;)

Hey there!

- Jeppe Nørregaard
- PostDoc
- Works with Leon on misinformation research
- All our work is in NLP and majority uses Deep Learning

- Today's content is on Teams: *day_2*

Today's Deal

Lecture 2

Machine Learning Introduction

1. Machine Learning - what is it and what is it not?
2. Bayes theorem
3. Naive Bayes
4. Machine Learning Theory
5. Perceptron

Machine Learning

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   - Maybe, but it is useful, so who cares?

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- We need you to get up to date with Machine Learning really quick, so we can get to Deep Learning :)

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$\ell(\mathcal{M}, \mathcal{D})$	Loss function	an evaluation of the performance of the model which we wish to *minimize*

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- Loss function depends a bit on purpose

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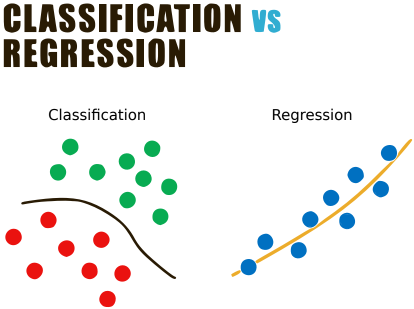

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Normality	For any x	$0 \leq P(x) \leq 1$	
Independence	x and y are independent iff.	$P(x, y) = P(x)\ P(y)$	$P(⚀, ⚀) = P(⚀) \times P(⚀) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

Proving Bayes Theorem

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\begin{aligned}
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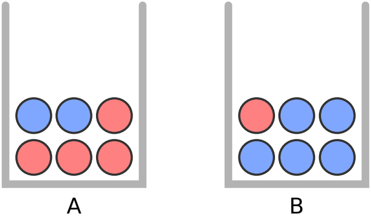
P(M I D) = P(D I M)P(M)



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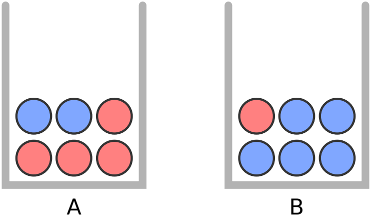
- What is the probability that I picked bag A?



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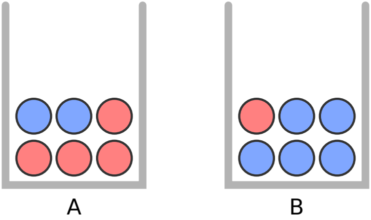

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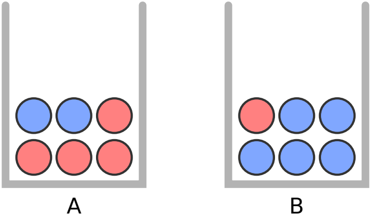

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  - $P(A) = \dfrac{\frac{2}{6}}{\frac{2}{6} + \frac{1}{12}}$

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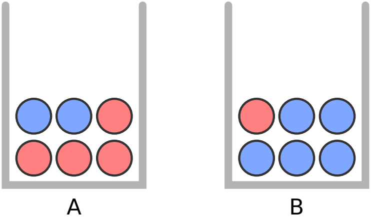

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  - $P(A) = \dfrac{P(red \mid A)\ P(A)}{P(red \mid A)\ P(A) + P(red \mid B)\ P(B)}$
  - $P(A) = \dfrac{\frac{2}{3}\ \frac{1}{2}}{\frac{2}{3}\ \frac{1}{2} + \frac{1}{6}\ \frac{1}{2}}$
  - $P(A) = \dfrac{\frac{2}{6}}{\frac{2}{6} + \frac{1}{12}}$
  - $P(A) = \dfrac{\frac{2}{6}}{\frac{5}{12}} = \dfrac{24}{30} = \dfrac{12}{15}$

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The *likelihood* of our model is "running our model". We designed it for prediction, so we can usually evaluate it quite easily.

Probability Theory and Bayes Theorem

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This is how the complete expression looks like. Notice that the denominator is independent of $\mathcal{M}$.

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$$P(\mathcal{M} \mid \mathcal{D}) \propto P(\mathcal{D} \mid \mathcal{M})\, P(\mathcal{M})$$

Maximizing this expression provides the same model! (we just don't know what the actual posteriors value was).

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$P(\mathcal{D} \mid \mathcal{M})$		The *likelihood* of our model is "running our model". We designed it for prediction, so we can usually evaluate it quite easily.
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$P(\mathcal{M} \mid \mathcal{D})$	$= \dfrac{P(\mathcal{D} \mid \mathcal{M})\ P(\mathcal{M})}{P(\mathcal{D})}$	The *posterior* is what we really would like to compute, but the denominator is annoying.
$P(\mathcal{D})$	$= \displaystyle\sum_i P(\mathcal{D} \mid \mathcal{M}_i)\ P(\mathcal{M}_i)$	The denominator is the sum (or integral) over the model space.
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We need to optimize the product of the likelihood and the prior!

Probability Theory and Bayes Theorem

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  - Common with frequentists approach

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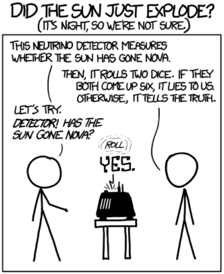

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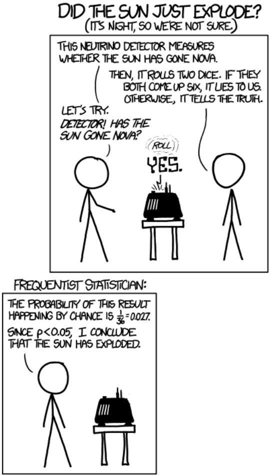

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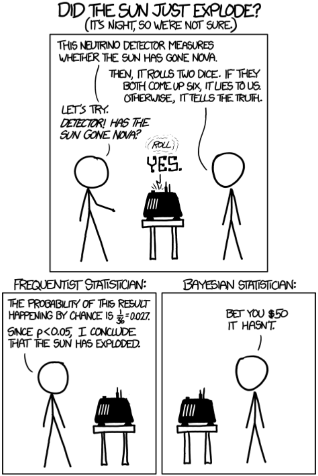

Bayes Rule as hypothesis probabilities

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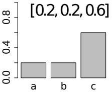

$P(M)$
Prior

Sums to 1

✓

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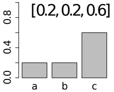

$P(M)$
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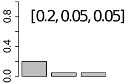
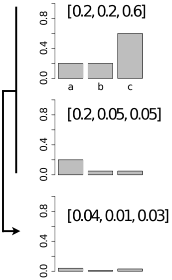


$P(\mathcal{D} \mid \mathcal{M})$
Likelihood

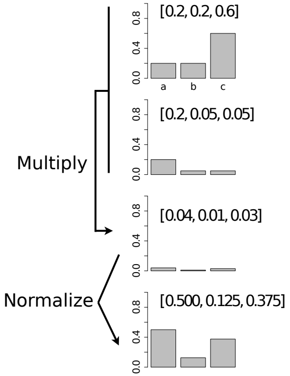
Bayes Rule as hypothesis probabilities

Multiply



		Sums to 1
$P(M)$	Prior	✓
$P(\mathcal{D} \mid \mathcal{M})$	Likelihood	✗
$P(\mathcal{D} \mid \mathcal{M})\,P(M)$	Unnorm. Posterior	✗

Bayes Rule as hypothesis probabilities



Sums to 1

$P(M)$
Prior ✓

$P(\mathcal{D} \mid \mathcal{M})$
Likelihood ✗

$P(\mathcal{D} \mid \mathcal{M})\, P(M)$
Unnorm. Posterior ✗

$P(\mathcal{M} \mid \mathcal{D})$
Posterior ✓

A more important case!

A patient takes a cancer test and the result comes back positive. The test returns a correct positive result in 98% of the cases in which the disease is actually present, and a correct negative result in 97% of the cases in which the disease is not present. Furthermore, 0.008 of the entire population have this cancer. What is the probability that the patient has cancer?

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- Prior: $P(\text{cancer}) = 0.008$
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$$P(\text{positive}) = P(\text{positive} \mid \text{cancer})\ P(\text{cancer}) + P(\text{positive} \mid \neg\text{cancer})\ P(\neg\text{cancer})$$

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$$= 0.98 \times 0.008 + (1 - 0.97) \times (1 - 0.008) = 0.0376$$

Probability Theory and Bayes Theorem

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- Posterior probability of having cancer:

$$P(\text{cancer} \mid \text{positive}) = \frac{P(\text{positive} \mid \text{cancer})\ P(\text{cancer})}{P(\text{positive})}$$

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$$
= \frac{0.98 \times 0.008}{0.0376} = 0.2085
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Naive Bayes

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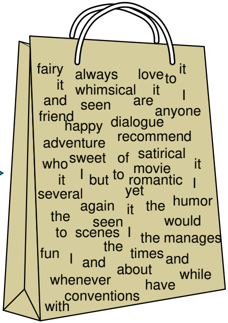
Bag-of-Words

I love this movie! It's sweet, but with satirical humor. The dialogue is great and the adventure scenes are fun... It manages to be whimsical and romantic while laughing at the conventions of the fairy tale genre. I would recommend it to just about anyone. I've seen it several times, and I'm always happy to see it again whenever I have a friend who hasn't seen it yet!



it	6
I	5
the	4
to	3
and	3
seen	2
yet	1
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Disregards order of words!

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  - Instead of making this decision, we make models learn the level they operate on
- We will learn more about this later, but for now our operating level is *words*

Naive Bayes

Documents:

- an1 *The domestic dog is a domesticated descendant of the wolf.*
- an2 *The cat is a domestic species of small carnivorous mammal.*
- pl1 *Saturn is the sixth planet from the Sun and the second-largest in the Solar System.*
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Naive Bayes

Bag-of-words

	an1	an2	pl1	an3	pl2
africa	0	0	0	1	0
and	0	0	1	1	1
are	0	0	0	1	0
asia	0	0	0	1	0
being	0	0	0	0	1
carnivorous	0	1	0	0	0
cat	0	1	0	0	0
descendant	1	0	0	0	0
desert	0	0	0	1	0
dog	1	0	0	0	0
domestic	1	1	0	0	0
domesticated	1	0	0	0	0
found	0	0	0	1	0
fourth	0	0	0	0	1
from	0	0	1	0	1
hopping	0	0	0	1	0
in	0	0	1	0	1
is	1	1	1	0	1
jerboas	0	0	0	1	0
larger	0	0	0	0	1
largest	0	0	1	0	0
mammal	0	1	0	0	0
mars	0	0	0	0	1
mercury	0	0	0	0	1
northern	0	0	0	1	0
of	1	1	0	0	0
only	0	0	0	0	1
planet	0	0	1	0	2
rodents	0	0	0	1	0
saturn	0	0	1	0	0
second	0	0	1	0	1
sixth	0	0	1	0	0
small	0	1	0	0	0
smallest	0	0	0	0	1
solar	0	0	1	0	1
species	0	1	0	0	0
sun	0	0	1	0	1
system	0	0	1	0	1
than	0	0	0	0	1
the	2	1	4	0	4
throughout	0	0	0	1	0
wolf	1	0	0	0	0

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- Make a simple model of the probability of words in a class

$$P(\text{word} \mid \text{class}) = \frac{\text{count of word in class}}{\text{total words in class}}$$

For example if "cat" occurs 3 times in class "anim" in our data, and there are a total of 20 words in that class, then we have

$$P(\text{cat} \mid \text{anim}) = \frac{3}{20}$$

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$$P(\text{the cat is small} \mid \text{anim}) \underbrace{\approx}_{\text{assumption}} P(\text{the} \mid \text{anim}) \times P(\text{cat} \mid \text{anim}) \times P(\text{is} \mid \text{anim}) \times P(\text{small} \mid \text{anim})$$

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$$P(\text{the cat is small} \mid \text{anim}) \underbrace{\approx}_{\text{assumption}} P(\text{the} \mid \text{anim}) \times P(\text{cat} \mid \text{anim}) \times P(\text{is} \mid \text{anim}) \times P(\text{small} \mid \text{anim})$$

- This assumption is why we call it naive
- *Prior*: Weight the classes according to how often we see them

Naive Bayes

- Make a simple model of the probability of words in a class

$$P(\text{word} \mid \text{class}) = \frac{\text{count of word in class}}{\text{total words in class}}$$

  For example if "cat" occurs 3 times in class "anim" in our data, and there are a total of 20 words in that class, then we have

$$P(\text{cat} \mid \text{anim}) = \frac{3}{20}$$

- *Likelihood*: Approximate the probability of a document to be the joint probability of the words in the document, **assuming independence**

$$P(\text{the cat is small} \mid \text{anim}) \underbrace{\approx}_{\text{assumption}} P(\text{the} \mid \text{anim}) \times P(\text{cat} \mid \text{anim}) \times P(\text{is} \mid \text{anim}) \times P(\text{small} \mid \text{anim})$$

- This assumption is why we call it naive
- *Prior*: Weight the classes according to how often we see them
- *Posterior*: compute from likelihood and prior according to Bayes theorem

Naive Bayes

Bag-of-words

	an1	an2	pl1	an3	pl2
africa	0	0	0	1	0
and	0	0	1	1	1
are	0	0	0	1	0
asia	0	0	0	1	0
being	0	0	0	0	1
carnivorous	0	1	0	0	0
cat	0	1	0	0	0
descendant	1	0	0	0	0
desert	0	0	0	1	0
dog	1	0	0	0	0
domestic	1	1	0	0	0
domesticated	1	0	0	0	0
found	0	0	0	1	0
fourth	0	0	0	0	1
from	0	0	1	0	1
hopping	0	0	0	1	0
in	0	0	1	0	1
is	1	1	1	0	1
jerboas	0	0	0	1	0
larger	0	0	0	0	1
largest	0	0	1	0	0
mammal	0	1	0	0	0
mars	0	0	0	0	1
mercury	0	0	0	0	1
northern	0	0	0	1	0
of	1	1	0	0	0
only	0	0	0	0	1
planet	0	0	1	0	2
rodents	0	0	0	1	0
saturn	0	0	1	0	0
second	0	0	1	0	1
sixth	0	0	1	0	0
small	0	1	0	0	0
smallest	0	0	0	0	1
solar	0	0	1	0	1
species	0	1	0	0	0
sun	0	0	1	0	1
system	0	0	1	0	1
than	0	0	0	0	1
the	2	1	4	0	4
throughout	0	0	0	1	0
wolf	1	0	0	0	0

Documents:

- an1 *The domestic dog is a domesticated descendant of the wolf.*
- an2 *The cat is a domestic species of small carnivorous mammal.*
- pl1 *Saturn is the sixth planet from the Sun and the second-largest in the Solar System.*
- an3 *Jerboas are hopping desert rodents found throughout Northern Africa and Asia.*
- pl2 *Mars is the fourth planet from the Sun and the second-smallest planet in the Solar System, being larger than only Mercury.*

Naive Bayes

Bag-of-words

	an1	an2	pl1	an3	pl2
africa	0	0	0	1	0
and	0	0	1	1	1
are	0	0	0	1	0
asia	0	0	0	1	0
being	0	0	0	0	1
carnivorous	0	1	0	0	0
cat	0	1	0	0	0
descendant	1	0	0	0	0
desert	0	0	0	1	0
dog	1	0	0	0	0
domestic	1	1	0	0	0
domesticated	1	0	0	0	0
found	0	0	0	1	0
fourth	0	0	0	0	1
from	0	0	1	0	1
hopping	0	0	0	1	0
in	0	0	1	0	1
is	1	1	1	0	1
jerboas	0	0	0	1	0
larger	0	0	0	0	1
largest	0	0	1	0	0
mammal	0	1	0	0	0
mars	0	0	0	0	1
mercury	0	0	0	0	1
northern	0	0	0	1	0
of	1	1	0	0	0
only	0	0	0	0	1
planet	0	0	1	0	2
rodents	0	0	0	1	0
saturn	0	0	1	0	0
second	0	0	1	0	1
sixth	0	0	1	0	0
small	0	1	0	0	0
smallest	0	0	0	0	1
solar	0	0	1	0	1
species	0	1	0	0	0
sun	0	0	1	0	1
system	0	0	1	0	1
than	0	0	0	0	1
the	2	1	4	0	4
throughout	0	0	0	1	0
wolf	1	0	0	0	0

Naive Bayes

Bag-of-words

	an1	an2	pl1	an3	pl2
africa	0	0	0	1	0
and	0	0	1	1	1
are	0	0	0	1	0
asia	0	0	0	1	0
being	0	0	0	0	1
carnivorous	0	1	0	0	0
cat	0	1	0	0	0
descendant	1	0	0	0	0
desert	0	0	0	1	0
dog	1	0	0	0	0
domestic	1	1	0	0	0
domesticated	1	0	0	0	0
found	0	0	0	1	0
fourth	0	0	0	0	1
from	0	0	1	0	1
hopping	0	0	0	1	0
in	0	0	1	0	1
is	1	1	1	0	1
jerboas	0	0	0	1	0
larger	0	0	0	0	1
largest	0	0	1	0	0
mammal	0	1	0	0	0
mars	0	0	0	0	1
mercury	0	0	0	0	1
northern	0	0	0	1	0
of	1	1	0	0	0
only	0	0	0	0	1
planet	0	0	1	0	2
rodents	0	0	0	1	0
saturn	0	0	1	0	0
second	0	0	1	0	1
sixth	0	0	1	0	0
small	0	1	0	0	0
smallest	0	0	0	0	1
solar	0	0	1	0	1
species	0	1	0	0	0
sun	0	0	1	0	1
system	0	0	1	0	1
than	0	0	0	0	1
the	2	1	4	0	4
throughout	0	0	0	1	0
wolf	1	0	0	0	0

Word counts in classes

	animal	planet
africa	1	0
and	1	2
are	1	0
asia	1	0
being	0	1
carnivorous	1	0
cat	1	0
descendant	1	0
desert	1	0
dog	1	0
domestic	2	0
domesticated	1	0
found	1	0
fourth	0	1
from	0	2
hopping	1	0
in	0	2
is	2	2
jerboas	1	0
larger	0	1
largest	0	1
mammal	1	0
mars	0	1
mercury	0	1
northern	1	0
of	2	0
only	0	1
planet	0	3
rodents	1	0
saturn	0	1
second	0	2
sixth	0	1
small	1	0
smallest	0	1
solar	0	2
species	1	0
sun	0	2
system	0	2
than	0	1
the	3	8
throughout	1	0
wolf	1	0

Naive Bayes

Bag-of-words

	an1	an2	pl1	an3	pl2
africa	0	0	0	1	0
and	0	0	1	1	1
are	0	0	0	1	0
asia	0	0	0	1	0
being	0	0	0	0	1
carnivorous	0	1	0	0	0
cat	0	1	0	0	0
descendant	1	0	0	0	0
desert	0	0	0	1	0
dog	1	0	0	0	0
domestic	1	1	0	0	0
domesticated	1	0	0	0	0
found	0	0	0	1	0
fourth	0	0	0	0	1
from	0	0	1	0	1
hopping	0	0	0	1	0
in	0	0	1	0	1
is	1	1	1	0	1
jerboas	0	0	0	1	0
larger	0	0	0	0	1
largest	0	0	1	0	0
mammal	0	1	0	0	0
mars	0	0	0	0	1
mercury	0	0	0	0	1
northern	0	0	0	1	0
of	1	1	0	0	0
only	0	0	0	0	1
planet	0	0	1	0	2
rodents	0	0	0	1	0
saturn	0	0	1	0	0
second	0	0	1	0	1
sixth	0	0	1	0	0
small	0	1	0	0	0
smallest	0	0	0	0	1
solar	0	0	1	0	1
species	0	1	0	0	0
sun	0	0	1	0	1
system	0	0	1	0	1
than	0	0	0	0	1
the	2	1	4	0	4
throughout	0	0	0	1	0
wolf	1	0	0	0	0

Word counts in classes

	animal	planet
africa	1	0
and	1	2
are	1	0
asia	1	0
being	0	1
carnivorous	1	0
cat	1	0
descendant	1	0
desert	1	0
dog	1	0
domestic	2	0
domesticated	1	0
found	1	0
fourth	0	1
from	0	2
hopping	1	0
in	0	2
is	2	2
jerboas	1	0
larger	0	1
largest	0	1
mammal	1	0
mars	0	1
mercury	0	1
northern	1	0
of	2	0
only	0	1
planet	0	3
rodents	1	0
saturn	0	1
second	0	2
sixth	0	1
small	1	0
smallest	0	1
solar	0	2
species	1	0
sun	0	2
system	0	2
than	0	1
the	3	8
throughout	1	0
wolf	1	0

Word probabilities in classes

	animal	planet
africa	0.03	0.00
and	0.03	0.05
are	0.03	0.00
asia	0.03	0.00
being	0.00	0.03
carnivorous	0.03	0.00
cat	0.03	0.00
descendant	0.03	0.00
desert	0.03	0.00
dog	0.03	0.00
domestic	0.07	0.00
domesticated	0.03	0.00
found	0.03	0.00
fourth	0.00	0.03
from	0.00	0.05
hopping	0.03	0.00
in	0.00	0.05
is	0.07	0.05
jerboas	0.03	0.00
larger	0.00	0.03
largest	0.00	0.03
mammal	0.03	0.00
mars	0.00	0.03
mercury	0.00	0.03
northern	0.03	0.00
of	0.07	0.00
only	0.00	0.03
planet	0.00	0.08
rodents	0.03	0.00
saturn	0.00	0.03
second	0.00	0.05
sixth	0.00	0.03
small	0.03	0.00
smallest	0.00	0.03
solar	0.00	0.05
species	0.03	0.00
sun	0.00	0.05
system	0.00	0.05
than	0.00	0.03
the	0.10	0.21
throughout	0.03	0.00
wolf	0.03	0.00

Naive Bayes

- **New datapoint:** *the cat is larger*

Word probabilities in classes

	animal	planet
africa	0.03	0.00
and	0.03	0.05
are	0.03	0.00
asia	0.03	0.00
being	0.00	0.03
carnivorous	0.03	0.00
cat	0.03	0.00
descendant	0.03	0.00
desert	0.03	0.00
dog	0.03	0.00
domestic	0.07	0.00
domesticated	0.03	0.00
found	0.03	0.00
fourth	0.00	0.03
from	0.00	0.05
hopping	0.03	0.00
in	0.00	0.05
is	0.07	0.05
jerboas	0.03	0.00
larger	0.00	0.03
largest	0.00	0.03
mammal	0.03	0.00
mars	0.00	0.03
mercury	0.00	0.03
northern	0.03	0.00
of	0.07	0.00
only	0.00	0.03
planet	0.00	0.08
rodents	0.03	0.00
saturn	0.00	0.03
second	0.00	0.05
sixth	0.00	0.03
small	0.03	0.00
smallest	0.00	0.03
solar	0.00	0.05
species	0.03	0.00
sun	0.00	0.05
system	0.00	0.05
than	0.00	0.03
the	0.10	0.21
throughout	0.03	0.00
wolf	0.03	0.00

Naive Bayes

- **New datapoint:** *the cat is larger*
- Get probabilities!

Word probabilities in classes

	animal	planet
africa	0.03	0.00
and	0.03	0.05
are	0.03	0.00
asia	0.03	0.00
being	0.00	0.03
carnivorous	0.03	0.00
cat	0.03	0.00
descendant	0.03	0.00
desert	0.03	0.00
dog	0.03	0.00
domestic	0.07	0.00
domesticated	0.03	0.00
found	0.03	0.00
fourth	0.00	0.03
from	0.00	0.05
hopping	0.03	0.00
in	0.00	0.05
is	0.07	0.05
jerboas	0.03	0.00
larger	0.00	0.03
largest	0.00	0.03
mammal	0.03	0.00
mars	0.00	0.03
mercury	0.00	0.03
northern	0.03	0.00
of	0.07	0.00
only	0.00	0.03
planet	0.00	0.08
rodents	0.03	0.00
saturn	0.00	0.03
second	0.00	0.05
sixth	0.00	0.03
small	0.03	0.00
smallest	0.00	0.03
solar	0.00	0.05
species	0.03	0.00
sun	0.00	0.05
system	0.00	0.05
than	0.00	0.03
the	0.10	0.21
throughout	0.03	0.00
wolf	0.03	0.00

Naive Bayes

- **New datapoint:** *the cat is larger*
- Get probabilities!

animal

word	*probability*
the	0.10
cat	0.03
is	0.07
larger	0.00

Word probabilities in classes

	animal	planet
africa	0.03	0.00
and	0.03	0.05
are	0.03	0.00
asia	0.03	0.00
being	0.00	0.03
carnivorous	0.03	0.00
cat	0.03	0.00
descendant	0.03	0.00
desert	0.03	0.00
dog	0.03	0.00
domestic	0.07	0.00
domesticated	0.03	0.00
found	0.03	0.00
fourth	0.00	0.03
from	0.00	0.05
hopping	0.03	0.00
in	0.00	0.05
is	0.07	0.05
jerboas	0.03	0.00
larger	0.00	0.03
largest	0.00	0.03
mammal	0.03	0.00
mars	0.00	0.03
mercury	0.00	0.03
northern	0.03	0.00
of	0.07	0.00
only	0.00	0.03
planet	0.00	0.08
rodents	0.03	0.00
saturn	0.00	0.03
second	0.00	0.05
sixth	0.00	0.03
small	0.03	0.00
smallest	0.00	0.03
solar	0.00	0.05
species	0.03	0.00
sun	0.00	0.05
system	0.00	0.05
than	0.00	0.03
the	0.10	0.21
throughout	0.03	0.00
wolf	0.03	0.00

Naive Bayes

- **New datapoint:** *the cat is larger*
- Get probabilities!

animal

word	*probability*
the	0.10
cat	0.03
is	0.07
larger	0.00

planet

word	*probability*
the	0.21
cat	0.00
is	0.05
larger	0.03

Word probabilities in classes

	animal	planet
africa	0.03	0.00
and	0.03	0.05
are	0.03	0.00
asia	0.03	0.00
being	0.00	0.03
carnivorous	0.03	0.00
cat	0.03	0.00
descendant	0.03	0.00
desert	0.03	0.00
dog	0.03	0.00
domestic	0.07	0.00
domesticated	0.03	0.00
found	0.03	0.00
fourth	0.00	0.03
from	0.00	0.05
hopping	0.03	0.00
in	0.00	0.05
is	0.07	0.05
jerboas	0.03	0.00
larger	0.00	0.03
largest	0.00	0.03
mammal	0.03	0.00
mars	0.00	0.03
mercury	0.00	0.03
northern	0.03	0.00
of	0.07	0.00
only	0.00	0.03
planet	0.00	0.08
rodents	0.03	0.00
saturn	0.00	0.03
second	0.00	0.05
sixth	0.00	0.03
small	0.03	0.00
smallest	0.00	0.03
solar	0.00	0.05
species	0.03	0.00
sun	0.00	0.05
system	0.00	0.05
than	0.00	0.03
the	0.10	0.21
throughout	0.03	0.00
wolf	0.03	0.00

Naive Bayes

- **New datapoint:** *the cat is larger*
- Get probabilities!

animal

word	*probability*
the	0.10
cat	0.03
is	0.07
larger	0.00

planet

word	*probability*
the	0.21
cat	0.00
is	0.05
larger	0.03

- Likelihood:

$$P(\mathrm{x} \mid \mathrm{animal}) = 0.10 \times 0.03 \times 0.07 \times 0.00 = 0.00$$

$$P(\mathrm{x} \mid \mathrm{planet}) = 0.21 \times 0.00 \times 0.05 \times 0.03 = 0.00$$

Word probabilities in classes

	animal	planet
africa	0.03	0.00
and	0.03	0.05
are	0.03	0.00
asia	0.03	0.00
being	0.00	0.03
carnivorous	0.03	0.00
cat	0.03	0.00
descendant	0.03	0.00
desert	0.03	0.00
dog	0.03	0.00
domestic	0.07	0.00
domesticated	0.03	0.00
found	0.03	0.00
fourth	0.00	0.03
from	0.00	0.05
hopping	0.03	0.00
in	0.00	0.05
is	0.07	0.05
jerboas	0.03	0.00
larger	0.00	0.03
largest	0.00	0.03
mammal	0.03	0.00
mars	0.00	0.03
mercury	0.00	0.03
northern	0.03	0.00
of	0.07	0.00
only	0.00	0.03
planet	0.00	0.08
rodents	0.03	0.00
saturn	0.00	0.03
second	0.00	0.05
sixth	0.00	0.03
small	0.03	0.00
smallest	0.00	0.03
solar	0.00	0.05
species	0.03	0.00
sun	0.00	0.05
system	0.00	0.05
than	0.00	0.03
the	0.10	0.21
throughout	0.03	0.00
wolf	0.03	0.00

Naive Bayes

- **New datapoint:** *the cat is larger*
- Get probabilities!

animal

word	*probability*
the	0.10
cat	0.03
is	0.07
larger	0.00

planet

word	*probability*
the	0.21
cat	0.00
is	0.05
larger	0.03

- Likelihood:

$$P(\mathrm{x} \mid \text{animal}) = 0.10 \times 0.03 \times 0.07 \times 0.00 = 0.00$$
$$P(\mathrm{x} \mid \text{planet}) = 0.21 \times 0.00 \times 0.05 \times 0.03 = 0.00$$

- We have a problem: any unseen word in class will "crash" the probabilities!

Word probabilities in classes

	animal	planet
africa	0.03	0.00
and	0.03	0.05
are	0.03	0.00
asia	0.03	0.00
being	0.00	0.03
carnivorous	0.03	0.00
cat	0.03	0.00
descendant	0.03	0.00
desert	0.03	0.00
dog	0.03	0.00
domestic	0.07	0.00
domesticated	0.03	0.00
found	0.03	0.00
fourth	0.00	0.03
from	0.00	0.05
hopping	0.03	0.00
in	0.00	0.05
is	0.07	0.05
jerboas	0.03	0.00
larger	0.00	0.03
largest	0.00	0.03
mammal	0.03	0.00
mars	0.00	0.03
mercury	0.00	0.03
northern	0.03	0.00
of	0.07	0.00
only	0.00	0.03
planet	0.00	0.08
rodents	0.03	0.00
saturn	0.00	0.03
second	0.00	0.05
sixth	0.00	0.03
small	0.03	0.00
smallest	0.00	0.03
solar	0.00	0.05
species	0.03	0.00
sun	0.00	0.05
system	0.00	0.05
than	0.00	0.03
the	0.10	0.21
throughout	0.03	0.00
wolf	0.03	0.00

Naive Bayes

- **New datapoint:** *the cat is larger*
- Get probabilities!

animal

word	*probability*
the	0.10
cat	0.03
is	0.07
larger	0.00

planet

word	*probability*
the	0.21
cat	0.00
is	0.05
larger	0.03

- Likelihood:

$$P(x \mid \text{animal}) = 0.10 \times 0.03 \times 0.07 \times 0.00 = 0.00$$
$$P(x \mid \text{planet}) = 0.21 \times 0.00 \times 0.05 \times 0.03 = 0.00$$

- We have a problem: any unseen word in class will "crash" the probabilities!
- We need some "base probability" of any word.

Word probabilities in classes

	animal	planet
africa	0.03	0.00
and	0.03	0.05
are	0.03	0.00
asia	0.03	0.00
being	0.00	0.03
carnivorous	0.03	0.00
cat	0.03	0.00
descendant	0.03	0.00
desert	0.03	0.00
dog	0.03	0.00
domestic	0.07	0.00
domesticated	0.03	0.00
found	0.03	0.00
fourth	0.00	0.03
from	0.00	0.05
hopping	0.03	0.00
in	0.00	0.05
is	0.07	0.05
jerboas	0.03	0.00
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- Multiply by prior:

$$P(\mathrm{animal} \mid \mathrm{x}) \propto P(\mathrm{x} \mid \mathrm{animal}) \times P(\mathrm{animal})$$

$$= 0.00000144 \times \frac{3}{5} = 0.00000086$$

$$P(\mathrm{planet} \mid \mathrm{x}) \propto P(\mathrm{x} \mid \mathrm{planet}) \times P(\mathrm{planet})$$

$$= 0.00000255 \times \frac{2}{5} = 0.00000102$$

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- The "added probability" is called α and can be interpreted as *having seen any word some small number of times in all classes*
- For example if $\alpha = 1$ then the interpretation is that any class has seen any word once + the number of occurrences in our dataset

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  - We divide by something that is suuuuper small: everything explodes

Log-probabilities

- Solution: we work in the log-domain

$$\begin{aligned}
\log\left[\left(\frac{1}{10000}\right)^{1000}\right] &= 1000\log\left(\frac{1}{10000}\right)\\
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- There is an information theoretical reason for log-probabilities
- You will often with with terms like: log-likelihood, log-posterior

$$\log P(\mathcal{M} \mid \mathcal{D}) \propto \log P(\mathcal{D} \mid \mathcal{M}) + \log P(\mathcal{M})$$

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- Consider again the (bad) model with $\alpha = 0$

Word probabilities in classes

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- The model works fine on the training data - so what's the problem?

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- How do we compute the generalization error? - we can't

- This is one of the most fundamental problems of machine learning
- We wish to have a model $\mathcal{M}$ where

$$\int \ell[\mathcal{M}(\mathrm{x}), \mathrm{t}] \cdot p(\mathrm{x}, \mathrm{t}) \, \mathrm{dx} \, \mathrm{dt} \tag{2}$$

  is as small as possible
- We call this the *generalization error*
- A model with low generalization error *generalizes well*
- How do we compute the generalization error? - we can't
- We approximate by

$$\int \ell[\mathcal{M}(\mathrm{x}), \mathrm{t}] \cdot p(\mathrm{x}, \mathrm{t}) \, \mathrm{dx} \, \mathrm{dt} \approx \sum_{(\mathrm{x}_i, \mathrm{t}_i) \in \mathcal{D}_{\mathrm{test}}} \ell[\mathcal{M}(\mathrm{x}_i), \mathrm{t}_i] \tag{3}$$

- We split our data into:

Training set

Used for optimizing our algorithms.
Abuse this as much as you want :)

Test set

Used for testing our algorithms.
Can be used for ABSOLUTELY NOTHING else.
Must represent new data.

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- Training and test set must always come from the same distribution

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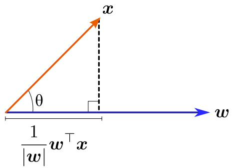
  - How about 10000 models?

Perceptron

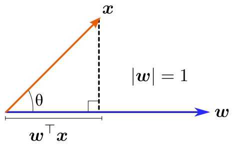
Perceptron

- Component 1: dot-product



Perceptron

- Component 1: dot-product
  - Dot-product measures how much a vector "aligns" with a weight vector



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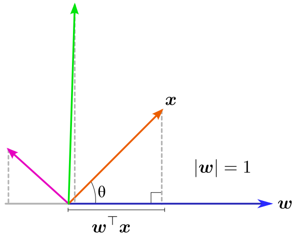

Perceptron

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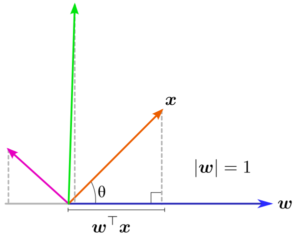
- Component 2: step function



Perceptron

- Component 1: dot-product
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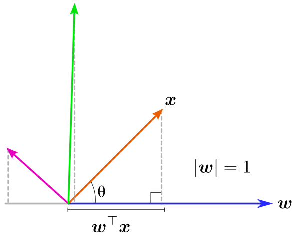
- Component 2: step function
  - Returns the sign of a value



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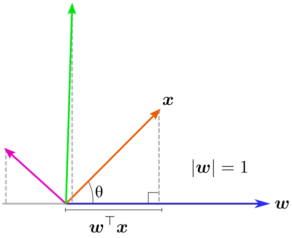
  - Returns the sign of a value
- We model the predicted class to be

$$y(\mathrm{x}) = \mathrm{sign}(\mathrm{w}^T\mathrm{x})$$

$$\mathrm{sign}(a) = \begin{cases} 1 & a > 0 \\ -1 & a < 0 \end{cases}$$



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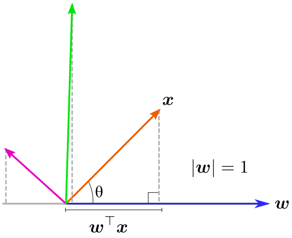
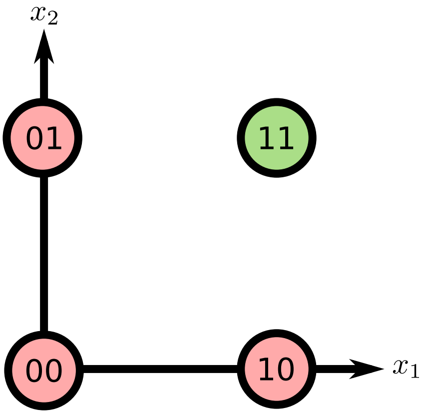
- What about 0? - do what you want



Example - **and**-operator

x_1	x_2	y
0	0	0
0	1	0
1	0	0
1	1	1

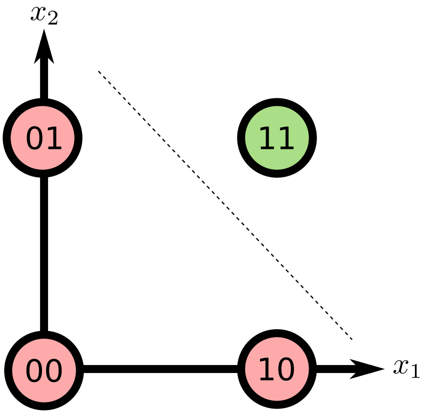


Perceptron

Example - **and**-operator

x_1	x_2	y
0	0	0
0	1	0
1	0	0
1	1	1

$$y = \sigma(w_0 + w_1 x_1 + w_2 x_2)$$



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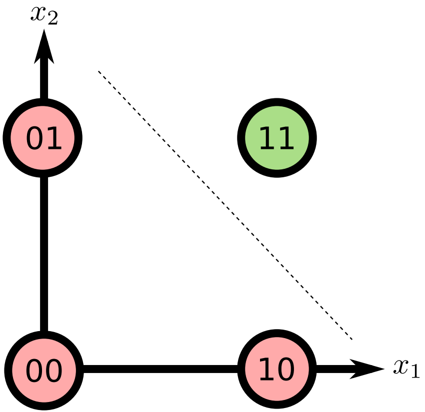
x_1	x_2	y
0	0	0
0	1	0
1	0	0
1	1	1

$$y = \sigma(w_0 + w_1 x_1 + w_2 x_2)$$

$$w_0 = -15$$

$$w_2 = 10$$

$$w_1 = 10$$



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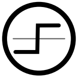
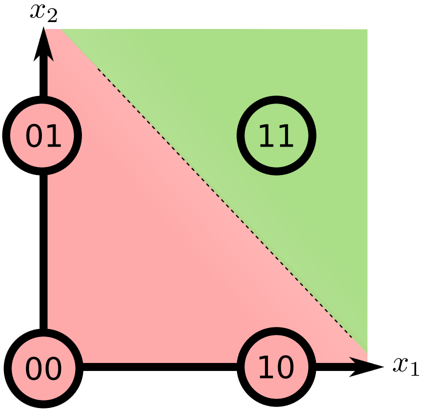




$$y = f(w_0 + w_1 x_1 + w_2 x_2)$$

$$w_0 = -15$$

$$w_2 = 10$$

$$w_1 = 10$$

Perceptron

Example - **or**-operator

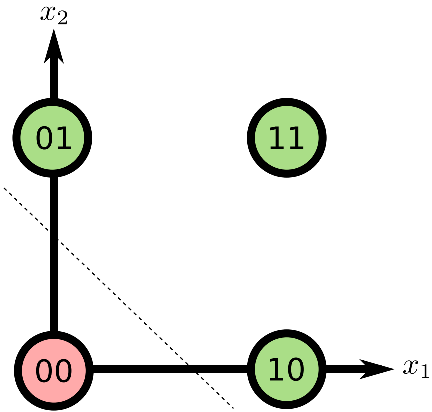
x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	1

$$y = \sigma(w_0 + w_1 x_1 + w_2 x_2)$$

$$w_0 = -5$$

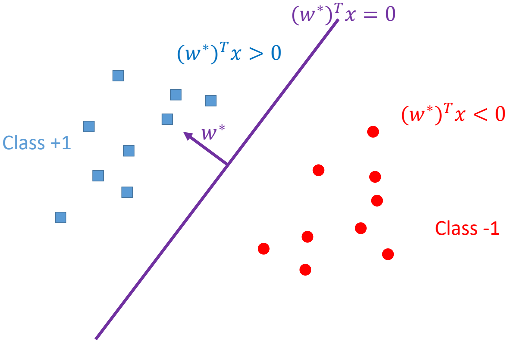
$$w_2 = 10$$

$$w_1 = 10$$



Perceptron

Task

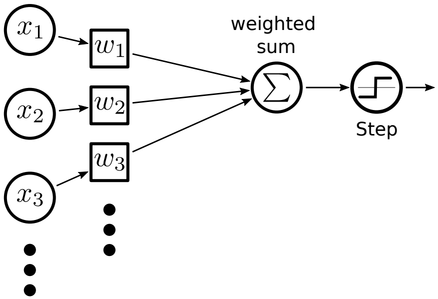


Perceptron

Graphical depiction of perceptron:

inputs

weights



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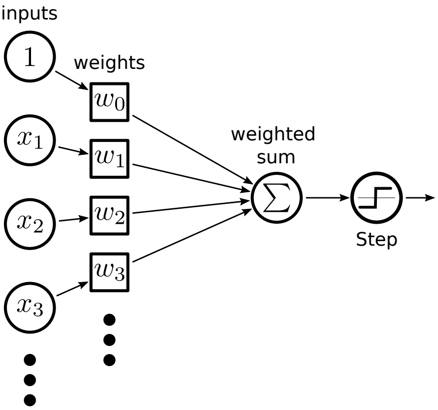

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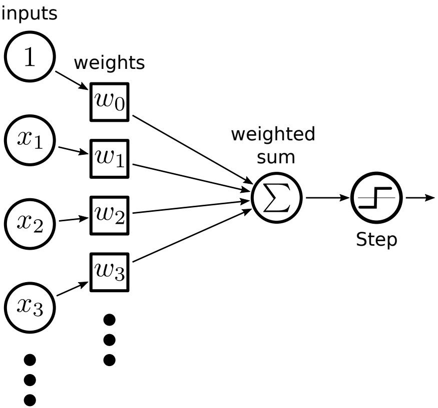

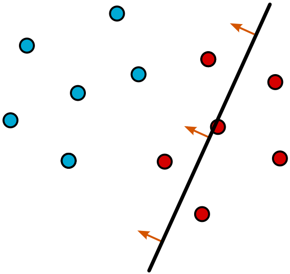

Perceptron

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We wish to optimize this perceptron somehow
(we don't want to set the weights manually)

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Perceptron learning algorithm:

1. Randomly initialize w

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2. For each *epoch*

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   - For each sample (x_i, t_i)

Perceptron

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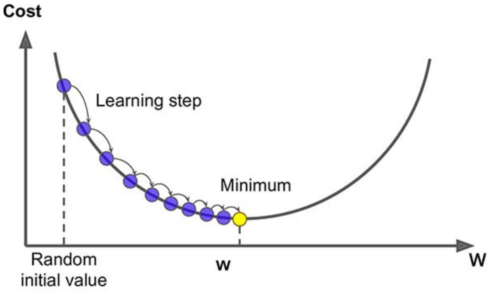
- Epoch: one training-iteration through entire dataset
- η: learning rate

Perceptron

Slowly get better and better solutions



Exercises

Exercise

- Script ex_2_1.py
  - Complete the implementation of Naive Bayes
  - Use log-probabilities to avoid underflow
  - Predict the probabilities of the test-set
  - How does α affect the prediction
- Script ex_2_2.py
  - Use sklearn's MultinomialNB and CountVectorizer to fit a Naive Bayes model on the 20-newsgroups dataset
  - Plot a visualization of the confusion matrix of the model's performance on the test set
  - Run predictions on the extra documents
  - Can you determine the best value for α?
- Script ex_2_3.py
  - Implement update-rule for perceptron to fit 2D problem
    - Remember to have a constant 1 as "first feature"
  - Can you fit to problems with more dimensions as well?
  - What is the decision boundary for the perceptron in higher dimensions?