Indirect Temperature Estimation Using Kalman Filter (Two Sensors)

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Probability Course Fall 2025 — Professor: Tran Vinh Linh
Interactive Demo: `https://leonathn.github.io/FinalProjectProbability/`

1. Problem Overview

Context: This experimental box was created for Performance Evaluation of Building Environment (Dr. Nguyen Hop Minh). The bulb temperature T_{bulb} is needed to feed CFD (Computational Fluid Dynamics) simulations, but no sensor can measure it directly at 200–300°C without melting. This Kalman Filter method estimates T_{bulb} from indirect air temperature measurements.

Two air-temperature sensors:

Sensor A (near-field)

Close to bulb, high noise: $z_A \approx T_{\text{air-near}}$

Sensor B (far-field)

Farther away, low noise: $z_B \approx T_{\text{air-room}}$

Both are **indirect proxies**. Distances d_A and d_B model heat diffusion.

2. Why Indirect Measurement?

- Bulb surface: **200–300°C** (sensors melt)
- Only air temperature available
- Heat diffusion + convection: noisy, time-varying

$\Rightarrow$ Requires **state estimation**, not direct measurement.

3. Diffusion Model

Heat diffusion approximation:

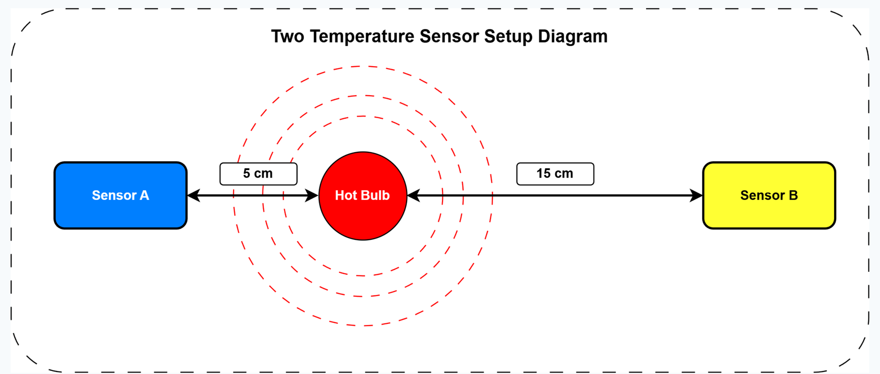
$$T_{\text{air}}(d) = T_{\text{room}} + \alpha \frac{T_{\text{bulb}} - T_{\text{room}}}{d}$$

Airflow, turbulence, convection → large uncertainty.

Hidden state: $x = T_{\text{bulb}}$

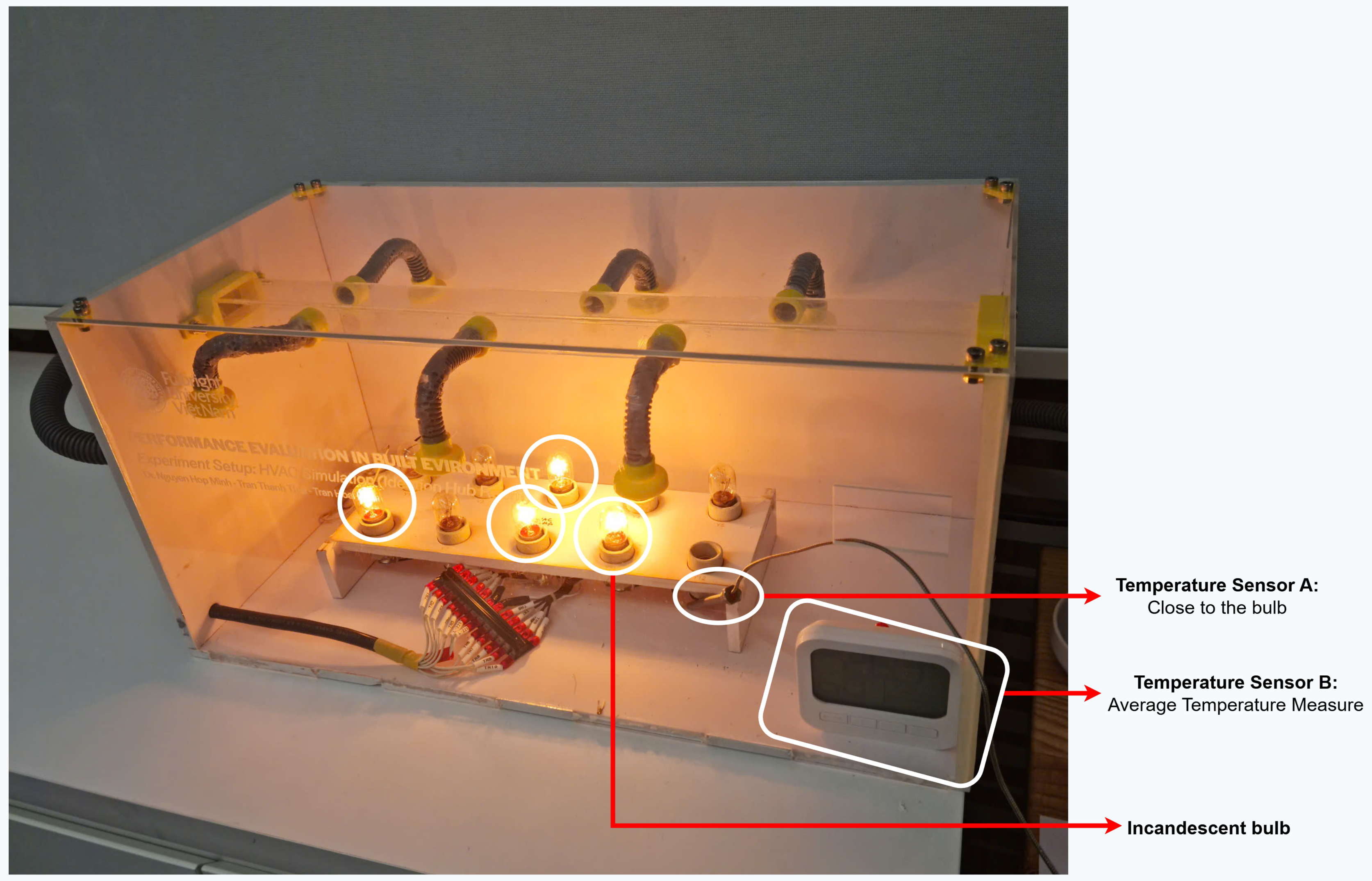
Measurements:

$$z_A = T_{\text{air}}(d_A) + \text{noise}$$
$$z_B = T_{\text{air}}(d_B) + \text{noise}$$



Schematic: Sensor A (close, high noise) and Sensor B (far, low noise).

Sensor Placement Diagram



Hot bulb (center), Sensor A close (d_A), Sensor B farther (d_B). Heat diffuses with decreasing intensity.

4. Kalman Filter Solution

Probabilistic Framework: Bayesian inference with Gaussian distributions.

Prior belief: $p(x) = \mathcal{N}(x; \hat{x}, P)$ where $x = T_{\text{bulb}}$ is hidden state.

Likelihood: $p(z|x) = \mathcal{N}(z; Hx, R)$ where z = sensor measurement.

Posterior: By Bayes' rule, $p(x|z) \propto p(z|x) \cdot p(x)$ yields:

$$p(x|z) = \mathcal{N}(x; \mu_{\text{post}}, \sigma^2_{\text{post}})$$

The Kalman Filter computes μ_{post} and σ^2_{post} in closed form:

Prediction Step (Prior Propagation)

$$\hat{x}_{\text{pred}} = \hat{x}_{\text{prev}} \quad \text{(propagate mean)}$$
$$P_{\text{pred}} = P_{\text{prev}} + Q \quad \text{(increase uncertainty)}$$

This gives prior $p(x) = \mathcal{N}(x; \hat{x}_{\text{pred}}, P_{\text{pred}})$ before measurement.

Update Step (Posterior via Bayes)

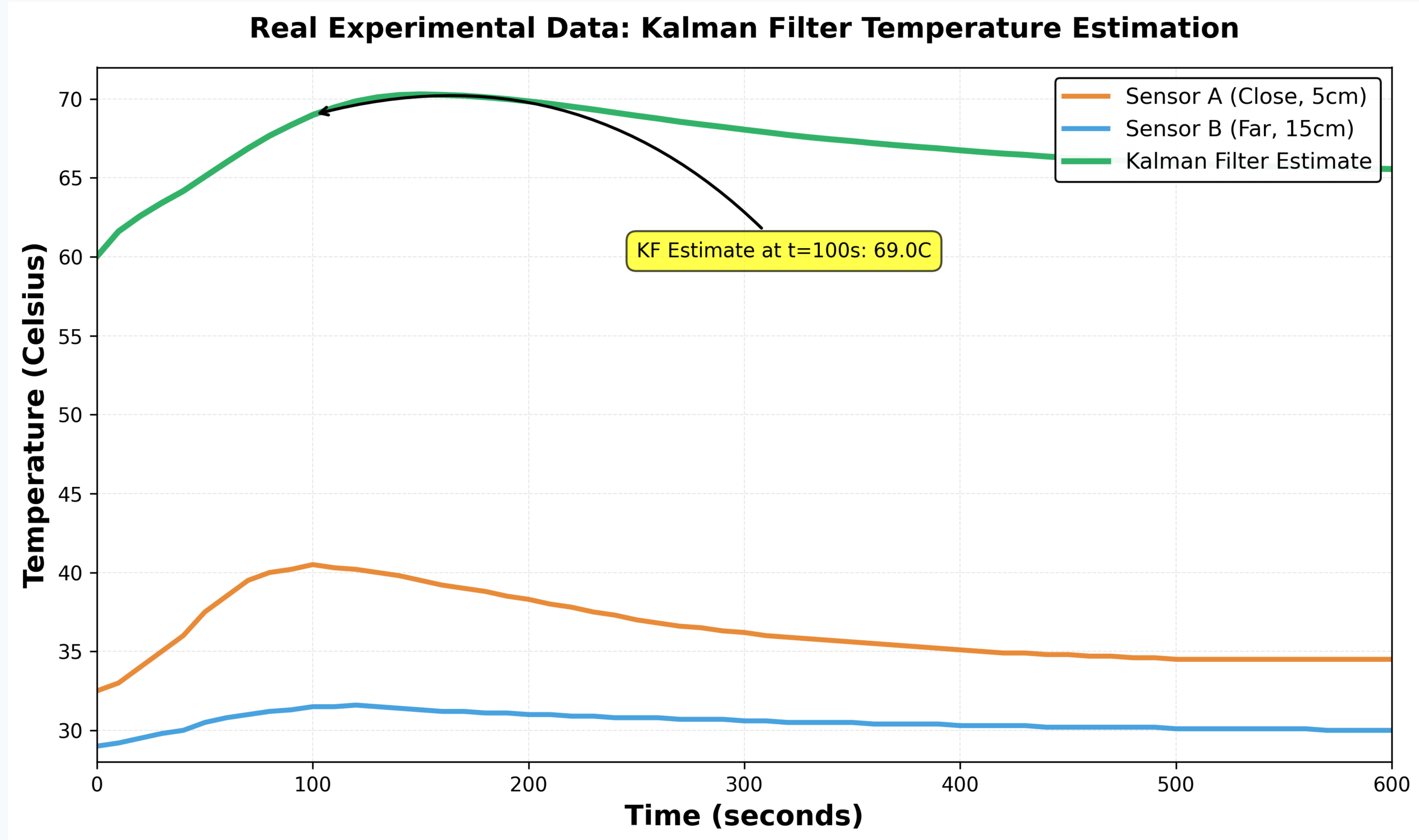
Sensor A: Combine prior with likelihood $p(z_A|x) = \mathcal{N}(z_A; H_A x, R_A)$

$$K_A = \frac{P_{\text{pred}}}{P_{\text{pred}} + R_A} \quad \text{(Bayes weight)}$$
$$\hat{x}_A = \hat{x}_{\text{pred}} + K_A(z_A - H_A \hat{x}_{\text{pred}}) \quad \text{(posterior mean)}$$
$$P_A = (1 - K_A H_A) P_{\text{pred}} \quad \text{(posterior variance)}$$

Sensor B: Update again with $p(z_B|x) = \mathcal{N}(z_B; H_B x, R_B)$

$$K_B = \frac{P_A}{P_A + R_B}, \quad \hat{x}_{\text{new}} = \hat{x}_A + K_B(z_B - H_B \hat{x}_A), \quad P_{\text{new}} = (1 - K_B H_B) P_A$$

Real Experimental Data



Setup: Heat source box, two ambient sensors

- $d_A = 5$cm (close), $d_B = 15$cm (far)
- $R_A = 2.0$ (noisier), $R_B = 0.5$ (stable)
- Process noise: $Q = 0.1$

Key Results at $t = 100$s:

- Sensor A: 40.5°C (close, noisy)
- Sensor B: 31.5°C (far, stable)
- **KF bulb estimate: 69.0°C**

Overall Statistics (600s):

- Sensor A: Mean=36.4°C, Std=2.1°C
- Sensor B: Mean=30.5°C, Std=0.6°C
- Bulb estimate: Mean=67.2°C, Std=2.2°C

Data from Dr. Nguyen Hop Minh's Building Environment Performance Evaluation box.

5. Discussion

Method Overview:

- **Challenge:** Estimate hidden bulb temperature from indirect air sensor measurements
- **Approach:** Sequential Bayesian inference via Kalman Filter with two complementary sensors

Experimental Results:

- Bulb estimate: Mean=67.2°C, Std=2.2°C (600s)
- KF fuses Sensor A (close, noisy) with Sensor B (far, stable)
- Kalman Gain adaptively weights sensors by uncertainty

Interactive Demo:
`https://leonathn.github.io/FinalProjectProbability`

6. References

[1] Kalman (1960). *J. Basic Eng.* [2] Welch & Bishop (2006). *UNC.* [3] Simon (2006). *Wiley.*