



## How tame will Leviathan become in institutional competition?

### *Competition among governments in the provision of public goods*

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**Abstract.** This article critically examines the hypothesis of *Brennan* and *Buchanan* that competition among governments in the provision of public goods can serve as a substitute for constitutional constraints on governments. Since *Leviathan*-type governments with free choice of tax instruments will be able to escape competitive pressure by shifting taxes to immobile factors, one could think of a rule of competition which prohibits taxes on immobile factors. Indeed, such a rule leads to a *Nash*-equilibrium where the tax burden lies on the mobile factor. However, net income of the citizens may or may not increase as a result from such a rule, depending on a number of variables presented in this article. A complete substitution of constitutional constraints by the rule of competition may, depending on the same variables, even decrease net income. Moreover, some potential for increases in net income may be forgone, since capital allocation and the supply of public goods will usually be inefficient in equilibrium. Finally, applying the rule in a real-world environment will be difficult and may even lead to further serious inefficiencies. For these reasons, such a rule will hardly ever be introduced. Hence, competition among governments cannot be viewed as a proper substitute for constitutional constraints. Whoever is afraid of *Leviathan* should thus not rely on competition among governments (alone).

## 1. Introduction

In their book *The Power to Tax*, Geoffrey Brennan and James Buchanan (1980: 168) claim that a decentralized federalism could serve as a proper substitute for constitutional constraints on the power of politicians. The argument behind this claim is that decentralized federal systems allow both citizens and capital to move out of jurisdictions with poor governmental performance or high tax rates. This directly relieves tax pressure on those who move out of the respective jurisdiction and, at the same time, forces government to improve its performance and to keep its size at a reasonable level. However, the hypothesis that institutional competition among governments improves the welfare of taxpayer-citizens is anything but new. Rather, it has roots in a famous article by Charles Tiebout (1956) in which he argues that decentralized provision of public goods may well function. Although this article has

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been published as early as 1956, it has been gaining considerable popularity specifically since the beginning of the 1980s.<sup>1</sup>

This paper critically examines one aspect of this hypothesis, namely that institutional competition among governments may indeed help to tame Leviathan. It suggests that such a taming effect can only be expected if a certain *rule of competition* among the several decentralized governments is applied. The paper is organized as follows. In the second chapter a federal state with several jurisdictions as well as one perfectly mobile and one totally immobile factor is modelled. The aim is to show how Leviathan-type governments would behave if they had a free choice with respect to the type of taxes they raise. It will be shown that, under these conditions, only the mobile factor will be protected against excessive taxation. In the third chapter a rule will be introduced that prohibits taxation of immobile factors in order to bring any taxation under competitive pressure. It will be shown that this results in a *Nash-equilibrium* with positive tax rates on the mobile factor and, moreover, that it will be the mobile factor that bears the taxes. It will then be argued, however, that such a rule will, under realistic circumstances, not increase net income of the immobile factor and that even some scope for an increase in its income may be forgone as a result of the rule. Also, taxation under this rule will cause further and possibly aggravating inefficiencies as well as some serious practical problems. If, for these reasons, the constitution of a federal system does without such a rule the immobile factor remains dependent on more traditional forms of protection against excessive taxation by Leviathan. Federalism, then, is no substitute for constitutional limits to Leviathan.

## 2. Leviathan with free choice of taxes

In the following model, a number of ( $m$ ) decentralized jurisdictions is assumed, where each government is too small to set tax prices.<sup>2</sup> Each government provides one public good of the amount ( $X^i$ ) which is not a consumption good but which is rather used as input for the production process of the respective jurisdiction. The production process of jurisdiction ( $i$ ) can be described by the following production function:

$$Y^i = F^i(L^i, K^i, n^i \cdot X^i), \quad \text{with } i = 1, 2, \dots, m. \quad (1)$$

where ( $L^i$ ) is the amount of the immobile factor (e.g. labor), ( $K^i$ ) is the amount of the mobile factor (e.g. capital), and ( $n^i$ ) is the number of firms that use the public input ( $X^i$ ). Since, in this chapter, ( $i$ ) is viewed as representative for all jurisdictions in the federation, we can drop the superscript. Also, we normalize the amount of labor ( $L$ ) and the number of firms ( $n$ ) to one. This simplifies the production function to:

$$Y = F(K, X). \quad (1a)$$

Firms are assumed to maximize net profits, i.e. profits after tax. The net-profit function of a representative firm in jurisdiction (i) is as follows:

$$G = F(K, X) - w \cdot L - r^* \cdot K - t_K \cdot K, \quad (2)$$

where ( $w$ ) is the wage rate, ( $L$ ) the amount of labor employed, ( $r^*$ ) the interest rate and ( $t_K$ ) the tax rate on capital. Since capital is perfectly mobile across the federation, there is only one federation-wide interest rate ( $r^*$ ). The firm finds the profit-maximizing capital input by differentiating Equation (2) with respect to ( $K$ ) and setting this first derivative equal to zero. The result is the well known *capital-arbitrage condition*:<sup>3</sup>

$$F_K = r^* + t_K. \quad (3)$$

A benevolent dictator would maximize consumption in the representative jurisdiction. Consumption ( $Z$ ) is total production ( $Y$ ), minus capital costs ( $r^* \cdot K$ ), the costs of the provision of the public good ( $P_X \cdot X$ ), and congestion costs ( $C$ ). For simplicity, it is assumed that public goods are bought abroad at given prices ( $P_X$ ). Congestion costs arise when firms use the public good. The firms' trucks, for example, congest highways and this reduces the jurisdiction's value of consumption in a somewhat broader sense. This is the easiest way to approach congestion costs, and it will be sufficient here.<sup>4</sup> Consumption will then be:

$$Z = F(K, X) - r^* \cdot K - C(K, X) - P_X \cdot X. \quad (4)$$

Congestion increases with any additional amount of capital invested in the respective jurisdiction, and it decreases when additional public inputs are supplied. The maximum in consumption can be found by differentiating (4) with respect to ( $K$ ) and ( $X$ ) and setting the results equal to zero. This yields:

$$F_X = P_X, \quad \text{and} \quad (5)$$

$$F_K = r^* + C_K. \quad (6)$$

The consumption-maximizing public-goods provision is where marginal product of the public input equals its price. The condition for an optimal capital input is, in principle, the same. The price, however, has two components, the first of which is the interest rate and the second is marginal congestion cost. Combining Equation (6) with (3) yields the following result (see H.-W. Sinn, 1997: 252):

$$t_K = C_K. \quad (7)$$

If a government would maximize welfare it would set taxes on the mobile factor equal to marginal congestion-costs. The core of the discussion about a competitive supply of public goods can be made clear with the help of this simple condition. If the public input is a pure Samuelsonian-type public good (Samuelson, 1954), then marginal congestion costs will be zero. Hence, the optimal tax rate on the mobile factor will be zero as well, given that government has an option to tax an immobile factor instead.<sup>5</sup> Thus, public-good's supply would not be competitive in the way that the users of the public good pay a certain price that, in turn, is somehow related to (marginal) costs of its provision. Instead the cost of the public good is shifted to whoever is immobile and, thus, taxable. True, this would not need to be a problem if government acted as a welfare maximizer. However, if it did there would be no need for competition among governments.

If, by contrast, the public input is not a pure *Samulsonian*-type public good, then the tax rate is set equal to marginal congestion costs. Moreover, as long as the congestion-costs function has a degree of homogeneity of one or more, tax revenues will be sufficiently high to cover the costs of the provision of the public input (see H.-W. Sinn, 1997: 254). In this case, though, we would lose the most prominent argument for a public provision of this input, since traditional private markets would provide the input efficiently too.<sup>6</sup> Why should a government be allowed to provide a good with the help of coercive tax power if markets would do this job at least as well and, moreover, on a voluntary basis?

For this reason, we will in the following concentrate on pure public goods. We will do so in order to see as to whether a Leviathan-type government can be forced to provide a public good under competitive pressure. We define a Leviathan-type government as a government that maximizes total tax revenue net of expenditures for public goods.<sup>7</sup> Consider two taxes which Leviathan may have access to. First, a labor-income tax which depends on the fixed number of workers ( $L$ ) in a representative jurisdiction, the wage rate of the workers, which is identical to their marginal productivity ( $F_L$ ), and the tax rate ( $t_L$ ). The second tax is a tax on the capital stock ( $t_K$ ). The rents of Leviathan are:

$$R = t_K \cdot K + t_L \cdot F_L(K, X) \cdot L - P_X \cdot X. \quad (8)$$

What happens if Leviathan levies a capital tax is illustrated in Figure 1. Here, marginal productivity of capital is depicted. With no taxation, capital will move into the jurisdiction until marginal productivity of capital equals the interest rate ( $r^*$ ). The capital stock will then be ( $K_0$ ). If Leviathan sets a

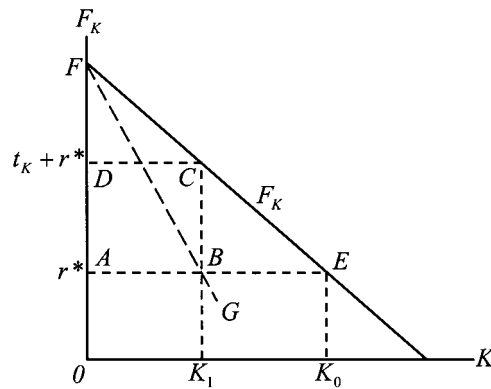


Figure 1. Effects of capital-income taxation.

positive tax rate, a part of the capital will move out and the remaining capital stock will have a higher productivity at the margin. Therefore, Leviathan will act just like a non-discriminating monopolist. The “demand curve” he faces is identical with the curve of marginal productivity of capital between the points F and E. In point E the tax rate is zero, therefore the government’s “demand curve” starts here. The marginal revenue curve is thus the line  $\overline{FG}$  in the figure. Since marginal costs of the provision of the public good are zero, a maximum of rents can be found where the marginal revenue line  $\overline{FG}$  intersects the “zero-tax line”  $\overline{r^*E}$ . Capital of the amount of  $(K_0 - K_1)$  will leave the jurisdiction, and tax revenue will be as much as the rectangular  $\overline{ABCD}$ .

With no capital tax, production will be equal to the area  $\overline{OK_0EF}$  of which  $\overline{OK_0EA}$  is capital income and  $\overline{AEF}$  the income of the immobile factor. When Leviathan raises a capital tax, production declines to  $\overline{OK_1CF}$ , of which  $\overline{OK_1BA}$  is capital income and  $\overline{ABCF}$  is the income of the immobile factor. Part of the capital moves out and earns an income of exactly  $\overline{K_1K_0EB}$  in another jurisdiction. The other part remains in the jurisdiction and earns the same income as before the tax was raised. There will be a welfare loss as big as the triangle  $\overline{BCE}$ . Both tax revenue  $\overline{ABCD}$  and welfare loss  $\overline{BCE}$  are born by the immobile factor alone. Hence, while Leviathan formally taxes the mobile factor, he economically taxes the immobile factor. This factor has to give away as much as the area  $\overline{AECD}$  from his former income  $\overline{AEF}$ .

This is a well known result from the theory of tax incidence. If there is one perfectly mobile factor and another factor which is totally immobile, then all taxes will be paid from the income of the immobile factor. This means that the income of the immobile factor will, under any circumstances, be the upper limit of whatever Leviathan may be able to realize as tax revenue. In reality, though, he will not be able to tax away the entire income of the people living in his jurisdiction. How much he will be able to tax and how much he has to

leave for the people depends on internal political factors like political competition among parties, constitutional rules, or, at least, the threat of a revolution. In what follows, we will not focus on what Leviathan can take away from the people but rather what he has to leave for the people. The reason is that it is not just taxes that Leviathan can take away from the people. He might, for example, reduce the provision of the public input and, in so doing, reduce marginal productivity of the immobile factor and hence its owners' income.

We will capture these factors in a single variable ( $\alpha$ ) with ( $0 \leq \alpha \leq 1$ ). This variable indicates that share of the immobile factor's highest achievable income, which Leviathan has to leave for the people without risking political trouble. The highest achievable income of the immobile factor is the income ( $F_L^{\text{opt}}(K^{\text{opt}}, X^{\text{opt}}) \cdot L$ ) which results from an efficient input of both capital ( $K^{\text{opt}}$ ) and the public good ( $X^{\text{opt}}$ ). The share ( $\alpha \cdot F_L^{\text{opt}} \cdot L$ ), in turn, is the minimum of what Leviathan has to leave for the people in order to stay in office. Therefore, it will always be ( $\alpha \cdot F_L^{\text{opt}} = F_L(K, X) - t_L$ ). On this basis, we can rewrite the rent function (8) of the government by subtracting from total production that income, which has to be left to the immobile factor and further by subtracting net capital income  $r^* \cdot K$  as well as the costs for the provision of the public good ( $P_X \cdot X$ ):

$$R = F(K, X) - r^* \cdot K - \alpha \cdot F_L^{\text{opt}} \cdot L - P_X \cdot X. \quad (9)$$

Remembering from the capital-arbitrage condition (3) that ( $r^* = F_K - t_K$ ), Equation (9) can finally be written as:

$$R = F(K, X) - F_K(K, X) \cdot K + t_K \cdot K - \alpha \cdot F_L^{\text{opt}} \cdot L - P_X \cdot X. \quad (9a)$$

In maximizing these rents, Leviathan is restricted by the capital-arbitrage condition (3). Maximizing (9a) subject to (3) with respect to ( $K$ ), ( $X$ ) and ( $t_K$ ) yields the following first-order conditions:

$$F_K - K - F_{KK} \cdot K - F_K + t_K - \lambda \cdot F_{KK} = 0; \quad (10)$$

$$F_X - F_{KX} \cdot K - P_X - \lambda \cdot F_{KX} = 0; \quad (11)$$

$$K = -\lambda. \quad (12)$$

Substituting (12) into (10) yields:

$$t_K = 0, \quad (10a)$$

and substituting (12) into (11) yields:

$$F_X = P_X. \quad (11a)$$

Leviathan does provide an efficient amount ( $X^{\text{opt}}$ ) of the public input and, indirectly by setting the capital tax rate equal to zero, an efficient amount of capital ( $K^{\text{opt}}$ ). The reason why the government avoids taxation of capital is that a positive tax rate would drive some capital out of the jurisdiction. That would reduce income of the immobile factor which, in turn, is ultimately the only source for any tax revenues (Nechyba 1997). The government does provide an efficient amount of the public good for much the same reason. On the one hand, it could directly increase political rents by reducing outlays for the public good. On the other hand, however, this would again reduce the income of the immobile factor which, in turn, is government's only tax base. Hence, as long as Leviathan has access to non-distorting taxes on the immobile factor he will reduce capital taxes to zero and also provide an optimal amount of the public good. He will then achieve a maximum of political rents by directly increasing the tax rate on the immobile factor up to the point where internal political pressure reaches its critical point. This critical point will be reached where the tax rate on the immobile factor's income is  $(1 - \alpha)$ , which can be seen by combining Equations (8) and (9):

$$t_L \cdot F_L \cdot L = F(K, X) - r^* \cdot K - \alpha F_L^{\text{opt}} \cdot L. \quad (13)$$

Since the tax rate on capital is zero in equilibrium the term  $(F(K, X) - r^* \cdot K)$  is nothing else than gross income of the immobile factor. We can thus substitute  $(F_L \cdot L)$  for  $(F(K, X) - r^* \cdot K)$ . Also,  $(F_L)$  will, in equilibrium, be identical with  $(F_L^{\text{opt}})$  since the government will care for optimal inputs ( $K^{\text{opt}}$ ) and ( $X^{\text{opt}}$ ). Taken together, (13) reduces to:

$$t_L = (1 - \alpha). \quad (14)$$

This is an important result with respect to the question raised in this paper. Contrary to what has frequently been suggested (e.g., S. Sinn, 1992), the share of income Leviathan may withdraw from the immobile factor has nothing to do with institutional competition and the mobility of capital at all. On the contrary: Since an increase in the mobility of capital narrows the tax base of Leviathan, government may even try to compensate for this loss of potential taxes by raising tax rates on the immobile factor. Whether or not he will do so once again depends on internal political and institutional conditions, as captured in variable  $(\alpha)$ . However, under no circumstances would institutional competition for mobile capital lead to *lower* taxes for the immobile factor.

For these reasons competition among governments will never be able to protect immobile factors from excessive taxation by Leviathan-type governments, as long as they have the option to choose the type of tax that best serves

their goals. If they have this option they will always avoid taxing mobile factors. The immobile factors, in turn, have no chance to escape taxation as long as there are no more traditional constraints on the government. Competition among governments does certainly restrict Leviathan's power over the mobile factor. He will indeed not be able to tax away even the slightest part of capital income  $\overline{OK_0EA}$ , if capital is really perfectly mobile. Note, however, that this inability to tax includes those taxes that might have been raised for good reasons, like the provision of public goods.

But Leviathan will not be restricted whatsoever in taxing away the income of the immobile factor. The share  $(1 - \alpha)$  of maximum income ( $F_L^{\text{opt}} \cdot L$ ) that he may be able to withdraw from the immobile factor is in no way related to the degree of competition he faces with respect to the mobile factor. Instead the degree of rivalry between political parties competing for political power, the institutional structure of the political system, constitutional limits to governmental revenues and expenditures, the redistributive power of pressure groups and many other factors do belong to the determinants of the maximum tax rate. Competition among governments, though, does not, at least as long as the government can freely choose between the factors it taxes.

Is there any remedy for this defect of a competitive provision of public goods? To answer this question one must see the crucial difference between competition among governments and competition in traditional markets. In traditional markets, suppliers have no coercive power, neither against "mobile" customers nor against anyone else. This is different in competition among governments. Governments have the option to switch over to the immobile factor when the other factors become "too" mobile. Here, they can exercise their coercive power without ever being bothered by too critical customers. It seems obvious that, if this switching from mobile to immobile factors makes the difference between market and governmental competition the remedy for the ensuing problems should be to prevent the government from switching. In other words the government should be constitutionally prohibited from imposing any tax on immobile factors.<sup>8</sup> If this is done government would really lose *all* of its coercive power since every unsatisfied customer could simply change the supplier of his public goods. Leviathan would transform into a usual market supplier under competition. But could such an institutional arrangement work? The next section will try to answer this question.

### 3. Leviathan and the prohibition of taxes on immobile factors

When a federal constitution prohibits the taxation of immobile factors, government is dependent on the mobile factor alone in order to raise taxes, finance public goods and keep some rents for the government. If the decent-



ralized governments have an option to tax an immobile factor as well, then these governments will set the tax rate on the mobile factor equal to zero. The reason is simply that the governments want to avoid an outflow of productive factors and, thus, a decline of the tax base. However, if the government is not allowed to tax the immobile factors, setting the tax rate on mobile factors equal to zero cannot be optimal anymore, since this would result in zero tax revenue as well. Rather, it would be optimal to accept a certain outflow of capital in order to realize at least some tax revenue. Since there are supposedly (m) jurisdictions in the federation, each of the (m) governments will act in the same way. Under these conditions, though, the result cannot be a net outflow of capital from every jurisdiction as long as the total capital stock ( $\bar{K}$ ) of the federation is fixed with ( $\bar{K} = \sum_i^m K^i$ ). Rather, after every government has set its optimal tax rate on the mobile factor there will be an equilibrium with some possible changes in the capital allocation across the jurisdictions, but without a net outflow of capital out of each jurisdiction. In what follows it will be assumed that jurisdictions are not too small, thus leading to a strategic interaction between the decentralized governments.<sup>9</sup>

In order to understand the resulting effects, we have to give up our assumption that all jurisdictions are identical. The easiest way to do so is to assume a federation with two jurisdictions (1) and (2). The objective function of the Leviathan governments are, in principle, the same as in the last section. However, we have to differentiate between the two jurisdictions and to remember that there is no tax on the immobile factor anymore. Thus, the objective functions are:

$$R^1 = t_{K^1} \cdot K^1 - P_X \cdot X^1 \quad (8a)$$

for jurisdiction (1) and

$$R^2 = t_{K^2} \cdot K^2 - P_X \cdot X^2 \quad (8b)$$

for jurisdiction (2). Assume now, for simplicity, that both jurisdictions have production functions with a linear slope of the marginal productivity lines ( $F_K^1$ ) and ( $F_K^2$ ).<sup>10</sup> These lines are shown in Figure 2, where ( $F_K^1$ ) is to be read from the left to the right and ( $F_K^2$ ) from the right to the left. As long as there are no capital taxes, capital will flow across the jurisdictions until marginal productivity in both jurisdiction is equal at ( $r^*$ ). The resulting capital allocation will then be ( $K_0^1, K_0^2$ ), and this allocation will be efficient.

This allocation, however, will change as soon as one or both jurisdictions impose a tax on capital. Specifically, the capital stock in jurisdiction (1) decreases to the extent of  $((\partial K^1 / \partial t_{K^1}) \cdot t_{K^1})$  as a result of a tax in jurisdiction (1), and it increases to the extent of  $((\partial K^1 / \partial t_{K^2}) \cdot t_{K^2})$  as a result of a tax in jurisdiction (2).<sup>11</sup> The resulting capital stock in jurisdiction (1) is thus:

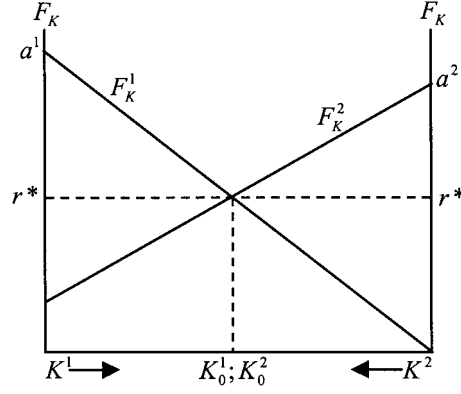


Figure 2. Capital allocation with no taxes on capital.

$$K^1 = K_0^1 + \frac{\partial K^1}{\partial t_{K^1}} \cdot t_{K^1} + \frac{\partial K^1}{\partial t_{K^2}} \cdot t_{K^2}. \quad (15a)$$

Accordingly, the capital stock in jurisdiction (2) is:

$$K^2 = K_0^2 + \frac{\partial K^2}{\partial t_{K^2}} \cdot t_{K^2} + \frac{\partial K^2}{\partial t_{K^1}} \cdot t_{K^1} \quad (15b)$$

Since the total capital stock in the federation is always the sum of the capital stocks in each jurisdiction, any increase in the capital stock in one jurisdiction is identical with a decrease in the capital stock in the other jurisdiction and vice versa. Hence, we can substitute  $(-\partial K^2 / \partial t_{K^2})$  for  $(\partial K^1 / \partial t_{K^2})$  in Equation (15a) and  $(-\partial K^1 / \partial t_{K^1})$  for  $(\partial K^2 / \partial t_{K^1})$  in Equation (15b). Equations (15a) and (15b) then change to:

$$K^1 = K_0^1 + \frac{\partial K^1}{\partial t_{K^1}} \cdot t_{K^1} - \frac{\partial K^2}{\partial t_{K^2}} \cdot t_{K^2} \quad (16a)$$

$$K^2 = K_0^2 + \frac{\partial K^2}{\partial t_{K^2}} \cdot t_{K^2} - \frac{\partial K^1}{\partial t_{K^1}} \cdot t_{K^1} \quad (16b)$$

Take now the capital-arbitrage condition from Section 2:

$$F_K = r^* + t_K \quad (3)$$

and remember, that marginal productivity is a function of the capital stock (K), which in turn is a function of the tax rate. Differentiating the capital-arbitrage condition with respect to the capital tax rate yields:

$$\frac{\partial K^1}{\partial t_{K^1}} = \frac{1}{F_{KK^1}} \quad (17a)$$

for jurisdiction (1) and

$$\frac{\partial K^2}{\partial t_{K^2}} = \frac{1}{F_{KK^2}} \quad (17b)$$

for jurisdiction (2). Substituting (17a) and (17b) into (16a) and (16b) yields:

$$K^1 = K_0^1 + \frac{1}{F_{KK^1}} \cdot t_{K^1} - \frac{1}{F_{KK^2}} \cdot t_{K^2} \quad (18a)$$

and

$$K^2 = K_0^2 + \frac{1}{F_{KK^2}} \cdot t_{K^2} - \frac{1}{F_{KK^1}} \cdot t_{K^1} \quad (18b)$$

Finally, substituting (18a) into (8a) leads to the modified objective functions of the government of jurisdiction (1):

$$R^1 = t_{K^1} \cdot K_0^1 + \frac{1}{F_{KK^1}} \cdot t_{K^1}^2 - \frac{1}{F_{KK^2}} \cdot t_{K^2} \cdot t_{K^1} - P_X \cdot X^1. \quad (19a)$$

Accordingly, substituting (18b) into (8b) leads to the modified objective function of the government of jurisdiction (2):

$$R^2 = t_{K^2} \cdot K_0^2 + \frac{1}{F_{KK^2}} \cdot t_{K^2}^2 - \frac{1}{F_{KK^1}} \cdot t_{K^1} \cdot t_{K^2} - P_X \cdot X^2. \quad (19b)$$

If we look at the problem from the viewpoint of government of jurisdiction (1), we take the tax rate of jurisdiction (2) as given and find the optimal tax rate of our own jurisdiction by differentiating (19a) with respect to  $(t_{K^1})$  and setting the result equal with zero. The government of jurisdiction (2) acts accordingly. The resulting first-order conditions represent the reaction functions of the two jurisdictions' governments:

$$t_{K^1} = -\frac{F_{KK^1}}{2} \cdot K_0^1 + \frac{F_{KK^1}}{2 \cdot F_{KK^2}} \cdot t_{K^2}; \quad (20a)$$

$$t_{K^2} = -\frac{F_{KK^2}}{2} \cdot K_0^2 + \frac{F_{KK^2}}{2 \cdot F_{KK^1}} \cdot t_{K^1}. \quad (20b)$$

As we would expect, an increase in the tax rate of jurisdiction (2) leads the government of jurisdiction (1) to raise its tax rate too. The extent to which jurisdiction one's tax rate reacts to an increase in the tax rate of jurisdiction

(2) depends on the relation between the slope of the marginal productivity lines. The steeper the slope of jurisdiction one's marginal productivity line compared to that of jurisdiction (2), the more will government (1) raise its tax rate in reaction to an increase in the tax rate of jurisdiction (2). In the special case of identical jurisdictions the slope of the reaction functions reduces to  $(-0.5)$ .

The *Nash*-equilibrium tax rate of jurisdiction (1) can now be found by substituting Equation (20b) into (20a) and rearranging. It is:

$$t_{K^1}^N = -\frac{2 \cdot K_0^1 + K_0^2}{3} \cdot F_{KK^1}. \quad (21a)$$

The *Nash*-equilibrium tax rate of jurisdiction (2) can finally be found in the same manner. It is:

$$t_{K^2}^N = -\frac{2 \cdot K_0^2 + K_0^1}{3} \cdot F_{KK^2}. \quad (21b)$$

Different from the last section the *Nash*-equilibrium tax rates on mobile capital will be above zero as long as  $(F_{KK^1})$  and  $(F_{KK^2})$  are both below zero. This in turn, will always be the case as long as there is another factor in the respective jurisdiction which is immobile. If, by contrast, there were no such immobile factor, then any capital outflow would be identical to a total factor variation. The capital outflow would then not change capital productivity ( $F_K$ ) and, thus,  $(F_{KK})$  would be zero. The governments would then have no chance to achieve any tax revenue from the mobile capital. Usually, however, there will at least be one immobile factor and thus  $(F_{KK^1})$  and  $(F_{KK^2})$  will be above zero. Since the tax rates are not related to the costs of the public goods, however, there is no guarantee that tax revenues in equilibrium will be sufficient to cover these costs.

Furthermore, the equilibrium tax rates will most probably distort capital allocation across the jurisdictions.<sup>12</sup> Tax rates will not be distorting if they do not change the initial capital allocation ( $K_0^1, K_0^2$ ). This, however, will only be the case as long as both jurisdictions apply the same tax rate in equilibrium, that is, as long as  $(t_{K^1}^N = t_{K^2}^N)$ . Any difference in equilibrium tax rates will lead to capital movements. There will be a relatively high capital stock in the "low-tax jurisdiction" and a relatively low capital stock in the "high-tax jurisdiction". The result will be that marginal productivity net of taxes will be lower in the low-tax jurisdiction compared to the high-tax jurisdiction. This is inefficient since a reallocation of capital from the low-tax jurisdiction to the high-tax jurisdiction would enhance total production in the federation as a whole. As will be shown in the Appendix, tax rates in *Nash* equilibrium will usually be different, resulting in an inefficient capital allocation.

At least in theory, however, cases are possible where tax rates in equilibrium are equal. One of these cases is that jurisdictions are perfectly identical with respect to size and production technologies.<sup>13</sup> With perfectly identical jurisdictions, tax rates will be equal in equilibrium and there will be no capital flow induced by capital taxation. In such a case it would indeed be possible to tax a mobile factor and, at the same time, to put competitive pressure onto the several decentralized governments of a federation without causing an inefficient capital allocation. Furthermore, the mobile factor could not shift the tax burden to the immobile factor anymore. Rather, the tax rate would directly reduce the federation-wide return on capital ( $r^*$ ). This can be seen by differentiating Equation (3) totally and setting ( $dF_K = 0$ ). We obtain (see Zodrow and Mieszkowski, 1986: 360):

$$dt_K = -dr^*. \quad (22)$$

However, in a less ideal world with different tax rates on capital in *Nash*-equilibrium, there would be a shift in the tax burden from one jurisdiction to the other instead of a shift from the mobile to the immobile factor. Nevertheless, there will in any case be a positive tax rate on capital which enables the governments to achieve tax revenues in order to finance their public-goods supply.

In order to see as to whether the government will supply an efficient amount of the public good, insert the capital-arbitrage condition into equation (8a). This yields:

$$R^1 = [F_K^1(K^1, X^1) - r^*] \cdot K^1 - P_X \cdot X^1. \quad (23)$$

Leviathan finds the rent-maximizing public input by differentiating (23) with respect to ( $X^1$ ) and setting equal to zero. The result is:

$$F_{KX} \cdot K = P_X \quad (24)$$

for each jurisdiction. Under the usual assumptions about the production functions and especially with decreasing returns on capital input, this will result in an inefficiently low supply of the public good. To see this consider the following specification of production function (1):

$$F = L^\alpha \cdot K^\beta \cdot n \cdot X^\gamma, \quad \text{with : } n = 1; \alpha + \beta + \gamma = 1. \quad (25)$$

The first derivative with respect to ( $X$ ) and the second derivative with respect first to ( $K$ ) and then to ( $X$ ) will then be:

$$F_X = \gamma \cdot L^\alpha \cdot K^\beta \cdot X^{\gamma-1}; \quad \text{and} \quad (26)$$

$$F_{KX} = \beta \cdot \gamma \cdot L^\alpha \cdot K^{\beta-1} \cdot X^{y-1}. \quad (27)$$

Multiplying (27) with (K) yields:

$$F_{KX} \cdot K = \beta \cdot \gamma \cdot L^\alpha \cdot K^\beta \cdot X^{y-1}. \quad (28)$$

If we combine Equation (26) with (28) and also consider Equation (24) we find:

$$P_X = \beta \cdot F_X = F_{KX} \cdot K \quad (29)$$

Remembering from Equation (5) that an efficient supply of the public good requires ( $P_X = F_X$ ) and remembering further that ( $0 \leq \beta \leq 1$ ) we find that public good's supply will fall short of its efficient level, except in the limiting case of ( $\beta = 1$ ) with constant returns to capital input. The reason is that, according to Equation (24), government will expand its public-good's supply until the marginal increase in *capital* income equals its price ( $P_X$ ), whereas it should increase public good's supply until the marginal increase in *total* income equals price ( $P_X$ ). Hence, government does not consider increases in labor income. This result stems directly from the rule that prohibits the taxation of the immobile factor. As was shown in Section 2 government will provide an efficient amount of the public input when it is not constrained with respect to the taxes it may raise. It does so since any inefficient supply of the public input would reduce the income of the immobile factor. This income, in turn, is the only source for tax revenues for the government. It will hence increase the tax rate ( $t_L$ ) up to the point where it reaches ( $1 - \alpha$ ). This, rather than reducing public good's supply, was shown to be the optimal strategy for the government.

This strategy, however, will no longer be viable once the rule of competition is implemented since this rule restricts the tax rate on the immobile factor to zero. Therefore, it becomes optimal for the government to save expenditures by reducing the supply of the public good and, hence, accepting a certain reduction in national income.<sup>14</sup> In that way, the government applies some kind of a "second-best rule" of taxation for Leviathan governments. This is so since Leviathan trades reductions in expenditures for public goods against reductions in total production, where the first increases and the second reduces political rents.

The relation between public good's supply and political rents is illustrated in Figure 3. Let ( $X^*$ ) be the public input that is chosen by the government according to condition (29), i.e. ( $X^* = X(\beta \cdot F_X)$ ). Further, let ( $K^*(t_K^N)$ ) be the capital stock that results from the tax rate on capital in *Nash*-equilibrium according to Equations (21a) and (21b). Then an efficient public input ( $X^{opt}$ )

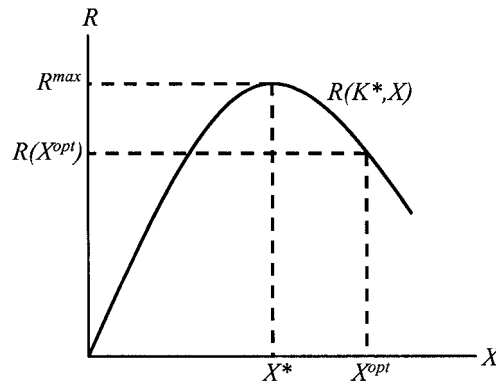


Figure 3. Rent-maximizing versus efficient public input under the rule of competition.

according to condition (5) would yield political rents below maximum. Reducing the public input to  $(X^*)$  would increase political rents up to the maximum level. However, a further reduction would again reduce political rents.

The question arises as to whether net income of the immobile factor will be higher or lower under the rule of competition. For a simple reason it can never be lower than that without the rule. Would Leviathan reduce the public input to such an extent that the income of the immobile factor dropped below its minimum level  $(\alpha \cdot F_L^{\text{opt}} \cdot L)$ , then the government would, as long as the restriction  $(\alpha)$  does still exist, be removed from office. At the same time, Leviathan will always try to draw  $((1 - \alpha) \cdot F_L^{\text{opt}} \cdot L)$  from the immobile factor. However, he may not always be able to do so under the rule of competition. Since he is not allowed to directly tax the immobile factor, he can only draw resources from that factor by reducing the public input from its efficient level  $(X^{\text{opt}})$  to the rent-maximizing level  $(X^*)$ , as shown in Figure 3.

But the rent-maximizing level of the public input may, depending on a number of parameters, still be high enough to yield an income  $(F_L(K^*, X^*) \cdot L)$  which is higher than the minimum income necessary for the government in order to stay in office. True, a further reduction in  $(X^*)$  would certainly further reduce the immobile factor's income, and it would also reduce expenditures for public goods. But it would reduce political rents as well. Therefore, no rent-maximizing government would do so. As a result, income of the immobile factor under the rule of competition would always be as high as, or higher than, the minimum net income required for avoiding political trouble. Or, more formally:

$$F_L(K^*, X^*) \cdot L \geq \alpha \cdot F_L^{\text{opt}} \cdot L. \quad (30)$$

Whether  $(F_L(K^*, X^*) \cdot L)$  is above or equal to the minimum income depends on the level of  $(X)$  that would be necessary to secure the minimum income  $(\alpha \cdot F_L^{\text{opt}} \cdot L)$ . Let that level be  $(X^{**})$ . Then we will have  $(F_L(K^*, X^*) \cdot L = \alpha \cdot F_L^{\text{opt}} \cdot L)$  for any  $(X^{**} > X^*)$ , that is whenever  $(X^{**})$  is between  $(X^*)$  and  $(X^{\text{opt}})$  in Figure 3. Note that, in this case a relaxation of the minimum share  $(\alpha)$  of what has to be left to the immobile factor would also reduce actual income of that factor since Leviathan would then reduce  $(X)$  as well. In case of  $(X^{**} > X^*)$  the rule of competition would hence not help taming Leviathan, since it is then again the internally determined factor  $(\alpha)$  alone which limits Leviathan's *power to tax*. If we interpret  $(\alpha)$  as a constitutional rule in the sense of Brennan and Buchanan, than removing this rule and relying on tax competition inside the federation would in fact reduce the income of the immobile factor in the case of  $(X^{**} > X^*)$ . Tax competition would thus not be a proper substitute for constitutional rules in this case.

In another possible case, though, we will have  $(F_L(K^*, X^*) \cdot L > \alpha \cdot F_L^{\text{opt}} \cdot L)$ , which means that Leviathan will not be able to reduce the level of the public input as much as to reach the minimum income level  $(\alpha \cdot F_L^{\text{opt}} \cdot L)$ . This will be so whenever  $(X^{**} < X^*)$ , that is when the rent-maximizing public input is higher than what is necessary for securing the minimum income level required for the government to stay in office. In that case, the constitutional rule which determines the minimum income share  $(\alpha)$  would indeed be superfluous under the rule of competition. The reason is that this income level would be exceeded anyway as a result from the rent-maximizing level  $(X^*)$  of the public input.

What implications do these results have for public decision-making in a Leviathan-state? As a result from the rule of competition Leviathan may or may not be constrained in his power to tax, depending on the level of  $(X^*)$  and  $(X^{**})$ , respectively. A complete substitution of a constitutional rule that limits the power to tax, as suggested by Brennan and Buchanan, indicated by a substantial reduction in  $(\alpha)$ , by the rule of competition, indicated by  $(t_L = 0)$ , could even lead to a reduction in the immobile factor's income. As shown, this will be the case for any  $(X^{**} > X^*)$ .

In any case will the rule of competition lead to an inefficient allocation of resources in the federation. The reason is that the inputs  $(K)$  and  $(X)$  will divert from their optimal levels in equilibrium. First, all governments in the federation will in all but one limiting case set taxes that distort capital allocation; and second, in practically any realistic case, each government will provide an amount  $(X)$  of the public input which falls short of its efficient level  $(X^{\text{opt}})$ . Public decision making will thus lead to inefficient results under the rule of competition. By contrast, we have shown in Section 2 that without a rule of competition, governmental decisions on tax rates and the supply



of public goods will, in principle, be efficient. Leviathan will choose non-distorting taxes on immobile factors, and he will also choose an optimal amount of the public input. However, in such a world without the rule of competition, he will still have access to the income of the immobile factor. The only constraint on his power to tax will be the constitutional rule, as summarized by the term  $(\alpha)$ .

In order to set additional constraints on Leviathan's power to tax, one may nevertheless be inclined to ignore the inefficiencies that result from the rule of competition. This may especially be tempting since these inefficiencies reduce solely the rents of the government and not the income of the private sector, as long as the constitutional rule  $(\alpha)$  remains in place. However, as long as there exist alternative instruments that limit Leviathan's power to tax without causing these inefficiencies, it may be possible to reduce the political rents and redistribute the thus freed resources to the people. As long as such instruments exist, the rule of competition will be inefficient in itself since it does not only cause a reduction in political rents, which may be ignored, but it also reduces the achievable income of the immobile factor. Hence, it is always paying to search for alternative constitutional limits on Leviathan's power to tax which do not cause significant inefficiencies.

What is more, there will be additional problems associated with the rule of competition. Some of these issues are mere practical ones. Others, though, will add further sources of inefficiencies, some of which are so significant that serious losses in national income have to be expected. The most important problems can be summarized as follows:

- The rule of competition does not introduce competition in the way it is usually understood by economists. The tax rate on the mobile factor is still in no way attached to (marginal) costs of the provision of the respective public good. To be sure, this can never be the case since, by definition, marginal costs of additional "consumers" (i.e. inflowing capital) of public goods are zero. The reason that a government can nevertheless charge a price which is above zero is simply that it still enjoys some monopoly power. This monopoly power stems from the fact that a partial outflow of the mobile factor raises its marginal productivity. Since government can tax this increase in marginal productivity away, tax rates in equilibrium are always above marginal costs of additional consumers. Thus the increase in marginal productivity resulting from an outflow of the mobile factor remains as the only source of taxation for the government. However, this remaining source of taxation rests on market power alone. In any case will governments act differently from firms under usual competitive pressure.

- Tax revenues that can be realized in *Nash*-equilibrium may not be sufficient to cover the costs of the provision of the public good. The size of the tax rate depends on marginal productivity of the invested capital in the respective jurisdictions, which has nothing to do with provision costs of the public good. Tax revenue may thus be too low. It may, however, be too high as well, leaving considerable rents for the government.
- If the tax base is mobile not only inside the respective federation but internationally as well, the rule may lead to considerable negative income effects, since the only legal source of taxation drives the mobile factor out of the federation. The only way to avoid this would be to cancel the rule and, thus, to allow the decentralized governments to directly tax immobile factors. This, again, abolishes competition among governments.
- A tax on the mobile factor alone is necessarily incompatible with benefit taxation since public goods and services are not only supplied to the mobile factor but to the immobile factor as well. However, benefit taxation is a precondition for a functioning competition among governments since only this guarantees that the respective costs and benefits are assigned to the same factor. Thus, if goods and services are supplied to both the mobile and the immobile factor it does not make much sense to tax the mobile factor alone. These kinds of problems are aggravated by the fact that in practice it is not always possible to distinguish services for mobile from those for immobile factors.

Taken together, it seems that no government can be expected to implement such a rule of competition. Nor would any member of the federation be willing to agree on such a rule on a constitutional basis as long as there exist alternative rules that limit the power to tax without causing all these inefficiencies. This applies even to persons that are exclusively dependent on income from the immobile factor. It is all the more important to develop proper constitutional constraints on governments' power to tax, that is constraints that do not cause (gross) inefficiencies. Governments provide public goods. It seems that it lies in the very nature of these goods that proper constitutional constraints will have little to do with competitive pressure. As a result, governments will in any realistic circumstances enjoy some direct monopoly power, and there is basically little to be done about it.

#### 4. Conclusions

We have shown that institutional competition between jurisdictions with free choice of taxes induces Leviathan-type politicians to avoid taxation of mobile

factors. Instead they tax immobile factors and, in this way, maintain their monopoly power to tax. In this way, no real competition among the suppliers of public goods arises. Whereas the mobile factors will be protected against Leviathan, the immobile factors will not. In order to overcome this defect of institutional competition, we have introduced a rule of competition that prohibits the taxation of immobile factors. Such a rule leads to a *Nash*-equilibrium in which there is a positive tax rate on the mobile factor and in which the tax burden cannot be shifted to another factor. The size of the tax rate, though, is in no way attached to (marginal) provision costs of the public good. Thus, tax revenues may or may not be sufficient to cover the costs of the provision of the public goods.

Moreover, capital allocation in *Nash*-equilibrium will practically always be inefficient and the amount of public goods provided will be inefficiently low. Finally, it is to be expected that there will be some further serious inefficiencies as well as practical problems. To the extent that, for these reasons, the rule is not strictly applied, government regains monopoly power over immobile factors. It then again avoids taxing the mobile factors and taxes the immobile factors instead. In doing so, however, it abolishes institutional competition.

Hence, we can under no realistic assumption expect institutional competition to work in a way competition in usual private-sector markets works. The crucial difference is that governments usually have free choice with respect to the type of taxes and the factors they tax. If one factor becomes “too” mobile in order to be taxed governments may switch to another, more immobile factor and, thus, bypass competition. Since a certain freedom of choice with respect to taxes and tax bases can, for a number of reasons, not be taken away from the governments, this deficiency of institutional competition cannot be overcome. To conclude, we should not expect Leviathan to be markedly tame in institutional competition, as some authors hope.

## Notes

1. See, for example, Breton (1987); Oates and Schwab (1991); S. Sinn (1992); Wildavsky (1990).
2. See for similar models: Fuest (1995); H.-W. Sinn (1996); Wilson (1986); Zodrow and Mieszkowski (1986).
3. See Boadway and Wildasin (1984: 387–401).
4. See for a detailed presentation of the theory of congestable goods Sandler and Tschirhart (1980).
5. See: Bucovetsky and Wilson (1991); Fuest (1995); Zodrow and Mieszkowski (1986).
6. This was shown by: Berglas (1976); (1981); Berglas and Pines (1980); see also H.-W. Sinn (1996).

7. See for Leviathan models of tax competition Edwards and Keen (1996).
8. A similar argument has recently been made by Hange and Wellisch (1998).
9. See, for models of strategic interaction in fiscal policy, Mintz and Tulkens (1986); Wildasin (1988); Laussel and Le Breton (1998).
10. Since, in what follows, we will work with the identity  $\Delta K^i = (\partial K^i / \partial t^j) \cdot t_K^j$  and since, with non-linear slopes of marginal productivity, the deviations in capital stocks  $\Delta K^i = K^i - K_0^i$  are not identical with, but can only be approximated by,  $\partial K^i / \partial t^j \cdot t_K^j$ , the conditions (20a,b), (21a,b) and the correspondent conditions in the Appendix will turn into approximations as well in the case of non-linear production functions. However, this would not change the general results of the following analysis.
11. Following Flatters and Henderson and Mieszkowski (1974: 105–106), the latter is called a fiscal externality; see also Wildasin (1986); Wilson (1986: 195–196).
12. Myers (1990) and Wrede (1998) have shown that, under certain conditions, local governments have an incentive to engage in voluntary interjurisdictional transfer payments which restore efficiency. However, these models are based on the assumption of welfare-maximizing governments or, in Wrede (1998), at least of “moderate Leviathans”, where one of the arguments of the moderate Leviathan’s objective function is utility of a representative citizen.
13. See, for a model with identical jurisdictions, Yang (1996); models with non-identical jurisdictions are: Bucovetsky (1991); Kanbur and Keen (1993); Tausch (1998); Wilson (1991).
14. A similar argument with respect to international tax coordination has been made by Fuest (1995).

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## Appendix

In this Appendix we will show under which conditions the *Nash*-equilibrium tax rates of the two jurisdictions in Section 3 will be equal, thus leading to an efficient capital allocation. Like in Section 3 and as shown in Figure 2 we assume the following linear production functions for jurisdictions (1) and (2):

$$F_{K^1} = a^1 + F_{KK^1} \cdot K^1; \quad \text{and} \quad (\text{A1a})$$

$$F_{K^2} = a^2 + F_{KK^2} \cdot K^2. \quad (\text{A1b})$$

Furthermore, we assume that the capital stocks in jurisdiction (1) and (2) sum up to the federation-wide capital stock ( $\bar{K}$ ):

$$\bar{K} = K^1 + K^2. \quad (\text{A2})$$

Finally, we define:

$$A \equiv a^2 - a^1. \quad (\text{A3})$$

We now calculate the initial (and efficient) capital stocks ( $K_0^1$ ) and ( $K_0^2$ ), which represent an efficient capital allocation resulting from equal capital tax rates in both jurisdictions. In order to find these initial capital stocks we set (A1a) equal to (A1b):

$$a^1 + F_{KK^1} \cdot K_0^1 = a^2 + F_{KK^2} \cdot K_0^2 \quad (\text{A4})$$

We can now, according to Equation (A2), substitute ( $\bar{K} - K_0^1$ ) for ( $K_0^2$ ). Remembering that, according to (A3), ( $a^1 - a^2 = -A$ ), we find the initial capital stock of jurisdiction (1):

$$K_0^1 = \frac{A + F_{KK^2} \cdot \bar{K}}{F_{KK^1} + F_{KK^2}}. \quad (\text{A5a})$$

In the same manner, we can find the initial capital stock of jurisdiction (2):

$$K_0^2 = \frac{-A + F_{KK^1} \cdot \bar{K}}{F_{KK^1} + F_{KK^2}}. \quad (\text{A5b})$$

We now substitute these initial capital stocks into the *Nash*-equilibrium tax rates (21a) and (21b) and obtain:

$$t_{K^1}^N = \left[ \frac{2}{3} \cdot \frac{A + F_{KK^2} \cdot \bar{K}}{F_{KK^1} + F_{KK^2}} + \frac{1}{3} \cdot \frac{-A + F_{KK^1} \cdot \bar{K}}{F_{KK^1} + F_{KK^2}} \right] \cdot F_{KK^1}; \quad \text{and} \quad (\text{A6a})$$

$$t_{K^2}^N = \left[ \frac{2}{3} \cdot \frac{-A + F_{KK^1} \cdot \bar{K}}{F_{KK^1} + F_{KK^2}} + \frac{1}{3} \cdot \frac{A + F_{KK^2} \cdot \bar{K}}{F_{KK^1} + F_{KK^2}} \right] \cdot F_{KK^2}. \quad (\text{A6b})$$

Rearranging (A6a) and (A6b) yields:

$$t_{K1}^N = \frac{1}{3} \cdot \frac{A \cdot F_{KK1} + 2 \cdot F_{KK1} \cdot F_{KK2} \cdot \bar{K} + F_{KK1}^2 \cdot \bar{K}}{F_{KK1} + F_{KK2}}; \quad \text{and} \quad (A7a)$$

$$t_{K2}^N = \frac{1}{3} \cdot \frac{A \cdot F_{KK2} + 2 \cdot F_{KK1} \cdot F_{KK2} \cdot \bar{K} + F_{KK2}^2 \cdot \bar{K}}{F_{KK1} + F_{KK2}}. \quad (A7b)$$

Since efficiency requires that ( $t_{K1}^N = t_{K2}^N$ ), we set (A7a) equal to (A7b) and obtain:

$$A \cdot F_{KK1} + F_{KK1}^2 \cdot \bar{K} = -A \cdot F_{KK2} + F_{KK2}^2 \cdot \bar{K}. \quad (A8)$$

Rearranging (A8) finally yields the following condition for an efficient capital allocation:

$$\frac{A}{\bar{K}} \cdot (F_{KK1} + F_{KK2}) + F_{KK1}^2 = F_{KK2}^2. \quad (A9)$$

The simplest way for condition (A9) to be given is that the jurisdictions are perfectly identical. In this case, the parameters ( $a^1$ ) and ( $a^2$ ) in the production functions (A1a) and (A1b) are also identical and, according to (A2), (A) in condition (A9) will be zero. Since the slope of the production function ( $F_{KK1}$ ) and ( $F_{KK2}$ ) will also be identical, condition (A9) will always be fulfilled with identical jurisdictions.

There are other cases with non-identical jurisdictions possible, however, that lead to an efficient capital allocation. If the slopes of the production functions are different, than an (A) that is different from zero could, under certain conditions, compensate for this difference in the slope of the marginal productivity lines. To see this, assume that ( $F_{KK2}^2 > F_{KK1}^2$ ). This means that the slope of the production function in jurisdiction (2) is steeper than that in jurisdiction (1). A reason could be that jurisdiction (2) is smaller than jurisdiction (1), thus leading to a stronger drop in capital productivity in the case of an inflow of capital. If, under this condition, the parameter ( $a^1$ ) were smaller than ( $a^2$ ) and, thus, (A) were positive to an extent which just compensates for the difference in ( $F_{KK1}^2$ ) and ( $F_{KK2}^2$ ), an efficient capital allocation would result (remember that  $F_{KK1}$  and  $F_{KK2}$  are negative!). However, this would only accidentally be the case, since there is no mechanism that leads (A) to compensate for differences in the slopes of the marginal productivity lines. We must therefore expect that capital allocation in *Nash*-equilibrium will most probably lead to an inefficient capital allocation. This especially applies for federations where jurisdictions are markedly different in size and production technologies.

