Bifurcations of zero balance state in one boundary-value problem with deviation in edge condition

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Boundary-value problem

$$\dot{u} = u'' + \gamma u - u^3, \tag{1}$$

$$\begin{aligned} u'(0,t) &= 0, \\ u'(1,t) &= \alpha\, u(x_0,t), \end{aligned} \tag{2}$$

$$\alpha, \gamma \in \mathbb{R}, \quad x_0 \in [0,1].$$

Form for u_0

$$\sqrt{\varepsilon}: \ \dot{u}_0 = u_0'' + \gamma u_0,$$

$$\begin{aligned} u_0'(0,t) &= 0, \\ u_0'(1,t) &= \alpha_{cr}\, u_0(x_0,t). \end{aligned}$$

Divergent loss of stability ($\lambda = 0$)

$$v'' + \gamma v = 0, \tag{3}$$

$$\begin{aligned} v'(0) &= 0, \\ v'(1) &= \alpha_{cr}\, v(x_0), \end{aligned} \tag{4}$$

Oscillating loss of stability ($\lambda = i\omega$)

$$v(x) = c\,\mathrm{ch}(\mu x),$$

$$v'(1) = \alpha_{cr} v(x_0) \tag{5}$$

$$\mu = \sqrt{-\gamma + i\omega}, \quad \omega \in \mathbb{R}.$$

System for numerical experiments

$$\dot{u}_j = n^2(u_{j+1} - 2u_j + u_{j-1}) + \gamma u_j - u_j^3, \quad j = \overline{1,n}, \tag{6}$$

$$\begin{aligned} u_0 &= u_1, \\ u_{n+1} &= u_n + \frac{\alpha}{n} u_k, \end{aligned} \tag{7}$$

$$k = k(x_0) \in \mathbb{N}, \quad k \in [1,n]$$

Normal form

$$u = \sqrt{\varepsilon} u_0 + \varepsilon u_1 + \varepsilon^{\frac{3}{2}} u_2 + O(\varepsilon^2),$$

$$\alpha = \alpha_{cr} + \varepsilon, \quad s = \varepsilon t.$$

$$u_0 = z(s) e^{i\omega t} v(x) + \overline{z}(s) e^{-i\omega t} \overline{v}(x).$$

Form for u_2

$$\varepsilon^{\frac{3}{2}}: \quad z' e^{i\omega t} v + \dot{u}_2 = u_2'' + \gamma u_2 - (z e^{i\omega t} v + \overline{z} e^{-i\omega t} \overline{v})^3$$

$$\begin{aligned} u_2'(0,t) &= 0, \\ u_2'(1,t) &= \alpha_{cr}\, u_2(x_0,t) + u_0(x_0,t) \end{aligned}$$

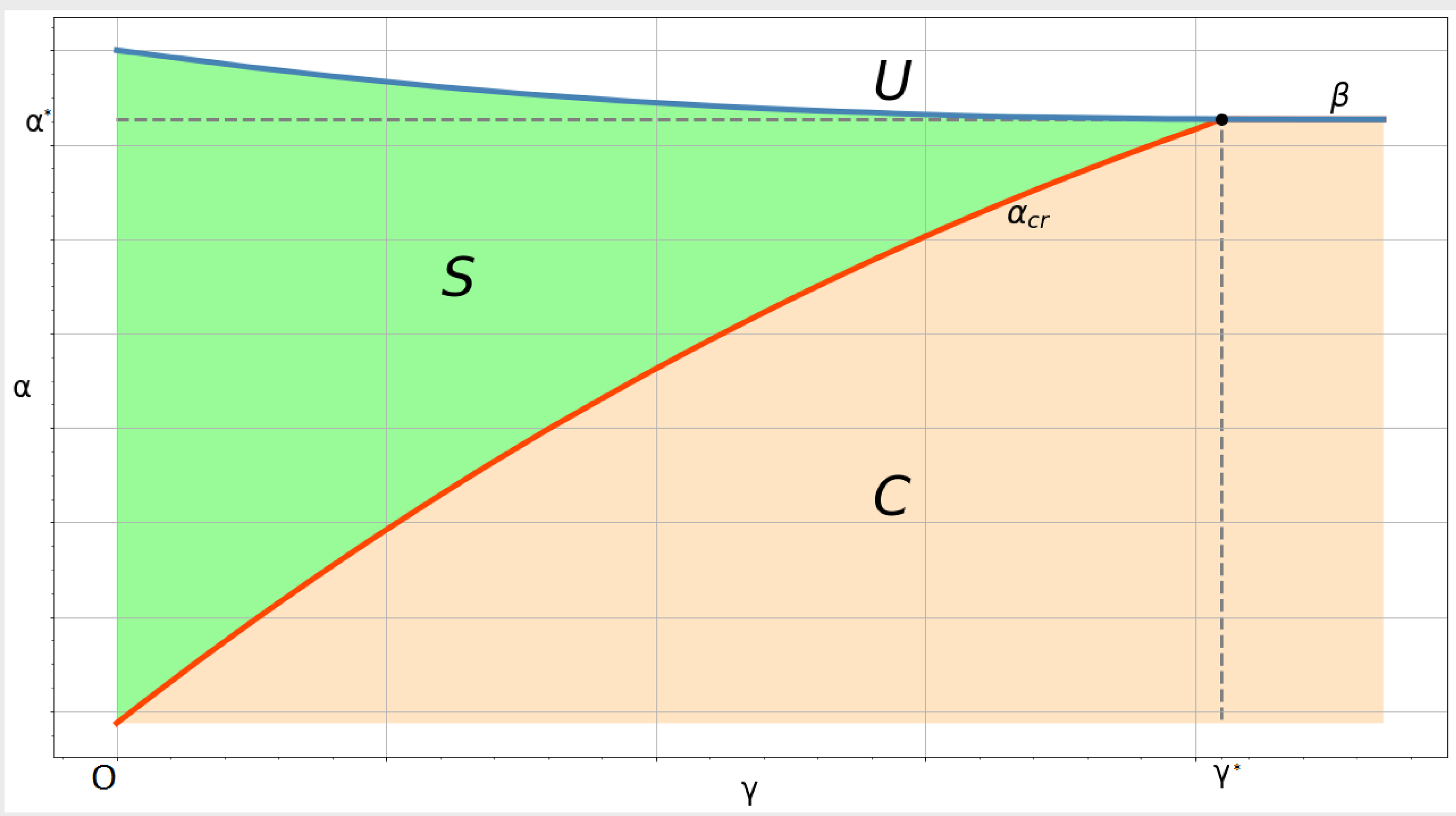
$$z' = \phi z + d z |z|^2$$

Main results: theorems

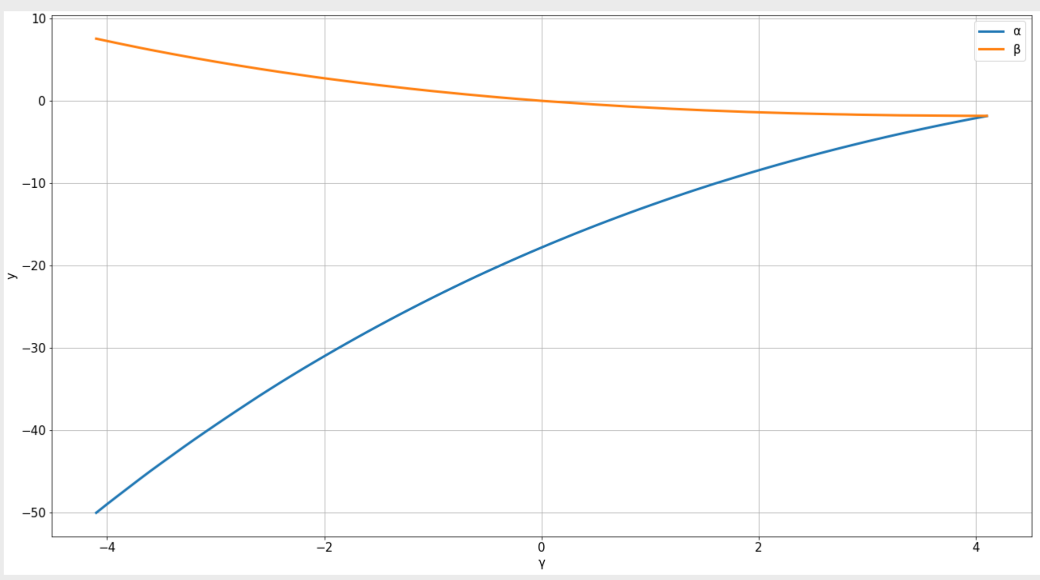
Theorem 1: In the case of $Re(\phi) > 0$, $Re(d) < 0$ $\exists \varepsilon_0 > 0$ $\forall \varepsilon \in (0, \varepsilon_0]$ there is observed an exponentially-orbitally stable cycle with asymptotic form $z(s) = \sqrt{-\frac{Re(\phi)}{Re(d)}} \exp\left(i\left(Im(\phi) - \frac{Im(d)Re(\phi)}{Re(d)}\right) s + i\gamma\right)$.

Theorem 2: For $\gamma < \gamma_*$ zero solution of (1), (2) will be asymptotically stable, if $\alpha < \beta$ and $\alpha > \alpha_{cr}$, where β, α_{cr} are roots of boundary-value problem (3), (4) and transcendental equation (5), respectively.

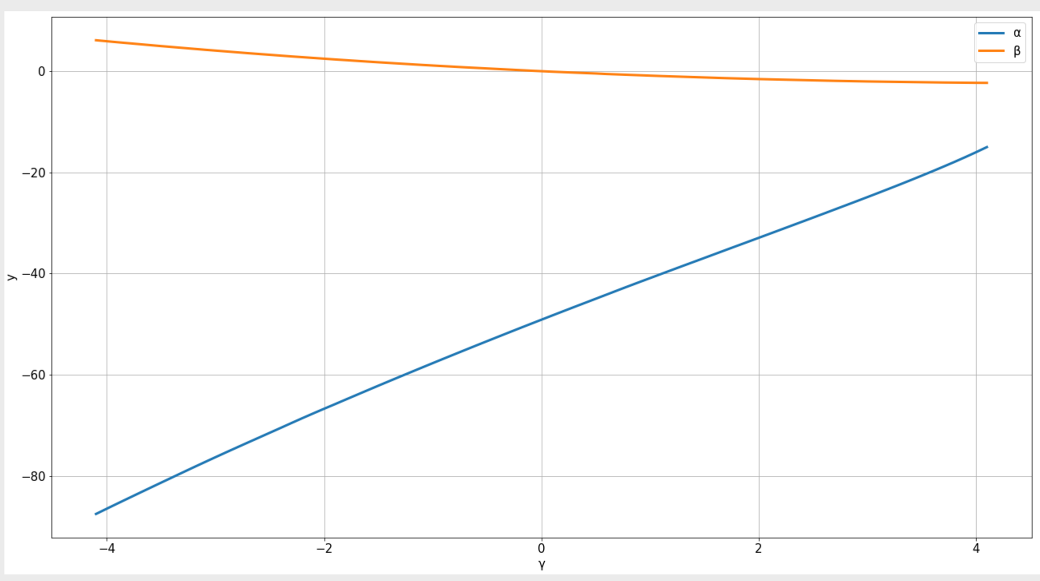
Numerical results: areas of stability for zero solution



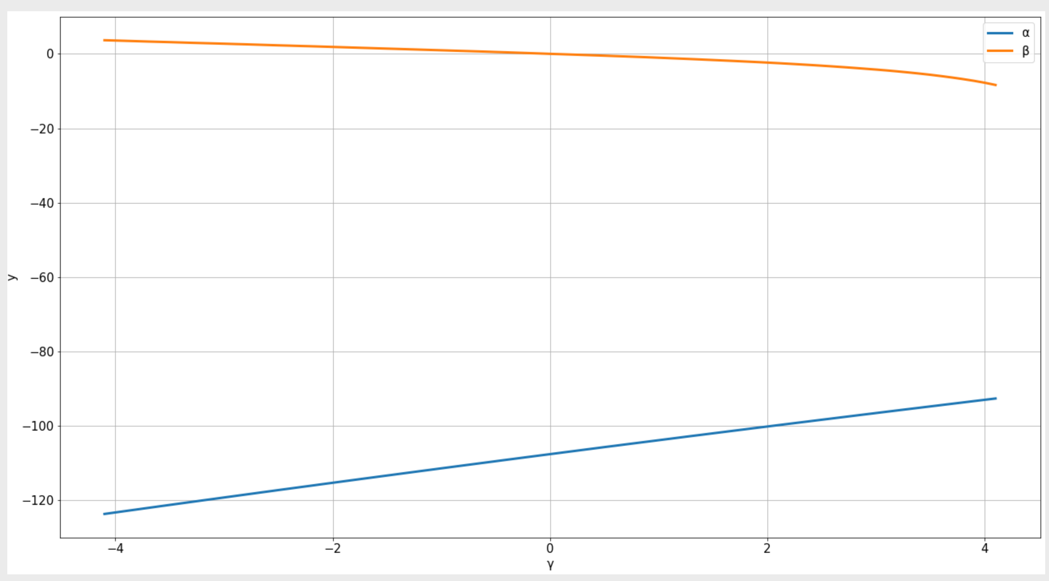
Numerical results



a) $x_0 = 0{,}0$



b) $x_0 = 0{,}33$



c) $x_0 = 0{,}67$