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The Effect of a Credit Crunch on Equilibrium Market Structure

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Abstract This article examines the impact of a credit crunch on market structure. We construct and simulate a dynamic model of a duopolistic industry in which firms' investments in capacity are constrained by the availability of credit. In such an industry, the dynamic interaction of credit limits and the competitive responses of firms turn out to be a powerful transmission mechanism by which the effects of shocks persist and are amplified. We show that a small, temporary shock to one firm's capacity can lead to its market exit, even if it is equally productive as the remaining incumbent. Consequently, if a recession is accompanied by a credit crunch, its cleansing effect might lead to monopolization of markets and welfare losses.

Keywords Dynamic oligopoly · Endogenous financial structure · Credit rationing · Ericson–Pakes framework · Experience based Markov equilibrium

JEL Classification L13 · G32 · C73

1 Introduction

In 2007 and 2008 there existed a widespread fear that several OECD countries were suffering from a credit crunch. Loan losses and lower asset prices ate significantly into the equity of the banking sector, a fact which many believed would cause banks to ration credit. According to standard macroeconomic models, a lesser amount of available credit leads to a reduction in investment, resulting in turn in lower production and lower welfare in the following periods (e.g. Bernanke et al. 1999). However, these

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models do not take into account the change in market structure which might result from the financial frictions. For a welfare analysis, market structure is important because it directly influences prices and the available choices for consumers. For example, if in a duopoly a smaller competitor is unable to replace its broken machinery because of a lack of credit, he might exit and leave the consumer with a monopoly supplier.

Including financial frictions in any oligopoly model is challenging because investment, financing decisions, and market competition are inherently interdependent and dynamic: past investment decisions determine today's market structure which in turn influences current investment decisions. But a firm can only invest if enough means are available. Investment funds can either come from current profits (determined by today's market structure), retained cash (determined by past financing decisions), or its debt capacity (determined by future profits). Additionally, a firm cannot retain more cash or pay back more debt than its current available funds. To complicate matters further, firms rationally anticipate future investment needs and shortages in funding.

This article contributes to the literature on financial constraints by proposing a computationally feasible model which takes all these factors into account for the case of a dynamic duopoly. To integrate financing and investment decisions, we introduce firms with an endogenous capital structure and an optimizing bank into an Ericson–Pakes framework. However, including an endogenous capital structure for each firm gives rise to a large state space which makes a calculation of the equilibrium intractable with conventional Gauss–Seidel and Gauss–Jacobi algorithms. Therefore, we use a variant of the novel algorithm based on the experience based Markov equilibrium (EBE) framework introduced by Fershtman and Pakes (2012) to solve our model.

Each firm is characterized by three state variables: capacity, debt level, and cash reserves. Firms accumulate capacity over time and compete repeatedly in the product market to earn profits. In every period, they aim to maximize the net present value of dividend payments. For this purpose, they optimally choose production, investment, the amount of cash to retain, and the size of debt repayments. Whatever is left of the profits after subtracting all incurred costs is distributed as a dividend to the shareholders. Firms can apply for a loan if the current cash flow is insufficient to cover expenses. The loan is provided by a risk-neutral bank given that its expected return exceeds an exogenously set minimum threshold. This threshold parametrizes the amount of credit rationing prevalent in the market.

Using this model, we show that credit rationing serves as a propagation mechanism which amplifies small idiosyncratic shocks to capacity. This mechanism can lead to the monopolization of the market. If a firm loses productive capacity through a depreciation shock, lower current profits are available for financing investment. With well-functioning credit markets, the firm can compensate for this loss in its cash flow by increasing the amount of credit financing. But if credit is rationed, firms might be unable to cover the cost of capacity addition. Without investment, the firm remains at the lower capacity level which is associated with less funds. If the firm is hit by another depreciation shock which further tightens credit constraints, its ability to react by an increase in investment is reduced even more. Eventually, this process can lead to the exit of the firm, even if it has the same total factor productivity as the remaining incumbent.

The monopolization of the market is made permanent by two other effects: first, with credit rationing, entrants face financing constraints, too. Therefore new firms cannot enter because they do not obtain sufficient credit to finance initial outlays. Therefore the monopolization due to the credit constraints is not quickly reversed by market entry. Second, the competing firm can expand its own capacity and market share. Increased capacity translates into higher profits which eases credit rationing in the competitor's investment process. In the following plays, the monopolist can then finance itself through cash retainment, increase capacity faster, and gain a dominant position in the market.

Given these theoretical results, a recession which is accompanied by a credit crunch might not be only "cleansing," i.e., destroy unproductive firms (as in Caballero and Hammour 1994), but also force viable competitors out of the market. The welfare of the consumer is reduced by higher prices caused by the ensuing monopolization. This observation gives a rationale for government interventions which aim to increase the credit volume available to companies. According to our model, such programs should seek in particular to support small firms to prevent their exit or facilitate their entry. The reason is that small companies (in contrast to large companies) do not have sufficient free cashflow to finance investment and therefore have to rely on a functioning credit market to fund start-up costs or growth plans. If they cannot finance investment with credit they exit or fail to enter the market, which reduces competition and consumer welfare.

This article contributes to the growing literature on modeling imperfect competition with heterogeneous firms. It is the first model to introduce financial frictions in an Ericson–Pakes framework.[1] We extend the dynamic duopoly model outlined in Besanko and Doraszelski (2004) and Besanko et al. (2010) by firms with an endogenous capital structure and an optimizing bank. The larger state space resulting from the endogenous capital structure makes it is necessary to use the new stochastic algorithm of Fershtman and Pakes (2012) to numerically compute the equilibrium. With this algorithm, we can solve the game much faster than with the commonly used Pakes and McGuire (1994) or Pakes and McGuire (2001) algorithms. To the best of our knowledge, the only other application of the Ericson–Pakes framework to finance is Kadyrzhanova (2009), which models the effect of corporate control imperfections on industry structure.

However, others have worked on financial frictions in dynamic firm models using the alternative framework of Hopenhayn (1992). In contrast to Ericson–Pakes type models, this framework considers only aggregate firm dynamics by assuming an infinite number of firms with an infinitely small market share. Therefore, it is impossible to consider oligopoly behavior in this framework. In addition, in the model of Hopenhayn (1992), all dynamics are driven by permanent firm specific shocks, because temporary shocks average out. In our model, in contrast to that, temporary idiosyncratic shocks are amplified through the capital structure and competitive behavior. The number of applications of the modeling framework of Hopenhayn (1992) in the finance literature is huge: for instance, Cooley and Quadrini (2001) investigate the effect of financial

[1] For a survey on this literature, see Doraszelski and Pakes (2007).

frictions on firm growth. Gomes (2001) explains the effect of financial frictions on investment. Hennessy and Whited (2005) consider a dynamic trade-off model of leverage, corporate saving, and real investment to explain debt dynamics.[2]

Our results are qualitatively similar to the effects described in Kiyotaki and Moore (1997), which characterizes the emergence of credit cycles. Their main idea is that in downturns, both earnings and the liquidation value of collateral are low because potential buyers are cash-strapped. Due to the lower collateral value credit constrained firms cannot borrow for investment, which in turn further reduces their future earnings. As the liquidation value of the collateral is again reduced by this reduction in expected profits, a reinforcing cycle ensues. In contrast, in our article, the firms cannot borrow further money because banks are cash strapped and the effect is transmitted via the expectations of the banking sector and oligopoly behavior.

The remainder of this article is organized as follows. We set up the model in Sect. 2. Sections 3 and 4 present the results and robustness checks. Section 5 concludes.

2 A Duopoly with Endogenous Capital Structure

2.1 Static Framework: The Optimization of the Firm

Assume that there are two firms which compete repeatedly in the same market and there exists one risk-neutral bank. Each firm $i \in \{1, 2\}$ is fully characterized by its state variables: its capacity ($\bar{q_i}$), its debt level (d_i), and its cash reserves (c_i). The value of the combined state variables of firm 1 and 2 are called the industry state $s = (\bar{q}_1, \bar{q}_2, d_1, d_2, c_1, c_2)$, which is common knowledge.

In every period, each firm can choose an action set $a = (INV, \Delta_{debt}, \Delta_{cash})$ to change the value of its state variable if it has enough funds to cover the associated costs: a firm can decide to add one unit of capacity ($INV = 1$) by incurring the expansion costs η or remain inactive ($INV = 0$). It can pay back the amount Δ_{debt} of debt, or it can increase its cash reserves by Δ_{cash}. Therefore the total costs of an action set are the sum of interest payments, the investment cost, the amount used for debt repayment, and the increase of the cash reserve:

$$cost(a, s)_i = r \cdot d_i + \eta \cdot INV_i + \Delta_{debt,i} + \Delta_{cash,i}$$

where r is the interest rate paid by the firm. The funds available to cover these costs are the current profits (π_i), the cash reserves, and the available line of credit ($credit(a, s)_i$). Thus an action is in the set of feasible actions $A(s)$ if

$$\pi_i + c_i + credit(a, s)_i - cost(a, s)_i \geq 0.$$

In every period, each firm acts in the interest of its shareholders and chooses the action set a^* which maximizes the expected discounted value of dividend payments[3]:

[2] Other articles modeling the intersection of investment and financial policy are, e.g., Acharya et al. (2007), Almeida and Campello (2007), Almeida et al. (2009) Moyen (2004), Titman and Tsyplakov (2007), and Adam et al. (2007). See Hubbard (1998) and Stein (2003) for reviews of this literature.

[3] For the sake of readability we suppress the subscript i in the following.

$$a^* = arg \max_{a \in A(s)} W(a, s).$$

where $W(a, s)$ is the expected value of dividends if action a is chosen in the industry state s. This means the firm optimizes with its actions the shareholder value of the company in a forward looking manner. The expected value of this dividend stream is given by

$$W(a, s) = div(a, s) + \beta E_{a'_*, s'}[W(a'_*, s')|a, s]$$

where $E[\cdot]$ is the expectation operator, β is the discount factor, s' is next period's state, and a'_* is next period's optimal action. The dividend payments are just the positive difference of current profits earned on the product market and the costs from the action set

$$div(a, s) = min\{\pi - costs(a^s, s), 0\}.$$

The law of motion for the state variables are defined in the next section.

2.2 Dynamic Framework: State to State Transition

The model contains three state variables: capacity, debt, and cash reserves. In the following, we outline the law of motion for each state variable in turn. At the end of this section, we describe what happens if firms are bankrupt, exit, or enter the market.

• *Capacity* A firm can choose to add one capacity unit ($INV = 1$) or remain inactive ($INV = 0$). With an exogenous probability δ, the current capacity is reduced by one unit because of depreciation. Therefore, the next period's capacity $\bar{q}'$ of a firm with capacity $\bar{q}$ is determined by:

$$\bar{q}' = \begin{cases} \bar{q} + 1 & \text{with probability } (1 - \delta) \text{ if } INV = 1 \\ \bar{q} & \text{with probability } \delta \text{ if } INV = 1 \\ & \text{and with probability } (1 - \delta) \text{ if } INV = 0 \\ \bar{q} - 1 & \text{with probability } \delta \text{ if } INV = 0. \end{cases}$$

If the firm decides to add capacity and no depreciation shock takes place, the capacity is increased by one. The capacity is decreased if there is no investment and a depreciation shocks hits the firm. It stays constant in all other cases.

As capacity is added and subtracted in discrete steps, it is treated as lumpy in our model. This is in line with the (s, S) modeling tradition of capacity adjustment (e.g. Caballero and Engel 1999; Caplin and Leahy 2010), prior work on Ericson–Pakes models (e.g. Besanko and Doraszelski 2004; Besanko et al. 2010), and empirical evidence. For example, Doms and Dunne (1998) show that in U.S. census data a significant amount of investment adjustment takes place in a relatively short period of time, while most periods are characterized by only minor changes. For example, 25 % of total investment derives from firms that adjust their capital stock in a given year by more than 30 %.

• *Debt and Cash Reserves* The costs associated with every action set a are financed by: current profits, a reduction of the cash reserves and/or with debt (in that order). Accordingly, the law of motion of the cash reserve is given by

$$c' = c + \Delta_{cash} - \min\{\max\{cost(a, s) - \pi, 0\}, c\}$$

Cash tomorrow is cash today plus the additional cash put into reserves less the amount necessary to cover the costs of the action set. If current profits and cash reserves are not sufficient to cover all costs, the firm can finance it with new debt. The law of motion of debt is

$$d' = d - \Delta_{debt} + \min\{\max\{cost(a, s) - \pi - c, 0\}, credit(a, s)\}$$

where $\max\{cost(a, s) - \pi - c, 0\}$ is new borrowing and $credit(a, s)$ is the credit limit. Debt in the next period is debt today minus the amount of debt repaid plus what is left to finance after the cash reserve is used up.

Although this hierarchy of finance looks strict, it is not: for example, firms can at the same time use cash and increase their cash reserves by choosing a high Δ_{cash}, thus increasing the percentage of debt financing. The only thing that is not possible is to rely on cash reserves and debt financing without using all current cash flow π. The hierarchy of finance approach is in line with the pecking order theory of Myers (1984).[4]

• *Market Exit and Entry* Two exemptions to the laws of motion outline above exist: the exit and the entry of a firm. A firm exits the market if it is either bankrupt or all its capacity is depreciated. Firm i is bankrupt if it is unable to pay its due interest payments out of current profits and retained cash, i.e.,

$$\pi + c - r \cdot d < 0.$$

The remainder of the cash reserve is given to the bank and the firm vanishes from the market. The bankruptcy process imposes an upper bound on the total amount of debt, precludes Ponzi games, and limits the size of the state space. If a firm exits, the possibility arises for an entrant to become the second player in the market. The new player has no capacity and no debt, but has the amount c^e of cash from equity investors.

2.3 The Optimization of the Financial Intermediary

The bank collects savings from depositors and gives out corporate loans. It earns profits on the interest rate spread, i.e., by paying savers an interest rate of r_{Bank} and charging

[4] It is necessary to introduce this hierarchy of finance for technical reasons. Restricting the choice space immensely simplifies the calculation of the equilibrium, because it reduces the number of follow-up states that must be considered to compute the continuation value of the firm. An alternative would be to rewrite the model in continuous time (Doraszelski and Satterthwaite 2010).

the firms a higher interest rate $r > r_{Bank}$. In every period, the bank receives the interest payments $r \cdot d$ and the repayments Δ_{debt} from the firm and gives out new loans.

In every period, the bank offers the firm a credit limit conditionally on the action taken by the firm, the industry state, and required minimum return R of the loan. This line of credit is determined by the difference of the discounted sum of payments which the bank receives from the firm with the credit ($V_{Bank}(a, s)$) and the amount in case the credit is not granted ($V_{Bank}(\tilde{a}_*, s)$) adjusted by the return R

$$credit(a, s) = \frac{V_{Bank}(a, s) - V_{Bank}(\tilde{a}_*, s)}{R}, \tag{1}$$

where $\tilde{a}_*$ is the optimal action the firm would take if no credit is given. Thus the bank is ready to grant a credit limit if it receives at least a return of R per unit of credit in expected repayment from the firm.

The discounted sum of payments ($V_{Bank}(s, a)$) it will receive from a firm if the firm takes action a in industry state s is given by

$$V_{Bank}(a, s) \equiv E\left[\sum_{t=0}^{\infty} \beta_{Bank}^t (r \cdot d_t + \Delta_{debt,t} - d_{new})\right]$$

where $\beta_{Bank} = \frac{1}{1+r_{Bank}}$ is the discount factor. In every period, the bank receives interest payments $r \cdot d$ and repayments Δ_{debt} from the firm. To account for the repayment to the savers, we subtract the net present value of all interest payments and the repayment of the principal at the time the loan is granted. By construction, this is exactly the value of the newly obtained credit $d_{new} = \min\{\max\{cost(a, s) - \pi - c, 0\}$.

Credit rationing is more severe if the required return R for loans is larger, i.e., the repayment per unit of credit must be higher. The functional form (1) described above is inspired by the credit crunch model of Holmstrom and Tirole (1997). Assume that there exist three types of agents: a continuum of firms, uninformed investors, and a continuum of banks. In each period, every firm has one project with a different return. The investor would like to invest in these projects, but is unable to prevent the firm from diverting the funds to only privately profitable projects. The bank can perfectly rule out such bad projects by monitoring the firms' efforts. However, it cannot credibly commit to do so because monitoring entails non-verifiable private costs. Consequently, the uninformed investor is only willing to employ the bank as a monitor if the bank invests a fixed amount of its own capital in the firm, too. This makes it privately optimal for the bank to control the firm. Since the bank has only a finite amount of equity, it can only fund a limited number of firms.

In order to choose which project to fund, the intermediary sorts the projects from the highest return to the lowest. Starting with the most profitable project, the bank gives loans to projects with lower and lower returns until all its equity is pledged. The excess return on the loan (corrected for its costs), which just attracts funding is denoted R. This return therefore entails a scarcity rent which might be high if a credit crunch has eaten up all of the bank's equity.

In our model, we set the return R exogenously, and assume it is time invariant, to parameterize the amount of credit rationing. This implies that throughout the economy, the returns distribution of projects is stable and the banks do not raise equity. If a firm in the considered duopoly can deliver in expected value the return R, it gets the loan, otherwise the loan is given to some other firm in the economy.

2.4 Timing

At the beginning of each period, the bank decides how much credit it offers to each firm and action. Furthermore, if a firm is unable to pay its due interest payment out of its current cash flow, the firm declares bankruptcy and exits. Next, each firm is privately informed about its cost of capacity addition η. Conditional on these investment costs and the amount of available credit, each firm takes its optimal action. Then both firms compete in the product market. At the end of the period, capacity is subject to depreciation and all decisions are implemented.

2.5 Equilibrium Concept and Computation

We focus our attention on a EBE. An EBE consists of (i) a subset of the set of possible states (the recurrent class), (ii) a vector of strategies which is optimal given the equilibrium continuation values from (iii), and (iii) a vector of continuation values for every state which is consistent with optimal actions defined in (ii). The concept of EBE as defined in Fershtman and Pakes (2012) is a similar, but a weaker concept than Markov perfect equilibrium because it is sufficient to calculate optimal policies on the recurrent class of states. A state is a member of the recurrent class if it is visited infinitely often in infinite time.

To solve for the EBE, we use a variant of the reinforcement learning algorithm outlined in Fershtman and Pakes (2012). We describe the computation and outline the merits and problems of this algorithm in Appendix 4.

2.6 Parameterization

• *State Space and Choice Set* To enable computation, we discretize the state space in all three dimensions of the state space to multiples of five units starting with a value of zero. The spacing is discretionary but this one is common in the literature (Besanko and Doraszelski 2004). Furthermore, we restrict the maximum capacity to 45, the debt to 195, and the cash to 95 units. These bounds are arbitrary but high enough so that they are never reached in equilibrium play. To ensure that firms stay within the state space, we have to restrict the potential choices of Δ_{debt} and Δ_{cash} to multiples of five with a finite upper bound.

• *Single Period Profit* Firms compete in quantities which are less than or equal to the firm's capacity. Consequently, we use the profit function ($\pi = \pi(\bar{q_i}, \bar{q_j})$) for capacity constraint quantity competition with the same parameters as Besanko and Doraszelski (2004). The derivation is outlined in Appendix 1 and illustrated in (Fig. 1). This specification is inessential because the same results ensue with capacity constraint price competition.

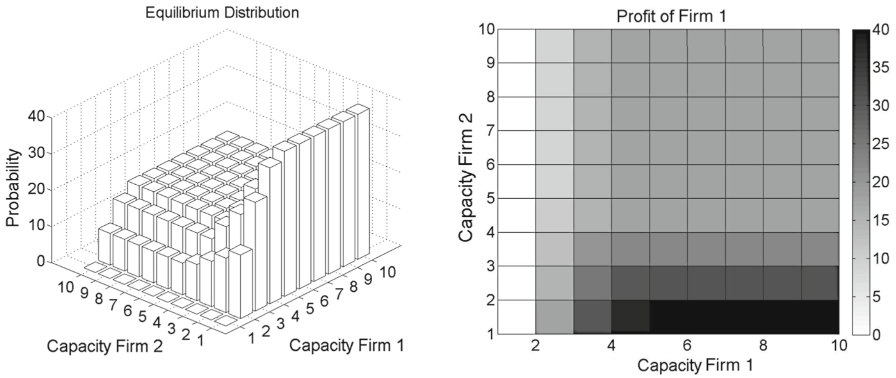

Fig. 1 Profit of firm 1

• *Investment* The investment costs η are random and are private information to the firm as in Besanko et al. (2010). They are determined by

$$\eta_{i,t} = 50 + 5 \cdot \psi_{i,t},$$

where

- the minimum construction costs are 50, which are the same for both firms and constant over time
- $5 \cdot \psi_{i,t}$ are project specific costs. $\psi_{i,t}$ is a random variable, drawn anew from a Beta (3,3) distribution with support [0,1] independently for each firm and each period. $\psi_{i,t}$ is private information for firm i and captures the idea that project opportunities are not the same for both firms and change over time.

Incorporating random investment costs and incomplete information is now common practice in the simulation of Ericson–Pakes models (e.g. Ryan 2012; Besanko et al. 2010). It is realistic that firms do not exactly know the expansion costs of the rival. Furthermore, random investment costs make it possible to use the purification techniques of Doraszelski and Satterthwaite (2010) to ensure the computability of the equilibrium.

Following Gomes (2001), who matches the investment and capital data obtained from Compustat, we set the probability of depreciation to $\delta = 12\,\%$.

• *Entry* The expected value of the amount of financing available to an entrant, the cash state of the entrant c^e, is set to 50 % of the expected average investment costs η. Thus c^e is determined by

$$c^e = 25 + 2.5 \cdot \psi^e_{i,t},$$

where ψ^e_i is a random variable drawn from a Beta(3,3) distribution with support [0,1] in every period. Again this random component of c^e ensures computability. We explore the sensitivity of our results to this assumption in the robustness section.

• *Financial Parameters* The yearly interest rate is set to $r = 6.5\,\%$ and the interest rate for savers to $r_{Bank} = 4.5\,\%$. This matches the real interest rate over the last century and the average interest rate spread of 2 % between 1968 and 1997 (Gomes 2001).

In the simulation we consider two degrees of credit rationing: $R = 5\,\%$ and $R = 120\,\%$. These two values are arbitrary but well illustrate the mechanisms at work. Without credit rationing the loan must deliver at least 5 % return in net present value terms compared to the case that the loan is not given. The firm must be able to pay interest for two years and return the principal to obtain such a loan. In the case of credit rationing, R is set to $120\,\%$. Such a return cannot be met by interest payments on the given credit alone, but there must be an additional future value for the bank. For example, the loan could ensure that a debt-laden firm survives and pays back more of its debt. Another possibility is that the loan helps a new firm to enter which relies heavily on the bank in future play.

3 The Effects of Financial Frictions on the Equilibrium Capacity Distribution

In this section, we show that credit rationing leads to the monopolization of markets and thus to welfare loss. The following section is concerned with the propagation mechanism which leads to the exit of one player, starting from a symmetric duopoly. Finally, we consider how credit rationing makes the monopolization permanent by preventing market entry.

3.1 Equilibrium Capacity Distribution and Welfare Results

To show the effect of financial frictions on the market structure in equilibrium, we discuss in the following the properties of the invariant equilibrium capacity distribution in the market. Table 1 and Fig. 2 picture the equilibrium capacity distribution for the case without credit rationing ($R = 5\,\%$) on the left hand side and with credit rationing ($R = 120\,\%$) on the right hand side. A higher probability of a certain industry structure indicates that this industry structure is more likely to occur in equilibrium play. For example, without credit rationing, the industry configuration with a firm 1 and a firm 2 of size 10 (3 capacity blocks) is played with a probability of 25 %. With credit rationing, the most likely market structure is with 15 % one large firm of four capacity blocks and the other firm with no capacity at all.

Credit rationing causes the equilibrium distribution to become skewed: one firm exits the market and the equally productive competitor becomes the monopolist. There is an equal probability that firm 1 or firm 2 is the monopolist, reflecting the symmetric set-up of the model. The large firm has four capacity blocks and the small firm has no capacity at all in the most likely industry structure. There is some probability mass in between the two extreme configurations, indicating that leadership changes from time to time. Without financial frictions, the most likely configuration is that both firms have an equal size with three capacity blocks. Due to the randomness in the investment and depreciation process there is also some probability for asymmetric market share configurations.

Table 1 Probability that a state is played in equilibrium (in percentage)

		j = 1 $\bar{q}_j = 0$	j = 2 $\bar{q}_j = 5$	j = 3 $\bar{q}_j = 10$	j = 4 $\bar{q}_j = 15$	j = 5 $\bar{q}_j = 20$	j = 6 $\bar{q}_j = 25$
(a) $R = 5\,\%$							
i = 1	$\bar{q}_i = 0$	0	0	0	0	0	0
i = 2	$\bar{q}_i = 5$	0	1	5	6	1	0
i = 3	$\bar{q}_i = 10$	0	5	25	15	2	0
i = 4	$\bar{q}_i = 15$	0	6	15	6	0	0
i = 5	$\bar{q} = 20$	0	1	2	0	0	0
i = 6	$\bar{q}_i = 25$	0	0	0	0	0	0
(b) $R = 120\,\%$							
i = 1	$\bar{q}_i = 0$	0	1	6	15	7	3
i = 2	$\bar{q}_i = 5$	1	1	3	2	0	0
i = 3	$\bar{q}_i = 10$	6	3	4	2	0	0
i = 4	$\bar{q}_i = 15$	15	2	2	0	0	0
i = 5	$\bar{q}_i = 20$	7	0	0	0	0	0
i = 6	$\bar{q}_i = 25$	3	0	0	0	0	0

i and *j* denote the number of capacity blocks held by firms *i* and *j*

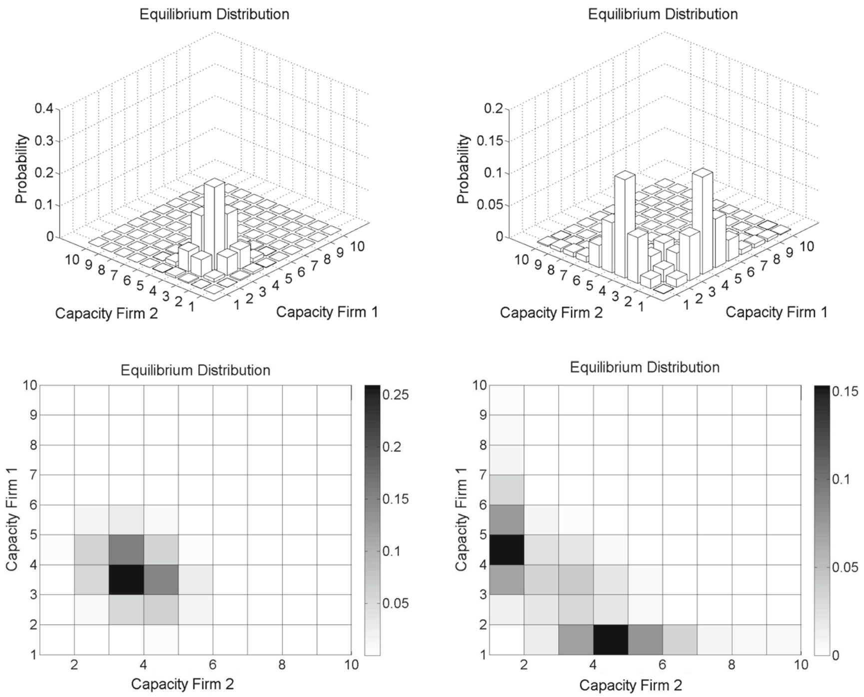

Fig. 2 Equilibrium distribution for $R = 5\,\%$ (*left*) and $R = 120\,\%$ (*right*). *Note* The capacity states of the two firms are depicted on the x- and y-axis. In the *upper panel* the probability of a state is displayed on the z-axis. In the *lower panel* the probability is shown by *different colors*

Table 2 Welfare effects of credit rationing

Surplus	Consumer	Producer	Bank	Total
Model with credit rationing	14.72	35.95	0.61	51.28
Model without credit rationing	24.19	35.03	2.38	61.60
Difference	−9.46	0.92	−1.77	−10.32

This is the expected welfare over all states. For the calculation of these measures please refer to Appendix 2

Table 3 Summary statistics

	Capacity	Price	Debt	Cash
Model with credit rationing	8.90	2.35	7.44	18.37
Model without credit rationing	11.25	1.83	26.53	13.20
Difference	−2.35	0.52	−19.09	5.17

These findings complement the results of Caballero and Hammour (1994) on the cleansing effect of recessions: they find that a fall in demand during a recession leads to job destruction, which they conjecture is due to the exit of unproductive firms. A recession is therefore "cleansing" for an economy. In our model, credit rationing (which often accompanies a fall in demand) leads to the exit of a firm which is equally productive as the remaining incumbent. Consequently, a recession with a credit crunch might only increase market power without the merits of increased average productivity. This effect bears some resemblance to the "scarring" effect of recessions outlined in Ouyang (2009). In that article, firms' learning-by-doing is reduced by the lower volumes produced in a recession killing potentially good firms in their infancy. In contrast, in the present article, firms are only constrained by financial factors.

The monopolization of the market leads to a welfare loss (Table 2) which is mainly borne by consumers and banks. The welfare loss of the consumer originates from lower capacities and higher prices as shown in the summary statistics of Table 3. Banks have lower profits because the amount of credit (on which they earn a fixed income) is smaller (the amount of debt is lower) with credit rationing. In contrast to the reduction of surplus for consumers and banks, the firm surplus stays approximately the same. The reason is simple: with credit rationing, a firm has with (approximately) equal probability monopoly and zero profits, whereas without financial frictions, it has duopoly profits for sure. Thus, in expected value, 50 % monopoly profits is a bit larger than 100 % of the duopoly profit. Therefore, if firms are risk neutral, they do not suffer from credit rationing.

All other statistics in Table 3 are in line with expectations: the average debt level is lower and the amount of retained cash is higher when credit rationing is present. If the financial market is not working properly firms get fewer and smaller loans and try to finance themselves through the retainment of cash.[5]

[5] Almeida et al. (2004) use a similar reasoning to justify cash flow sensitivities of cash as a sensible measure for financial constraints.

3.2 Credit Rationing as Propagation Mechanism

Approximately every eight periods a depreciation shock hits each firm. This initially small shock sets a process in motion which results in a skewed equilibrium distribution given that credit rationing is present.

The propagation mechanism works as follows: a shock reduces the capacity of firm i. Lower capacity translates into lower current profits. Because of credit rationing, this loss in profit cannot be compensated by taking out more loans. With lower cash flow and insufficient available credit, the probability that a firm can afford the costs of capacity expansion is reduced. A reduction in investment together with an unaltered probability of depreciation results in less capacity, which again triggers less investment. The competitor benefits from this mechanism: the original shock reduces the capacity on the market and increases the price level. Therefore, the competitor has more profit available for saving and investment. With this additional profit, he can increase his investment in order to further tighten the credit constraints of the smaller firm.

To illustrate, we now compare the different investment probabilities with and without credit constraints given that a firm is hit by a depreciation shock. Assume that in a market with credit rationing, the capacity of a firm in state (3,3) is reduced by one unit so the firm finds itself in state (2,3). Then the investment probability with credit rationing is 27 %, much smaller than in the case without, where the firm invests with a probability of 66 % (Tables 4a, b). After the first shock, the larger competitor

Table 4 Investment probability for firm 1 in percentage

		j = 1 $\bar{q}_j = 0$	j = 2 $\bar{q}_j = 5$	j = 3 $\bar{q}_j = 10$	j = 4 $\bar{q}_j = 15$	j = 5 $\bar{q}_j = 20$	j = 6 $\bar{q}_j = 25$
$R = 5\,\%$							
i = 1	$\bar{q}_i = 0$	*94*	*98*	94	92	96	*100*
i = 2	$\bar{q}_i = 5$	*98*	74	66	49	54	*73*
i = 3	$\bar{q}_i = 10$	51	41	7	5	8	13
i = 4	$\bar{q}_i = 15$	23	6	1	0	0	0
i = 5	$\bar{q}_i = 20$	14	2	1	0	0	*0*
i = 6	$\bar{q}_i = 25$	*0*	*0*	1	0	0	*0*
$R = 120\,\%$							
i = 1	$\bar{q}_i = 0$	*72*	39	7	2	3	4
i = 2	$\bar{q}_i = 5$	63	44	27	15	18	14
i = 3	$\bar{q}_i = 10$	25	27	8	3	5	4
i = 4	$\bar{q}_i = 15$	6	10	1	0	0	*0*
i = 5	$\bar{q}_i = 20$	5	5	1	0	*0*	*0*
i = 6	$\bar{q}_i = 25$	3	3	1	*0*	*0*	*0*

This is the average investment probability in each capacity state. Investment probabilities also depend on the debt state and the amou nt of retained cash

States written in italics are played in equilibrium with a probability below 0.1 % and are likely calculated with error

i and j denote the number of capacity blocks held by firm i and j

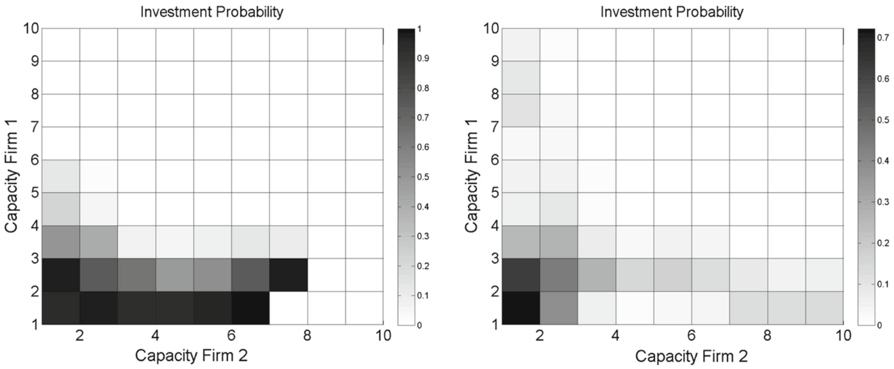

Fig. 3 Investment probabilities for Firm 1, $R = 5\,\%$ (*left*) and $R = 120\,\%$ (*right*)

Table 5 Sum of current profits, retained cash and credit with $R = 120$ % for firm 1

		j = 1 $\bar{q_j} = 0$	j = 2 $\bar{q_j} = 5$	j = 3 $\bar{q_j} = 10$	j = 4 $\bar{q_j} = 15$	j = 5 $\bar{q_j} = 20$	j = 6 $\bar{q_j} = 25$
i = 1	$\bar{q_i} = 0$	*55*	40	30	28	28	29
i = 2	$\bar{q_i} = 5$	59	39	35	30	30	24
i = 3	$\bar{q_i} = 10$	49	48	40	33	31	29
i = 4	$\bar{q_i} = 15$	45	49	43	32	30	*27*
i = 5	$\bar{q_i} = 20$	44	52	45	43	*0*	*0*
i = 6	$\bar{q_i} = 25$	44	61	48	*42*	*0*	*0*

This is the average amount available for investment in each capacity state. This amount is also dependent on the debt state and the amount of cash retained

States written in italics are played in equilibrium with a probability below 0.1 % and are likely calculated with error

i and j denote the number of capacity blocks held by firms i and j

has on average an investment probability of 27 %. This is still a reduction, albeit a much smaller one, from the investment probability of 41 % without financial frictions. Therefore, the key observation is here that the investment probability of the smaller firm is reduced by $1 - \frac{0.27}{0.66} = 59\,\%$ while the investment probability of the larger firm is reduced only by 34 % compared to the case without credit rationing.[6] This relatively larger reduction in the ability to invest makes it more likely that the smaller firm exits the market: if it fails to reinvest, the firm might be hit by another depreciation shock. Furthermore, if the competitor invests (and the other firm fails to do so), the capacity state evolves to (2,4), reducing the investment probability of the smaller firm further to 15 % (Fig. 3; Table 5).

The effect is driven by the differing optimal probabilities that investment is carried out with and without credit rationing (Table 4). Because the analysis is ceteris paribus, this difference can only originate from the amount of credit available to the firms: with $R = 5\,\%$, abundant credit is extended in any state (Table 6a) and the firm does not need

[6] The reduction in the neutral (2,2) state is 40.5 to 44 % with credit rationing from 74 % without.

Table 6 Credit for firm 1

		j = 1 $\bar{q_j} = 0$	j = 2 $\bar{q_j} = 5$	j = 3 $\bar{q_j} = 10$	j = 4 $\bar{q_j} = 15$	j = 5 $\bar{q_j} = 20$	j = 6 $\bar{q_j} = 25$
(a) $R = 5\,\%$							
i = 1	$\bar{q_i} = 0$	*899*	*436*	338	236	229	*308*
i = 2	$\bar{q_i} = 5$	*338*	228	198	76	66	*37*
i = 3	$\bar{q_i} = 10$	47	44	14	4	6	4
i = 4	$\bar{q_i} = 15$	44	8	0	0	0	0
i = 5	$\bar{q_i} = 20$	27	1	1	0	0	*0*
i = 6	$\bar{q_i} = 25$	*1*	*0*	0	0	*0*	*0*
(b) $R = 120\,\%$							
i = 1	$\bar{q_i} = 0$	*27*	12	2	0	0	1
i = 2	$\bar{q_i} = 5$	20	11	6	3	3	2
i = 3	$\bar{q_i} = 10$	3	3	1	0	1	0
i = 4	$\bar{q_i} = 15$	1	1	0	0	0	*0*
i = 5	$\bar{q_i} = 20$	1	0	0	0	*0*	*0*
i = 6	$\bar{q_i} = 25$	1	0	0	*0*	*0*	*0*

This is the average amount of credit offered in each capacity state. The amount of credit is dependent also on the debt state and the amount of cash retained

States written in italics are played in equilibrium with a probability below 0.1 % and are likely calculated with error

i and j denote the number of capacity blocks held by firms i and j

to delay any investment. With credit rationing, only 6 units of credit are offered by the financial intermediary (Table 6b). This lack of credit drives down the equilibrium investment probability.

The described qualitative results are similar to those of Kiyotaki and Moore (1997), however the mechanism is different. In their model, a negative productivity shock reduces the net worth of the credit constrained firm. This leads to a reduction in investment in the productive factor which is also a collateral for credit. The resulting shortfall in demand for the productive factor reduces its value as collateral. Consequently, the firm cannot get as much credit as before. This reduces the demand for the productive factor further, drives down the net worth of the constrained firm again and the firm enters a reinforcing credit cycle.

In our model, firms hit by a depreciation shock are unable to tap the credit market to finance investment. The bank does not offer enough credit because the expected net present value of the investment is not high enough to satisfy the return requirements. Without investment, the firm's capital stock depreciates further resulting again in reduced credit and reduced current profits. In the end, this mechanism can lead to the exit of one firm.

3.3 Credit Rationing as Entry Barrier

If one firm exits, the possibility arises for an entrant to become the second firm in the market. However, the monopolization of the market is (relatively) stable because credit rationing also serves as a barrier to entry. Thus, financial frictions lead to lower entry rates which in turn results in a skewed capacity distribution.

In the basic configuration, investment costs are uniformly distributed between 50 and 55 and the amount of start-up financing provided by the equity markets is between 20 and 25. Therefore, the firm has to take at least an amount of 25 as credit to enter the market.[7] If credit rationing is present, the entrant only receives such an amount of credit in case the competitor is out of the market or small, with only one or two capacity blocks (Table 6). Hence, the investment probabilities are only high in these states, but not when the competitor has more capacity (Table 4). The investment probability is 72 % for firm 1, given that no firm is in the market. With a competitor's capacity of one capacity block, the investment probability decreases to 38 %. The investment probability becomes negligible if the capacity of the competitor is higher. On the equilibrium path, the monopolist is out of the market or small with only a probability of 1 % (Table 1b). Credit rationing therefore serves as an effective barrier to entry.

In contrast, without credit rationing, the probability to invest for an entrant is always above 90 % irrespective of the competitor's capacity (Table 4). Consequently, a firm with zero capacity always enters the market immediately.

4 Robustness Checks

To show that the outlined results are stable despite the multiplicity of assumptions made, we vary three key parameters in our model: the severeness of credit rationing R, the maximum possible amount of retained cash, and the amount of start-up financing c^e. Furthermore, we explore the implications of incomplete information on the equilibrium capacity distribution.

- *Severeness of Credit Rationing* In Fig. 4, we gradually increase the severeness of credit rationing. With an increase in the minimum return R, the probability of an asymmetric equilibrium capacity distribution increases. This is intuitive: the more banks tighten the credit constraint, the more adverse is the effect.
- *Maximum Amount of Retainable Cash* In the preceding analysis, firms were able to accumulate a large amount of cash. The limit was set to 100, the equivalent of two capacity blocks or approximately five periods of profit. However, in reality, shareholders might have an incentive to limit the amount of cash a company can hold, to mitigate moral hazard problems: if a manager must regularly apply for funds, the capital market controls their proper use (Jensen 1986; Easterbrook 1984).

Figure 5 presents the equilibrium distribution with varying amounts of maximum retainable cash. The effects of credit rationing become more severe if a company can retain less cash.

- *Increase in Start-up Financing* Throughout the main part of the analysis, entrants only had a limited amount ϕ^e of start-up financing. This can be thought of as the amount a start-up can raise on the equity market. This small scale is intuitive because the monitoring service of the bank is only needed if investment projects without monitoring

[7] Profits in the outside state are zero.

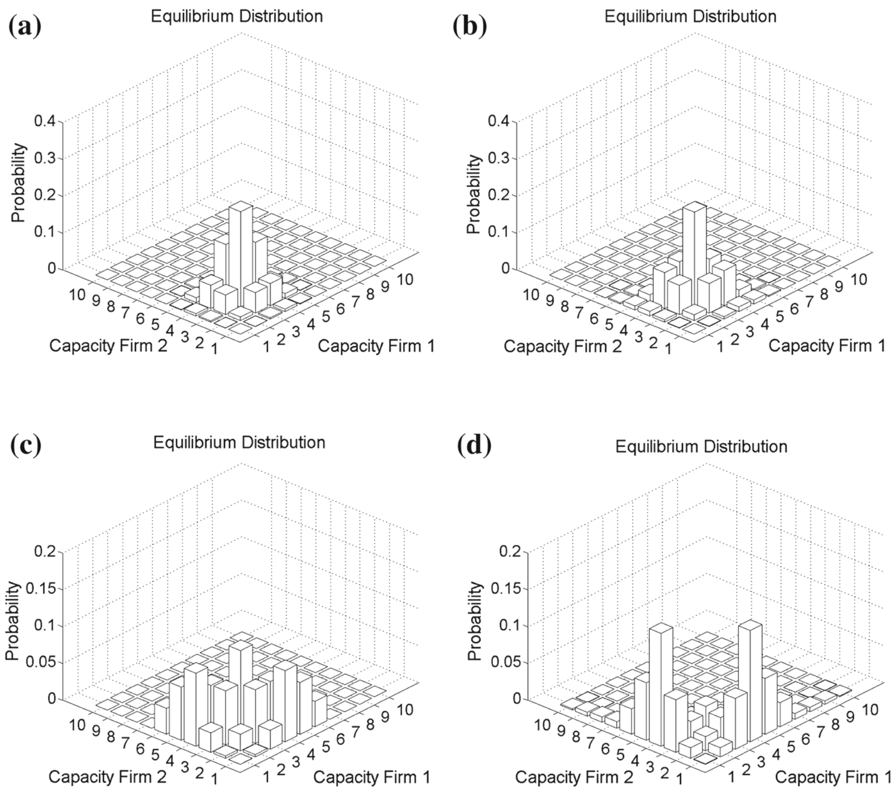

Fig. 4 Equilibrium distribution with varying amount of credit rationing. **a** $R = 5\,\%$, Cash = 100. **b** $R = 40\,\%$, Cash = 100. **c** $R = 80\,\%$, Cash = 100. **d** $R = 120\,\%$, Cash = 100

achieve a negative net present value due to severe moral hazard. Consequently, naive investors are not willing to finance start-ups on a large scale in such a market.

In our analysis, the size of start-up financing is 50 % of the investment an entrant needs to enter the market. If we increase this proportion, the effects of credit rationing are smaller. This result is illustrated in Fig. 6.

• *Incomplete Information* In the whole analysis, we allow the firms to condition on the complete industry state $s = (\bar{q_i}, \bar{q_j}, d_i, d_j, c_i, c_j)$. The assumption that each firm knows the exact financial structure of its competitor might be rather extreme. Fortunately, the Fershtman and Pakes (2012) algorithm allows of introducing incomplete information in the Ericson–Pakes framework. If we let firms condition their strategy only on their own financial structure and the two capacity states, the results are qualitatively similar to the full information case (Fig. 7).

5 Conclusion

In this article, we identify the effects of credit rationing on the equilibrium market structure in a duopoly. We employ the Experience Based Markov Equilibrium frame-

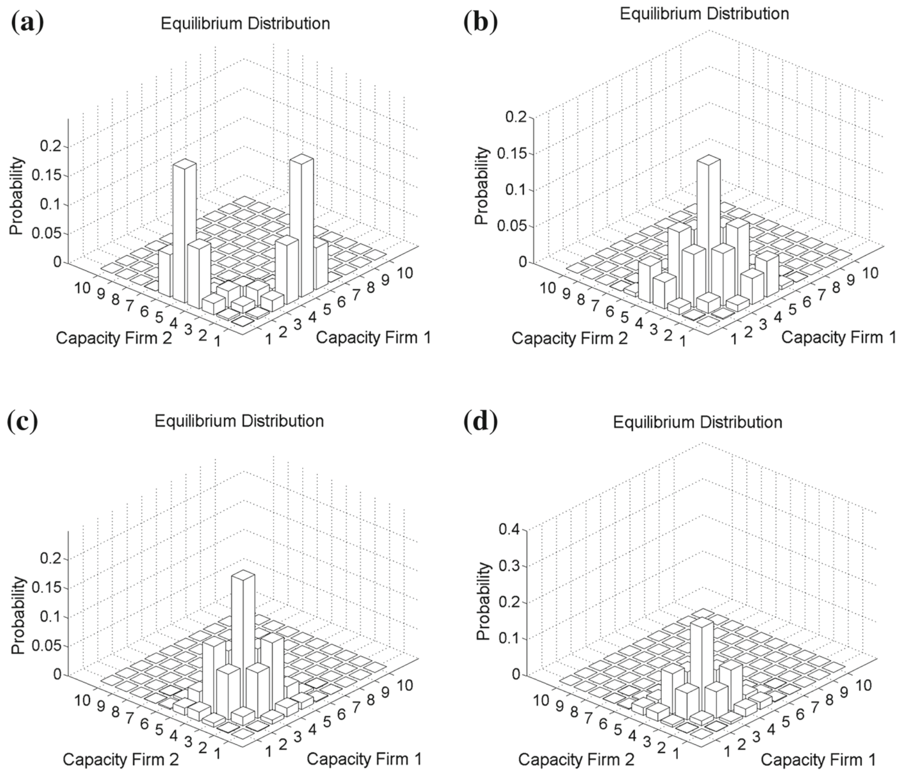

Fig. 5 Equilibrium distribution with varying amount of retainable cash. **a** $R = 50\,\%$, Cash = 0. **b** $R = 50\,\%$, Cash = 20. **c** $R = 50\,\%$, Cash = 80. **d** $R = 50\,\%$, Cash = 100

work presented in Fershtman and Pakes (2012) to extend the model of Besanko and Doraszelski (2004) by an optimizing bank and firms which actively choose their capital structure.

In our model, firms can retain cash, borrow from banks, or use current cash flow to finance themselves. Due to a shortage of capital, banks might be unable to fund every profitable project. Therefore, credit rationing might prevail. If then a small shock reduces the capacity of a firm, this firm might find itself unable to finance capacity expansion. If a firm then stays at a lower capacity level, it has also less funds to finance investment in future periods. Eventually, this lack of investment can lead to the exit of one firm and monopolization of the market. The monopolization is made permanent because new entrants also suffer from credit rationing and cannot enter to fill the void.

This article shows that in equilibrium, the exit of firms during a recession might not be driven by insufficient productivity but by a lack of credit financing. Therefore, policy makers should put emphasis on the functioning of the credit market during recessions to prevent welfare losses through an increase in market power. For exam-

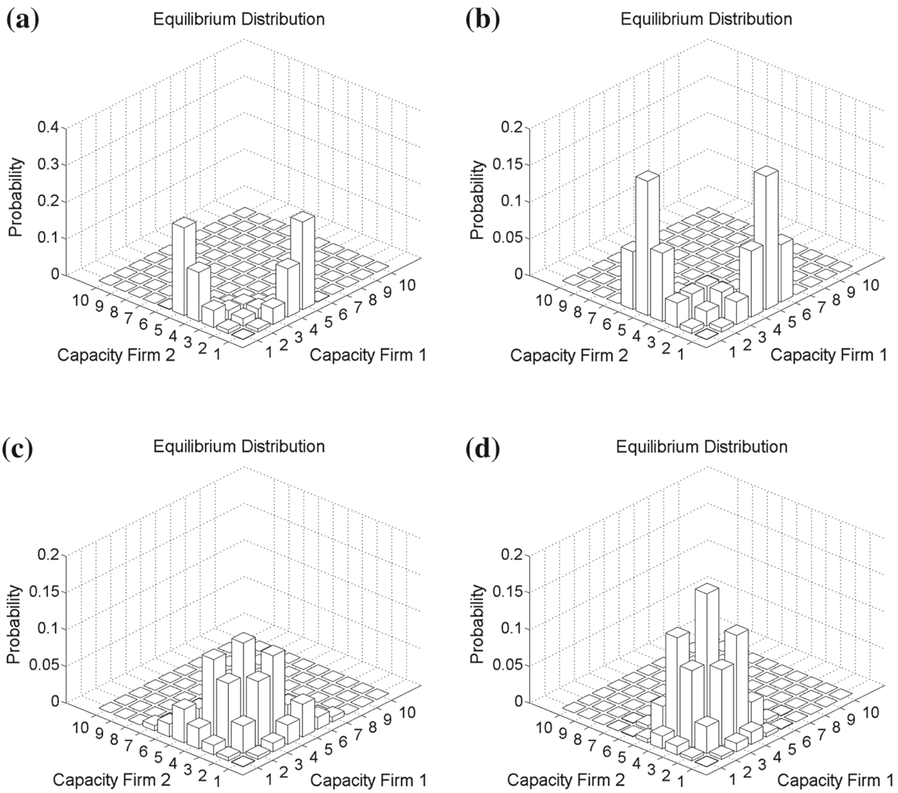

Fig. 6 Equilibrium distribution with varying start-up financing. **a** $R = 120\,\%$, $\phi^e = 20$. **b** $R = 120\,\%$, $\phi^e = 25$. **c** $R = 120\,\%$, $\phi^e = 35$. **d** $R = 120\,\%$, $\phi^e = 50$

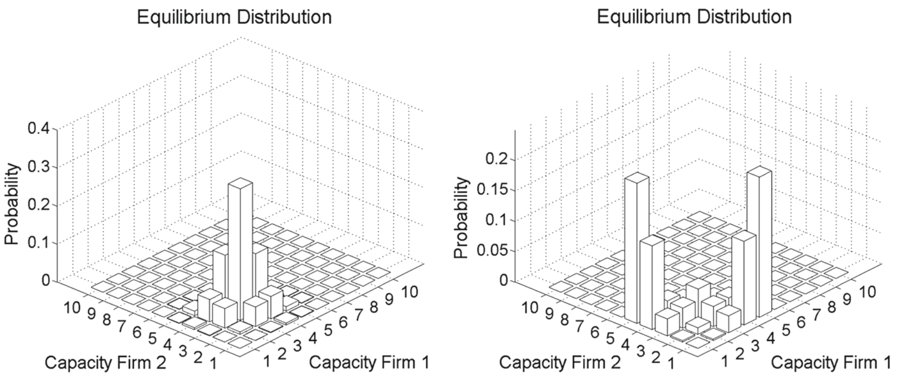

Fig. 7 Equilibrium distribution with incomplete information for $R = 5\,\%$ (*left*) and $R = 120\,\%$ (*right*)

ple the government could introduce credit support programs for small companies and start-ups. According to our model, such programs could foster competition and thus increase overall welfare.

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Appendices

Appendix 1: Single Period Profit Function

The mode of product market competition is capacity constraint quantity competition as in Besanko and Doraszelski (2004).

The inverse demand function $P(q_i, q_j)$ with P as market price and q_i as quantity produced by firm i is given by

$$P(q_i, q_j) = \frac{a}{b} - \frac{q_i + q_j}{b}.$$

Suppose that firm i and firm j's capacities are given by $(\bar{q}_i, \bar{q}_j)$ and that they compete in the product market by setting quantities (q_i, q_j).

The profit-maximization problem for firm i with $i, j \in (1, 2), i \neq j$ is then given by

$$\max_{0<q_i<\bar{q}_i} P(q_i, q_j)q_i$$

This maximization problem for i and the symmetric problem for j lead to symmetric reaction functions which are known to have a unique Nash equilibrium (Vives 2001). The single period profit function of firm i in the Nash equilibrium of the capacity constrained quantity setting game is therefore

$$\pi_i(\bar{q}_i, \bar{q}_j) = P(q_i^*, q_j^*)q_i^*.$$

Table 7 Profit for firm 1

		j = 1 $\bar{q_j} = 0$	j = 2 $\bar{q_j} = 5$	j = 3 $\bar{q_j} = 10$	j = 4 $\bar{q_j} = 15$	j = 5 $\bar{q_j} = 20$	j = 6 $\bar{q_j} = 25$
i = 1	$\bar{q_i} = 0$	0	0	0	0	0	0
i = 2	$\bar{q_i} = 5$	18	15	13	10	9	9
i = 3	$\bar{q_i} = 10$	30	25	20	15	15	15
i = 4	$\bar{q_i} = 15$	38	30	23	18	18	18
i = 5	$\bar{q_i} = 20$	40	31	23	18	18	18
i = 6	$\bar{q_i} = 25$	40	31	23	18	18	18

i and j denote the number of capacity blocks held by firms i and j

The demand parameters used in the simulation are $a = 40$ and $b = 10$ and are thus the same as in Besanko and Doraszelski (2004). These parameters ensure that a company can have more capacity than the entire market demand (Table 7).

Appendix 2: Welfare Measures

To evaluate the implication of credit rationing on welfare, we calculate the expected consumer surplus and the expected producer surplus of the firms and of the bank.

Expected consumer surplus is calculated by integrating the demand function

$$CS = E\left[\int_{p_{Market}}^{p_{max}(s)} D(t)dt\right]$$

where $D(\cdot)$ is the demand function, p_{max} is the choke price and p_{Market} is the prevailing market price. The expectation is taken with respect to the probability of the state in equilibrium.

As marginal costs are normalized to zero, expected producer surplus for every state is calculated as the sum of profits minus the financing costs:

$$PS = E\left[\pi(\bar{q}_1, \bar{q}_2) + \pi(\bar{q}_2, \bar{q}_1) - d_1 \cdot r - d_2 \cdot r\right].$$

Expected bank surplus is the interest rate differential multiplied by the sum of debt:

$$BS = E\left[(r - r_{Bank}) \cdot (d_1 + d_2)\right].$$

Appendix 3: Transitory Dynamics

Figure 8 shows the distribution after 10, 20, and 50 periods, starting from the initial value of zero capacities for both players. The distribution without credit rationing evolves directly towards symmetry and stays there forever. With credit rationing, first a symmetric configuration with equal capacity for both firms is reached and then the distribution becomes asymmetric. There is a large probability that one firm exits the market on the equilibrium path.

Appendix 4: The Reinforcement Learning Algorithm

In this section, we outline the reinforcement learning algorithm used in this article. It is a variant of the algorithm described in Fershtman and Pakes (2012), to which the reader should refer for an extensive description.

Intuitively, the algorithm employed works as follows: a firm starts in state s and time t. For every potential action a and state s, the firm holds beliefs $W(a_t, s_t)$ about the expected discounted sum of cash flows the action will yield. The firm then chooses the best action a^* according to its beliefs and receives an instant payoff of $div(a_t^*, s_t)$. The actions together with the law of motions of the state variables prescribe the next

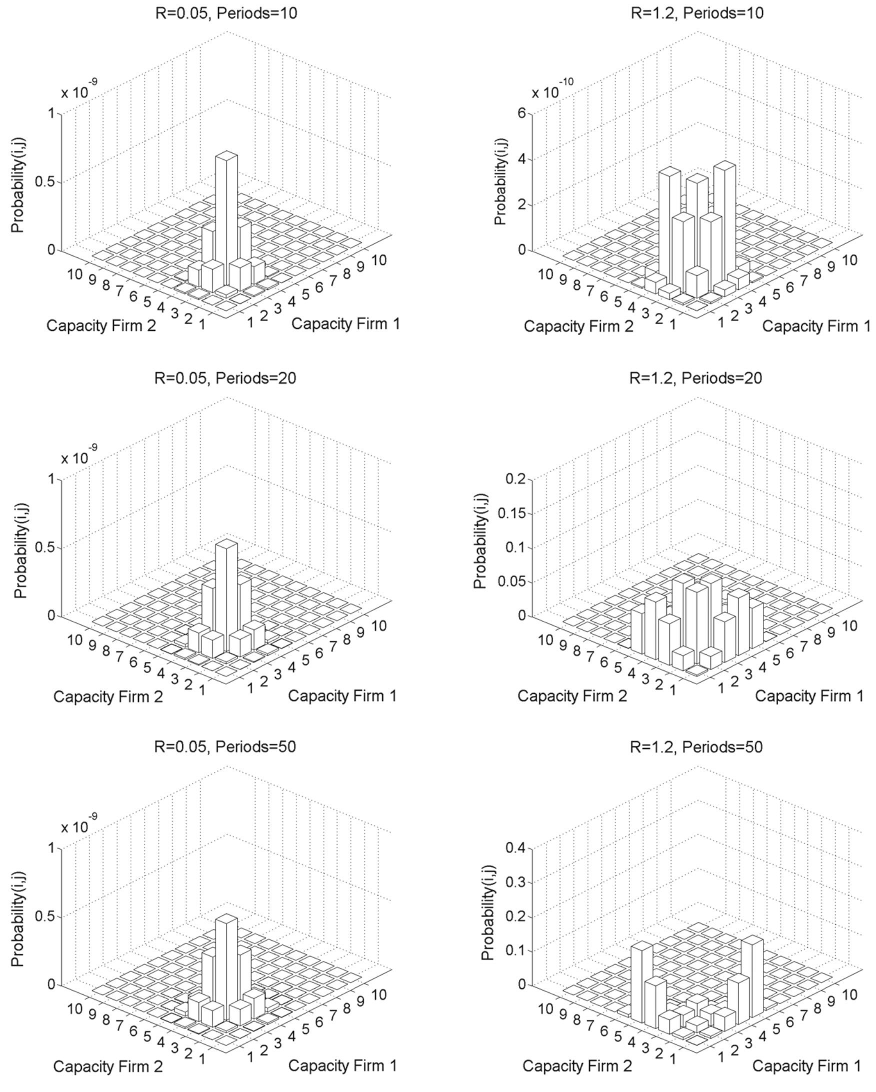

Fig. 8 Transitory Dynamics for $R = 5\,\%$ (*left*) and $R = 120\,\%$ (*right*). *Note* the capacity states of the two firms are depicted on the x and y axes. The probability of a state is displayed on the z-axis. On the *left hand side*, the evolution without credit rationing is pictured. On the *right hand side*, severe credit rationing prevails

state. In the next state, the firm chooses again its best actions according to its belief $W(s_{t+1}, a_{t+1})$. At this point, the algorithm can update the belief $W(s_t, a_t^*)$ because $div(a_t, s_t)$ and $W(a_{t+1}^*, s_{t+1})$ are part of the discounted sum of cash flows originating in s_t if a_t is chosen. This procedure is repeated for *Iter* periods. The optimal actions a_t in every period are stored in memory for use in the equilibrium testing procedure. To test for an equilibrium, the algorithm simulates a large number of periods with the stored optimal actions and checks whether the resulting beliefs $W^{test}(s, a)$ are the same as the beliefs $W(s, a)$ which justified the actions in the first place.

A Tentative Example Assume that the player is in state s. Assume further that the player is under the impression that storing five more units of cash starting from state s gives—for the sake of illustration—a continuation value of 2000. This is better than the alternative of not doing so as he believes that this gives him a continuation value of 1500. He stores five more units and then finds out during the play that this decision only resulted in a continuation value of 1600. So he adjusts his expectation of storing five units in state s downwards to 1800. He does not adjust it downwards to 1600 as he cannot perfectly distinguish if this was just a matter of bad luck or truly the consequence of his actions. The benefit of this solution algorithm is that it can accommodate larger state spaces than the commonly used Pakes and McGuire (1994) algorithm. This algorithm only calculates policies on the recurrent state space and therefore ignores states which are never played in equilibrium. To give an idea, the state space in our calculation has about 6.25×10^{12} states. By selecting only those states relevant to the equilibrium, we only have to calculate equilibrium policies for around one million states, which is still large but manageable. The idea of calculating policies in an Ericson–Pakes model only for a sample of states is gaining prominence in numerical analysis, e.g., another algorithm using this method is Farias et al. (2012).

There are also several known problems for this kind of algorithms and numerical simulations of imperfect competition in general:

1. It is not guaranteed that an equilibrium exists. Even if one exists, the algorithm does not necessarily converge to it.
2. There might be multiple equilibria for reasonable parameter values. Besanko et al. (2009) offer a possible solution, however, we did not explore this issue up to now.
3. There might be more than one recurrent class associated with a set of policies.
4. It is not clear that the off-equilibrium beliefs are irrelevant for the equilibrium play. This is known as the problem of insufficient exploration.

In line with common practice, we check if the algorithm converges to the same equilibrium for different starting values. Although this appears to be the case, the above mentioned issues should be kept in mind.

Scheme of the Algorithm The algorithm requires the following inputs:

- A set of beliefs about the continuation value for every action in every state $W(a, s)$
- A counter $h(a, s)$ for every state and action which measures how often the action was taken
- An arbitrary initial state $\breve{s}$ and an arbitrary initial action $\breve{a}$
- An instant return function $div(a_t, s_t)$ for every action and state
- A function assigning the next period's state s' conditional on today's state s and action a, $f(\cdot)$.
- Technical parameters: length of iteration (*Iter*), ϵ precision of the approximation, and the discount factor β

Algorithm for calculating EBE

1: $s_t = \breve{s}$, $a_t = \breve{a}$ {Set initial state and initial actions }
2: **repeat**
3: t:=0 {Set index t for best simulation}
4: **while t <= Iter** {Begin learning process, last for T periods}
5: $t = t + 1$
6: $s_{t+1} = f(a_t, s_t)$ {Assign next state in $t + 1$ according to the optimal actions and state in t }
7: Load $W(\cdot, s_{t+1})$ for all a_{t+1} from memory if already visited, otherwise assign initial values.
8: $a^*_{t+1} = arg\max_{a_{t+1}} W(a_{t+1}, s_{t+1})$
{Calculate the optimal action}
9: $h(a^*_{t+1}, s_{t+1}) = h(a, s) + 1$
{Increase the counter of the state s_{t+1} and action a_{t+1} by one. }
10: $\hat{W}(a^*_t, s_t) = div(a^*_t, s_t,) + \beta W(a^*_{t+1}, s_{t+1})$
{Calculate the continuation value in t according to the next period's action and state. $W(a_{t+1}, s_{t+1})$ is a draw of the integral governing the continuation value. }
11: $W(a^*_t, s_t) = W(a^*_t, s_t) + \frac{1}{h(a^*_{t+1}, s_{t+1})}[W(a^*_t, s_t) - \hat{W}(a^*_t, s_t)]$
{save the updated belief $W(a^*_t, s_t)$ to memory and store the optimal a^*_t action}
12: set $s_t = s_{t+1}$ and $a^*_t = a^*_{t+1}$
13: **end**
14: $t = 0$
15: **while t <= T**{Begin test procedure}
16: $s_{t+1} = f(a^*_t, s_t)$ {Assign new state}
17: Load a^*_{t+1} and $W(a^*_{t+1}, s_{t+1},)$ from memory
18: $h(a^*_{t+1}, s_{t+1}) = h(a, s) + 1$
{Increase the counter of the state s_{t+1} and action a_{t+1} by one. }
19: $\hat{W}^{test}(a^*_t, s_t) = div(a^*_t, s_t) + \beta W(a^*_{t+1}, s_{t+1})$
{Calculate the continuation value in t according to the next period's action and state. }
20: $W^{test}(a^*_t, s_t) =$
$W^{test}(a^*_t, s_t) + \frac{1}{a^*_{t+1}, h(s_{t+1})}[W^{test}(a^*_t, s_t) - \hat{W}^{test}(a^*_t, s_t)]$
{Update the belief about the continuation value. Also do the procedure for the square of $W^{test}(.)$ to calculate the sampling variance.}
21: store $W^{test}(a^*_t, s_t)$
22: set $s_t = s_{t+1}$ and $a^*_t = a^*_{t+1}$
23: **end**
24: $Bias(s, a) = \frac{W^{test}(a,s)}{W(a,s)}^2 - Var(\frac{W^{test}(a,s)}{W(a,s)})$
{Calculate for every state and action visited on the equilibrium path a bias statistic. The variance term is used to adjust for sampling variance. }
25: $T = \left\| \sum_a \frac{h(a,s)}{\sum_a h(a,s)} Bias(a, s)|| \right\|_{L^2_{P(s)}}$
{The test statistic is then an L^2 norm in the bias term where $P(s)$ is a measure for the fraction of time s is visited on the equilibrium path. The test statistic measures if the stored optimal action can replicate the continuation values. }
26: until $T < \epsilon$ {The algorithm if T is below the required precision ϵ. }

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