Bifurcations of zero balance state in one boundary-value problem with deviation in edge condition

Leonid Ivanovsky

P.G. Demidov Yaroslavl State University,   Scientific Center in Chernogolovka of RAS

Boundary-value problem

$$\dot u = u'' + \gamma u - u^3, \tag{1}$$

$$\begin{aligned} u'(0,t) &= 0, \\ u'(1,t) &= \alpha\, u(x_0,t), \end{aligned} \tag{2}$$

$$\alpha, \gamma \in \mathbb{R}, \quad x_0 \in [0,1].$$

System for numerical experiments

$$\dot u_j = n^2(u_{j+1} - 2u_j + u_{j-1}) + \gamma u_j - u_j^3, \quad j = \overline{1,n}, \tag{4}$$

$$\begin{aligned} u_0 &= u_1, \\ u_{n+1} &= u_n + \frac{\alpha}{n} u_k, \end{aligned} \tag{5}$$

$$k = k(x_0) \in \mathbb{N}, \quad k \in [1,n]$$

Normal form

$$\alpha = \alpha_{cr} + \varepsilon$$

$$u = \sqrt{\varepsilon} u_0 + \varepsilon u_1 + \varepsilon^{\frac{3}{2}} u_2 + O(\varepsilon^2) \tag{3}$$

$$\sqrt{\varepsilon}: \ \dot u_0 = u_0'' + \gamma u_0$$

$$\begin{aligned} u_0'(0,t) &= 0 \\ u_0'(1,t) &= \alpha_{cr}\, u_0(x_0,t) \end{aligned}$$

Form for u_0

$$u_0 = z(s) e^{i\omega t} w(x) + \overline{z}(s) e^{-i\omega t} \overline{w}(x) \tag{6}$$

$$w(x) = c\,\mathrm{ch}(\mu x),$$

$$s = \varepsilon t,$$

$$\mu = \sqrt{\gamma + i\omega}, \quad \omega \in \mathbb{R}.$$

Main result

Theorem: In the case of $Re(\phi) > 0$, $Re(d) < 0$ $\ \exists \varepsilon_0 > 0 \ \ \forall \varepsilon \in (0, \varepsilon_0]$ there is observed an exponentially-orbitally stable cycle with asymptotic form $z(s) = \sqrt{-\frac{Re(\phi)}{Re(d)}} \exp\left(i\left(Im(\phi) - \frac{Im(d)Re(\phi)}{Re(d)}\right) s + i\gamma\right)$.

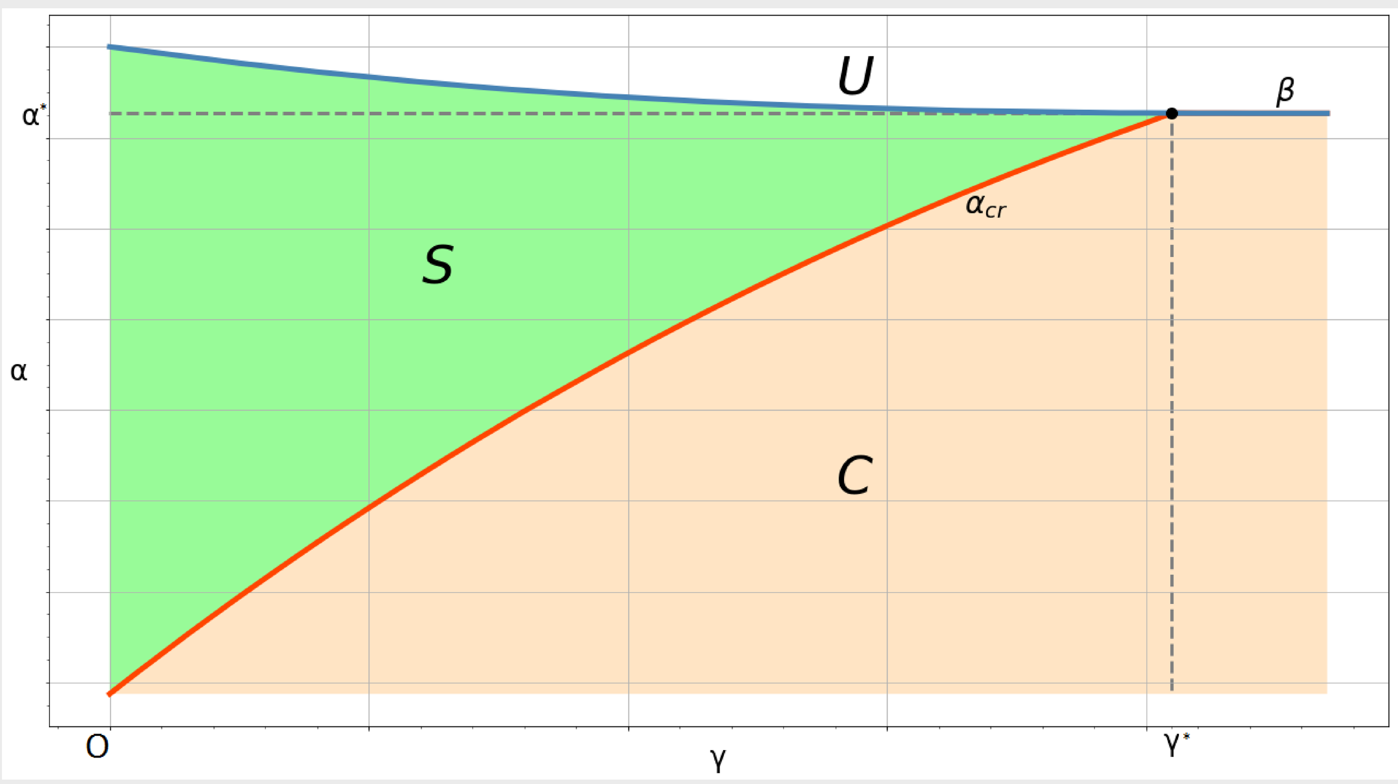
Form for u_2

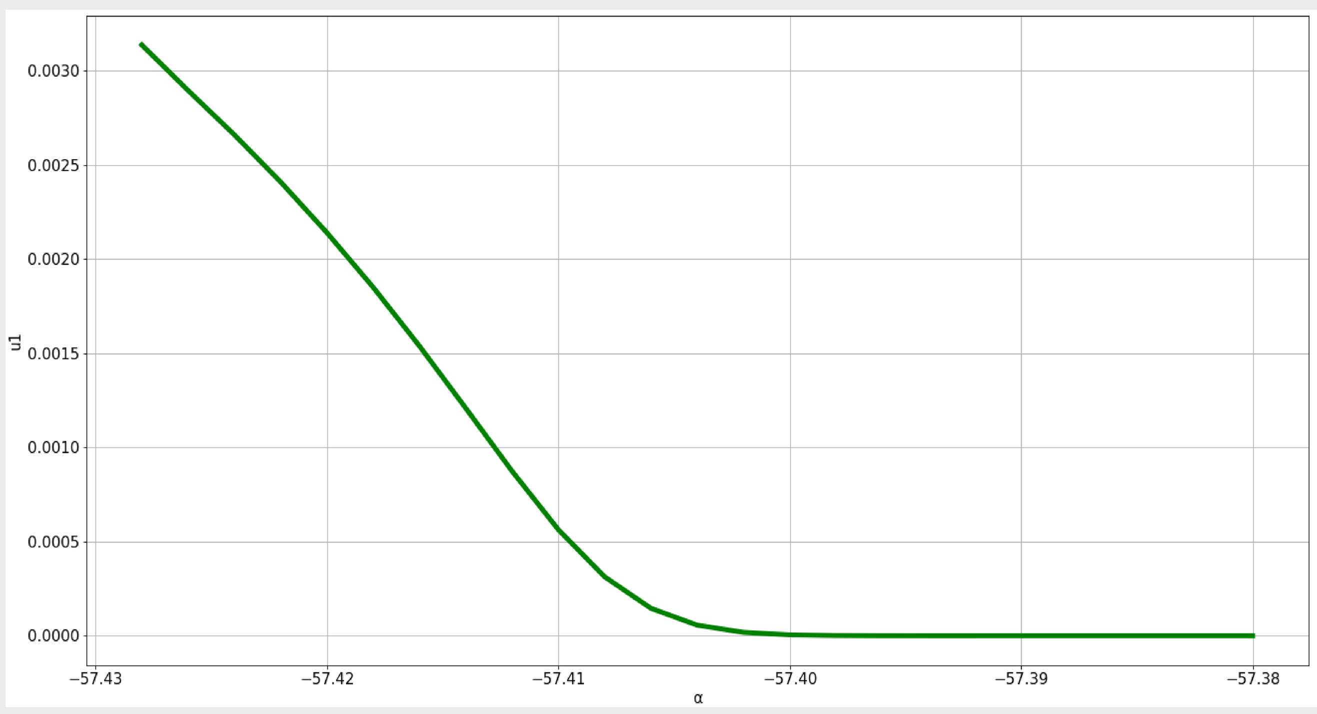
$$\varepsilon^{\frac{3}{2}}: \quad z' e^{i\omega t} w + \dot u_2 = u_2'' + \gamma u_2 - (z e^{i\omega t} w + \overline{z} e^{-i\omega t} \overline{w})^3$$

$$\begin{aligned} u_2'(0,t) &= 0 \\ u_2'(1,t) &= \alpha_{cr}\, u_2(x_0,t) + u_0(x_0,t) \end{aligned}$$

$$z' = \phi z + d z |z|^2$$

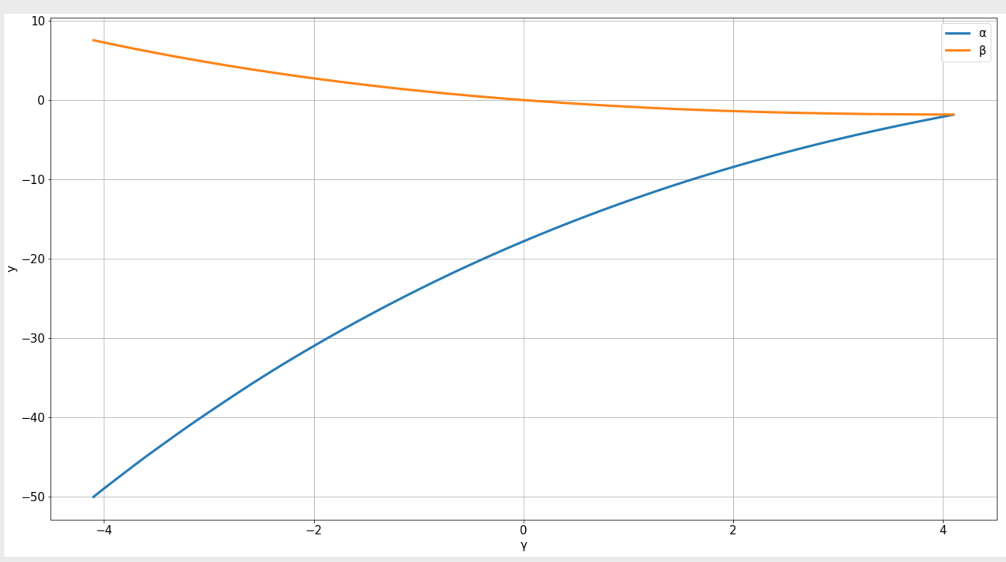
Numerical results



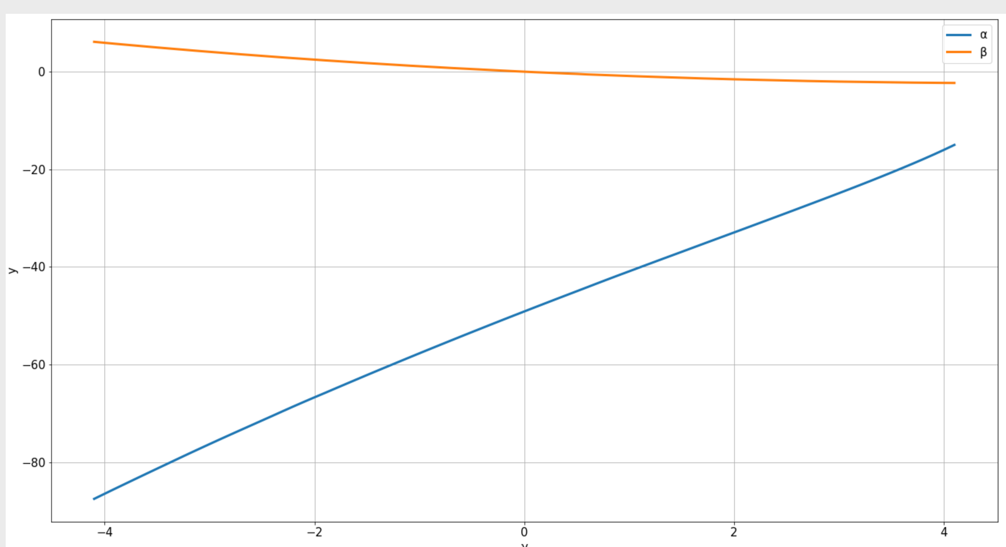


$x_0 = 0{,}49, \ \gamma = 2{,}4$

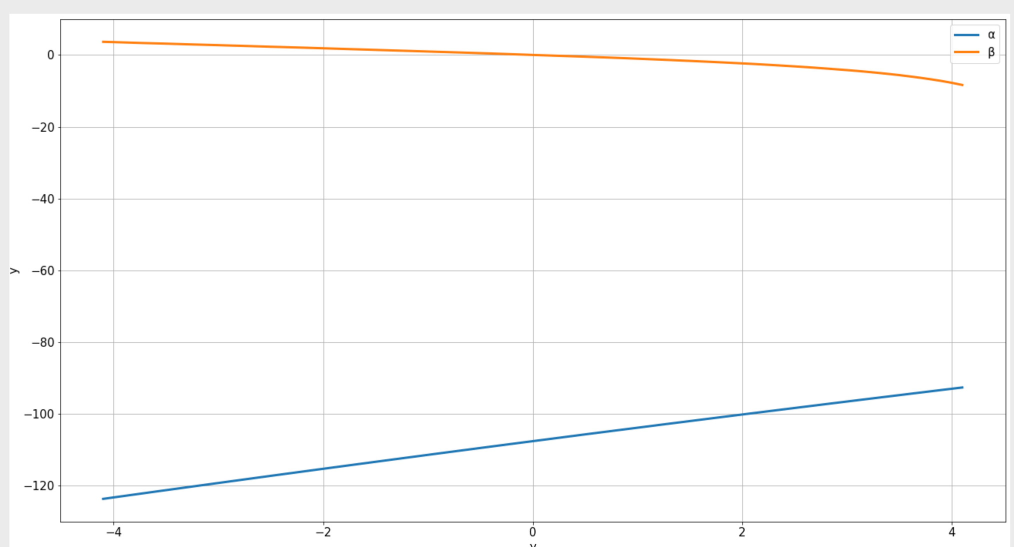
Numerical results



a) $x_0 = 0{,}0$



b) $x_0 = 0{,}33$



c) $x_0 = 0{,}67$