# Algoritmi di Crittografia

## Corso di Laurea Magistrale in Informatica

A.A. 2018/2019

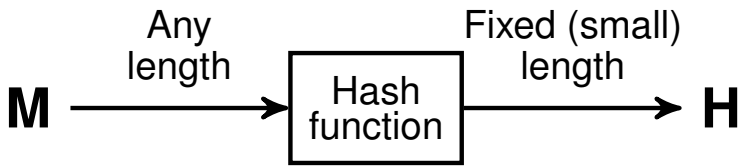
# Algoritmi di Crittografia

1. Hash Functions
   - General concepts
   - Building hash functions
   - Real hash functions

# Algoritmi di Crittografia

# Hash functions

- In general terms, a *hash function* is a function that takes an arbitrary long input **M** and returns a fixed size output




- "Long input" may refer, say, to a long piece of text or to a very large number (depending on the application)
- The output can be generally seen as a sequence of bits, although in some applications it is rather interpreted as an integer number (i.e, an index to a table)
- For all the conceivable applications, though, the main property of a hash function is that it must appear *random*, i.e., returns "unexpected" values

# Cryptographic hash functions

- *Cryptographic Hash Functions* (*CHF*s) are often referred to as (message) *fingerprints* or *digests*
- However, there are many more applications of CHFs than those implied by these terms
- With respect to hash functions used in other application settings, a CHF must exhibit additional properties
- The fundamental (and, in some sense "ideal") requirement for crypto is that hash functions must be *one-way*
- Being one-way for a function $f$ means that it "easy" to compute $f(x)$, for any $x$, but "hard" to invert, i.e., given $y$ compute $x$ such that $f(x) = y$
- This is also known as (*first*) *pre-image resistance*

# Some cryptographic applications of hash Functions

- User authentication
- Digital signatures
- Message Authenticity and integrity
- File identifiers
- Program obfuscation

# Other important properties of a CHF $f$

- *Second pre-image resistance*: given $x$, it is computationally hard to find $x' \neq x$ such that $f(x) = f(x')$
- *Collision resistance*: find a pair $x \neq x'$ such that $f(x) = f(x')$
- (*Lossy*) *compression*: since the size of hash values is fixed, for the majority of inputs $f$ does behave like a lossy compressor

# Significance of the fundamental properties

- First pre-image resistance is required in application contexts like *password authentication* and *key derivation*
- We have seen a notable example of key derivation when we discussed the Fortuna generator
- Also second pre-image resistance is crucial to protect user authentication through passwords
- In fact, even is the colliding value is not the “true” password, it is equally good to grant system access
- A collision resistant hash function $f$ is suitable for use in all the applications where it is crucial to detect possible modifications in the input (be it unintentional or malicious): digital signatures, intrusion detection systems, forensic analysis, and many others

# Attacks to find collisions

- Technically, there should be no significantly better attack to find a collision other than the generic *birthday attack*
- Let $n$ be the length of the hash values. Thus there are $2^n$ possible hash values, i.e., days
- Then, we expect to find a collision (with prob. $> \frac{1}{2}$) provided that we choose $2^{n/2}$ random inputs, i.e., persons
- To avoid to check $O(2^n)$ pairs, we can store a lookup table with $2^{n/2}$ messages indexed by the corresponding hash values (a lot of memory...)
- A different attack, known as $\rho$ *method*, avoid storing such a huge table by paying a little more in terms of time

# Relative difficulty

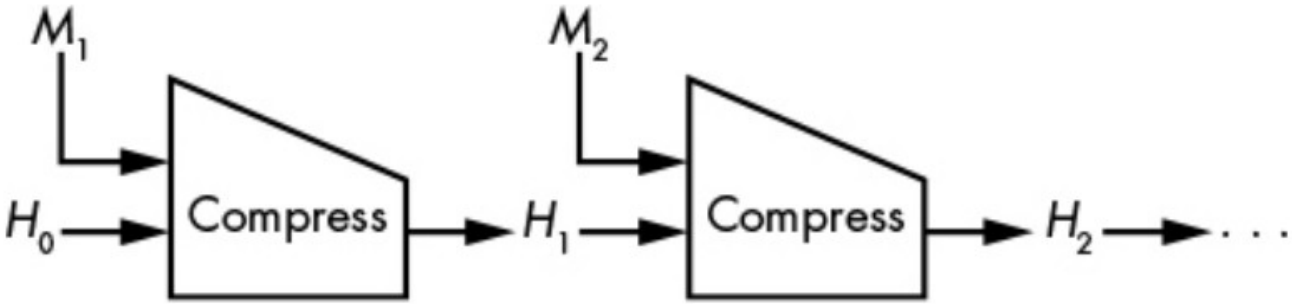
- It is not difficult to show that pre-image resistance is at least as hard as second pre-image resistance
- In fact, to find an input that collide with a given input $x$, we can first compute $H = f(x)$ and the find a pre-image $y$ of $H$
- In turn, second pre-image resistance is at least as difficult as collision resistance
- In fact, to find a collision you can first create a random input $x$ and then solve the second pre-image problem to find some $y$ that collide with $x$
- Generic attacks to find a pre-image have average complexity $2^{n/2}$, rather than $2^n$

# Algoritmi di Crittografia

# An iterative structure

- Almost all hash functions used in real applications embody a technique that reminds the design of block ciphers
- The input $M$ is split in blocks, say $M_1, M_2, \ldots, M_N$, with the last block padded and the blocks are processed in rounds
- There is an internal state that evolves with rounds, starting with a fixed initial state $H_0$
- The generic round computes $H_{i+1} = c(H_i, M_{i+1})$, $i = 0, \ldots, N-1$, where $c$ is an accurately designed *compression function*, while $H_n$ is the computed hash value
- Clearly, the size of the internal state is the output size (say, 128, 1690, ... bits)
- This design is known as the *Merkle-Damgård* construction
- Actually, there are hash functions families that do not use compression, but we shall not consider them here

# The Merkle-Damgård construction




The Merkle–Damgård construction with a generic compression function
Source: J.P. Aumasson, Serious Cryptography, No Starch Press (2018)

# Padding

- Padding must ensure that different messages get different hash values
- A padding scheme, specified by ISO/IEC 9797-1 is the following
- First compute the length $\ell$ of the unpadded message $M$ (length expressed in bits)
- Then prepare the padded message by inserting (in this order):
  1. the value $\ell$ expressed in big-endian binary in $n$ bits ($n$ is the block length)
  2. the unpadded message
  3. as many (possibly none) bits with value 0 as are required to bring the total length to a multiple of $n$ bits
- For instance, with $n = 128$, a 230 bit message $M$ would be transformed to $\underbrace{\backslash x00 \ldots \backslash x00}_{15} \backslash xe6 M \underbrace{00 \ldots 0}_{26}$

# A (perhaps unexpectedly) insecure hash function

- Take and block cipher you trust, say *AES* with 256 bit keys
- Hash functions (at least the ones we are studying here) do not require a key, so take $K = \underbrace{00\ldots0}_{256}$
- Also, take $H_0 = \underbrace{00\ldots0}_{128}$ and define:

$$H_{i+1} = \mathbf{E}(H_i \oplus M_{i+1}), \quad i = 1, \ldots, N$$

where $N$ is the number of blocks of the (already padded) message and $\mathbf{E}$ is the AES encryption function

- So, if our message (after padding) is made of just two blocks, i.e., $M = M_1||M_2$, then we have:

$$\begin{aligned} H_1 &= \mathbf{E}(H_0 \oplus M_1) = \mathbf{E}(M_1) \\ H_2 &= \mathbf{E}(H_1 \oplus M_2) \end{aligned}$$

# A (perhaps unexpectedly) insecure hash function

- Now assume we can manage to have, after padding, a different message $M' = M'_1 || M'_2$ defined as follows:

$$\begin{aligned} M'_1 &= M_2 \oplus H_1 \\ M'_2 &= H_2 \oplus M_2 \oplus H_1 \end{aligned}$$

- This is not immediate, but can be done (remember we can adapt $M$ as well, since we are looking for arbitrary collisions)

# A (perhaps unexpectedly) insecure hash function

- Then we have:

$$\begin{aligned} H'_1 &= \mathbf{E}(H'_0 \oplus M'_1) \\ &= \mathbf{E}(H_0 \oplus M_2 \oplus H_1) \\ &= \mathbf{E}(M_2 \oplus H_1) \end{aligned}$$

and

$$\begin{aligned} H'_2 &= \mathbf{E}(H'_1 \oplus M'_2) \\ &= \mathbf{E}(\mathbf{E}(M_2 \oplus H_1) \oplus H_2 \oplus M_2 \oplus H_1) \\ &= \mathbf{E}(\mathbf{E}(M_2 \oplus H_1) \oplus \mathbf{E}(H_1 \oplus M_2) \oplus M_2 \oplus \mathbf{E}(M_1)) \\ &= \mathbf{E}(M_2 \oplus H_1) \\ &= H_2 \end{aligned}$$

which is also the hash value of the first message

# Security aspects

- Merkle and Damgård proved that the security (i.e., resistance to pre-image and collision attacks) of the hash function is guaranteed if the compression function is secure
- The converse is not necessarily true as the hash can be secure even in cases where the compression function is not secure
- In spite of the latter circumstance, a discovered weakness of the compression function is a serious “alarm bell” ringing about the security of the hash function
- The main concern about hash functions based on the Merkle-Damgård construction if the so-called *length-extension bug*
- In fact, for this construction, if $M' = M||E$, then it holds that:

$$f(M') = c(f(M), E)$$

where (recall) $c$ is the compression function

# The Davis-Meyer compression "strategy"

- There is a simple solution to the problem of defining a secure compression function, namely to use a "secure" block cipher
- Well, actually, a block cipher does not "shrink" the size of the input...
- However, a block cipher takes 2 inputs, namely a block $B$ and the key $K$, and returns the transformed block
- Hence, if we regard the input as $B||K$, then we can think of a block cipher as of a compressor
- Here clearly, we use the fact that the hash function does not require a key
- The Davis-Meyer compression strategy is thus the following:

$$c(H_i, M_{i+1}) = \mathbf{E}(M_{i+1}, H_i) \oplus H_i$$

where, in $\mathbf{E}(M_{i+1}, H_i)$, the message block $M_{i+1}$ as the role of the key

# The Davis-Meyer compression "strategy" (cnt.d)

- The reason for the final $\oplus H_i$ should be clear
- Without it, the chain could be easily inverted using the decryption function and (again) the message blocks as keys
- Many real hash functions use the Davis-Meyer compression

# Algoritmi di Crittografia

1. Hash Functions
   - General concepts
   - Building hash functions
   - Real hash functions

# MD5

- Developed by Ron Rivest!
- Hash values are 128 bit long
- Splits the message in blocks of 512 bits and pads the last block
- The internal state (128 bits) is regarded as four 32-bit words
- The compression function is based on four rounds that mix the state and the message blocks
- The “mixing” is quite complex, as it is composed of a number of basic operations (additions, boolean bitwise operations, rotations)

# Insecurity of MD5

- 128 bits lead immediately to an attack based on the birthday paradox that only uses $2^{64}$ MD5 computations
- However, the compression function of MD5 admits easier to find collisions
- Although a collision in the compression function dows not imply necessarily hash vulnerability, it has been recently shown that attacks can be made which use much less than $2^{64}$ MD5 computations
- Summing up: MD5 should not be used

# SHA-1

- 160 bit internal state (hence hash values)
- Combines Merkle and Damgård construction with a Davis-Meyer compression function
- The block cipher used to compress is a specially designed one
- Used the cipher, the internal state is modified as follows:

$$H = \mathbf{E}(M, H) + H$$

- Observe that the encryption does work on the whole message (not on single blocks), and that integer addition “replaces” the exclusive or
- Actually, the 160 bit internal state is regarded as five 32 bit words

# Security issues

- Considered insecure
- Web servers using SHA-1 are marked as insecure by chrome (and chromium alike) web browsers
- Theoretical 80 bit security against birthday attacks recently reduced to 63 bits
- Real (and successful) attacks are known, though (two different pdf files with the same SHA-1 digest)
- Summing up: like MD5, SHA-1 should not be used (attacks cannot but improve...)

# SHA-2 family

- Includes 2 pairs of algorithms: SHA-256 and SHA-224 from the one hand, and SHA-512 and SHA-384 from the other
- SHA-256 and SHA-224 works with 256 bit internal states
- SHA-224 simply truncates the last state (i.e., the hash value to be returned) to 224 bits but is otherwise identical to SHA-256
- Similarly, SHA-384 truncates the last state to 384 bits but is otherwise identical to SHA-512
- The reason for all these different hash lengths has to do with the fact that the SHA-2 family has been designed to be used with triple-DES (112 bit keys) as well as with AES (128, 192, or 256 bit keys)

# Security issues

- As far as we know, the SHA-2 family members guarantee the security tied to the respective bit lengths: for instance, SHA-512 seems to guarantee 256 bit security against collision attacks
- Some concerns have been raised that have to with the fact that the structure of the SHA-2 family is really close to that of SHA-1 (which is deemed insecure)
- However, no theoretical nor practical attacks to any SHA-2 member has been reported yet
- The relative inefficiency of SHA-2 computations, is a limiting factor affecting (negatively) the efficiency of the attacks