tools for artistic expression. One way to create art with a computer is through *computer graphics*. *Computer graphics* can be defined as

> "The science and art of communicating visually via a computer's display and its interaction devices."
> (Hughes et al. 2014)

Advancements in this field have made computer-generated imagery a pervasive component of our day-to-day lives, ranging from special effects in blockbuster movies to the *Graphical User Interfaces* (GUIs) on our phones.

Thus, the world of computer graphics is a vast one. We should keep in mind that the problems and solutions presented here refer to the context of "geometry-based 3D graphics", as we'll call them. Our graphics will be "geometry-based" in the sense that we'll first describe the objects we want to draw on the screen with *geometric models*[1] (lines, polygons, polygonal meshes, ...), to then *sample* them for visualization. We can imagine our process to be as follows.

1. Create a *geometric model* of the object,
2. Describe the *material* it is made of,
3. Place the object in a *virtual scene* with *light sources*,
4. Use a *virtual camera* to "take a picture" of the object (this is the so-called *sampling* phase).

Our graphics will be 3D, meaning that the geometrical models, other than width and height, will also have *depth*. Our virtual camera will then communicate that depth to the viewer through the use of *perspective*.

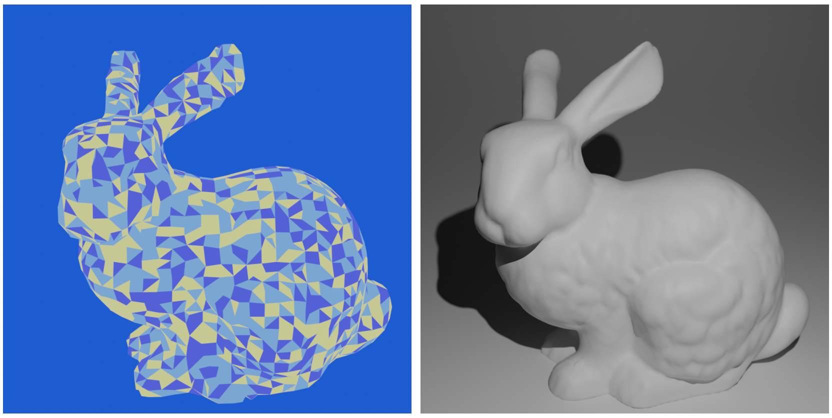

Fig. 1: The *Stanford Bunny Model*, drawn on the left as a *triangle mesh* (random color for each triangle). Model courtesy of the Stanford Computer Graphics Laboratory, rendered with Blender.

To restrict ourselves even further, we'll keep our focus mainly on steps 2. and 4. of the just mentioned process. Our aim will be to derive a single *mathematical model* than can describe as many materials as feasible, in

1 In the field of computer graphics, the word "model" is used to refer both to *geometric models* and *mathematical models*. A *geometric model* describes an object we want to "take a picture" of (for example, the *Stanford Bunny*). A *mathematical model* describes some physical or computational process that we use to take that picture (for example, the equations we use to tell how much light is reflected by the bunny's surface).

relation to how they *reflect light*. We'll then use this model to record reflection effects with our virtual camera.

The final images will be generated so:

1. Light gets emitted from a light source,

2. Light hits a surface and is reflected in various amounts and directions, depending on the *material* of the surface,

3. Light that's reflected towards the virtual camera gets recorded in the final image.

We'll now give a more rigorous definition of the virtual "picture-taking" step that we're interested in. We'll refer to it as *rendering*.

1.2 RENDERING AND PBR

In the book *Physically Based Rendering, From Theory to Implementation* - an important resource for this thesis - *rendering* (or, as it has been historically referred to, *image synthesis*) is defined as

> "The process of converting a description of a three-dimensional scene into an image."
> (Pharr, Jakob, and Humphreys 2023)

Note that a *3D scene* is comprised not only of 3D objects, but also light sources and a camera. Generally speaking, the final image of this process can be generated using any rules we choose. This flexibility enables a wide range of different rendering techniques, each of which will communicate a different *message* to the viewer.

This is our final goal in graphics; to *communicate* with viewers through *meaningful images*. As Andrew Glassner put it:

> "The field of *image synthesis*, also called *rendering*, is a field of transformation: it turns the rules of geometry and physics into pictures that mean something to people."
> (Glassner 1995)

If our aim is to create art - as in the case of *animated movies*[2] - rendering can be an important tool for creative expression. The end result of

2 In 3D animated films, each of the frames shown on screen was generated by some advanced rendering software. Achieving high levels of realism demands intense computation; applications used in movie production spend *hours* computing a *single frame*.

a rendering process will influence the emotional responses elicited in spectators. This interplay of different fields of study - math, physics, art, computer science, psychology, to name a few - is what makes rendering, and computer graphics in general, fascinating disciplines that are worth exploring.

Fig. 2: Stylized render from Blender Studio's *Project Gold*. ©Blender Foundation

So, in theory, there are no rules on how to *render*[3] a picture. However, what we are often interested in is making our images *physically plausible*, that is, *realistic*. This is why various rendering applications - such as *RenderMan*, used to make Pixar movies - are, we say, written to be *physically-based*.

Physically Based Rendering (or *PBR* for short) is a collection of rendering techniques based on physical models of real-world light and materials.

Our objective in this thesis will be physical plausibility, so we'll stick with PBR. As an alternative approach one could, for example, render *stylized* images to favour the expressiveness of their art over its realism.

1.3 HISTORICAL NOTES

In this paragraph, we'll go through a short summary of some of the key steps that have been taken in the research progress of physically based rendering. We will also see how PBR has been gradually adopted in film production.

3 The word "render" can be used to express two different concepts. When used as a *verb*, it is the process followed by the computer to produce our image. When used as a *noun*, it refers to the final image produced.

Physically based rendering research started to catch on only after the 1980s. An important seminal work was Whitted's 1980 paper, *An Improved Illumination Model for Shaded Display* (Whitted 1980), which has since served as a foundation for modern *ray-traced* computer graphics which feature *global illumination*.

PBR generally makes substantial use of *ray tracing*, and for this reason, in Chapter 3, we'll give a brief description of the main ideas behind this technique. For now, let's just say that ray tracing involves - very surprisingly - tracing rays of light in a 3D scene, to determine the colors of objects. As for *global illumination*, we'll elaborate on it in Chapter 3 as well.

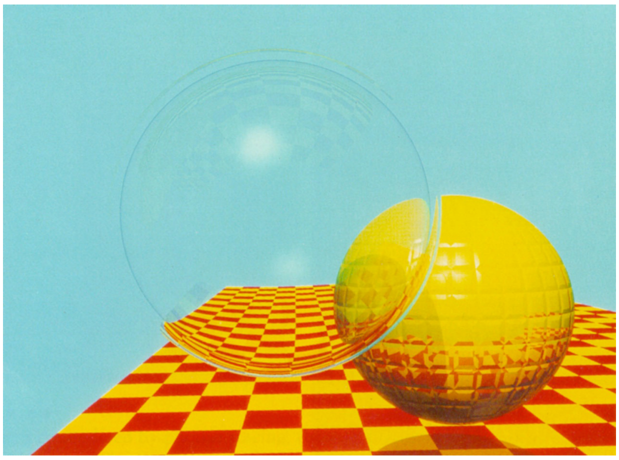

Fig. 3: One of the renders from Whitted's 1980 paper. ©ACM

We should mention that an important early contribution to PBR techniques was also given by Cook and Torrance, when they proposed a new reflection model based on *microfacet theory* (Cook and Torrance 1982). The techniques for realistic reflections described in Chapter 4 will be based on said theory.

Many other historic steps were necessary in order to make today's PBR techniques a reality. One of these was the work of Kajiya, who in 1986 introduced the *rendering equation* (Kajiya 1986). This equation describes in a concise and elegant way the problem solved by any realistic rendering application. More on the rendering equation will be said in Chapter 3.

Kajiya also introduced *path tracing*, which we can consider as an advanced form of *ray tracing* that properly takes into account *global illumination* effects, through the use of *Monte Carlo integration*.

Again, we'll cover this topic better in Chapter 3. To understand what we are talking about, let's put it this way. In order to do proper realistic rendering, we need to approximate the values of many (nested) definite integrals. *Monte Carlo integration* is a technique, based on random numbers, for doing just that.

Monte Carlo integration really transformed the field, allowing the creation of images unlike any before. Many important contributions were made to Monte Carlo-based efforts during the years, one of the most important ones being Veach's 1997 PhD thesis. Veach advanced key theoretical foundations of Monte Carlo rendering, and also developed new algorithms that improved its efficiency, like *bidirectional path tracing* (Pharr, Jakob, and Humphreys 2023).

It took some time to incorporate physically based rendering techniques in film production, due to its computational costs. An early example of a movie made with Monte Carlo global illumination was the short film *Bunny* (1998), by Blue Sky Studios. Its visual look was substantially different from other films and shorts of the past. Before then, renderers were mostly based on *rasterization*, a rendering technique that's faster than ray tracing, but, in general, less realistic. *Reyes* is an important example of a *rasterization-based* architecture that had been used to generate photorealistic images (Pharr, Jakob, and Humphreys 2023).

After *Bunny*, another watershed moment came in 2001, when Marcos Fajardo presented at the SIGGRAPH conference an early version of his *Arnold* renderer. *Arnold* was able to render scenes with complex geometry, textures, and global illumination in just tens of minutes (Pharr, Jakob, and Humphreys 2023). With the help of Sony Pictures Imageworks, *Arnold* was developed into a production-capable rendering system. It is now available as a commercial product.

In the early 2000s, Pixar's *RenderMan* renderer started to support hybrid rasterization and ray-tracing algorithms. It also introduced a number of innovative algorithms for computing global illumination solutions in complex scenes (Pharr, Jakob, and Humphreys 2023).

RenderMan was recently rewritten to be a physically based ray tracer. It was first employed in its new form for the production of the 2013 movie *Monsters University* (Hery and Villemin 2013).

Fig. 4: Monsters University, ©Disney/Pixar 2013

1.4 THESIS OUTLINE

The rest of the thesis will be organized as follows.

Chapter 2 will be devoted to the properties and behaviour of light, in a physical sense. Establishing a solid theoretical basis, borrowed from physics, is fundamental for PBR.

Chapter 3 will describe the algorithm at the base of our PBR rendering techniques; that is, *ray tracing*.

Chapter 4 is where we'll get into the main topic. We'll analyze some of the most popular theoretical models behind light reflection in state-of-the-art rendering software.

In Chapter 5 an implementation of the previously discussed techniques will be presented, in the form of a program called *BoxOfSunlight*. The focus here will be on the architectural choices made during development that make said program stand out.

In Chapter 6, *BoxOfSunlight* will be validated. We'll see how even just by changing the values of a few input parameters, we can simulate the looks of a wide range of materials. This will also be a chance to compare different reflection models; that is, to see which one works best inside a program like ours.

Finally, Chapter 7 will be dedicated to some of the shortcomings of *BoxOfSunlight*, and will contain ideas on how it could be extended into a more complete *physically based rendering engine*.

2

PHYSICS OF LIGHT

This chapter establishes the physical foundations of our Physically Based Rendering techniques. By its end, we'll have acquired important intuitions relating to three topics in the physics of light, namely, the *dual nature of light*, the *electromagnetic spectrum*, and *light-matter interactions*. The very last subject will be a physical interpretation of the *reflection of light*.

2.1 WAVE-PARTICLE DUALITY

When we create images, we are effectively using light as a tool to communicate information to the viewers. This raises an apparently trivial question.

What *is* light?

We'll give the physicist's answer. It might seem non-sensical at first, since it's actually made up of *two*, very different, answers. We say in fact that light has a *dual nature*, in the sense that there are two ways, that is, *models*, to define what light is. One represents light as a *wave*, the other as a stream of *particles*. Now the interesting part: light is actually *both* of these at the same time. There are situations in which light behaves just like a smooth, continuous wave, and you couldn't possibly think of it as being *granular*. But then, we also find cases indicating that light *is* actually composed of small, individual "energy packets", and where the wave theory is definitely not an option. In the following sub-sections, we'll dive (although a bit superficially) into a discussion about both models, and extract some facts that will be useful to us[1].

1 Various examples and definitions that follow were taken from *light*. Encyclopaedia Britannica (Stark 2025)

2.1.1 *Wave Nature of Light*

Obviously, light is important for far more reasons than just visual perception and communication. It is a central component to our lives (actually, to *life* in general as we know it), and for this reason we're usually somewhat familiar with the physical concept of it. Frequently, we say that light is an *electromagnetic wave* (or *radiation*). It's easier to visualize what a *wave* is by thinking of *mechanical waves*. For example, we can imagine light as a phenomenon similar to water waves. We say that they are similar since they are both *disturbances* that *propagate* through space. Water waves propagate as physical displacements of water, a *material medium*. By contrast, electromagnetic waves don't require any material substance to propagate.

Using a proper definition, we say that electromagnetic waves are *oscillations* in the *strengths* of *electric* and *magnetic fields*.

There's a lot to unpack here. Most importantly, we haven't talked about *fields* yet. We can think of a field as of a function that associates some physical quantity to every point in space (and time). For example, each point in Earth's atmosphere has a temperature associated with it. This temperature can be expressed as a function of spatial coordinates and time: $T(x, y, z, t)$, where T is the temperature field, x, y, and z are the spatial coordinates, and t is the time. We say that temperature is a *scalar* field. On the other hand, *electric* and *magnetic fields* are *vector fields*, since they associate *vectors* to points in space and time. We indicate the electric field as $E(x, y, z, t)$ and the magnetic field as $B(x, y, z, t)$ (E and B for short).

We won't get much deeper into the topic of fields. For us, it's sufficient to know that E and B are abstractions, in the following sense. Two electrically charged bodies, just like two magnets, exhibit forces (repellent or attractive) on each other. The forces depend on the characteristics of *both* bodies. Now, If we focus our attention on only *one* of the bodies, between the two electric charges or the two magnets, we can consider it as the source of a *field*, *electric* or *magnetic* respectively, which extends in the surrounding space. From this point of view, the force exerted on the second body of the couple is caused by the *field*. Thinking of fields instead of forces is a more abstract approach, since, putting it simply, we need to consider only one of the two bodies to describe its influence on the space surrounding it (the field it produces). Thus, we can see that E and B are abstractions which allow for more complex reasoning.

We know that E and B are needed to properly define light, which is an *oscillation* in the strengths of both fields. The definition of light might

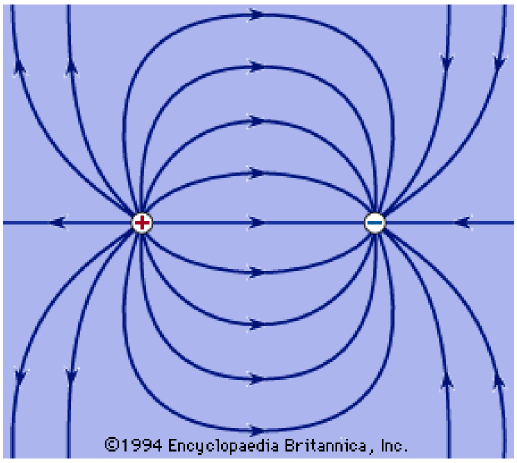

Fig. 5: "Electric field lines near equal but opposite charges." Source: *electric field*. Encyclopaedia Britannica. URL: https://www.britannica.com/science/electric-field (last accessed: 12.03.2025)

still be a bit confusing, though, so our next aim will be to *see* what an electromagnetic wave actually looks like (that is, its *graph*). In order to get there, we should start with a reminder on why we are considering the electric field E and the magnetic field B *together*.

Developments in 19th century physics, which famously culminated in *Maxwell's equations*, proved that E and B are intimately coupled; when one of them changes, the other one does as well. For example, Faraday's well-known *law of induction* describes how a moving magnet can generate electric current[2]. For this reason, we unite E and B in a single concept: the *electromagnetic field*, which describes both electric and magnetic influences produced by bodies in space.

Now, both E and B can be modeled as *time-harmonic fields* (Glassner 1995); that is, they can be described as *traveling harmonic waves*. A harmonic wave can be specified as a sinusoidal function over distance and time, which outputs the *displacement* (in our case, the strengths of the two fields). An example of a harmonic wave traveling in only the x direction is given by

$$y(x,t) = A\cos(\frac{2\pi}{\lambda}x - \frac{2\pi}{T}t) \tag{2.1}$$

where:

2 From *Faraday's law of induction*. Encyclopaedia Britannica. URL: https://www.britannica.com/science/Faradays-law-of-induction (last accessed: 12.03.2025)

- y is the vertical displacement at distance x and time t,
- A is the maximum displacement of the wave, or *amplitude*,
- λ, the *wavelength* of the wave, is the physical distance between successive crests,
- T is the *period*.

If we were standing still, on a point along the wave's path, and were to measure the time for one crest to reach us after the previous, that would get us the *period*.

Another important quantity is the wave's *frequency* ν; it is defined as $\nu = \frac{1}{T}$, and we can think of it as the "speed" at which the wave repeats. It is measured in Hertz (Hz).

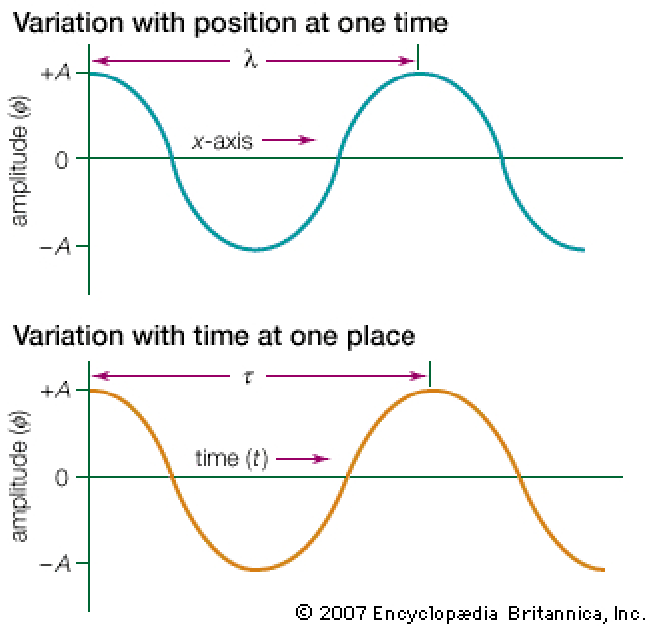

Fig. 6: "Snapshots of a harmonic wave can be taken at a fixed time to display the wave's variation with position (top) or at a fixed location to display the wave's variation with time (bottom)." Source: Encyclopaedia Britannica (Stark 2025)

Frequently, instead of writing $\frac{2\pi}{\lambda}$, we substitute it with k, the so-called *wave number*. We also use $\omega = \frac{2\pi}{T}$, which is called the *angular frequency*[3].

3 As done in *The Feynman Lectures on Physics*, Vol. I, Ch. 29 (Feynman, Leighton, and Sands 2011)

Moreover, we should mention that in the more general case, we can add a *phase shift* ϕ inside the cosine[4]. The effect of this is to, basically, shift the wave function along the x axis.

With these modifications, 2.1 becomes:

$$y(x,t) = A\cos(kx - \omega t + \phi) \tag{2.2}$$

Putting it all together, we can thus say that an *electromagnetic wave*, that is, *light*, is composed of two harmonic waves, one representing the strength of E, the other the strength of B. We also add that E and B are always perpendicular to each other, and that the magnitude and direction of propagation of an electromagnetic wave can be calculated with the cross product $E \times B$ (Glassner 1995). We can now see what the graph of an electromagnetic wave looks like by plotting E and B along the x axis (Figure 7). We define their values as

$$\begin{aligned} \vec{E}(x,t) &= E_0\cos(kx - \omega t + \phi)\hat{y} \\ \vec{B}(x,t) &= B_0\cos(kx - \omega t + \phi)\hat{z} \end{aligned} \tag{2.3}$$

To be more precise, what we are considering here is a *linearly polarized* wave, which has the property that the fields oscillate in fixed directions.

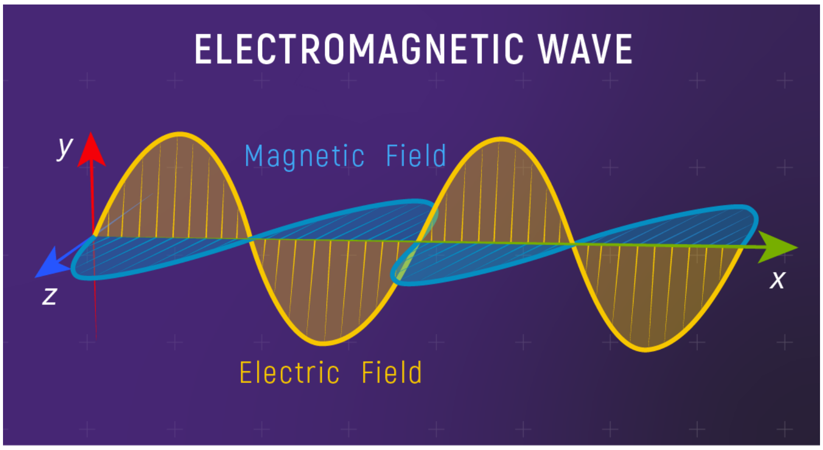

Fig. 7: Graph of a light wave. Source: NASA, ESA, CSA, Leah Hustak (STScI).

Before we move on, it's important that we address a situation that we'll encounter many times. Let's take two or more waves that are overlapping (they're "on top of each other"). Such waves are said to be in *superposition*.

4 As done in *The Feynman Lectures on Physics*, Vol. I, Ch. 21 (Feynman, Leighton, and Sands 2011)

The *superposition principle* states that, when two or more waves overlap, they produce a *new* wave, with a net displacement that is equal to the algebraic sum of the individual displacements. What this means is that the values of a wave function, be it $R(x,t)$, could actually be the result of summing up *many* different harmonic wave functions:

$$R(x,t) = A_1\cos(k_1x - \omega_1t + \phi_1) + A_2\cos(k_2x - \omega_2t + \phi_2) + ... \\ + A_n\cos(k_nx - \omega_nt + \phi_n)$$

In fact, the electromagnetic waves that we'll talk about should be in general interpreted in this form. In rendering, we'll think of every electromagnetic as being composed of (infinitely) many overlapping harmonic waves. As we know, each of these harmonic waves has its own wavelength λ. Putting it simply, we can thus say that a single ray of light "contains" many wavelengths. We'll find this to be an important fact.

There are good reasons to model light as a wave. A classic demonstration of the wave nature of light is the *double-slit experiment*, first performed by Young in 1801. In this famous experiment, two parallel slits on an opaque surface are equally illuminated by a light source, and the light that passes through the slits is observed on a screen. When the slits are close together, light that strikes the screen creates a pattern of alternating bright and dark bands (Glassner 1995) (see Figure 8).

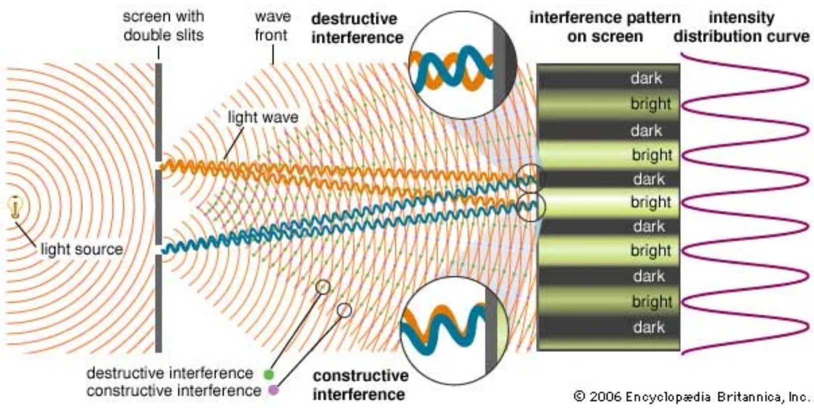

Fig. 8: "Young's double-slit experiment." Source: Encyclopaedia Britannica (Stark 2025)

The easiest way to explain this result is by positing that the light exiting the two slits has a wavelike nature. As we know, we can describe light waves with the two functions $\vec{E}(x,t)$ and $\vec{B}(x,t)$ (from (2.3)), periodic over distance and time. However, to simplify this discussion, let's express light as a single periodic function $A(x,t)$, instead of two (with $A(x,t) = A_0\cos(kx - \omega t + \phi)$).

We start by noting that when waves pass through a slit in a barrier, the slit acts as a source of cylindrical waves (the waves propagate symmetrically beyond it). This is due to a phenomenon called *diffraction* (Glassner 1995).

In the experiment, both slits are at the same distance from the light source, so the waves leaving the double slits have the same *phase* (they're in "sync", so to speak). This means that, at any time t, the *same wave* is generated in synchrony at both slits. Now, even though the waves leaving the two slits are the same, the two distances x_1, x_2 that they travel to reach a same point on the screen will be, in general, different ($x_1 \neq x_2$). Consequently, their wave functions in that point might differ, that is, $A(x_1) \neq A(x_2)$ (with t fixed). In some points, for example, one wave arrives at its maximum and the other at its minimum, causing their sum to be 0; the waves here leave a dark spot, and we say that they *destructively interfere*. In some other points, however, both wave functions might arrive at their maximum, creating a bright spot. In this case, they *constructively interfere*. Between these two extremes we get different intensities due to different amounts of *interference* between the two waves.

This is exactly what causes the light-and-dark pattern that we wanted to explain (Glassner 1995).

2.1.2 *Particle Nature of Light*

Not all physical phenomena related to light can be explained by thinking of waves. Most famously, the wave model disagrees with the *photoelectric effect*, observed first by Heinrich Hertz in 1887.

Consider an experimental setup which consists in a beam of light shining onto a piece of metal, called the *cathode*. Next to the cathode, we also set up a detection device that can measure the energy of electrons striking it. What happens in this scenario is that, as soon as we shine light on the cathode, the detector starts reporting electrons. Clearly, the light that strikes the cathode triggers an expulsion of electrons from the metal. For each electron, we find its energy E to be[5]

$$E = h\nu - p \tag{2.4}$$

5 Note that the energy of the electron E is a result of its *movement* as it's expelled, and is called *kinetic energy*.

where ν is the frequency of the incident wave of light, p is a constant characteristic of the metal, and h is a factor that seems to be constant for all metals and all wavelengths (Glassner 1995). This expulsion of electrons caused by light is what we call *the photoelectric effect*.

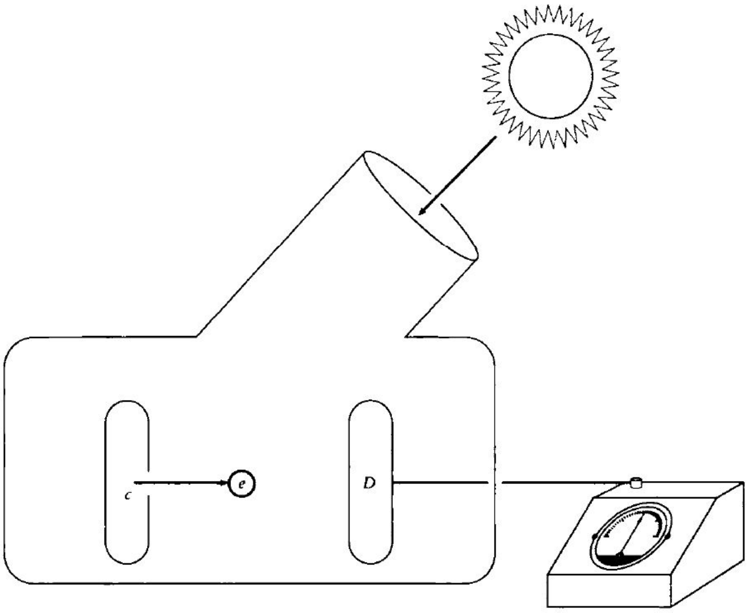

Fig. 9: "Apparatus for observing the photoelectric effect. The cathode is *c*, the detector is *D*." (Glassner 1995)

If we experiment with different light "strengths", or, to be more precise, different wave *amplitudes*, strange things start to happen. First of all, the energy E of the electrons is found to be independent of the light beam's amplitude. For example, illuminating the metal with a 40-watt light bulb rather then an otherwise identical 20-watt light bulb causes more electrons to be freed, but their individual energy does not change. According to the wave model, this doesn't make much sense, since a stronger wave should impart more energy to the electrons. Another surprising fact is that even extremely low amplitudes of light free some electrons. Again, thinking of the light beam as a wave, we would be led to believe the incident energy to be so low that it gets spread *all over* the cathode. Consequently, there wouldn't be any point with enough energy to free an electron[6] (Glassner 1995).

6 More precisely, the energy wouldn't be higher than the constant p from (2.4)

In 1905, Einstein advanced the hypothesis that the energy flowing along the incident beam is quantized into small, individual packets called *photons*. He supposed that to liberate an electron from a surface, there's a minimum amount of energy that's required. A photon that contains this amount of energy transfers it to the electron at the time of collision, resulting in the electron being freed. On the other hand, photons with energies lower than this minimum cannot induce electron emission (Glassner 1995).

Going back to the two phenomena mentioned earlier, we can now see that each photon actually interacts with each electron *independently*. Therefore, if we increase the incident beam's energy, that is, the *number* of photons, we don't increase the energy of the emitted electrons, we just produce *more* of them. On the other hand, even a beam with low energy (less photons) will trigger the emission of some electrons, if the energy carried by individual photons is over a certain threshold.

Einstein's interpretation of the *photoelectric effect* was based on the work of Max Planck, who in 1900 had proposed a mathematical construct to calculate *blackbody radiation*. (We'll mention more about *blackbodies* later.) Planck had considered electromagnetic energy as *released* and *absorbed* in discrete packets, with the energy of a packet being directly proportional to the *frequency* of the radiation:

$$E = h\nu \tag{2.5}$$

Where the constant h has since been called *Planck's constant*. Einstein's contribution, which earned him the Nobel price in 1921[7], was interpreting *light itself* as composed of energy packets, that is, *photons*.

So, the particle model is better suited to explain phenomena such as the *photoelectric effect* and *blackbody radiation*. It is still, however, indisputable that light is also a wave, due to the manifestation of *interference effects* such as the ones present in Young's experiment. But how can both theories be right?

Quantum mechanics solves this dilemma by stating that light exhibits *wave-particle duality*. Actually, this is also true for *electrons* and other discrete bits of matter; that is, they also possess wave properties such as *wavelength* and *frequency*. For example, in modern versions of the double-slit experiment, electrons have been shown to form the same interference pattern as the one demonstrated originally by Young (Stark 2025).

7 See `https://www.nobelprize.org/prizes/physics/1921/summary/` (last accessed: 12.03.2025)

But how does this work? Briefly put, each particle has a *wave function* associated with it, which should be interpreted *statistically*, as suggested originally by the German physicist Max Born. In fact, the *wave function* is required to calculate the *probability* of finding a particle at any point in space (Stark 2025).

We now close our discussion on the dual nature of light. We started with what seemed like a simple question and ended up in the realm of modern physics, which have challenged not only earlier theories in the field, but our own perception of reality. It seems that to answer the question "what is light?" we need embrace a vision of the Universe where concepts that baffle the intellect, like particles being also waves, and waves being also particles, are a reality.

2.2 THE ELECTROMAGNETIC SPECTRUM

From here on, we'll alternate freely between the wave and particle models. Let's now consider the first. Changing the wavelength of an electromagnetic radiation, we can span a broad *spectrum*, from very long waves to very short. This spectrum can be divided in *bands*, each with a different name. For the sake of curiosity, we'll provide a short summary of them, based on the NASA Science article series *The Electromagnetic Spectrum* (National Aeronautics and Space Administration, Science Mission Directorate 2010).

2.2.1 *From Radio Waves to Gamma Rays*

The longest waves in the spectrum are *radio waves*. Their existence was proved by Hertz in the late 1880s, and their wavelengths range "from the length of a football to larger than our planet". As suggested by the name, they are used to transmit *radio signals*.

At the higher frequency end of the radio spectrum, *microwaves* can be found. They are distinguished from radio waves because of the technologies used to access them. They are used by microwave ovens to cook food.

Really hot objects, like fire, emit *visible light*. Other objects, such as humans, are not as hot and only emit only so-called *infrared waves*. Infrared waves are just beyond the visible spectrum of light; although the human eye cannot see them, we do sense them as *heat*.

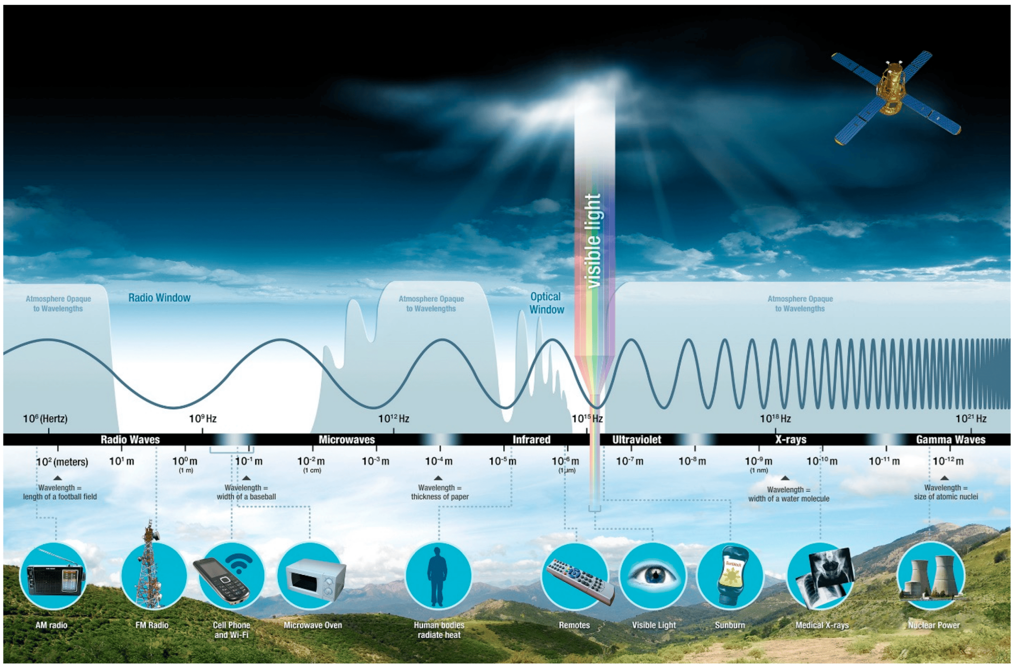

Fig. 10: Introduction to the Electromagnetic Spectrum. Source: NASA Science (National Aeronautics and Space Administration, Science Mission Directorate 2010)

Visible light falls roughly in the wavelengths

$$380 < \lambda < 700$$

measured in *nanometers* (nm).
There is a direct correlation between the wavelength of a visible ray of light and the color that our eyes see in response. Short wavelengths look blueish (the shortest waves look violet), and long wavelengths red. In the middle, there's green.

Ultraviolet light (*UV*) has shorter wavelengths than visible light. The Sun is a source of ultraviolet radiation, some of which can be harmful to living organisms. Luckily, the vast majority of these are absorbed by Earth's atmosphere.

X-rays possess extremely small wavelengths, between 0.03 and 3 nanometers (that is smaller than a single atom of many elements). They are used for medical diagnosis, since bones are dense and absorb more x-rays than skin does, as rays pass through the body.

The smallest wavelengths (and highest energies) are reserved to *gamma rays*. On Earth, they are generated from events such as nuclear explosions and lightning.

We'll now focus a bit more on *visible light*, to explain how we perceive colors and, therefore, how we should treat them in a computer program.

2.2.2 *Spectral Distributions and Color*

When we'll be defining physical quantities for the measurement of light, we'll find it necessary to talk in terms of *spectral distributions*. We'll also refer to these, simply, as *spectra* (*spectrum* for the singular). A *spectral distribution* is a distribution function that gives us the amount of light as a function of wavelength (Pharr, Jakob, and Humphreys 2023).

To make an example, let's consider the energy of a ray of light. We know by now that it can be decomposed into discrete packets, each of which is carried by a photon. We are also aware of the fact that, given a light wave's frequency ν, the energy E of its photons is (from (2.5))

$$E = h\nu$$

We've mentioned earlier that a light wave can be considered as the sum of many harmonic waves, each with its own wavelength λ. A light wave's frequency ν can be calculated from its wavelength λ as

$$\nu = \frac{c}{\lambda} \tag{2.6}$$

where c is the speed of light in a vacuum (Stark 2025). Consequently, a light wave "contains" many frequencies, just like it does with wavelengths. This means that it's composed of many types of photons; that is, photons with different energies. These individual energy values are summed up to obtain the total energy carried by light.

Let's now see how the light's energy could be used to determine its color. To do so, we follow a simplified overview from the book *An Introduction to Ray Tracing* (Glassner 2019).

We know that light is composed of photons at many different frequencies (or, equivalently, wavelengths). When these photons reach our eyes, each of them will be "deciphered" as a *different color* from the visible spectrum (based on the wavelength). What happens next is that these base colors are blended together by the eye. The resulting coloring will be tinted towards the base colors for which there were more photons (of that wavelength). This means that, for example, if most of the light's energy was carried by photons with very long wavelengths, it will appear more red (it will be more intense at the corresponding wavelength). On the contrary, if energy came mostly from short-wavelength photons, the light will look more blueish.

So, in our case, we don't really care for the light's *total* energy: it only reveals the light's "total" intensity. What we actually want to know is how much energy there is *per-wavelength* (we're asking ourselves "how red is this light?", or "how blue?"). Thus, we want to be able to *distribute* the total energy between the range of wavelengths that the light ray is made of. This is exactly the purpose of a *spectral distribution function*; in this case, it's called *spectral energy distribution* (or *SED*)[8].

Putting it simply, we thus say that colors are the result of a "blending" operation, which can be predicted through spectral distributions. At this point, we might be led to think that colors *are* spectral distributions. However, we should be careful not to mix up a purely physical concept such as spectra with the *experience* of colors, which is strongly related to human *perception*. The human visual system actually employs some tricks that we can exploit for our own rendering purposes. Most notably, we can avoid representing each color as a distribution over all wavelengths.

Since the 1800s, numerous experiments have proven that all colors perceived by the human eye can be represented using three scalar values. This is called the *tristimulus theory*, and it's been confirmed by the study

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8 From the NASA IPAC Teacher Archive Research Program (NITARP) wiki, at: `https://coolwiki.ipac.caltech.edu/index.php/SED_plots_introduction` (last accessed: 12.03.2025)

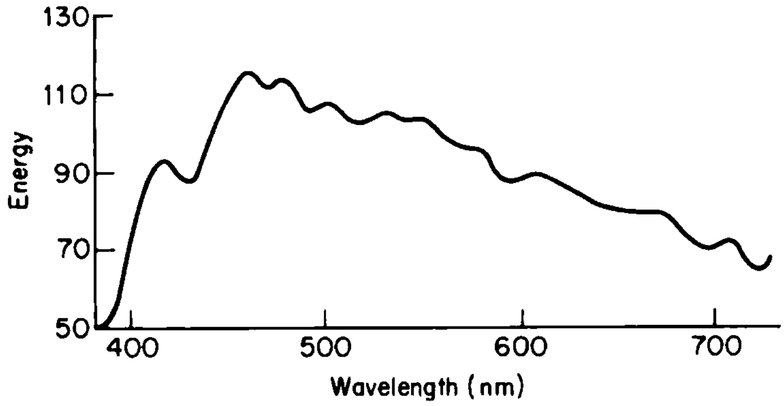

Fig. 11: "The spectrum of CIE Standard Illuminant D6500, which approximates sunlight on a cloudy day." (Glassner 2019)

of *cone cells* (Pharr, Jakob, and Humphreys 2023). *Cone cells* are light-sensitive cells on the retina which allow us to see colors. There are three types of cone cells, which we can call *short*, *medium* and *long*, after the wavelengths of light they are most sensitive to. To each of these cone cells, we can associate a *response function*, which describes how strongly one type of cone “responds” to beams of light of different wavelengths. Let’s call these functions $k_s(\lambda)$, $k_m(\lambda)$ and $k_l(\lambda)$ (for short, medium and long cones respectively). Since each ray of light with a specific (or "single") wavelength λ produces three response values on the retina, one for each type of cone, we can visualize this response as a single point in a 3D space:

$$[k_s(\lambda), k_m(\lambda), k_l(\lambda)]^t$$

We call this space a *color space*[9]. Light rays which don’t have a specific wavelength (they have "multiple" wavelengths) produce colors that are just linear combinations of the "simpler" ones; that is, the ones that *did* come from light rays with well-defined wavelengths (so, the colors are still just points in the color space) (Gortler 2012). Note that, since we’re just speaking of coordinates in a 3D space, we can easily describe the same color using a different basis (any three linearly independent base colors).

9 We should mention that what we’re talking about here is called *retinal color*. The *perceived color*, the one we experience and base judgements upon, is actually the result of some non-trivial processing steps done by the human visual system. For example, our brains make us perceive the colors of objects as more or less constant, even under different illuminants. (Gortler 2012)

It is well known that colors of computer pixels are the result of blending together red, green and blue (*RGB*), and now we can clearly see why this works. In fact, red, green and blue constitute the basis of the *RGB color space*.

In our implementation of Physically Based Rendering techniques, we'll be using RGB to represent *spectral quantities*; that is, values that vary with wavelength, like the energy of a light ray. Advanced rendering engines, such as *pbrt*, actually employ *spectral rendering*, where full spectra are used for higher precision (Pharr, Jakob, and Humphreys 2023). We'll consider the loss of information caused by RGB color spaces as negligible for visual aspects.

Before we conclude this section, we should also mention that digital images are, in general, not stored using the RGB format. More commonly, computers use the *sRGB color space*, which involves a non-linear transformation from RGB, for the purpose of storing color values more efficiently in the computer's memory (Gortler 2012). This, however, in rendering, will be our concern only at the moment of reading or storing an image from/to memory, while our physically based calculations will be done in the linear, RGB space.

2.3 LIGHT-MATTER INTERACTIONS

Now we have a pretty good mental model of light, but we should remember that we also need one for materials. To build material models, we need to talk about *matter*. Matter can either *emit* light, as a light source, or *respond* to incident energy (for example, through *reflection*). We'll consider both cases.

For the moment, however, we should settle on a model to describe matter. We'll be considering the simplest form of it: the atom.

It is well-known that the classical model of the atom includes a center, the *nucleus*, composed by small particles called *protons* (with positive charge $+e$) and *neutrons*. The nucleus is also orbited by another group of much smaller particles, called *electrons* (with negative charge $-e$).

In quantum mechanics, these particles don't have a precise location. Basically, unless a particle is observed, it is *nowhere*; particles are only associated with *probabilities*. For example, instead of individual electron particles, we have a *cloud of electrons*, with the cloud being more dense where electrons are more likely to be found (Glassner 1995).

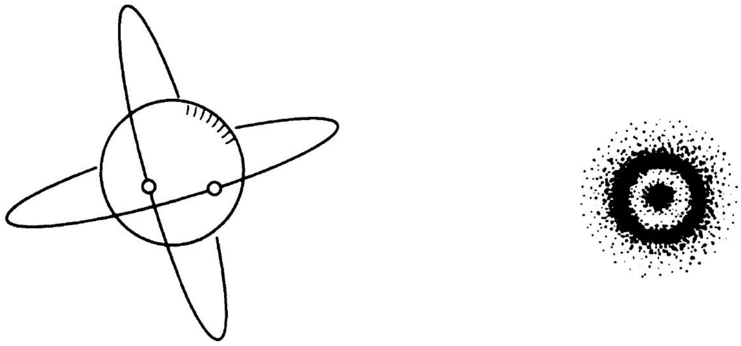

Fig. 12: The classical atom model (left) compared to a more modern view (right). (Glassner 1995)

Electrons are very susceptible to external influences, which make them move between different *states* by *absorbing* and *releasing* energy, often in the form of photons (Glassner 1995). The state of an electron at any time is given by a set of four *quantum numbers*. The *principal quantum number* n characterizes the electron's energy, with $n \epsilon \{1, 2, 3, ...\}$. Higher values of n indicate that the electron is more likely to be found far away from the nucleus[10].

The electrons of a neutral atom normally inhabit *ground states*. Apart from these, however, there are many higher-order *excited-state energy levels*. Above, we find the *ionization continuum*, where electrons become disassociated from the atoms and are free to move away (Glassner 1995). We have already seen this in the photoelectric effect, where electrons where expelled from atoms with kinetic energy E.

Finally, we can say a couple things about molecules. A molecule is an electrically neutral, stable combination of two or more atoms. The energy transitions for electrons change when they are involved in a bonding process that brings atoms together into molecules. Also, the size (much greater compared to atoms) and translational/vibrational energy of molecules affects what energy is absorbed and emitted by their atoms (Glassner 1995).

10 From *orbital*. Encyclopaedia Britannica. URL: `https://www.britannica.com/science/orbital` (last accessed: 12.02.2025)

2.3.1 *Emission*

We can classify the emission of light into two distinct types: *thermal* and *luminescent* (Glassner 1995).

Thermal emissions are caused by heat. We can explain this phenomenon as follows. It is known that, when the temperature of an object is above absolute zero, its atoms are moving. As is described by *Maxwell's equations*, the motion (more precisely, the *acceleration*) of atomic particles that hold electrical charges causes objects to emit electromagnetic radiation (Pharr, Jakob, and Humphreys 2023). In fact, we've already mentioned that humans emit light at infrared frequencies, and that, to emit light that is visible, an object needs to be much warmer. This is because, to put it simply, the atoms need to move *faster*. An incandescent light bulb is an example of a *thermal radiator*; it contains a small filament which, when heated by the flow of electricity, emits electromagnetic radiation (Pharr, Jakob, and Humphreys 2023).

When talking about thermal emission, it is common to run into the term *blackbody* (actually, we already have while discussing the particle nature of light). Blackbodies are (theoretical) perfect absorbers; they absorb all incident light, without reflecting any of it. They are also perfect emitters; that is, they convert *power* to electromagnetic radiation as efficiently as physically possible[11]. Even though true blackbodies are not physically realizable, the idea of them is still quite useful. This is because *Planck's law* specifies a way to, given a temperature, determine the emission of a blackbody as a function of wavelength. Taking a non-black body emitter, if the shape of its emitted spectral distribution is similar to the one of a blackbody at some temperature, we say that the emitter has the corresponding *color temperature*. Color temperatures over 5000 K are generally described as "cool," while those at 2700–3000K "warm". For example, the CIE standard illuminant A, which represents the average incandescent light, corresponds to a blackbody radiator of about 2856 K (Pharr, Jakob, and Humphreys 2023).

Luminescent emission is due to energy stored (perhaps for a very short time) in the material, and is determined primarily by factors different that temperature, although the temperature can affect the material (Glassner 1995). In the context of *luminescent emission*, it's important that we mention *phosphors*. Speaking broadly, a *phosphor* is a material that absorbs some form of energy, like an electromagnetic wave or an electron beam, and

11 This makes sense if we think of emission as of the reverse operation of absorption. (Pharr, Jakob, and Humphreys 2023)

then emits it as light over some period of time (Glassner 1995). When certain phosphors *luminesce* from *electron excitation*, the process is called *electroluminescence*. This is used in the production of TV screens and computer monitors[12].

Given our model of the atom, we can actually explain luminescence. We already know that electrons tend to move through different states, absorbing and emitting energy in the process, and that this energy is often in the form of photons. When a photon is absorbed, it generally disappears completely (a photon cannot exist at rest, and cannot transfer only *some* of its energy), and the electron transitions into what we called an *excited-state energy level*. Generally, this can't be kept up for long, so after a while the electron will drop back to its *ground state*, emitting the difference in energy between the two states as a *new* photon. Most times, this happens under 10^{-8} seconds. This number is used to divide phosphors into two categories. If the material responds to incident energy by reradiating most of it within 10^{-8} seconds, we call it *fluorescent* (this is what happens, for example, in *fluorescent lamps*[13]). If the emission persists longer, we're talking about *phosphorescence* (Glassner 1995).

2.3.2 *Response*

We now consider some common events that can be triggered when light strikes matter.

To do so, we'll first move back to a simpler model of light; we won't be thinking of waves or particles, but of *rays*. This is the point of view of *geometrical optics*, where we describe the interaction of light with objects much larger than its wavelength, allowing for this abstract idea of *rays of light* (Pharr, Jakob, and Humphreys 2023).

To predict how matter will respond to light, we can use a property called the *refractive index*. This is a complex number, where its real part describes how much the matter slows down the speed of light (which, as we know, is equal to c in a vacuum), and its imaginary part determines whether the light is *absorbed* as it propagates in the material (Hoffman 2012). We should mention that by *absorption* we mean the conversion of the energy of light into some other energy form, internal to the atoms.

12 From *phosphor*. Encyclopaedia Britannica. URL: `https://www.britannica.com/science/phosphor` (last accessed: 12.03.2025)

13 From *fluorescent lamp*. Encyclopaedia Britannica. URL: `https://www.britannica.com/technology/fluorescent-lamp` (last accessed: 12.03.2025)

One example for this is *thermal energy*, which derives from the *movement* of atoms[14].

We'll now consider matter as a *medium* through which light is propagating (just like it does through air, for example). We can distinguish two types of media: *homogeneous* and *heterogeneous*.

We say that a medium is *homogeneous* if it presents a uniform index of refraction (at the scale of the light's wavelength). Water and glass are examples of this. We also say that they're *transparent* media, since they don't absorb visible light in any significant way. In fact, their refraction indices have a very low imaginary part for visible light wavelengths (Hoffman 2012).

Fig. 13: Light in transparent media like water and glass just keeps on propagating in a straight line at the same intensity and color. (Hoffman 2012)

On the other hand, if the homogeneous medium *does* absorb light from the visible spectrum significantly, what happens is that the light's intensity dies down as it moves farther into the medium. However, the direction of light does not change.

We should note that these properties are always in function of *scale*. For instance, on the scale of many feet of distance, water actually absorbs quite a bit of light, especially red colors (Hoffman 2012).

Let's turn now to *heterogeneous media*, which have variations in the index of refraction. If the index of refraction changes slowly and continuously, then the light bends in a curve. If it changes abruptly, the light *scatters*, meaning that is splits into multiple directions. We can think about light encountering microscopic particles, which induce regions in space where

14 From The IBM article *What is thermal energy?*, at https://www.ibm.com/think/topics/thermal-energy (last accessed: 12.03.2025)

Fig. 14: Over large distances, the slight absorptivity of water becomes noticeable. (Image by Andreas Schau, from Pixabay).

the index of refraction becomes different. The type of particle affects the distribution of the scattered light's directions (Hoffman 2012).

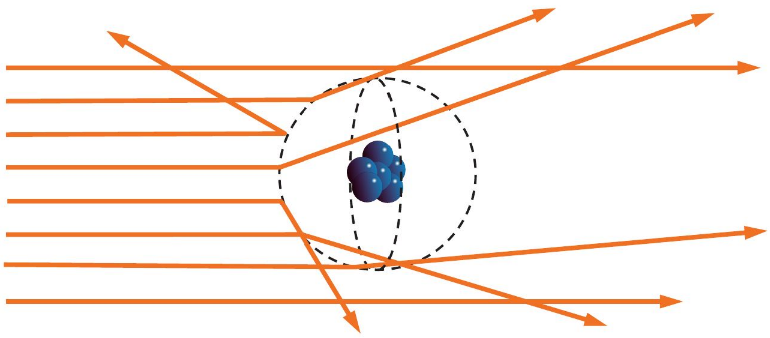

Fig. 15: "Particles cause light to scatter in all directions." (Hoffman 2012)

Media that are *translucent* (or *opaque*) contain such a high density of scattering elements that, as a result, light gets scattered in completely random directions.

Most media both scatter and absorb light, at least to some extent.

Now, we want to know what happens when light, propagating through air, hits an object's surface. An analytical solution to the problem can be

Fig. 16: Example of an opaque medium. (Hoffman 2012)

found if we consider the surface of the object as infinite and perfectly flat (relatively to the light's wavelength). In this case, we get special solutions to Maxwell's equations, called the *Fresnel equations*. These equations tell us that the material causes light to split only in two directions: *reflection* and *refraction*. They also describe the portions of reflected and refracted light (Hoffman 2012).

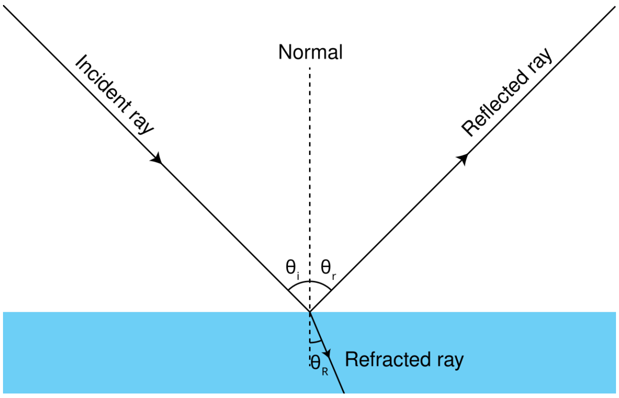

Fig. 17: An incident ray gets split into a reflected ray and a refracted ray. Source: Wikimedia Commons.

The angle formed by the reflected ray of light with the surface *normal*[15], that is, the *angle of reflection*, is equal to the angle of the incoming ray. The *angle of refraction* depends on the refractive index, and can be found through *Snell's law*.

15 the *normal* to a surface in a given point is a vector perpendicular to the surface in that point

2.3.3 *Reflection and Subsurface Scattering*

If real-world surfaces were really perfectly flat, computing realistic reflections would be extremely simple, in the sense that we would just reflect the incident light vector. In most cases, however, surfaces present irregularities larger than the light's wavelength, but too small to be seen by the eye. To account for this, we can model the surface as a large collection of small optically flat surfaces, which will cause light to be reflected in various directions, and various amounts for each direction. The resulting surface *roughness* will cause reflections to be more blurry.

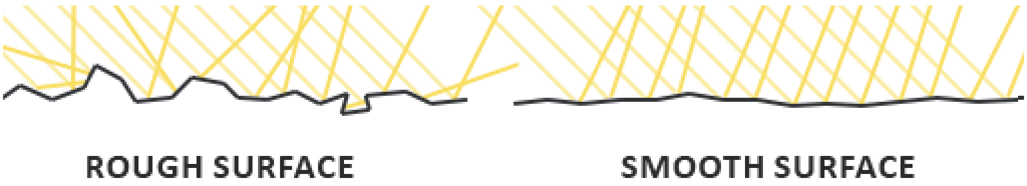

Fig. 18: Light gets reflected off a surface, modeled as a collection of smaller, optically flat surfaces. Image courtesy of Joey de Vries, at `https://learnopengl.com/PBR/Theory`. `(last accessed: 12.03.2025)` (license: `https://creativecommons.org/licenses/by/4.0/`)

Let's now consider the refracted light; what happens to it depends on the type of material. Metals immediately absorb refracted light (more precisely, their free electrons do). As for non-metals (also called *dielectrics* or *insulators*), light behaves the way we would expect, in part getting absorbed and in part scattered. Most times, the refracted light gets scattered so much that it's re-emitted out of the surface, in general, at a different point. This is called *subsurface scattering* (Hoffman 2012).

2.3.4 *Reflection, as seen by Electrodynamics*

Here, we'll form a quick intuition on how the phenomenon of reflection fits into the framework of *classical electrodynamics*. This means that we'll pretend again that light is simply a wave. Addressing the reflection of *photons* is a problem that belongs to *quantum electrodynamics* (Feynman, Leighton, and Sands 2011), and would constitute a topic too complex for this thesis.

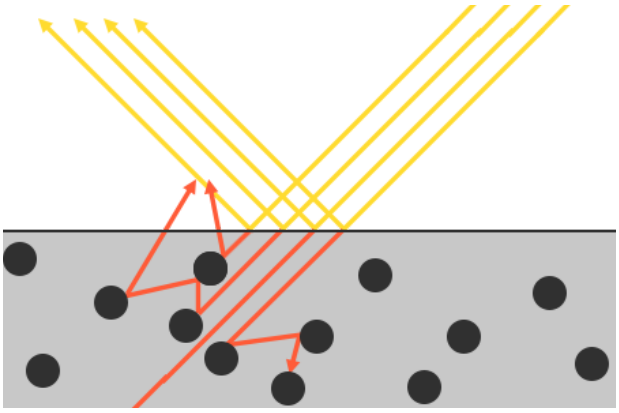

Fig. 19: The refracted light scatters until it re-emerges from the surface. Image courtesy of Joey de Vries, at `https://learnopengl.com/PBR/Theory`. (`last accessed: 12.03.2025`) (license: `https://creativecommons.org/licenses/by/4.0/`)

We'll be going through a very brief summary of the Feynman lectures on electrodynamics (volume I, lectures 28-30) (Feynman, Leighton, and Sands 2011).

Remember that Maxwell's equations tell us that an accelerating charge produces an electromagnetic field. To be more precise, we consider the case where a charge is moving nonrelativistically at a very large distance.

We can also have many charges instead of one. If we can make them move together, all in the same way, we can get the resulting field just by summing the effects of the individual charges. This is a consequence of the *superposition principle*, which we have already encountered.

Let's be more concrete now, and conceive an experiment. We have two pieces of wire connected to a generator (as shown in Figure 20). The generator makes a *potential difference*, which first pulls electrons from piece A and pushes them into B, to then almost immediately reverse the effect, and make the electrons move from B into A. This way, the charges accelerate up and down in the wires. This is the same as having a single charge (summing the effects of the individual charges) accelerating up and down as though A and B were a *single* wire. When the wire is very short compared to the distance traveled by light in one oscillation period, it is called an *electric dipole oscillator*.

Our accelerating charge generates an electric field, which we pick up with an identical instrument; that is, another pair of wires just like A and B. We call this a *detector*. The incoming electric field will produce a force

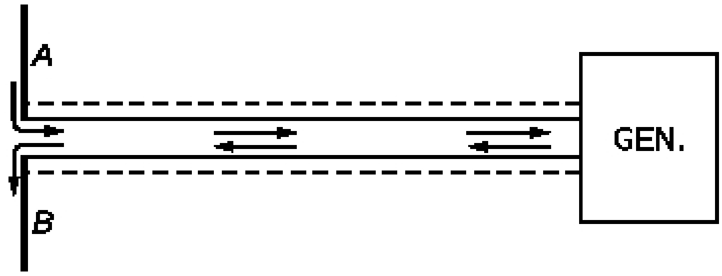

Fig. 20: "A high-frequency signal generator drives charges up and down on two wires." (Feynman, Leighton, and Sands 2011)

which will pull the electrons up on both wires or down on both wires. The resulting signal is then detected by a *rectifier* mounted between A and B, and amplified by an *amplifier*, so that we can perceive it in some form (like sound).

With this setup, we can observe that the field is strongest when the detector is parallel to the generator, and is 0 when they are perpendicular. In fact, what matters when calculating the field is the acceleration of the charge *projected* on the plane *perpendicular* to the *line of sight*.

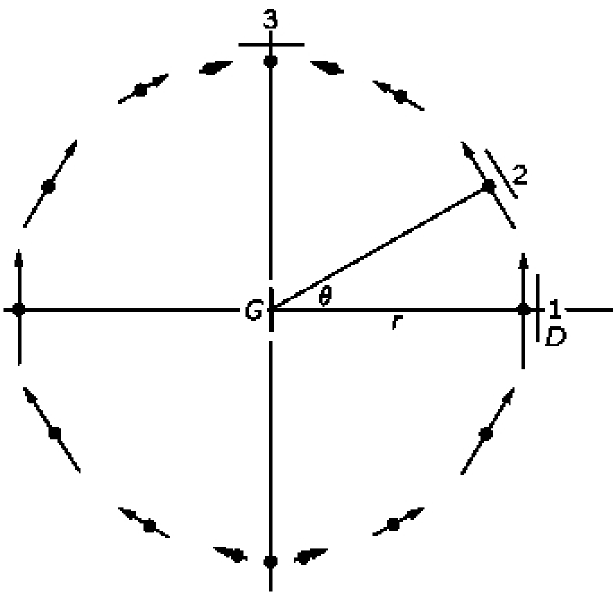

Fig. 21: The detector D finds a strong field when parallel to the detector G at point 1. The field is 0 when the detector is at 3. (Feynman, Leighton, and Sands 2011)

We now consider a situation where there are n oscillators, equally spaced at a distance d, all with the same amplitude, and lying on the same line (Figure 22). Their phases are different to the observer, both because of some intrinsic shift in phase from one to the next, and because we are looking at them at different angles, resulting in different distances traveled by the individual waves to reach us.

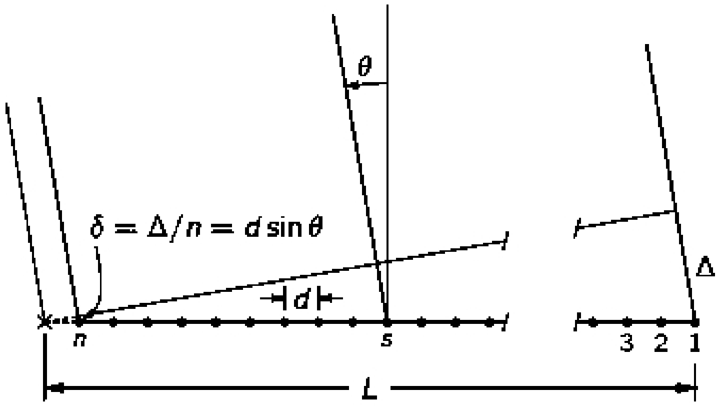

Fig. 22: "A linear array of n equal oscillators." (Feynman, Leighton, and Sands 2011)

The intrinsic shift in phase, one to the next, is α. If we are observing in a given direction θ from the normal, there is an additional phase difference contribution $2\pi d \sin\theta/\lambda$, because of the time delay between each successive oscillator. The sum of the waves becomes

$$\begin{aligned} R = & A[\cos\omega t + \cos(\omega t + \phi) + \cos(\omega t + 2\phi) + ... \\ & + \cos(\omega t + (n-1)\phi)] \end{aligned} \tag{2.7}$$

where $\phi = \alpha + 2\pi d \sin\theta/\lambda = \alpha + kd\sin\theta$ is the net phase difference between one oscillator and the next one.

If we consider the intensity of the resulting wave (2.7), we notice that we have a maximum when $\phi = 0$ (all the oscillators are in phase). We also get maxima, in general, when $\phi = 2\pi m$, where m is any integer (successive waves are out of phase by a multiple of 2π).

Now suppose that we have a source of electromagnetic radiation that's far away, practically at infinity, and that its light is coming in at an angle θ_{in}. The corresponding electric field will cause the electrons to move up and down in the wires, and in moving, they will generate *new* waves. This phenomenon is what we call *scattering*. In the case of a material hit by a light wave, its electrons will also start moving just like for the oscillators, generating new waves.

In *diffraction gratings*, instead of wires we have a flat piece of glass with tiny notches in it, each of which represents a source of slightly different scattered waves. Diffraction gratings can be used to split light based on its wavelength (that is, its color). In one of its forms, a diffraction grating is composed of just a plane glass sheet (transparent and colorless) with scratches on it, all equally spaced.

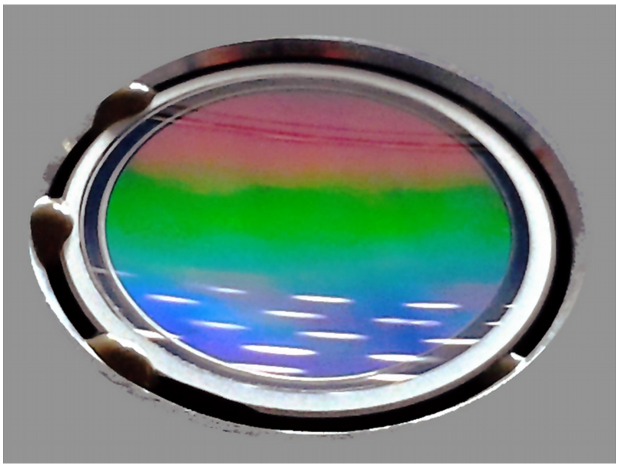

Fig. 23: A parabolic reflective diffraction grating. Source: Wikimedia Commons. (license: https://creativecommons.org/licenses/by-sa/3.0/deed.en)

So, we have a light source at infinity, which sends light to the scratches at an angle θ_{in}. We are interested in the scattered beam at the angle θ_{out}, which is the angle from which we are observing the grating (we called it θ earlier). The incident light will hit the scratches one after the other, with some time delay, resulting in a phase shift between adjacent scratches. This can be calculated as $\alpha = -2\pi d \sin\theta_{in}/\lambda$. Putting it all together, we have:

$$\phi = 2\pi d \sin\theta_{out}/\lambda - 2\pi d \sin\theta_{in}/\lambda$$

To have maximum intensity, we know that ϕ should satisfy $\phi = 2\pi m$:

$$2\pi m = 2\pi d \sin\theta_{out}/\lambda - 2\pi d \sin\theta_{in}/\lambda$$

Multiplying both sides by $\lambda/2\pi$, we get

$$\lambda m = d \sin\theta_{out} - d \sin\theta_{in}$$

Now, if d is less than $\lambda/2$, this equation can have no solution except $m = 0$. In this case, we have

$$\sin\theta_{in} = \sin\theta_{out}$$

Which may mean two things: either $\theta_{out} = \pi - \theta_{in}$ or $\theta_{out} = \theta_{in}$.

- In the first case, it's as if the light goes "right through" the grating.
- In the second, we have *reflection*: the *angle of incidence* is equal to the *angle of scattering*.

We can now end this chapter with a solid physical interpretation of reflection, which goes as follows. When light hits a surface, it generates motion in the atoms (or electrons, to be more precise) of the object, causing them, in turn, to regenerate *new* electromagnetic waves. These waves will add up, resulting in a *single* wave, produced by the object as a whole. If the spacing of the scatterers is small compared with one wavelength, the reflected wave will be strongest in intensity (only) at the angle $\theta_{out} = \theta_{in}$ (ignoring light that goes "right through"). Thus, we consider the direction of scattering as symmetric to the direction of incident light, with respect to the surface normal.

3

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BRDF MODELS

5

A RAY TRACER IMPLEMENTATION

6

RESULTS

7

CONCLUSIONS AND FUTURE WORK

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