The loss of stability for null solution in parabolic boundary-value problem with the deviate in edge condition

Let us consider the following parabolic boundary-value problem:

$$u' = d\ddot{u} - \gamma u + F(u), \tag{1}$$

At points $x = 0$ and $x = x_0$ there are set zero and linear edge conditions:

$$u' \mid_{x=0} = 0,$$

$$u' \mid_{x=x_0} = \alpha\, u \mid_{x=0} .$$

Function $u = u(x,t)$, parameters $\alpha, \gamma \in \mathbb{R}$, $d > 0$. This differential equation is based on the similar equation in [1].

By means of following substitutions

$$t_1 = dt, \qquad u(x,t) = w(x) \exp\left(\lambda - \frac{\gamma}{d} t\right)$$

in the case of $x_0 = 0$ system (1) is transformed to simplified parabolic boundary-value problem

$$\begin{array}{l} w'' - \lambda w = 0, \\ w'(0) = 0, \\ w'(1) = \alpha\, w(0), \end{array} \tag{2}$$

which has a solution

$$w(x) = c\, ch(\mu\, x),$$

where c is a constant and $\mu^2 = \lambda$. In the case of $\mu \in \mathbb{C}$, $\mu = \tau + i\omega$, the solution of differential equation (2) can be transformed to the following system:

$$\begin{cases} f(\tau,\omega) = 0 \\ g(\tau,\omega) - \alpha = 0. \end{cases} \tag{3}$$

$$f(\tau,\omega) = \tau \operatorname{cth} \tau + \omega \operatorname{ctg} \omega,$$

$$g(\tau,\omega) = \tau \operatorname{sh} \tau \cos\omega - \omega \operatorname{sh} \tau \sin\omega.$$

Functions f, g are even for variables τ and ω.

The task of research was to find values of parameter $\alpha = \alpha_{cr}$, when for the solution of system (3), $\mathrm{Re}(\lambda) = \gamma$. In this case for equation (2) there will be the loss of stability for null solution.

The research was carried out by means of special software. All calculations of roots for system (3) are performed on a large number of independent streams on GPU. As a result of research there were get values of parameters $\alpha = \alpha_{cr}$ and λ, when the null solution of equation (2) will lose the stability.

Список литературы

[1] *Landis E.M.* Second Order Equations of Elliptic and Parabolic Type. M., 1971.