The loss of stability for null solution in parabolic boundary-value problem with the deviate in edge condition

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Parabolic boundary-value problem

$$u' = d\ddot{u} - \gamma u + F(u), \tag{1}$$

$$u' \mid_{x=0} = 0,$$
$$u' \mid_{x=1} = \alpha\, u \mid_{x=x_0} .$$

$$\alpha, \gamma \in \mathbb{R}, \quad d > 0, \quad x_0 \in [0,1].$$

Substitutions

$$t_1 = dt,$$

$$u(x,t) = w(x) \exp\left(\lambda - \frac{\gamma}{d} t\right).$$

Simplified parabolic boundary-value problem

$$w'' - \lambda w = 0, \tag{2}$$

$$w'(0) = 0,$$
$$w'(1) = \alpha\, w(x_0).$$

Deviate in edge condition

$$x_0 = 0 :$$

$$w(x) = c\, ch(\mu\, x),$$

$$\mu^2 = \lambda.$$

System of equations

$$\lambda \in \mathbb{C} : \quad \mu = \tau + i\omega.$$

$$\begin{cases} f(\tau,\omega) = 0 \\ g(\tau,\omega) - \alpha = 0. \end{cases} \tag{3}$$

$$f(\tau,\omega) = \tau \operatorname{cth} \tau + \omega \operatorname{ctg} \omega,$$

$$g(\tau,\omega) = \tau \operatorname{sh} \tau \cos \omega - \omega \operatorname{sh} \tau \sin \omega.$$

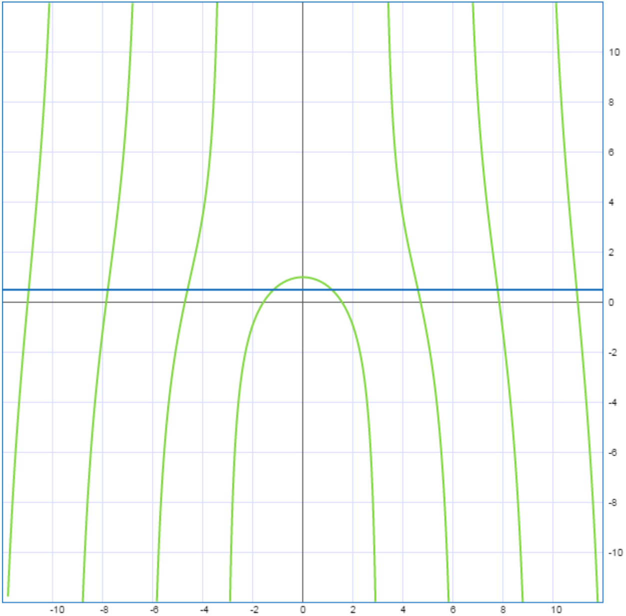
Loss of stability for null solution

Remark

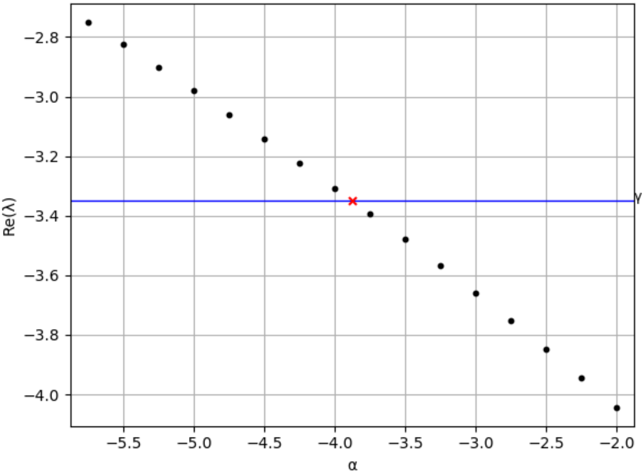
In the case $\mathrm{Re}(\lambda) = \gamma$ there will be the loss of stability for null solution of equation (2).

Numerical research

Search of roots

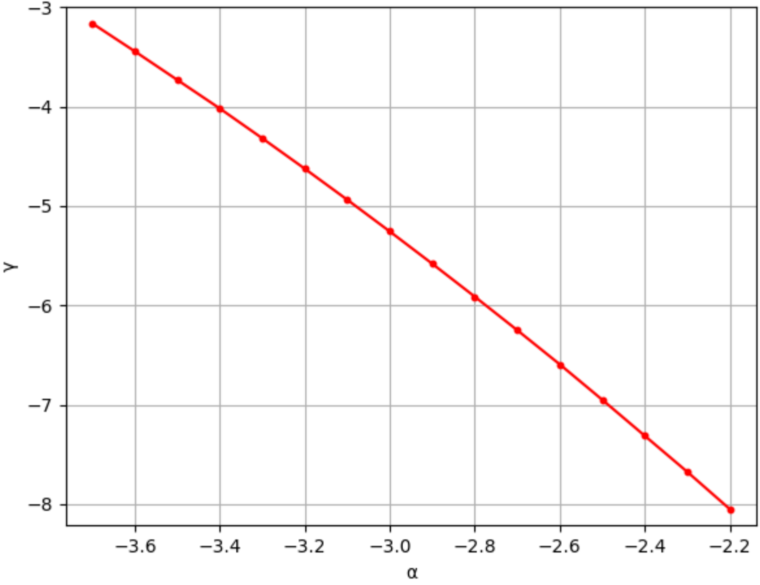


Critical value of α



$$\gamma = -3.33$$

Plot



Parabolic boundary-value problem

$$u' = d\ddot{u} - \gamma u - u^3,$$

$$u' \,|_{x=0} = 0,$$

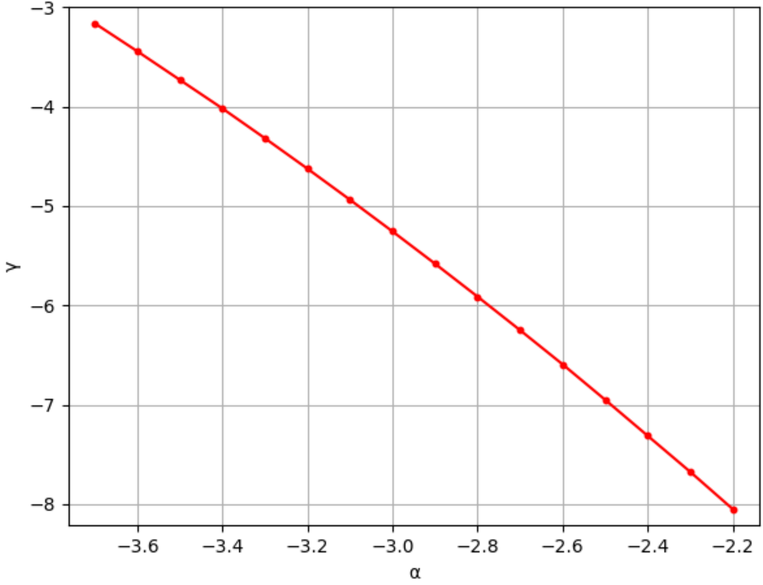
$$u' \,|_{x=1} = \alpha \, u \,|_{x=0} \; .$$

Dynamic system

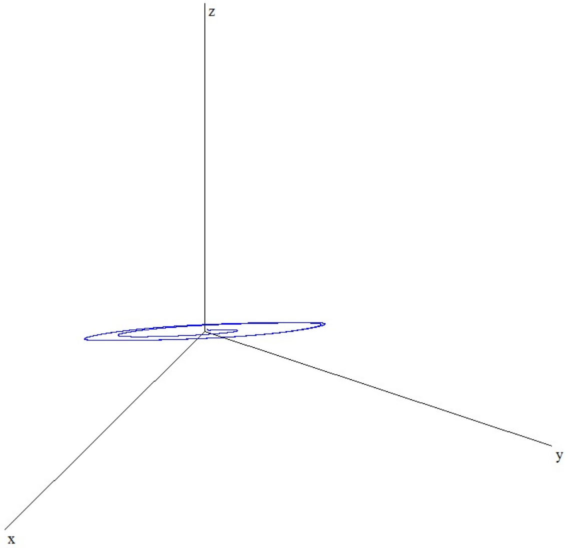
$$\dot{u}_j = p^2(u_{j-1} - 2u_j + u_{j+1}) - \gamma\, u_j - v_j^3, \qquad j = \overline{1, p}, \tag{4}$$

$$\begin{aligned} u_0 &= u_1, \\ u_{p+1} &= u_p + \frac{\alpha}{p}\, u_1. \end{aligned}$$

Plot

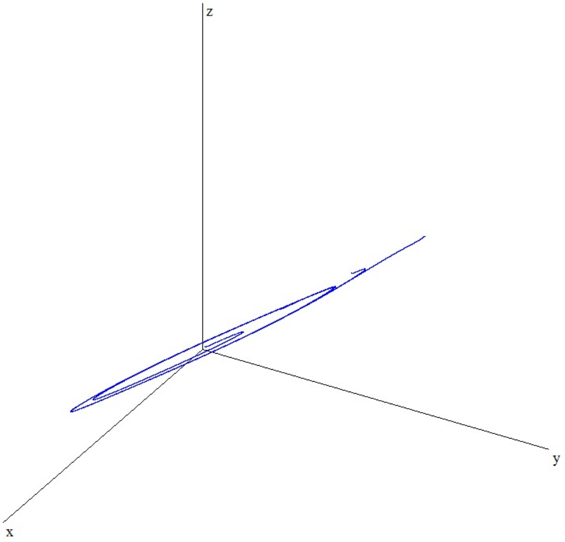


Andronov-Hopf bifurcation



$$\gamma = -5.58, \quad \alpha = -2.9$$

Andronov-Hopf bifurcation



$$\gamma = -5.58, \quad \alpha = -2.9$$