$$\dot u = u'' + \gamma u - u^3,$$

$$u'(0,t) \,=0, \qquad u'(1,t) \,= \alpha\, u(x_0,t),$$

$u = u(x,t),\ t \geqslant 0,\ x \in [0,1],\ \alpha, \gamma \in \mathbb{R},\ x_0 \in [0,1).$

$$\dot u = u'' + \gamma u,$$

$$u'(0,t) \,=0, \qquad u'(1,t) \,= \alpha\, u(x_0,t).$$

$u(x,t) = e^{\lambda t}\, v(x).$

$$v'' + (\gamma - \lambda) v = 0,$$

$$v'(0) \,=0, \qquad v'(1) \,= \alpha\, v(x_0).$$

$\mu = \sqrt{-\gamma + \lambda},\ v(x) = c\,\mathrm{ch}\mu x,\ c \in \mathbb{R}.$

$$\mu\,\mathrm{sh}\mu \;=\; \alpha\mathrm{ch}\mu x_0,$$

$\lambda = 0:\ \ \mu = \sqrt{-\gamma},$

$$\alpha_u = \frac{\sqrt{-\gamma}\ \mathrm{sh}\sqrt{-\gamma}}{\mathrm{ch}\sqrt{-\gamma} x_0}.$$

$\lambda = i\omega:\ \ \mu = \sqrt{-\gamma + i\omega},$

$$\alpha_c = \frac{\sqrt{-\gamma + i\omega}\ \mathrm{sh}\sqrt{-\gamma + i\omega}}{\mathrm{ch}\sqrt{-\gamma + i\omega} x_0}.$$

$\gamma = 0,\ x_0 = 0:$

$$\begin{cases} \mathrm{tg}\ y + \mathrm{th}\ y = 0, \\ \alpha_c = y(\mathrm{sh}\, y \cos y - \mathrm{ch}\, y \sin y), \end{cases}$$

$$y = \sqrt{\frac{\omega}{2}}.$$

$\gamma = 0,\ x_0 \neq 0:$

$$\begin{cases} \dfrac{\mathrm{sh}\, y \cos y + \mathrm{ch}\, y \sin y}{\mathrm{sh}\, y \cos y - \mathrm{ch}\, y \sin y} - \mathrm{tg}\, y x_0\, \mathrm{th}\, y x_0 = 0, \\ \alpha_c = \dfrac{y\,\mathrm{sh}\, y \cos y - y\,\mathrm{ch}\, y \sin y}{\mathrm{ch}\, y x_0 \cos y x_0}. \end{cases}$$

$\gamma \neq 0,\ x_0 \neq 0.$

$$\dot{u}_j = n^2(u_{j+1} - 2u_j + u_{j-1}) + \gamma u_j, \quad j = \overline{1,n}.$$

$$u_0 = u_1, \quad u_{n+1} = u_n + \frac{\alpha_c}{n}\, u_1.$$

$$u = \sqrt{\varepsilon} u_0 + \varepsilon u_1 + \varepsilon^{\frac{3}{2}} u_2 + O(\varepsilon^2),$$

$\varepsilon = |\alpha - \alpha_{cr}|,\ \varepsilon \ll 1,\ s = \varepsilon t.$

$\lambda = 0 : \mu = \sqrt{-\gamma},\ \alpha_{cr} = \alpha_u,\ \varepsilon = \alpha - \alpha_d.$

$$u_0 = u_0'' + \gamma u_0,$$

$$u_0'(0,t) = 0, \qquad u_0'(1,t) = \alpha_u u_0(x_0,t),$$

$$u_0 = \rho(s)\,\mathrm{ch}\,\sqrt{-\gamma} x.$$

$$\dot{u}_2 + \frac{\partial u_0}{\partial s} = u_2'' + \gamma u_2 - u_0^3,$$

$$u_2'(0,t) = 0, \qquad u_2'(1,t) = \alpha_u u_2(x_0,t) + u_0(x_0,t).$$

$u_2 = e^{\lambda t} v_2(x),\ \lambda = 0.$

$$v_2'' + \gamma v_2 - \rho^3\,\mathrm{ch}^3 \sqrt{-\gamma} x - \rho'\,\mathrm{ch}\,\sqrt{-\gamma} x = 0,$$

$$v_2'(0) = 0, \qquad v_2'(1) = \alpha_u v_2(x_0) + \rho(s)\,\mathrm{ch}\,\sqrt{-\gamma} x_0.$$

$$v_2(x) = c\ \mathrm{ch}\,\sqrt{-\gamma} x - \frac{\rho^3}{32}\,\mathrm{ch}\,3\sqrt{-\gamma} x + \frac{3\rho^3 + 4\rho'}{8\sqrt{-\gamma}} x\ \mathrm{sh}\,\sqrt{-\gamma} x.$$

$$\rho' = \phi\rho + d\rho^3,$$

$$\phi = \frac{2\mu \mathrm{ch}\mu x_0}{\mu \mathrm{ch}\mu + \mathrm{sh}\mu - \alpha_d x_0 \mathrm{sh}\mu x_0},$$

$$d = \frac{-3\gamma \mathrm{sh}3\mu - 12\mathrm{sh}\mu - 12\mu\mathrm{ch}\mu - \alpha_d \mu \mathrm{ch}3\mu x_0 + 12\alpha_d x_0 \mathrm{sh}\mu x_0}{16(\mathrm{sh}\mu + \mu\mathrm{ch}\mu - \alpha_d x_0 \mathrm{sh}\mu x_0)}.$$

$$u = \pm\sqrt{\varepsilon} A_u\ \mathrm{ch}\,\sqrt{-\gamma} x + O(\varepsilon),$$

$$A_u = \sqrt{\left|\frac{\phi_0}{d_0}\right|}.$$

$\lambda = i\omega : \mu = \sqrt{-\gamma + i\omega},\ \alpha_{cr} = \alpha_c,\ \varepsilon = \alpha_c - \alpha.$

$$u_0 = u_0'' + \gamma u_0,$$

$$u_0'(0,t) = 0, \qquad u_0'(1,t) = \alpha_c u_0(x_0,t).$$

$$u_0 = z(s)e^{i\omega t}\operatorname{ch}\mu x + \overline{z(s)}e^{-i\omega t}\overline{\operatorname{ch}\mu x}.$$

$$\dot{u}_2 + \frac{\partial u_0}{\partial s} = u_2'' + \gamma u_2 - u_0^3,$$

$$u_2'(0,t) = 0, \qquad u_2'(1,t) = \alpha_c u_2(x_0,t) + u_0(x_0,t).$$

$u_2 = e^{i\omega t} v_2(x).$

$$v_2'' + (\gamma - i\omega)v_2 - z' w(x) - 3z|z|^2 w|w|^2 = 0,$$

$$v_2'(0) = 0, \qquad v_2'(1) = \alpha_u v_2(x_0) + z(s)w(x_0),$$

$$w(x) = \operatorname{ch}\sqrt{-\gamma + i\omega}\, x.$$

$$z' = \phi z + dz|z|^2.$$

$$\phi_0 = \operatorname{Re}\left(\frac{2\mu \operatorname{ch}\mu x_0}{\mu \operatorname{ch}\mu + \operatorname{sh}\mu - \alpha_c x_0 \operatorname{sh}\mu x_0}\right),$$

$$d_0 = \operatorname{Re}\left(\frac{3\mu(G(\mu + 2\operatorname{Re}\mu) + G(\mu + 2i\operatorname{Im}\mu) + 2G(\overline{\mu}))}{2(\mu \operatorname{ch}\mu + \operatorname{sh}\mu - \alpha_c x_0 \operatorname{sh}\mu x_0)}\right),$$

$$G(y) = \frac{\alpha_c - y \operatorname{sh} y}{y^2 + \gamma - i\omega}.$$

$$u = \pm\sqrt{\varepsilon} A_c \operatorname{ch}\sqrt{-\gamma + i\omega}\, x + O(\varepsilon),$$

$$A_c = \sqrt{-\frac{\phi_0}{d_0}}.$$

$$\dot{u} = u'' + \gamma u,$$

$$u'(0,t) = 0, \qquad u'(1,t) = \alpha\, u(x_0,t) + \beta u^3(x_0,t),$$

$u = u(x,t),\ t \geqslant 0,\ x \in [0,1],\ \alpha, \gamma \in \mathbb{R},\ \beta \in \mathbb{R} \setminus \{0\},\ x_0 \in [0,1).$
$\lambda = 0 : \mu = \sqrt{-\gamma},\ \alpha_{cr} = \alpha_u,\ \varepsilon = \alpha - \alpha_d.$

$$\dot{u}_2 + \frac{\partial u_0}{\partial s} = u_2'' + \gamma u_2,$$

$$u_2'(0,t) = 0, \qquad u_2'(1,t) = \alpha_u u_2(x_0,t) + u_0(x_0,t) + \beta u_0^3(x_0,t).$$

$u_2 = e^{\lambda t} v_2(x),\ \lambda = 0.$

$$v_2'' + \gamma v_2 - \rho' \operatorname{ch} \sqrt{-\gamma} x = 0,$$

$$v_2'(0) = 0, \quad v_2'(1) = \alpha_u v_2(x_0) + \rho \operatorname{ch} \sqrt{-\gamma} x_0 + \beta \rho^3 \operatorname{ch}^3 \sqrt{-\gamma} x_0.$$

$$v_2(x) = c \operatorname{ch} \sqrt{-\gamma} x + \frac{\rho'}{2\sqrt{-\gamma}} \operatorname{sh} \sqrt{-\gamma} x + \frac{\rho' x}{2} \operatorname{ch} \sqrt{-\gamma} x.$$

$$\phi_0 = Q \operatorname{ch} \sqrt{-\gamma} x_0, \quad d_0 = \beta Q \operatorname{ch}^3 \sqrt{-\gamma} x_0,$$

$$Q = \frac{2\sqrt{-\gamma}}{\sqrt{-\gamma} \operatorname{ch} \sqrt{-\gamma} + \operatorname{sh} \sqrt{-\gamma} - \alpha_u x_0 \sqrt{-\gamma} x_0}.$$

$\lambda = i\omega : \mu = \sqrt{-\gamma + i\omega},\ \alpha_{cr} = \alpha_c,\ \varepsilon = \alpha_c - \alpha.$

$$\dot{u}_2 + \frac{\partial u_0}{\partial s} = u_2'' + \gamma u_2,$$

$$u_2'(0,t) = 0, \qquad u_2'(1,t) = \alpha_c u_2(x_0,t) - u_0(x_0,t) + \beta u_0^3(x_0,t).$$

$u_2 = e^{i\omega t} v_2(x).$

$$v_2'' + (\gamma - i\omega) v_2 - z' w(x) = 0,$$

$$v_2'(0) = 0, \qquad v_2'(1) = \alpha_u v_2(x_0) - z w(x_0) + 3\beta z |z|^2 w |w|^2.$$

$$\phi_0 = -2 \operatorname{Re}(Q \operatorname{ch} \mu x_0),$$

$$d_0 = 1.5\beta \operatorname{Re}(Q(\operatorname{ch}(\mu + 2 \operatorname{Re}\mu) x_0 + \operatorname{ch}(\mu + 2i \operatorname{Im}\mu) x_0 + 2 \operatorname{ch} \overline{\mu} x_0)).$$

$$Q = \frac{\mu}{\mu \operatorname{ch} \mu + \operatorname{sh} \mu - \alpha_c x_0 \operatorname{sh} \mu x_0}.$$

$$\dot{u} = u'' + \gamma u - u^3,$$

$$u'(0,t) = 0, \qquad u'(1,t) = \alpha \int\limits_0^1 u(y,t)dy,$$

$u = u(x,t),\ t \geqslant 0,\ x \in [0,1],\ \alpha, \gamma \in \mathbb{R}.$

$$\dot{u} = u'' + \gamma u,$$

$$u'(0,t) = 0, \qquad u'(1,t) = \alpha \int\limits_0^1 u(y,t)dy.$$

$u(x,t) = e^{\lambda t} v(x).$

$$v'' + (\gamma - \lambda)v = 0,$$

$$v'(0) = 0, \qquad v'(1) = \alpha \int\limits_0^1 v(y)dy.$$

$\mu = \sqrt{-\gamma + \lambda},\ v(x) = c\,\mathrm{ch}\mu x,\ c \in \mathbb{R}.$

$$\alpha = \mu^2 = -\gamma + \lambda.$$

$\lambda = 0:\ \mu = \sqrt{-\gamma},$

$$\alpha_u = -\gamma.$$

$$\dot{u}_j = n^2(u_{j+1} - 2u_j + u_{j-1}) + \gamma u_j, \quad j = \overline{1,n}.$$

$$u_0 = u_1, \quad u_{n+1} = u_n + \frac{\alpha}{n^2} \sum_{k=1}^{n} u_k.$$

$\varepsilon = \alpha - \alpha_u.$

$$\dot{u}_0 = u_0'' + \gamma u_0,$$

$$u_0'(0,t) = 0, \qquad u_0'(1,t) = \alpha_u \int\limits_0^1 u_0(y,s)dy,$$

$$u_0 = \rho(s)\,\mathrm{ch}\,\sqrt{-\gamma}x.$$

$$\dot{u}_2 + \frac{\partial u_0}{\partial s} = u_2'' + \gamma u_2 - u_0^3,$$

$$u_2'(0,t) \ = 0, \qquad u_2'(1,t) = \alpha_u \int\limits_0^1 u_2(y,t)dy \ + \int\limits_0^1 u_0(y,s)dy.$$

$u_2 = e^{\lambda t} v_2(x), \ \lambda = 0.$

$$v_2'' + \gamma v_2 - \rho' \operatorname{ch} \sqrt{-\gamma} x - \frac{3\rho^3 \operatorname{ch} \sqrt{-\gamma} x}{4} - \frac{\rho^3 \operatorname{ch} 3\sqrt{-\gamma} x}{4} = 0,$$

$$v_2'(0) \ = 0, \quad v_2'(1) = \alpha_u \int\limits_0^1 v_2(y)dy + \frac{\rho \operatorname{sh} \sqrt{-\gamma}}{\sqrt{-\gamma}}.$$

$$v_2(x) = c \ \operatorname{ch} \sqrt{-\gamma} x + -\frac{\rho^3}{32} \operatorname{ch} 3\sqrt{-\gamma} x + \frac{3\rho^3 + 4\rho'}{8\sqrt{-\gamma}} x \operatorname{sh} \sqrt{-\gamma} x.$$

$$\rho' = \rho + d_0 \rho^3,$$

$$d_0 = -\frac{5\gamma \operatorname{sh} 3\sqrt{-\gamma}}{48 \operatorname{sh} \sqrt{-\gamma}} - \frac{3}{4}.$$