Bifurcations of zero balance state in one boundary-value problem with linear deviation in boundary condition

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Let us consider dynamic properties of boundary-value problem

$$\dot{u} = u'' + \gamma u - u^3, \tag{1}$$

with linear deviation in one boundary condition

$$u'(0,t) = 0, \qquad u'(1,t) = \alpha\, u(x_0,t), \tag{2}$$

where $u(x,t)$ is a smooth function, $t \geq 0,\ x \in [0,1]$, parameters α, γ are real numbers and $x_0 \in [0,1)$.

In boundary-value problem (1), (2) there are implemented two cases of stability loss of zero balance state — divergent, when the zero value appears in the spectrum of stability, and oscillating, when the spectrum of stability has a pair of complex eigenvalues with zero real parts. Our task of research was to find critical values of parameters α, γ and detect regimes which derive from zero balance state of boundary-value problem (1), (2).

As a result of numerical research there were found areas of values γ and α, where various bifurcations of zero solution took place. When parameter α was close to the critical value, by means of normal form there were determined conditions of appearance of nonuniform balance states and cycles.

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