ECON 144 HW 5

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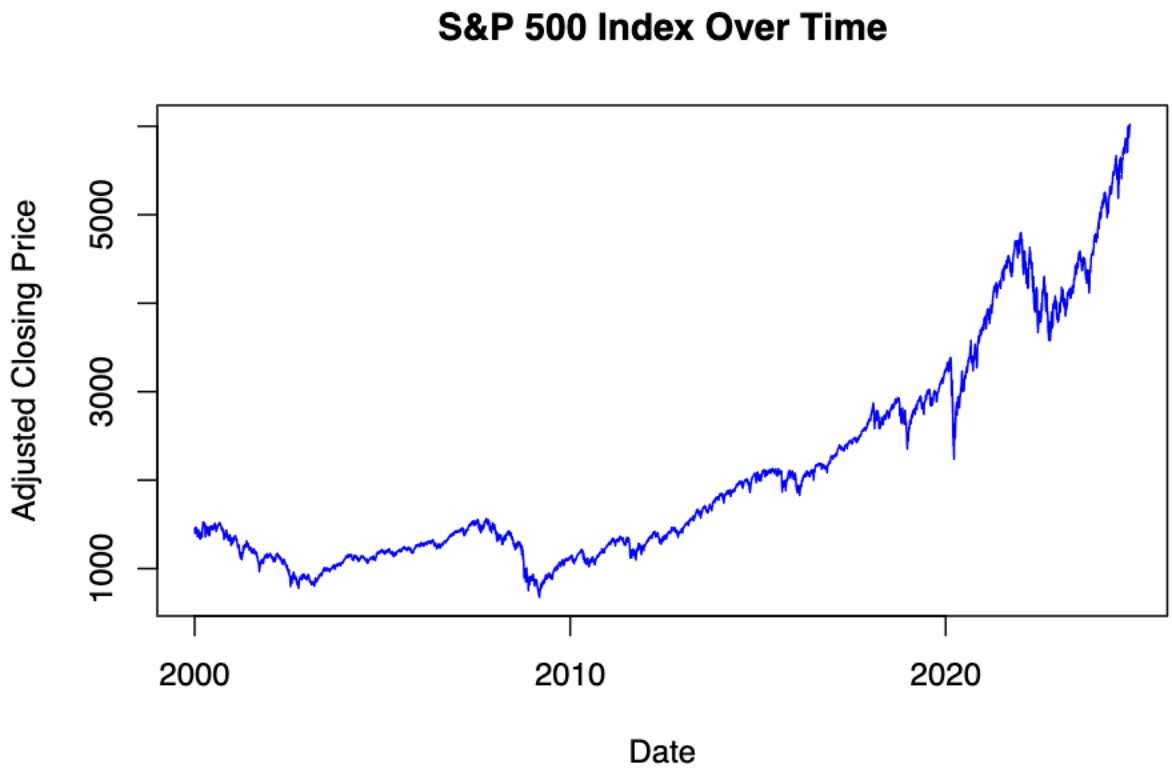
Problem 14.3

```
# Step 1: Fetch S&P 500 data from Yahoo Finance
getSymbols("^GSPC", src = "yahoo", from = "2000-01-01")
```

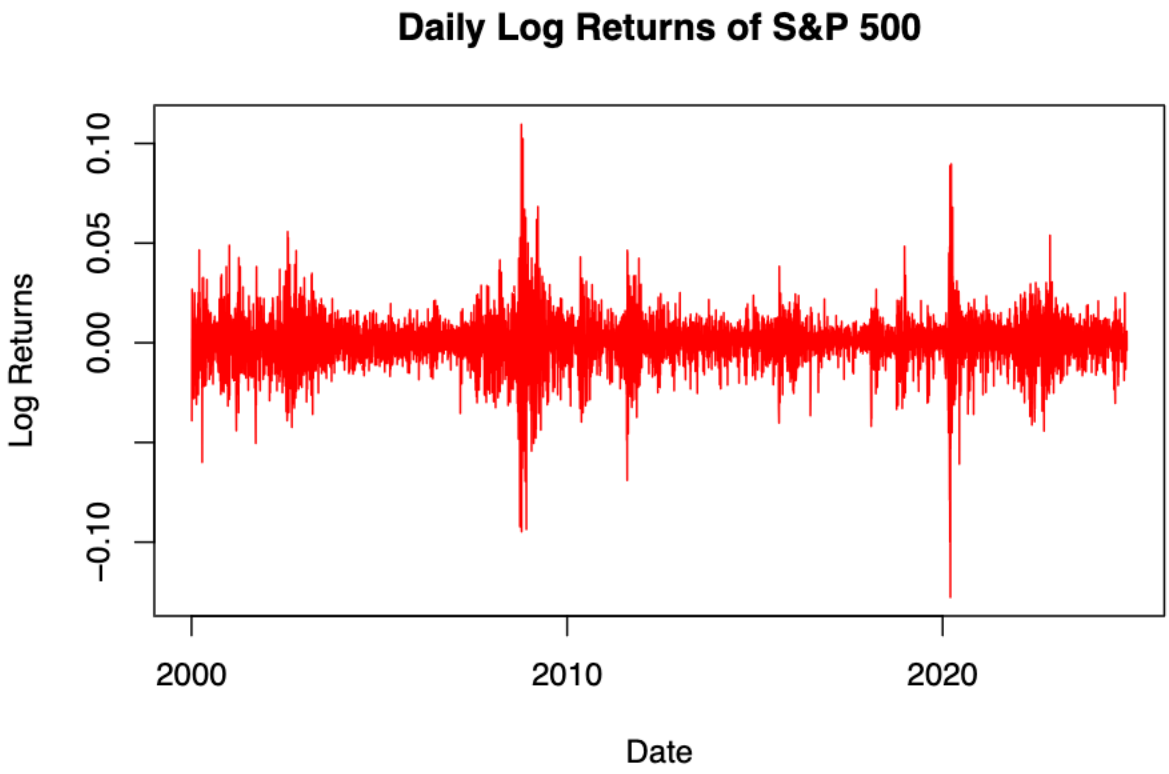
```
## [1] "GSPC"
```

```
# Extract the adjusted closing prices
sp500 <- GSPC$GSPC.Adjusted

# Convert to a time series object
sp500_ts <- as.numeric(sp500)  # Ensure numeric format
sp500_dates <- index(GSPC)     # Get the dates
sp500_df <- data.frame(Date = sp500_dates, SP500 = sp500_ts)
```

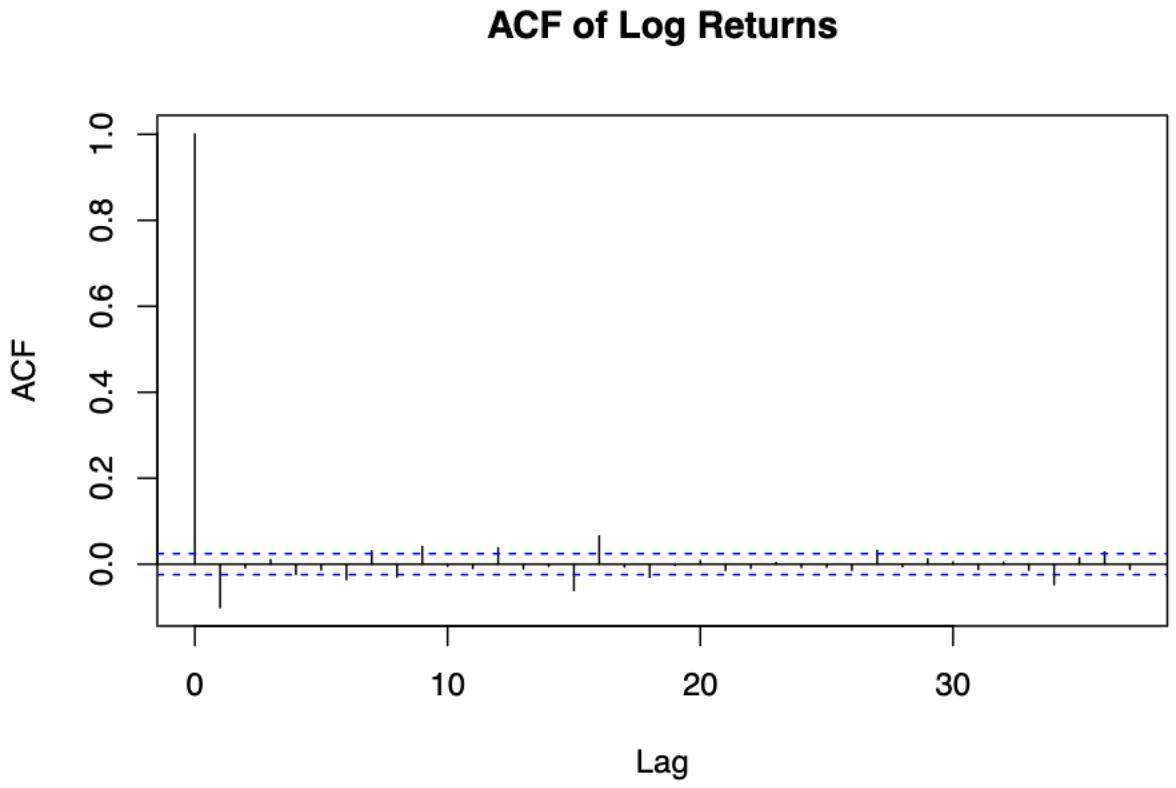
```
# Step 2: Plot the S&P 500 Index
plot(sp500_dates, sp500_ts, type = "l", main = "S&P 500 Index Over Time",
     xlab = "Date", ylab = "Adjusted Closing Price", col = "blue")
```



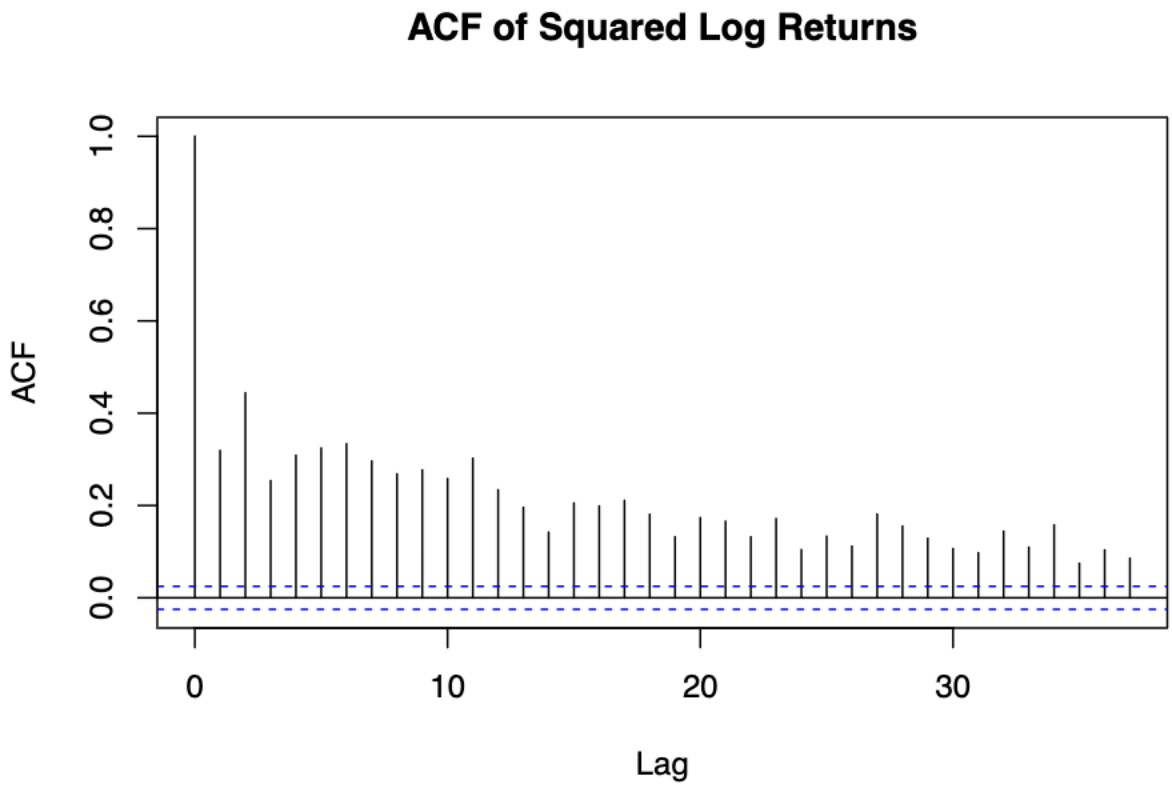

```r
# Step 3: Compute daily returns
returns <- diff(log(sp500_ts))  # Log returns to stabilize variance

# Plot returns
plot(sp500_dates[-1], returns, type = "l", main = "Daily Log Returns of S&P 500",
     xlab = "Date", ylab = "Log Returns", col = "red")
```




```r
# Step 4: Compute autocorrelation functions (ACF) for returns and squared returns
acf(returns, main = "ACF of Log Returns")
```




```r
acf(returns^2, main = "ACF of Squared Log Returns")
```




```r
# Step 5: Fit an ARCH model to volatility
# Specify an ARCH(1) model
arch_model <- garch(returns, order = c(0, 1))  # ARCH(1)
```

```
##
##  ***** ESTIMATION WITH ANALYTICAL GRADIENT *****
##
##
##      I     INITIAL X(I)        D(I)
##
##      1     1.424260e-04     1.000e+00
##      2     5.000000e-02     1.000e+00
##
##     IT   NF      F         RELDF    PRELDF    RELDX   STPPAR   D*STEP   NPRELDF
##      0    1 -2.470e+04
##      1    7 -2.471e+04  6.87e-04  9.23e-04  1.0e-04  2.3e+11  1.0e-05  1.05e+08
##      2    8 -2.472e+04  1.36e-04  1.83e-04  8.6e-05  2.0e+00  1.0e-05  1.45e+02
##      3    9 -2.472e+04  1.33e-05  1.48e-05  9.8e-05  2.0e+00  1.0e-05  1.42e+02
##      4   16 -2.480e+04  3.26e-03  4.38e-03  2.9e-01  2.0e+00  4.1e-02  1.42e+02
##      5   17 -2.484e+04  1.58e-03  1.73e-03  1.8e-01  3.9e-01  4.1e-02  1.90e-03
##      6   19 -2.489e+04  2.13e-03  3.07e-03  2.6e-01  4.3e-01  9.1e-02  3.38e-03
##      7   20 -2.491e+04  8.69e-04  6.89e-04  5.9e-02  0.0e+00  2.8e-02  6.89e-04
##      8   22 -2.492e+04  4.43e-04  3.12e-04  9.9e-02  0.0e+00  5.5e-02  3.12e-04
##      9   23 -2.493e+04  1.61e-04  1.27e-04  7.3e-02  0.0e+00  4.9e-02  1.27e-04
##     10   24 -2.493e+04  1.92e-05  1.64e-05  2.9e-02  0.0e+00  2.1e-02  1.64e-05
##     11   25 -2.493e+04  8.55e-07  7.85e-07  6.3e-03  0.0e+00  4.8e-03  7.85e-07
```

```
##     12   26 -2.493e+04  1.07e-08  1.02e-08  5.5e-04  0.0e+00  4.2e-04  1.02e-08
##     13   27 -2.493e+04  8.15e-11  7.54e-11  3.8e-06  0.0e+00  2.9e-06  7.54e-11
##
##  ***** RELATIVE FUNCTION CONVERGENCE *****
##
##  FUNCTION    -2.492691e+04   RELDX        3.824e-06
##  FUNC. EVALS      27         GRAD. EVALS      14
##  PRELDF       7.542e-11      NPRELDF      7.542e-11
##
##      I      FINAL X(I)        D(I)          G(I)
##
##      1    9.587932e-05     1.000e+00    -7.424e+01
##      2    3.809622e-01     1.000e+00     2.360e-03
```

```r
summary(arch_model)
```

```
##
## Call:
## garch(x = returns, order = c(0, 1))
##
## Model:
## GARCH(0,1)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -9.20869 -0.42366  0.05682  0.52234  8.97091
##
## Coefficient(s):
##     Estimate  Std. Error  t value Pr(>|t|)
## a0 9.588e-05   1.160e-06    82.69   <2e-16 ***
## a1 3.810e-01   1.432e-02    26.60   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Diagnostic Tests:
##  Jarque Bera Test
##
## data:  Residuals
## X-squared = 13920, df = 2, p-value < 2.2e-16
##
##
##  Box-Ljung test
##
## data:  Squared.Residuals
## X-squared = 5.7052, df = 1, p-value = 0.01691
```

```r
# Try ARCH models with different lags
for (p in 1:5) {
  model <- garch(returns, order = c(0, p))
  print(paste("ARCH(", p, ") AIC:", AIC(model)))
}
```

```
##
```

```
##  ***** ESTIMATION WITH ANALYTICAL GRADIENT *****
##
##
##      I     INITIAL X(I)        D(I)
##
##      1     1.424260e-04     1.000e+00
##      2     5.000000e-02     1.000e+00
##
##     IT   NF      F         RELDF    PRELDF    RELDX   STPPAR   D*STEP   NPRELDF
##      0    1 -2.470e+04
##      1    7 -2.471e+04  6.87e-04  9.23e-04  1.0e-04  2.3e+11  1.0e-05  1.05e+08
##      2    8 -2.472e+04  1.36e-04  1.83e-04  8.6e-05  2.0e+00  1.0e-05  1.45e+02
##      3    9 -2.472e+04  1.33e-05  1.48e-05  9.8e-05  2.0e+00  1.0e-05  1.42e+02
##      4   16 -2.480e+04  3.26e-03  4.38e-03  2.9e-01  2.0e+00  4.1e-02  1.42e+02
##      5   17 -2.484e+04  1.58e-03  1.73e-03  1.8e-01  3.9e-01  4.1e-02  1.90e-03
##      6   19 -2.489e+04  2.13e-03  3.07e-03  2.6e-01  4.3e-01  9.1e-02  3.38e-03
##      7   20 -2.491e+04  8.69e-04  6.89e-04  5.9e-02  0.0e+00  2.8e-02  6.89e-04
##      8   22 -2.492e+04  4.43e-04  3.12e-04  9.9e-02  0.0e+00  5.5e-02  3.12e-04
##      9   23 -2.493e+04  1.61e-04  1.27e-04  7.3e-02  0.0e+00  4.9e-02  1.27e-04
##     10   24 -2.493e+04  1.92e-05  1.64e-05  2.9e-02  0.0e+00  2.1e-02  1.64e-05
##     11   25 -2.493e+04  8.55e-07  7.85e-07  6.3e-03  0.0e+00  4.8e-03  7.85e-07
##     12   26 -2.493e+04  1.07e-08  1.02e-08  5.5e-04  0.0e+00  4.2e-04  1.02e-08
##     13   27 -2.493e+04  8.15e-11  7.54e-11  3.8e-06  0.0e+00  2.9e-06  7.54e-11
##
##  ***** RELATIVE FUNCTION CONVERGENCE *****
##
##  FUNCTION    -2.492691e+04   RELDX        3.824e-06
##  FUNC. EVALS      27         GRAD. EVALS      14
##  PRELDF       7.542e-11      NPRELDF      7.542e-11
##
##      I      FINAL X(I)        D(I)          G(I)
##
##      1    9.587932e-05     1.000e+00    -7.424e+01
##      2    3.809622e-01     1.000e+00     2.360e-03
##
## [1] "ARCH( 1 ) AIC: -38335.5263445595"
##
##  ***** ESTIMATION WITH ANALYTICAL GRADIENT *****
##
##
##      I     INITIAL X(I)        D(I)
##
##      1     1.349299e-04     1.000e+00
##      2     5.000000e-02     1.000e+00
##      3     5.000000e-02     1.000e+00
##
##     IT   NF      F         RELDF    PRELDF    RELDX   STPPAR   D*STEP   NPRELDF
##      0    1 -2.493e+04
##      1    7 -2.497e+04  1.70e-03  1.90e-03  1.1e-04  4.1e+11  1.1e-05  3.87e+08
##      2    8 -2.501e+04  1.66e-03  2.11e-03  2.2e-04  2.8e+00  2.2e-05  3.74e+02
##      3   15 -2.522e+04  8.07e-03  1.40e-02  4.3e-01  2.0e+00  8.8e-02  3.23e+02
##      4   17 -2.522e+04  6.88e-05  2.58e-03  1.5e-01  2.0e+00  4.0e-02  3.43e-01
##      5   18 -2.524e+04  9.05e-04  6.78e-04  7.2e-02  2.0e+00  2.0e-02  1.62e-02
##      6   20 -2.524e+04  2.09e-05  1.17e-04  2.7e-02  2.0e+00  7.3e-03  1.69e-02
```

```
##      7   21 -2.524e+04  5.48e-05  6.16e-05  2.7e-02  2.0e+00  7.3e-03  8.27e-02
##      8   22 -2.524e+04  5.77e-07  1.34e-05  2.7e-02  2.0e+00  7.3e-03  1.79e-02
##      9   25 -2.531e+04  2.41e-03  2.37e-03  1.8e-01  4.8e-01  5.8e-02  2.78e-03
##     10   26 -2.536e+04  2.23e-03  2.41e-03  1.5e-01  7.6e-01  5.8e-02  3.29e-03
##     11   27 -2.539e+04  1.07e-03  9.15e-04  1.1e-01  3.4e-01  5.8e-02  9.91e-04
##     12   28 -2.540e+04  5.36e-04  4.39e-04  9.2e-02  1.4e-01  5.8e-02  4.44e-04
##     13   29 -2.541e+04  1.30e-04  1.04e-04  5.3e-02  0.0e+00  3.7e-02  1.04e-04
##     14   30 -2.541e+04  1.40e-05  1.17e-05  2.0e-02  0.0e+00  1.4e-02  1.17e-05
##     15   31 -2.541e+04  1.93e-06  1.37e-06  6.3e-03  0.0e+00  4.7e-03  1.37e-06
##     16   32 -2.541e+04  1.14e-06  8.59e-07  3.9e-03  0.0e+00  4.0e-03  8.59e-07
##     17   33 -2.541e+04  3.48e-07  2.90e-07  3.0e-03  0.0e+00  2.2e-03  2.90e-07
##     18   34 -2.541e+04  3.76e-08  3.31e-08  7.2e-04  0.0e+00  7.5e-04  3.31e-08
##     19   35 -2.541e+04  1.50e-09  1.42e-09  2.5e-04  0.0e+00  1.9e-04  1.42e-09
##     20   36 -2.541e+04  1.12e-11  1.09e-11  1.8e-05  0.0e+00  1.7e-05  1.09e-11
##
##  ***** RELATIVE FUNCTION CONVERGENCE *****
##
##  FUNCTION    -2.540674e+04   RELDX        1.825e-05
##  FUNC. EVALS      36         GRAD. EVALS      21
##  PRELDF       1.086e-11      NPRELDF      1.086e-11
##
##      I      FINAL X(I)        D(I)          G(I)
##
##      1    6.042884e-05     1.000e+00    -3.695e+00
##      2    2.380968e-01     1.000e+00    -1.754e-03
##      3    3.658986e-01     1.000e+00    -1.010e-04
##
## [1] "ARCH( 2 ) AIC: -39295.015992183"
##
##  ***** ESTIMATION WITH ANALYTICAL GRADIENT *****
##
##
##      I     INITIAL X(I)        D(I)
##
##      1     1.274338e-04     1.000e+00
##      2     5.000000e-02     1.000e+00
##      3     5.000000e-02     1.000e+00
##      4     5.000000e-02     1.000e+00
##
##     IT   NF      F         RELDF    PRELDF    RELDX   STPPAR   D*STEP   NPRELDF
##      0    1 -2.505e+04
##      1    7 -2.511e+04  2.51e-03  2.76e-03  1.3e-04  4.1e+11  1.3e-05  5.66e+08
##      2    8 -2.517e+04  2.50e-03  3.32e-03  2.6e-04  3.0e+00  2.6e-05  4.44e+02
##      3    9 -2.517e+04  5.07e-06  5.96e-06  2.0e-04  2.0e+00  2.6e-05  3.49e+02
##      4   16 -2.543e+04  9.92e-03  1.74e-02  4.4e-01  2.0e+00  1.1e-01  3.47e+02
##      5   28 -2.544e+04  7.18e-04  2.87e-03  7.6e-05  2.5e+00  2.0e-05  4.50e-01
##      6   29 -2.545e+04  3.87e-04  2.89e-04  6.1e-05  2.0e+00  2.0e-05  7.24e-01
##      7   30 -2.545e+04  3.19e-05  3.97e-05  6.3e-05  2.0e+00  2.0e-05  7.84e-01
##      8   31 -2.545e+04  1.67e-06  1.77e-06  6.4e-05  2.0e+00  2.0e-05  7.72e-01
##      9   37 -2.546e+04  7.65e-05  1.50e-04  6.5e-02  2.0e+00  1.9e-02  7.71e-01
##     10   39 -2.546e+04  1.44e-06  2.82e-06  1.0e-02  2.0e+00  2.7e-03  4.42e-03
##     11   41 -2.546e+04  1.08e-05  1.34e-05  1.1e-02  2.0e+00  2.7e-03  6.37e-03
##     12   42 -2.546e+04  2.67e-05  3.77e-05  1.1e-02  2.0e+00  2.7e-03  1.31e-02
##     13   45 -2.546e+04  2.76e-04  3.86e-04  1.0e-01  1.8e+00  2.5e-02  4.98e-03
```

```
##     14   47 -2.556e+04  3.56e-03  4.06e-03  2.5e-01  5.7e-01  1.0e-01  5.10e-03
##     15   48 -2.560e+04  1.63e-03  1.78e-03  1.4e-01  0.0e+00  9.2e-02  1.78e-03
##     16   49 -2.561e+04  5.04e-04  4.59e-04  3.3e-02  0.0e+00  2.1e-02  4.59e-04
##     17   51 -2.561e+04  1.33e-04  1.03e-04  4.6e-02  0.0e+00  3.3e-02  1.03e-04
##     18   52 -2.561e+04  1.88e-05  1.61e-05  2.3e-02  0.0e+00  1.6e-02  1.61e-05
##     19   53 -2.561e+04  2.21e-06  1.53e-06  6.7e-03  0.0e+00  4.5e-03  1.53e-06
##     20   54 -2.561e+04  1.41e-06  1.09e-06  5.4e-03  0.0e+00  4.3e-03  1.09e-06
##     21   55 -2.561e+04  3.81e-07  2.99e-07  2.8e-03  0.0e+00  2.1e-03  2.99e-07
##     22   56 -2.561e+04  8.74e-08  7.08e-08  1.6e-03  0.0e+00  1.0e-03  7.08e-08
##     23   57 -2.561e+04  1.52e-08  1.23e-08  4.3e-04  0.0e+00  4.3e-04  1.23e-08
##     24   58 -2.561e+04  2.21e-09  1.90e-09  2.3e-04  0.0e+00  1.5e-04  1.90e-09
##     25   59 -2.561e+04  1.51e-10  1.36e-10  4.3e-05  0.0e+00  4.1e-05  1.36e-10
##     26   60 -2.561e+04  4.31e-12  4.04e-12  1.2e-05  0.0e+00  8.6e-06  4.04e-12
##
##  ***** RELATIVE FUNCTION CONVERGENCE *****
##
##  FUNCTION    -2.561365e+04   RELDX        1.243e-05
##  FUNC. EVALS      60         GRAD. EVALS      27
##  PRELDF       4.044e-12      NPRELDF      4.044e-12
##
##      I      FINAL X(I)        D(I)          G(I)
##
##      1    4.292115e-05     1.000e+00    -7.219e+00
##      2    1.679820e-01     1.000e+00     1.328e-03
##      3    3.280021e-01     1.000e+00    -1.752e-03
##      4    2.513187e-01     1.000e+00    -1.877e-04
##
## [1] "ARCH( 3 ) AIC: -39708.6700001193"
##
##  ***** ESTIMATION WITH ANALYTICAL GRADIENT *****
##
##
##      I     INITIAL X(I)        D(I)
##
##      1     1.199377e-04     1.000e+00
##      2     5.000000e-02     1.000e+00
##      3     5.000000e-02     1.000e+00
##      4     5.000000e-02     1.000e+00
##      5     5.000000e-02     1.000e+00
##
##     IT   NF      F         RELDF    PRELDF    RELDX   STPPAR   D*STEP   NPRELDF
##      0    1 -2.516e+04
##      1    7 -2.524e+04  3.16e-03  3.42e-03  1.4e-04  4.4e+11  1.4e-05  7.59e+08
##      2    8 -2.533e+04  3.52e-03  4.56e-03  2.8e-04  3.5e+00  2.8e-05  4.81e+02
##      3    9 -2.533e+04  8.61e-06  1.94e-05  1.7e-04  2.0e+00  2.8e-05  3.35e+02
##      4   10 -2.533e+04  9.14e-06  9.29e-06  1.7e-04  2.0e+00  2.8e-05  3.31e+02
##      5   17 -2.559e+04  9.95e-03  1.83e-02  4.1e-01  2.0e+00  1.1e-01  3.27e+02
##      6   19 -2.562e+04  1.12e-03  2.85e-03  1.6e-01  2.0e+00  5.5e-02  3.03e-01
##      7   20 -2.562e+04  2.39e-04  5.90e-04  1.7e-01  2.0e+00  5.5e-02  6.37e-01
##      8   25 -2.563e+04  1.03e-04  3.33e-04  2.3e-05  4.7e+01  5.5e-06  3.26e-01
##      9   26 -2.563e+04  2.15e-05  2.46e-05  1.7e-05  2.0e+00  5.5e-06  1.46e-01
##     10   27 -2.563e+04  2.82e-07  2.71e-07  1.8e-05  2.0e+00  5.5e-06  1.45e-01
##     11   33 -2.563e+04  1.36e-05  1.91e-05  1.9e-02  2.0e+00  5.6e-03  1.45e-01
##     12   35 -2.563e+04  1.80e-04  2.49e-04  7.9e-02  1.8e+00  2.2e-02  3.82e-03
```

```
##     13   36 -2.563e+04  4.29e-05  1.19e-04  6.6e-02  2.0e+00  2.2e-02  2.84e-01
##     14   38 -2.568e+04  1.91e-03  2.60e-03  1.4e-01  5.8e-01  6.2e-02  3.32e-03
##     15   48 -2.570e+04  6.70e-04  1.27e-03  4.0e-05  2.5e+00  1.3e-05  1.40e-01
##     16   49 -2.570e+04  9.82e-05  1.47e-04  2.4e-05  2.0e+00  1.3e-05  4.09e-02
##     17   50 -2.570e+04  1.01e-05  9.14e-06  2.5e-05  2.0e+00  1.3e-05  4.23e-02
##     18   58 -2.574e+04  1.64e-03  3.00e-03  1.6e-01  1.8e+00  9.7e-02  4.24e-02
##     19   59 -2.575e+04  1.52e-04  1.33e-04  3.6e-02  0.0e+00  2.3e-02  1.33e-04
##     20   60 -2.575e+04  2.88e-05  2.48e-05  2.6e-02  0.0e+00  1.7e-02  2.48e-05
##     21   61 -2.575e+04  2.47e-06  1.94e-06  6.9e-03  0.0e+00  4.7e-03  1.94e-06
##     22   62 -2.575e+04  5.07e-07  4.12e-07  3.1e-03  0.0e+00  2.1e-03  4.12e-07
##     23   63 -2.575e+04  1.06e-07  7.98e-08  1.5e-03  0.0e+00  9.6e-04  7.98e-08
##     24   64 -2.575e+04  3.33e-08  2.72e-08  9.8e-04  0.0e+00  6.4e-04  2.72e-08
##     25   65 -2.575e+04  5.09e-09  4.27e-09  3.2e-04  0.0e+00  2.3e-04  4.27e-09
##     26   66 -2.575e+04  4.15e-10  3.80e-10  8.6e-05  0.0e+00  5.9e-05  3.80e-10
##     27   67 -2.575e+04  8.64e-12  8.09e-12  1.5e-05  0.0e+00  9.9e-06  8.09e-12
##
##  ***** RELATIVE FUNCTION CONVERGENCE *****
##
##  FUNCTION    -2.574887e+04   RELDX        1.501e-05
##  FUNC. EVALS      67         GRAD. EVALS      28
##  PRELDF       8.093e-12      NPRELDF      8.093e-12
##
##      I      FINAL X(I)        D(I)          G(I)
##
##      1    3.422278e-05     1.000e+00    -9.023e+00
##      2    1.208398e-01     1.000e+00     1.895e-03
##      3    2.479561e-01     1.000e+00    -1.661e-03
##      4    2.259294e-01     1.000e+00    -3.440e-03
##      5    2.038904e-01     1.000e+00    -1.882e-04
##
## [1] "ARCH( 4 ) AIC: -39978.9612141236"
##
##  ***** ESTIMATION WITH ANALYTICAL GRADIENT *****
##
##
##      I     INITIAL X(I)        D(I)
##
##      1     1.124416e-04     1.000e+00
##      2     5.000000e-02     1.000e+00
##      3     5.000000e-02     1.000e+00
##      4     5.000000e-02     1.000e+00
##      5     5.000000e-02     1.000e+00
##      6     5.000000e-02     1.000e+00
##
##     IT   NF      F         RELDF    PRELDF    RELDX   STPPAR   D*STEP   NPRELDF
##      0    1 -2.527e+04
##      1    7 -2.535e+04  3.25e-03  3.44e-03  1.3e-04  5.4e+11  1.3e-05  9.27e+08
##      2    8 -2.546e+04  4.41e-03  5.17e-03  2.5e-04  4.6e+00  2.5e-05  5.14e+02
##      3   10 -2.546e+04  1.05e-04  5.70e-04  1.2e-04  2.1e+00  1.2e-05  3.28e+02
##      4   11 -2.547e+04  2.04e-04  2.55e-04  6.2e-05  2.0e+00  1.2e-05  2.96e+02
##      5   12 -2.547e+04  7.10e-06  6.51e-06  6.6e-05  2.0e+00  1.2e-05  3.00e+02
##      6   20 -2.570e+04  9.05e-03  1.82e-02  3.9e-01  2.0e+00  1.2e-01  2.98e+02
##      7   22 -2.573e+04  1.09e-03  4.19e-03  3.8e-02  2.0e+00  1.2e-02  2.11e-01
##      8   23 -2.575e+04  7.53e-04  5.40e-04  3.7e-02  2.0e+00  1.2e-02  1.38e-01
```

```
##      9   24 -2.575e+04  8.33e-05  2.07e-04  3.5e-02  2.0e+00  1.2e-02  1.14e-01
##     10   25 -2.575e+04  3.95e-05  5.78e-05  3.6e-02  2.0e+00  1.2e-02  8.80e-03
##     11   27 -2.575e+04  5.75e-06  1.08e-05  1.7e-02  2.0e+00  5.7e-03  3.02e-02
##     12   33 -2.575e+04  4.64e-07  9.08e-07  9.0e-07  3.5e+01  2.1e-07  4.46e-03
##     13   34 -2.575e+04  6.00e-10  6.03e-10  5.7e-07  2.0e+00  2.1e-07  6.76e-03
##     14   35 -2.575e+04  8.35e-10  8.35e-10  1.2e-06  2.0e+00  4.2e-07  6.76e-03
##     15   43 -2.575e+04  2.19e-06  4.37e-06  1.1e-02  2.0e+00  4.0e-03  6.76e-03
##     16   44 -2.575e+04  4.91e-06  7.15e-06  1.2e-02  2.0e+00  4.0e-03  1.95e-03
##     17   47 -2.577e+04  6.64e-04  1.20e-03  1.7e-01  1.9e+00  6.2e-02  3.89e-02
##     18   48 -2.580e+04  1.09e-03  1.63e-03  1.1e-01  2.0e+00  6.2e-02  2.70e+00
##     19   55 -2.580e+04  2.32e-06  4.50e-06  1.3e-06  1.0e+02  4.4e-07  3.19e-03
##     20   65 -2.582e+04  9.08e-04  1.44e-03  1.4e-01  9.1e-01  6.9e-02  3.09e-03
##     21   66 -2.583e+04  2.49e-04  2.13e-04  7.0e-02  0.0e+00  3.7e-02  2.13e-04
##     22   67 -2.583e+04  1.57e-05  1.35e-05  1.7e-02  0.0e+00  1.0e-02  1.35e-05
##     23   68 -2.583e+04  6.70e-07  6.24e-07  4.2e-03  0.0e+00  2.4e-03  6.24e-07
##     24   69 -2.583e+04  3.48e-08  2.70e-08  9.2e-04  0.0e+00  5.2e-04  2.70e-08
##     25   70 -2.583e+04  9.05e-09  8.13e-09  6.2e-04  0.0e+00  3.3e-04  8.13e-09
##     26   71 -2.583e+04  4.88e-10  4.09e-10  1.1e-04  0.0e+00  6.0e-05  4.09e-10
##     27   72 -2.583e+04  5.30e-11  4.48e-11  5.9e-05  0.0e+00  2.4e-05  4.48e-11
##
##  ***** RELATIVE FUNCTION CONVERGENCE *****
##
##  FUNCTION    -2.582721e+04   RELDX        5.893e-05
##  FUNC. EVALS      72         GRAD. EVALS      28
##  PRELDF       4.478e-11      NPRELDF      4.478e-11
##
##      I      FINAL X(I)        D(I)          G(I)
##
##      1    2.947807e-05     1.000e+00    -4.961e+01
##      2    1.082614e-01     1.000e+00    -2.423e-02
##      3    1.985115e-01     1.000e+00    -1.325e-02
##      4    1.828927e-01     1.000e+00    -1.469e-02
##      5    1.814562e-01     1.000e+00     1.158e-02
##      6    1.476214e-01     1.000e+00    -8.585e-03
##
## [1] "ARCH( 5 ) AIC: -40135.4747282945"
```

```r
# Step 6: Fit a GARCH model
# Specify a GARCH(1,1) model
spec_garch <- ugarchspec(variance.model = list(model = "sGARCH", garchOrder = c(1, 1)),
                         mean.model = list(armaOrder = c(0, 0), include.mean = TRUE))
garch_model <- ugarchfit(spec = spec_garch, data = returns)
summary(garch_model)
```

```
##    Length     Class      Mode
##         1 uGARCHfit        S4
```

```r
# Compare AIC values
for (q in 1:3) {
  for (p in 1:3) {
    spec <- ugarchspec(variance.model = list(model = "sGARCH", garchOrder = c(p, q)),
                       mean.model = list(armaOrder = c(0, 0), include.mean = TRUE))
    model <- ugarchfit(spec = spec, data = returns)
```

```
    print(paste("GARCH(", p, ",", q, ") AIC:", infocriteria(model)[1]))
  }
}
```

```
## [1] "GARCH( 1 , 1 ) AIC: -6.45583962236738"
## [1] "GARCH( 2 , 1 ) AIC: -6.45716074438333"
## [1] "GARCH( 3 , 1 ) AIC: -6.45670059168907"
## [1] "GARCH( 1 , 2 ) AIC: -6.45538046892068"
## [1] "GARCH( 2 , 2 ) AIC: -6.45724195440042"
## [1] "GARCH( 3 , 2 ) AIC: -6.45727113839456"
## [1] "GARCH( 1 , 3 ) AIC: -6.45514153554737"
## [1] "GARCH( 2 , 3 ) AIC: -6.45692702908259"
## [1] "GARCH( 3 , 3 ) AIC: -6.45697974614959"
```

Problem 14.4

```
# Step 1: Fetch S&P 500 data and calculate returns
getSymbols("^GSPC", src = "yahoo", from = "2000-01-01")
```

```
## [1] "GSPC"
```

```
sp500 <- GSPC$GSPC.Adjusted

# Calculate log returns
returns <- diff(log(sp500))[-1]  # Exclude the first NA
```

```
# Step 2: Fit a GARCH(1,1) model
spec_garch <- ugarchspec(variance.model = list(model = "sGARCH", garchOrder = c(1, 1)),
                         mean.model = list(armaOrder = c(0, 0), include.mean = TRUE))
garch_model <- ugarchfit(spec = spec_garch, data = returns)
summary(garch_model)
```

```
##    Length     Class      Mode
##         1 uGARCHfit        S4
```

```
# Step 3: Generate volatility forecasts
forecast <- ugarchforecast(garch_model, n.ahead = 2)

# Extract forecasts
volatility_1step <- sigma(forecast)[1]  # One-step ahead volatility
volatility_2step <- sigma(forecast)[2]  # Two-step ahead volatility

cat("One-step ahead volatility:", volatility_1step, "\n")
```

```
## One-step ahead volatility: 0.006958087
```

```r
cat("Two-step ahead volatility:", volatility_2step, "\n")
```

```
## Two-step ahead volatility: 0.007063033
```

```r
# Step 4: Construct 95% interval forecasts
# Assumption: Returns are conditionally normal
mean_forecast <- fitted(forecast)  # Mean return forecast
z_critical <- qnorm(0.975)         # 1.96 for 95% confidence interval

# One-step ahead forecast
lower_1step <- mean_forecast[1] - z_critical * volatility_1step
upper_1step <- mean_forecast[1] + z_critical * volatility_1step

# Two-step ahead forecast
lower_2step <- mean_forecast[2] - z_critical * volatility_2step
upper_2step <- mean_forecast[2] + z_critical * volatility_2step

# Display results
cat("One-step ahead 95% CI: [", lower_1step, ",", upper_1step, "]\n")
```

```
## One-step ahead 95% CI: [ -0.01301784 , 0.01425736 ]
```

```r
cat("Two-step ahead 95% CI: [", lower_2step, ",", upper_2step, "]\n")
```

```
## Two-step ahead 95% CI: [ -0.01322353 , 0.01446305 ]
```

Problem 14.5

```r
# Step 1: Download CPI and GDP data from FRED
getSymbols("CPIAUCSL", src = "FRED")  # US CPI (All Urban Consumers, Seasonally Adjusted)
```

```
## [1] "CPIAUCSL"
```

```r
getSymbols("GDP", src = "FRED")       # US GDP (Chained 2012 Dollars)
```

```
## [1] "GDP"
```

```r
# Convert data to time series
cpi <- CPIAUCSL  # CPI
gdp <- GDP       # GDP
```

```r
# Step 2: Calculate inflation rate from CPI
# Inflation rate = log difference of CPI
inflation_rate <- diff(log(cpi)) * 100  # Convert to percentage

# Step 3: Calculate GDP growth rate
# GDP growth = log difference of GDP
gdp_growth <- diff(log(gdp)) * 100  # Convert to percentage
```

```r
# Step 4: Calculate unconditional means
mean_inflation <- mean(inflation_rate, na.rm = TRUE)
mean_gdp_growth <- mean(gdp_growth, na.rm = TRUE)

cat("Unconditional mean of inflation rate:", mean_inflation, "\n")
```

```
## Unconditional mean of inflation rate: 0.287984
```

```r
cat("Unconditional mean of GDP growth rate:", mean_gdp_growth, "\n")
```

```
## Unconditional mean of GDP growth rate: 1.546277
```

```r
# Step 5: Fit GARCH models for inflation rate
# Specify a GARCH(1,1) model for inflation rate
spec_inflation <- ugarchspec(variance.model = list(model = "sGARCH", garchOrder = c(1, 1)),
                             mean.model = list(armaOrder = c(0, 0), include.mean = TRUE))
garch_inflation <- ugarchfit(spec = spec_inflation, data = na.omit(inflation_rate))
summary(garch_inflation)
```

```
##    Length     Class      Mode
##         1 uGARCHfit        S4
```

```r
# Fit GARCH models for GDP growth
spec_gdp <- ugarchspec(variance.model = list(model = "sGARCH", garchOrder = c(1, 1)),
                       mean.model = list(armaOrder = c(0, 0), include.mean = TRUE))
garch_gdp <- ugarchfit(spec = spec_gdp, data = na.omit(gdp_growth))
summary(garch_gdp)
```

```
##    Length     Class      Mode
##         1 uGARCHfit        S4
```

```r
# Step 6: Generate 1-step-ahead volatility forecasts
# Inflation rate forecast
forecast_inflation <- ugarchforecast(garch_inflation, n.ahead = 1)
volatility_inflation <- sigma(forecast_inflation)

# GDP growth forecast
forecast_gdp <- ugarchforecast(garch_gdp, n.ahead = 1)
volatility_gdp <- sigma(forecast_gdp)

cat("1-step-ahead volatility forecast for inflation rate:", volatility_inflation, "\n")
```

```
## 1-step-ahead volatility forecast for inflation rate: 0.161745
```

```r
cat("1-step-ahead volatility forecast for GDP growth:", volatility_gdp, "\n")
```

```
## 1-step-ahead volatility forecast for GDP growth: 0.6058046
```

```r
# Step 7: Construct 95% confidence intervals
# Critical z-value for 95% CI
z_critical <- qnorm(0.975)

# Inflation rate CI
mean_inflation_forecast <- fitted(forecast_inflation)
lower_inflation <- mean_inflation_forecast - z_critical * volatility_inflation
upper_inflation <- mean_inflation_forecast + z_critical * volatility_inflation

# GDP growth CI
mean_gdp_forecast <- fitted(forecast_gdp)
lower_gdp <- mean_gdp_forecast - z_critical * volatility_gdp
upper_gdp <- mean_gdp_forecast + z_critical * volatility_gdp

cat("95% CI for inflation rate: [", lower_inflation, ",", upper_inflation, "]\n")
```

```
## 95% CI for inflation rate: [ -0.08872718 , 0.5453017 ]
```

```r
cat("95% CI for GDP growth rate: [", lower_gdp, ",", upper_gdp, "]\n")
```

```
## 95% CI for GDP growth rate: [ 0.2852091 , 2.659919 ]
```

Problem 12.2 & Problem 12.3

```r
# Step 1: Load the dataset (replace 'us_gasoline.csv' with the actual file name)
us_gasoline <- read_excel("~/Downloads/us_gasoline.xlsx")

# Step 2: Convert to a tsibble (time-series tibble)
us_gasoline <- us_gasoline %>%
  mutate(Week = as.Date(Week, format = "%m/%d/%y")) %>%
  as_tsibble(index = Week)
```

```r
# Step 3: Fit a dynamic harmonic regression model
gasoline_dhr <- us_gasoline %>%
  model(
    harmonic = TSLM(Gasoline ~ trend() + fourier(K = 2))  # Adjust K if needed
  )
```

```
## Warning: 1 error encountered for harmonic
## [1] K must be not be greater than period/2
```

```r
# Summary of the harmonic model
report(gasoline_dhr)
```

```
## Series: Gasoline
## Model: NULL model
## NULL model
```

```r
# Step 4: Fit a regression model
gasoline_reg <- us_gasoline %>%
  model(
    regression = TSLM(Gasoline ~ trend() + season())
  )
```

```
## Warning: 1 error encountered for regression
## [1] contrasts can be applied only to factors with 2 or more levels
```

```r
# Summary of the regression model
report(gasoline_reg)
```

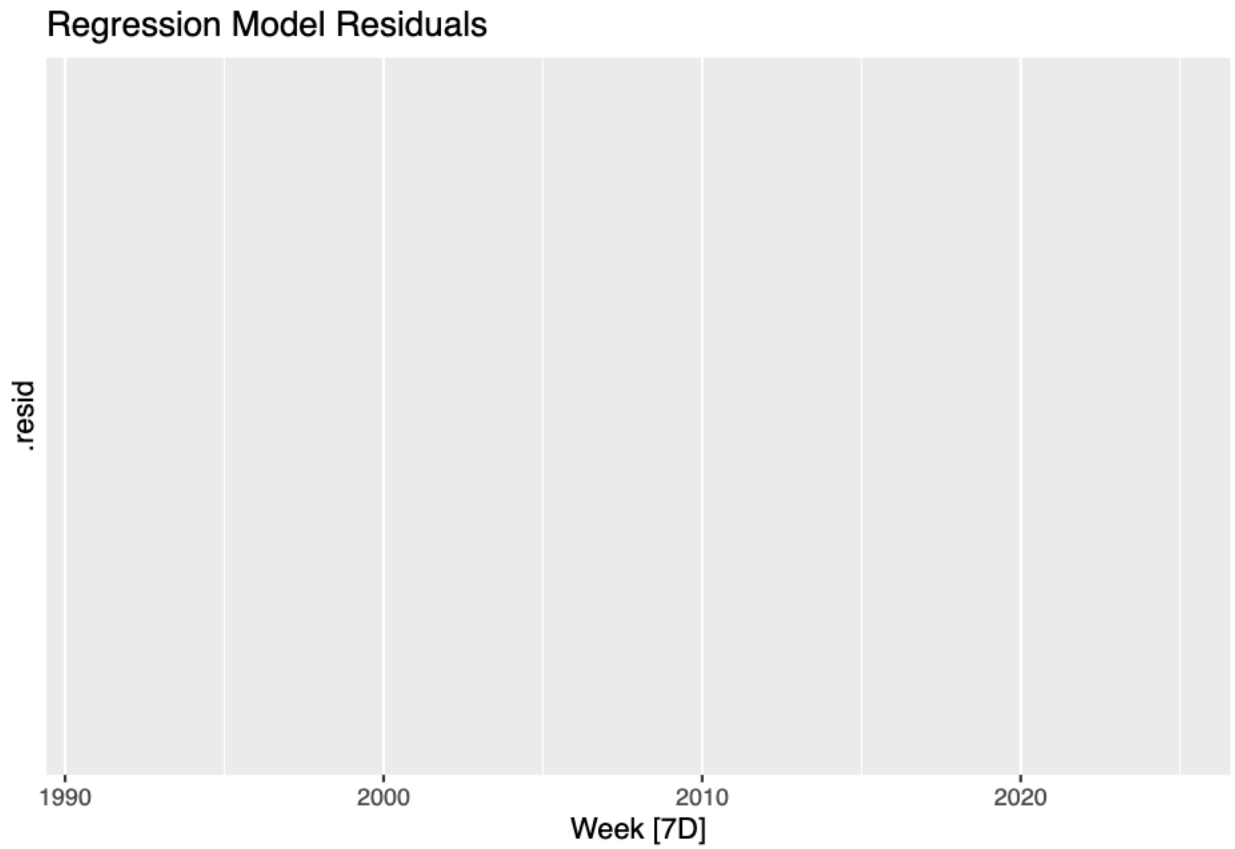
```
## Series: Gasoline
## Model: NULL model
## NULL model
```

```r
# Step 5: Residual diagnostics
# Extract residuals from both models
gasoline_dhr_residuals <- augment(gasoline_dhr) %>% filter(.model == "harmonic")
gasoline_reg_residuals <- augment(gasoline_reg) %>% filter(.model == "regression")

# Plot residuals
autoplot(gasoline_dhr_residuals, .resid) + ggtitle("Harmonic Model Residuals")
```

```
## Warning: Removed 1764 rows containing missing values or values outside the scale range
## (`geom_line()`).
```


```
autoplot(gasoline_reg_residuals, .resid) + ggtitle("Regression Model Residuals")
```

```
## Warning: Removed 1764 rows containing missing values or values outside the scale range
## (`geom_line()`).
```




```
# Ljung-Box test for residual autocorrelation
ljung_box_dhr <- gasoline_dhr_residuals %>% features(.resid, ljung_box)
ljung_box_reg <- gasoline_reg_residuals %>% features(.resid, ljung_box)

ljung_box_dhr
```

```
## # A tibble: 1 x 3
##   .model   lb_stat lb_pvalue
##   <chr>      <dbl>     <dbl>
## 1 harmonic      NA        NA
```

```
ljung_box_reg
```

```
## # A tibble: 1 x 3
##   .model     lb_stat lb_pvalue
##   <chr>        <dbl>     <dbl>
## 1 regression      NA        NA
```

```
# Step 6: Fit alternative models (ARIMA, ETS, NNETAR)

# ARIMA Model
gasoline_arima <- us_gasoline %>%
  model(ARIMA = ARIMA(Gasoline))
```

```r
# ETS Model
gasoline_ets <- us_gasoline %>%
  model(ETS = ETS(Gasoline))

# NNETAR Model
gasoline_nnetar <- us_gasoline %>%
  model(NNETAR = NNETAR(Gasoline))

# Compare models using AIC
model_comparisons <- glance(gasoline_dhr, gasoline_reg, gasoline_arima, gasoline_ets, gasoline_nnetar)
model_comparisons
```

```
## # A tibble: 0 x 1
## # i 1 variable: .model <chr>
```

```r
# Step 7: Forecast using NNETAR
nnetar_forecast <- gasoline_nnetar %>%
  forecast(h = 12)  # Forecast for 12 weeks ahead

# Plot forecast
autoplot(nnetar_forecast) +
  ggtitle("NNETAR Forecast for Gasoline Supply")
```