Stability loss of the trivial solution of boundary-value problem with linear deviate in boundary condition

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Boundary-value problem

$$\dot{u} = \beta u'' - \gamma u, \tag{1}$$
$$u'\mid_{x=0} = 0, \quad u'\mid_{x=1} = \alpha\, u\mid_{x=x_0}, \tag{2}$$

$$\alpha, \gamma \in \mathbb{R}, \quad \beta > 0, \quad x_0 \in [0, 1].$$

Eigenvalue problem

$$u(x,t) = w(x)\exp\left(\lambda - \frac{\gamma}{\beta}\right)t$$

$$w'' - \lambda w = 0, \tag{3}$$
$$w'(0) = 0, \quad w'(1) = \alpha\, w(x_0). \tag{4}$$

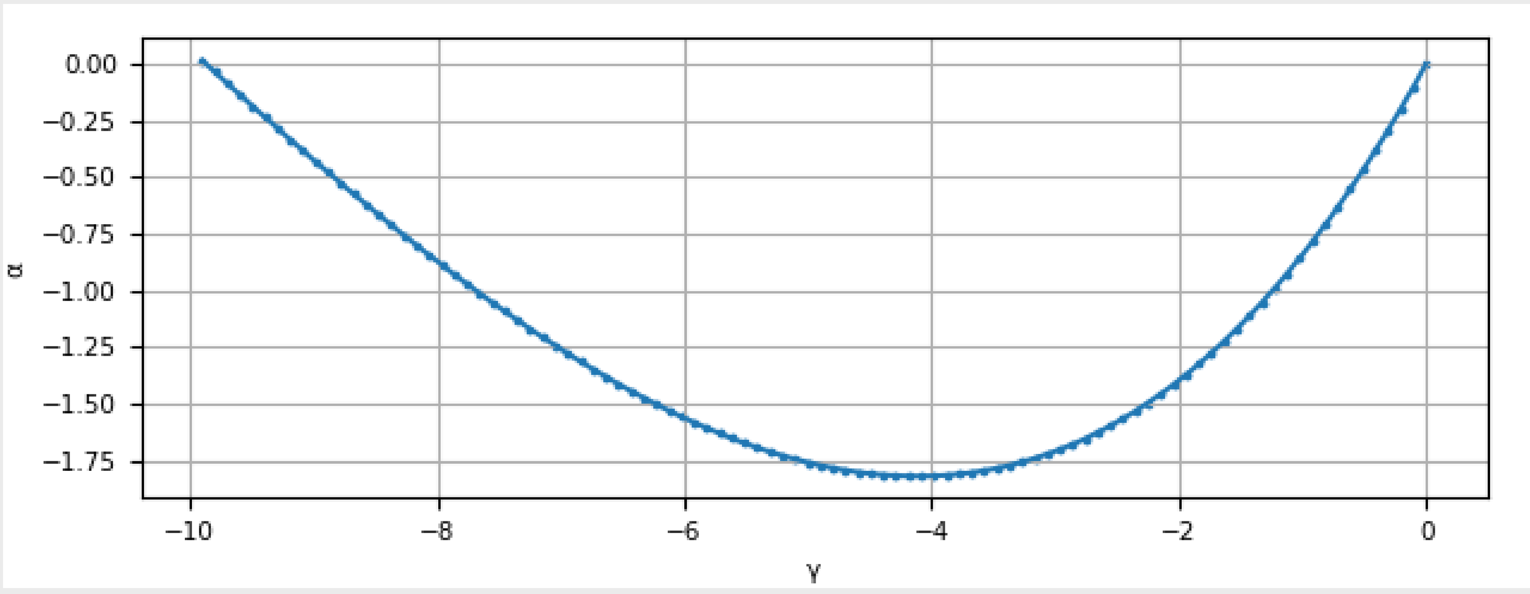
$$w(x) = c\,\mathrm{ch}(\mu x),$$
$$x = 1: \quad \mu\,\mathrm{sh}\mu = \alpha\,\mathrm{ch}(\mu x_0),$$

$$\mu = \sqrt{\lambda}, \quad \lambda \in \mathbb{C}.$$

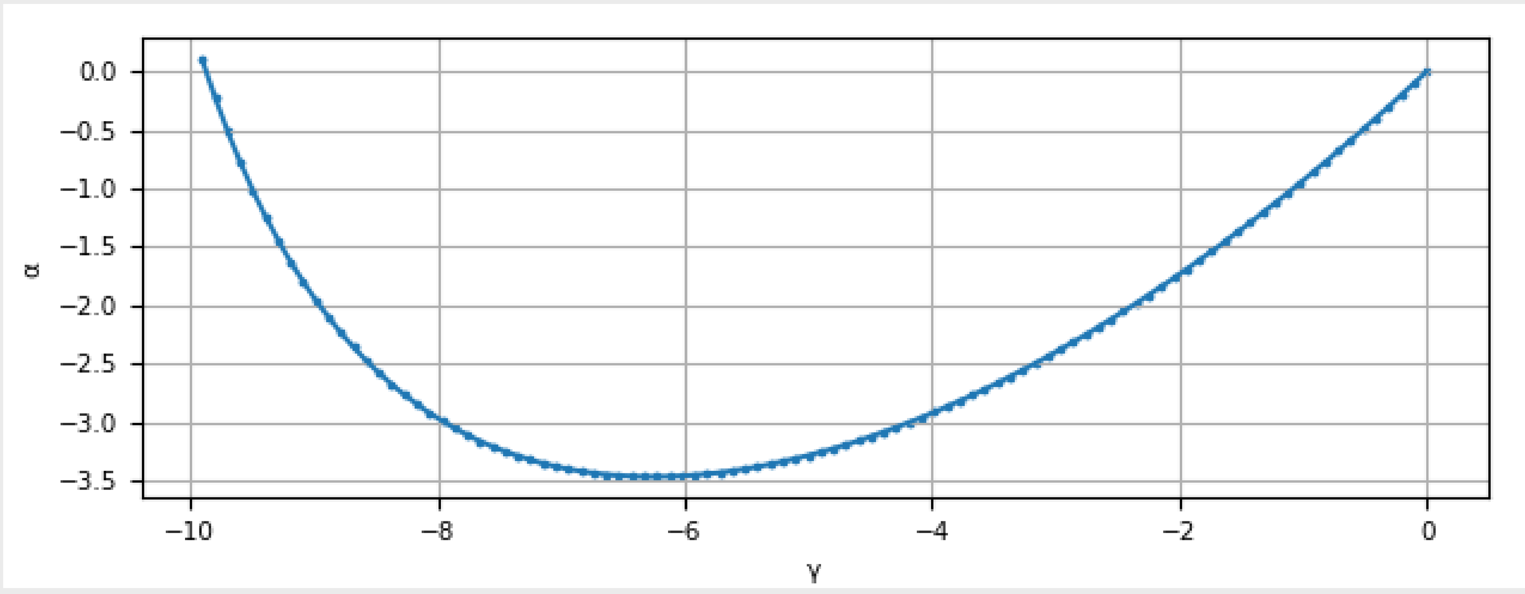
Main result

Theorem: Suppose $x_0 \in [0,1]$, $\beta > 0$, $\gamma > 0$. Then there exists $\alpha = \alpha_{cr}$, for that $\mathrm{Re}(\lambda_*) = \dfrac{\gamma}{\beta}$ and for the rest eigenvalues of problem (3), (4) $\mathrm{Re}(\lambda) < \dfrac{\gamma}{\beta}$.

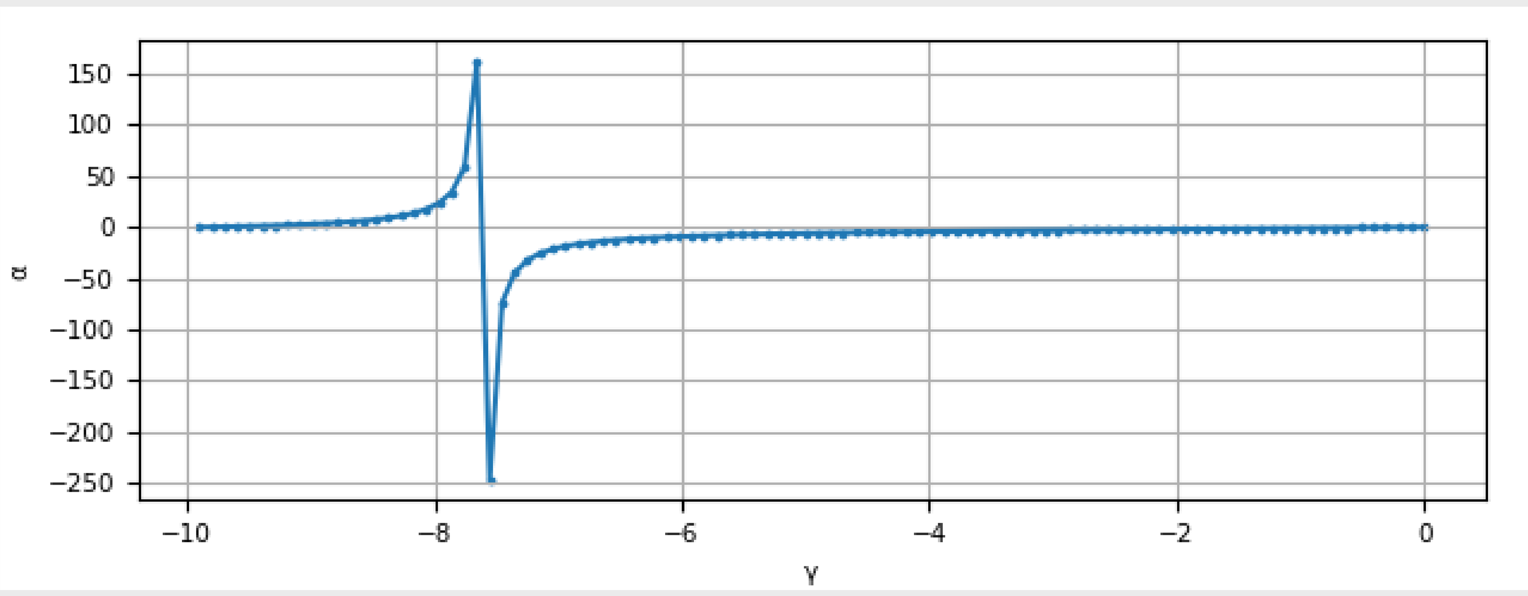
Numerical result: potential α_{cr}



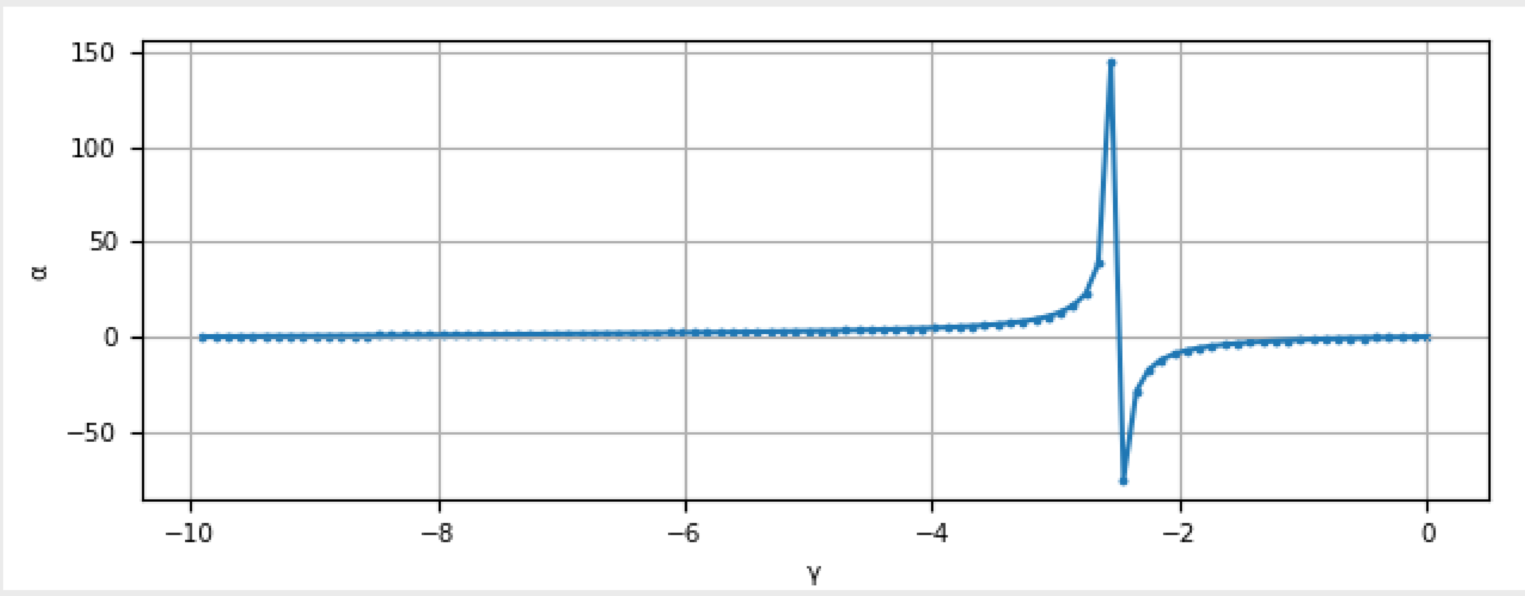
a) $x_0 = 0$



b) $x_0 = 0{,}45$



c) $x_0 = 0{,}57$



d) $x_0 = 0{,}99$

Numerical result: eigenfunctions