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## Modelling unsteady flows in elastic pipes: energy conservation law

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Energy conservation law is presented for compressible flows in an elastic pipe for a small ratio between the pipe diameter a characteristic length scale. Conservation of the flow energy together with mass and momentum conservation constitute the system of equations to be solved for description of a fluid transport along the pipe.

Energy conservation law for finite volumes in absence of external inflows has the form [1]

$$\frac{d}{dt}\int\limits_{V^*}\rho\left(\frac{\mathbf{v}^2}{2}+U\right)d\tau=\int\limits_{V}\rho\mathbf{F}\cdot\mathbf{v}\,d\tau+\int\limits_{\Sigma}\mathbf{p}_N\cdot\mathbf{v}\,d\sigma, \tag{1}$$

where $\rho$ is the density, $U=U(T)$ is the volumetric density of internal energy of a volume $V$, $T=T(x,y,z,t)$ is the fluid temperature, $\rho$ is the fluid density, $\mathbf{v}=u\mathbf{i}+v\mathbf{j}+w\mathbf{k}$ is the fluid velocity, $\mathbf{p}_N=-p\mathbf{N}$ for the ideal fluid, $p$ is a pressure, $\mathbf{N}$ is an external normal vector to a surface $\Sigma$, bounding $V$, $\mathbf{F}$ in an external forces vector acting on $V$ and $V^*=V$ in the given moment of time. Further we use a constitutive law for an arbitrary two-parametric medium

$$p=f(\rho,T). \tag{2}$$

The energy conservation law (1) for smooth processes takes the form [1]

$$\frac{\partial}{\partial t}\rho\left(\frac{\mathbf{v}^2}{2}+U\right)+\operatorname{div}\rho\left(\frac{\mathbf{v}^2}{2}+U\right)\mathbf{v}=\rho\mathbf{F}\cdot\mathbf{v}-\operatorname{div}p\mathbf{v}. \tag{3}$$

Assume, that $F_x$ is a friction force plus a projection of gravity force (see, Fig. 1 in [2])

$$F_x=-\rho g S_f+\rho g\sin\theta,$$

where $g$ is the gravity, and $S_f$ is given by the Manning-Stricler law [2].

Define an average

$$\overline{\alpha}=\frac{1}{A}\int\limits_{\Omega(x,t)}\alpha\,d\sigma,$$

where $A=A(x,t)$ is a square of pipe's cross section $\Omega(x,t)$. We consider that

$$\overline{\rho\left(\frac{u^2}{2}+U\right)u}=\overline{\rho}\left(\frac{\overline{u}^2}{2}+\overline{U}\right)\overline{u},\quad \overline{\rho F_x u}=\overline{\rho}\overline{F_x}\overline{u},\quad \overline{pu}=\overline{p}\,\overline{u},\quad \overline{U}=U(\overline{T}).$$

Then Eq.(2) has the form

$$\overline{p}=f(\overline{\rho},\overline{T}), \tag{4}$$

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and

$$\overline{F_x} = -\overline{\rho} g S_f + \overline{\rho} g \sin\theta. \tag{5}$$

It can be shown that (3) transforms into

$$\frac{\partial}{\partial t}\left[\overline{\rho}\left(\frac{\overline{u}^2}{2}+\overline{U}\right)A\right]+\frac{\partial}{\partial x}\left[\overline{\rho}\left(\frac{\overline{u}^2}{2}+\overline{U}+\overline{p}\right)\overline{u}A\right]=\overline{\rho}\,\overline{F_x}\,\overline{u}A-\overline{p}\frac{\partial A}{\partial t}. \tag{6}$$

The square $A$ is defined from Hooke's law (see [2]) and is given by

$$\frac{\partial A}{\partial x}=\frac{\partial S_0}{\partial x}+\frac{2}{\sqrt{\pi}}\frac{A^{3/2}}{eE\cos\phi}\frac{\partial \overline{p}}{\partial x} \tag{7}$$

and

$$\frac{\partial A}{\partial t}=\frac{2}{\sqrt{\pi}}\frac{A^{3/2}}{eE\cos\phi}\frac{\partial \overline{p}}{\partial t}, \quad A(0,x)=S_0(x), \tag{8}$$

where $\phi$ is the angle between pipe's axis and its generating line, $e$ is the constant width of the pipe, and $E$ is the Young module.

Finally we have the equations (4), (5), (6), where $A(x,t)$ is defined from (7), (8).

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## References

[1] *Sedov, L.I.*, Mechanics of continuous media, World Scientific, 1997.

[2] *Bourdarias, C., Gerbi, S* A conservative model for unsteady flows in deformable closed pipe and its implicit second order finite volume discretisation // Comput. Fluids. 2008. V. 37. № 10. P. 1255-1237.

# The loss of stability for null solution in parabolic boundary-value problem with the deviate in edge condition

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Let us consider the parabolic boundary-value problem with zero and linear edge conditions. There were researched the questions of the loss of stability for null solution of due to a numerical research of special system. The special attention is paid to values of certain initial parameters, when the null solution of parabolic boundary-value problem will lose the stability.

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Let us consider the following parabolic boundary-value problem:

$$u' = d\ddot{u} - \gamma u + F(u), \tag{1}$$

At points $x = 0$ and $x = x_0$ there are set zero and linear edge conditions:

$$u' \mid_{x=0} = 0,$$

$$u' \mid_{x=x_0} = \alpha\, u \mid_{x=0} .$$

Function $u = u(x,t)$, parameters $\alpha, \gamma \in \mathbb{R}$, $d > 0$. This parabolic boundary-value problem is based on the similar equation in [1].

By means of following substitutions

$$t_1 = dt, \qquad u(x,t) = w(x) \exp\left(\lambda - \frac{\gamma}{d}t\right)$$

in the case of $x_0 = 0$ system (1) is transformed to simplified parabolic boundary-value problem with edge conditions.

$$\begin{aligned} &w'' - \lambda w = 0, \\ &w'(0) = 0, \\ &w'(1) = \alpha\, w(0). \end{aligned} \tag{2}$$

This differential equation has a solution

$$w(x) = c\, ch(\mu\, x),$$

where $c$ is a constant and $\mu^2 = \lambda$. In the case of $\mu \in \mathbb{C}$, $\mu = \tau + i\omega$, the solution of parabolic boundary-value problem (2) can be transformed to the following system:

$$\begin{cases} f(\tau, \omega) = 0 \\ g(\tau, \omega) - \alpha = 0. \end{cases} \tag{3}$$

$$f(\tau, \omega) = \tau \operatorname{cth} \tau + \omega \operatorname{ctg} \omega,$$

$$g(\tau, \omega) = \tau \operatorname{sh} \tau \cos \omega - \omega \operatorname{sh} \tau \sin \omega.$$

Functions $f, g$ are even for variables $\tau$ and $\omega$.

The task of research was to find values of parameter $\alpha = \alpha_{cr}$, when for the solution of system (3), $\mathrm{Re}(\lambda) = \gamma$. In this case for parabolic boundary-value problem (2) there will be the loss of stability for null solution.

The research was carried out by means of special software. All calculations of roots for system (3) are performed on a large number of independent streams on GPU. As a result of research there were get values of parameters $\alpha = \alpha_{cr}$ and $\lambda$, when the null solution of parabolic boundary-value problem (2) will lose the stability.