EIGENFUNCTION

In[1]:=
```
w[x_] := c Cosh[μ x]
```

In[2]:=
```
μ := τ + i ω
```

INTEGRALS

In[3]:=
```
I1[x_] := -Collect[Integrate[
    (z'[s] w[ξ] + 3 z[s] Abs[z[s]]^2 w[ξ] Abs[w[ξ]]^2) (Sinh[μ ξ]/μ), {ξ, 0, x}],
  {Abs[z[s]]^2 z[s], z'[s]}
 ]
```

In[4]:=
```
Simplify[I1[x] /. {Im[c]^2 + Re[c]^2 → Abs[c]^2}]
```

Out[4]=
$$\frac{1}{16\,(\tau+i\,\omega)^2}\,c\left(-\frac{1}{\tau\,(2\,\tau+i\,\omega)\,(\tau+2\,i\,\omega)\,\omega}3\,(\tau+i\,\omega)\,\text{Abs}[c]^2\,\text{Abs}[z[s]]^2\,\left(-\tau\,(\tau+2\,i\,\omega)\,\omega-\tau\,(\tau+2\,i\,\omega)\,(-2\,i\,\tau+\omega)+2\,\omega\,(-2\,\tau^2-3\,i\,\tau\,\omega+\omega^2)-2\,\omega\,(-2\,\tau^2-3\,i\,\tau\,\omega+\omega^2)\,\text{Cos}[2\,x\,\omega]\,\text{Cosh}\left[2\,x\,(\tau+i\,\omega)\right]+\tau\,(\tau+2\,i\,\omega)\,\omega\,\text{Cosh}[4\,x\,\tau+2\,i\,x\,\omega]+(-2\,i\,\tau+\omega)\,(\tau\,(\tau+2\,i\,\omega)\,\text{Cos}[2\,x\,\omega]+2\,i\,\omega^2\,\text{Sin}[2\,x\,\omega]\,\text{Sinh}\left[2\,x\,(\tau+i\,\omega)\right])\right)z[s]-8\,\text{Sinh}\left[x\,(\tau+i\,\omega)\right]^2 z'[s]\right)$$

In[5]:=
```
I2[x_] :=
 Collect[Integrate[
   (z'[s] w[ξ] + 3 z[s] Abs[z[s]]^2 w[ξ] Abs[w[ξ]]^2) (Cosh[μ ξ]/μ), {ξ, 0, x}],
  {Abs[z[s]]^2 z[s], z'[s]}
 ]
```

In[6]:=
```
Simplify[I2[x] /. {Im[c]^2 + Re[c]^2 → Abs[c]^2}]
```

Out[6]=
$$\frac{1}{16\,(\tau+i\,\omega)^2}\,c\left(\frac{1}{\tau}3\,(\tau+i\,\omega)\,\text{Abs}[c]^2\,\text{Abs}[z[s]]^2\left(-\left(\left(2\,i\,\left(\tau\,(\tau+2\,i\,\omega)+\omega^2\,\text{Cosh}\left[2\,x\,(\tau+i\,\omega)\right]\right)\right)\text{Sin}[2\,x\,\omega]\right)\Big/\left(\omega\,(-i\,\tau+2\,\omega)\right)\right)+\frac{2\,(\tau+i\,\omega)\,\text{Cos}[2\,x\,\omega]\,\text{Sinh}\left[2\,x\,(\tau+i\,\omega)\right]}{\tau+2\,i\,\omega}+\left(2\,\omega\,(-2\,i\,\tau+\omega)\,\text{Sinh}[2\,x\,\tau]+\tau\left(\left(-2\,i\,\tau+\omega\right)\text{Sin}[2\,x\,\omega]-i\,\omega\,\text{Sinh}[4\,x\,\tau+2\,i\,x\,\omega]\right)\right)\Big/\left(\omega\,(-2\,i\,\tau+\omega)\right)\right)z[s]+4\,\left(2\,x\,(\tau+i\,\omega)+\text{Sinh}\left[2\,x\,(\tau+i\,\omega)\right]\right)z'[s]\right)$$

In[7]:=

```
i1[x_] :=
 1/(16 (τ + i ω)^2) c (-1/(τ (2 τ + i ω) (τ + 2 i ω) ω) 3 (τ + i ω) Abs[c]^2 Abs[z[s]]^2 (-τ (τ + 2 i ω) ω -
      τ (τ + 2 i ω) (-2 i τ + ω) + 2 ω (-2 τ^2 - 3 i τ ω + ω^2) - 2 ω (-2 τ^2 - 3 i τ ω + ω^2)
       Cos[2 x ω] Cosh[2 x (τ + i ω)] + τ (τ + 2 i ω) ω Cosh[4 x τ + 2 i x ω] +
      (-2 i τ + ω) (τ (τ + 2 i ω) Cos[2 x ω] + 2 i ω^2 Sin[2 x ω] Sinh[2 x (τ + i ω)])) )
   z[s] - 8 Sinh[x (τ + i ω)]^2 z'[s])
```

In[8]:=

```
i2[x_] := 1/(16 (τ + i ω)^2) c (1/τ
   3 (τ + i ω) Abs[c]^2 Abs[z[s]]^2 (-(2 i (τ (τ + 2 i ω) + ω^2 Cosh[2 x (τ + i ω)]) Sin[2 x ω])/(ω (-i τ + 2 ω)) +
     (2 (τ + i ω) Cos[2 x ω] Sinh[2 x (τ + i ω)])/(τ + 2 i ω) +
     (2 ω (-2 i τ + ω) Sinh[2 x τ] + τ ((-2 i τ + ω) Sin[2 x ω] - i ω Sinh[4 x τ + 2 i x ω]))/(ω (-2 i τ + ω)))
   z[s] + 4 (2 x (τ + i ω) + Sinh[2 x (τ + i ω)]) z'[s])
```

DERIVATES OF INTEGRALS

In[10]:= `D[i1[x], x]`

Out[10]=

```
1/(16 (τ + i ω)^2)
c (-1/(τ (2 τ + i ω) (τ + 2 i ω) ω) 3 (τ + i ω) Abs[c]^2 Abs[z[s]]^2 (4 ω^2 (-2 τ^2 - 3 i τ ω + ω^2)
      Cosh[2 x (τ + i ω)] Sin[2 x ω] - 4 (τ + i ω) ω (-2 τ^2 - 3 i τ ω + ω^2) Cos[2 x ω]
      Sinh[2 x (τ + i ω)] + (-2 i τ + ω) (-2 τ (τ + 2 i ω) ω Sin[2 x ω] + 4 i (τ + i ω)
        ω^2 Cosh[2 x (τ + i ω)] Sin[2 x ω] + 4 i ω^3 Cos[2 x ω] Sinh[2 x (τ + i ω)]) +
     τ (τ + 2 i ω) (4 τ + 2 i ω) ω Sinh[4 x τ + 2 i x ω]) z[s] -
  16 (τ + i ω) Cosh[x (τ + i ω)] Sinh[x (τ + i ω)] z'[s])
```

In[11]:=
```
D[i2[x], x]
```

Out[11]=
$$\frac{1}{16\left(\tau+i\omega\right)^2}$$
$$c\left(\frac{1}{\tau}3\left(\tau+i\omega\right)\text{Abs}[c]^2\,\text{Abs}[z[s]]^2\left(\frac{4\left(\tau+i\omega\right)^2\text{Cos}[2x\omega]\,\text{Cosh}\left[2x\left(\tau+i\omega\right)\right]}{\tau+2i\omega}-\right.\right.$$
$$\frac{4i\,\text{Cos}[2x\omega]\left(\tau\left(\tau+2i\omega\right)+\omega^2\,\text{Cosh}\left[2x\left(\tau+i\omega\right)\right]\right)}{-i\tau+2\omega}+$$
$$\frac{1}{\omega\left(-2i\tau+\omega\right)}\left(4\tau\omega\left(-2i\tau+\omega\right)\text{Cosh}[2x\tau]+\right.$$
$$\left.\tau\left(2\omega\left(-2i\tau+\omega\right)\text{Cos}[2x\omega]-i\left(4\tau+2i\omega\right)\omega\,\text{Cosh}[4x\tau+2ix\omega]\right)\right)-$$
$$\frac{4\left(\tau+i\omega\right)\omega\,\text{Sin}[2x\omega]\,\text{Sinh}\left[2x\left(\tau+i\omega\right)\right]}{\tau+2i\omega}-$$
$$\left.\frac{4i\left(\tau+i\omega\right)\omega\,\text{Sin}[2x\omega]\,\text{Sinh}\left[2x\left(\tau+i\omega\right)\right]}{-i\tau+2\omega}\right)z[s]+$$
$$\left.4\left(2\left(\tau+i\omega\right)+2\left(\tau+i\omega\right)\text{Cosh}\left[2x\left(\tau+i\omega\right)\right]\right)z'[s]\right)$$

In[12]:=
```
pi1[x_] := 1/(16 (τ + i ω)^2) c
 (-1/(τ (2 τ + i ω) (τ + 2 i ω) ω) 3 (τ + i ω) Abs[c]^2 Abs[z[s]]^2 (4 ω^2 (-2 τ^2 - 3 i τ ω + ω^2)
      Cosh[2 x (τ + i ω)] Sin[2 x ω] - 4 (τ + i ω) ω (-2 τ^2 - 3 i τ ω + ω^2) Cos[2 x ω]
      Sinh[2 x (τ + i ω)] + (-2 i τ + ω) (-2 τ (τ + 2 i ω) ω Sin[2 x ω] + 4 i (τ + i ω)
         ω^2 Cosh[2 x (τ + i ω)] Sin[2 x ω] + 4 i ω^3 Cos[2 x ω] Sinh[2 x (τ + i ω)]) +
    τ (τ + 2 i ω) (4 τ + 2 i ω) ω Sinh[4 x τ + 2 i x ω]) z[s] -
  16 (τ + i ω) Cosh[x (τ + i ω)] Sinh[x (τ + i ω)] z'[s])
```

In[13]:=
```
pi2[x_] := 1/(16 (τ + i ω)^2) c
 (1/τ 3 (τ + i ω) Abs[c]^2 Abs[z[s]]^2 (4 (τ + i ω)^2 Cos[2 x ω] Cosh[2 x (τ + i ω)]/(τ + 2 i ω) -
      4 i Cos[2 x ω] (τ (τ + 2 i ω) + ω^2 Cosh[2 x (τ + i ω)])/(-i τ + 2 ω) +
      1/(ω (-2 i τ + ω)) (4 τ ω (-2 i τ + ω) Cosh[2 x τ] +
        τ (2 ω (-2 i τ + ω) Cos[2 x ω] - i (4 τ + 2 i ω) ω Cosh[4 x τ + 2 i x ω])) -
      4 (τ + i ω) ω Sin[2 x ω] Sinh[2 x (τ + i ω)]/(τ + 2 i ω) -
      4 i (τ + i ω) ω Sin[2 x ω] Sinh[2 x (τ + i ω)]/(-i τ + 2 ω)) z[s] +
    4 (2 (τ + i ω) + 2 (τ + i ω) Cosh[2 x (τ + i ω)]) z'[s])
```

In[14]:=
```
q2 := -(pi1[0] + μ I2[0]) / μ
```

In[15]:=
```
q2
```

Out[15]= 0

In[16]:=
```
FullSimplify[α i1[x0] Cosh[μ x0] + α i2[x0] Sinh[μ x0] - pi1[1] Cosh[μ] -
  pi2[1] Sinh[μ] - μ i1[1] Sinh[μ] - μ i2[1] Cosh[μ] + z'[s] w[x0]]
```

Out[16]=
$$\frac{1}{16} c \left(-\frac{1}{\tau (2\tau + i\omega)(\tau + 2i\omega)\omega} 3i\,\text{Abs}[c\,z[s]]^2 \right.$$
$$(\alpha(-2(2\tau + i\omega)(\tau + 2i\omega)\cosh[(\tau - i\omega)x_0] + 2(\tau^2 + 4i\tau\omega - \omega^2)\cosh[(\tau + i\omega)x_0] + i(\tau + 2i\omega)\omega\cosh[(3\tau + i\omega)x_0] + \tau(2\tau + i\omega)\cosh[(\tau + 3i\omega)x_0]) +$$
$$\cosh[\tau](-6i\tau^3 + 13\tau^2\omega - 3i\tau\omega^2 + 4\omega^3 + 2(3\tau + i\omega)(\tau + 2i\omega)\omega\cosh[2\tau])$$
$$\sin[\omega] + \tau(\tau + 3i\omega)(-2i\tau + \omega)\cosh[\tau]\sin[3\omega] -$$
$$\tau(2\tau + i\omega)(\tau + 3i\omega)\cos[3\omega]\sinh[\tau] + \cos[\omega](2\tau^3 - 7i\tau^2\omega +$$
$$23\tau\omega^2 + 8i\omega^3 + 2(\tau + 2i\omega)\omega(-3i\tau + \omega)\cosh[2\tau])\sinh[\tau])\,z[s] -$$
$$\frac{1}{\tau + i\omega} 8\,((\tau + i\omega)(\cosh[\tau + i\omega] - 2\cosh[(\tau + i\omega)x_0]) + \sinh[\tau + i\omega] -$$
$$\left. \alpha\sinh[(\tau + i\omega)x_0]\,x_0)\,z'[s] \right)$$

In[17]:=
```
num = 1/16 c (-1/(τ (2 τ + i ω) (τ + 2 i ω) ω) 3 i Abs[c z[s]]^2
  (α (-2 (2 τ + i ω) (τ + 2 i ω) Cosh[(τ - i ω) x0] + 2 (τ^2 + 4 i τ ω - ω^2) Cosh[(τ + i ω)
        x0] + i (τ + 2 i ω) ω Cosh[(3 τ + i ω) x0] + τ (2 τ + i ω) Cosh[(τ + 3 i ω) x0]) +
    Cosh[τ] (-6 i τ^3 + 13 τ^2 ω - 3 i τ ω^2 + 4 ω^3 + 2 (3 τ + i ω) (τ + 2 i ω) ω Cosh[2 τ])
     Sin[ω] + τ (τ + 3 i ω) (-2 i τ + ω) Cosh[τ] Sin[3 ω] -
    τ (2 τ + i ω) (τ + 3 i ω) Cos[3 ω] Sinh[τ] + Cos[ω]
     (2 τ^3 - 7 i τ^2 ω + 23 τ ω^2 + 8 i ω^3 + 2 (τ + 2 i ω) ω (-3 i τ + ω) Cosh[2 τ]) Sinh[τ])
  z[s] - 1/(τ + i ω) 8 ((τ + i ω) (Cosh[τ + i ω] - 2 Cosh[(τ + i ω) x0]) +
    Sinh[τ + i ω] - α Sinh[(τ + i ω) x0] x0) z'[s])
```

Out[17]=
$$\frac{1}{16} c \left(-\frac{1}{\tau (2\tau + i\omega)(\tau + 2i\omega)\omega} 3i\,\text{Abs}[c\,z[s]]^2 \right.$$
$$(\alpha(-2(2\tau + i\omega)(\tau + 2i\omega)\cosh[(\tau - i\omega)x_0] + 2(\tau^2 + 4i\tau\omega - \omega^2)\cosh[(\tau + i\omega)x_0] + i(\tau + 2i\omega)\omega\cosh[(3\tau + i\omega)x_0] + \tau(2\tau + i\omega)\cosh[(\tau + 3i\omega)x_0]) +$$
$$\cosh[\tau](-6i\tau^3 + 13\tau^2\omega - 3i\tau\omega^2 + 4\omega^3 + 2(3\tau + i\omega)(\tau + 2i\omega)\omega\cosh[2\tau])$$
$$\sin[\omega] + \tau(\tau + 3i\omega)(-2i\tau + \omega)\cosh[\tau]\sin[3\omega] -$$
$$\tau(2\tau + i\omega)(\tau + 3i\omega)\cos[3\omega]\sinh[\tau] + \cos[\omega](2\tau^3 - 7i\tau^2\omega +$$
$$23\tau\omega^2 + 8i\omega^3 + 2(\tau + 2i\omega)\omega(-3i\tau + \omega)\cosh[2\tau])\sinh[\tau])\,z[s] -$$
$$\frac{1}{\tau + i\omega} 8\,((\tau + i\omega)(\cosh[\tau + i\omega] - 2\cosh[(\tau + i\omega)x_0]) + \sinh[\tau + i\omega] -$$
$$\left. \alpha\sinh[(\tau + i\omega)x_0]\,x_0)\,z'[s] \right)$$

In[18]:=
```
q1 = num / (μ Sinh[μ] - α Cosh[μ x0])
```

Out[18]=
$$\left(c\left(-\frac{1}{\tau\,(2\tau+i\omega)\,(\tau+2i\omega)\,\omega}\right.\right.$$
$$3i\,\text{Abs}[c\,z[s]]^2\,(\alpha\,(-2\,(2\tau+i\omega)\,(\tau+2i\omega)\,\text{Cosh}[(\tau-i\omega)\,x_0]+$$
$$2\,(\tau^2+4i\tau\omega-\omega^2)\,\text{Cosh}[(\tau+i\omega)\,x_0]+$$
$$i\,(\tau+2i\omega)\,\omega\,\text{Cosh}[(3\tau+i\omega)\,x_0]+\tau\,(2\tau+i\omega)\,\text{Cosh}[(\tau+3i\omega)\,x_0])+$$
$$\text{Cosh}[\tau]\,(-6i\tau^3+13\tau^2\omega-3i\tau\omega^2+4\omega^3+2\,(3\tau+i\omega)\,(\tau+2i\omega)\,\omega\,\text{Cosh}[2\tau])$$
$$\text{Sin}[\omega]+\tau\,(\tau+3i\omega)\,(-2i\tau+\omega)\,\text{Cosh}[\tau]\,\text{Sin}[3\omega]-$$
$$\tau\,(2\tau+i\omega)\,(\tau+3i\omega)\,\text{Cos}[3\omega]\,\text{Sinh}[\tau]+\text{Cos}[\omega]\,(2\tau^3-7i\tau^2\omega+$$
$$23\tau\omega^2+8i\omega^3+2\,(\tau+2i\omega)\,\omega\,(-3i\tau+\omega)\,\text{Cosh}[2\tau])\,\text{Sinh}[\tau])\,z[s]-$$
$$\frac{1}{\tau+i\omega}8\,((\tau+i\omega)\,(\text{Cosh}[\tau+i\omega]-2\,\text{Cosh}[(\tau+i\omega)\,x_0])+\text{Sinh}[\tau+i\omega]-$$
$$\left.\left.\alpha\,\text{Sinh}[(\tau+i\omega)\,x_0]\,x_0)\,z'[s]\right)\right)\Big/$$
$$(16\,(-\alpha\,\text{Cosh}[(\tau+i\omega)\,x_0]+(\tau+i\omega)\,\text{Sinh}[\tau+i\omega]))$$

```
ExpandAll[q1]
```

$$\frac{\text{num}}{-\alpha\,\text{Cosh}[(\tau+i\omega)\,x_0]+(\tau+i\omega)\,\text{Sinh}[\tau+i\omega]}$$

In[21]:=
```
FullSimplify[q1]
```

Out[21]=
$$\left(c\left(-\frac{1}{\tau\,(2\tau+i\omega)\,(\tau+2i\omega)\,\omega}\right.\right.$$
$$3i\,\text{Abs}[c\,z[s]]^2\,(\alpha\,(-2\,(2\tau+i\omega)\,(\tau+2i\omega)\,\text{Cosh}[(\tau-i\omega)\,x_0]+$$
$$2\,(\tau^2+4i\tau\omega-\omega^2)\,\text{Cosh}[(\tau+i\omega)\,x_0]+$$
$$i\,(\tau+2i\omega)\,\omega\,\text{Cosh}[(3\tau+i\omega)\,x_0]+\tau\,(2\tau+i\omega)\,\text{Cosh}[(\tau+3i\omega)\,x_0])+$$
$$\text{Cosh}[\tau]\,(-6i\tau^3+13\tau^2\omega-3i\tau\omega^2+4\omega^3+2\,(3\tau+i\omega)\,(\tau+2i\omega)\,\omega\,\text{Cosh}[2\tau])$$
$$\text{Sin}[\omega]+\tau\,(\tau+3i\omega)\,(-2i\tau+\omega)\,\text{Cosh}[\tau]\,\text{Sin}[3\omega]-$$
$$\tau\,(2\tau+i\omega)\,(\tau+3i\omega)\,\text{Cos}[3\omega]\,\text{Sinh}[\tau]+\text{Cos}[\omega]\,(2\tau^3-7i\tau^2\omega+$$
$$23\tau\omega^2+8i\omega^3+2\,(\tau+2i\omega)\,\omega\,(-3i\tau+\omega)\,\text{Cosh}[2\tau])\,\text{Sinh}[\tau])\,z[s]-$$
$$\frac{1}{\tau+i\omega}8\,((\tau+i\omega)\,(\text{Cosh}[\tau+i\omega]-2\,\text{Cosh}[(\tau+i\omega)\,x_0])+\text{Sinh}[\tau+i\omega]-$$
$$\left.\left.\alpha\,\text{Sinh}[(\tau+i\omega)\,x_0]\,x_0)\,z'[s]\right)\right)\Big/$$
$$(16\,(-\alpha\,\text{Cosh}[(\tau+i\omega)\,x_0]+(\tau+i\omega)\,\text{Sinh}[\tau+i\omega]))$$

In[35]:=
```
q1 /. {τ + i ω → M, τ - i ω → Conjugate[M], τ + 2 i ω → M1, τ + 3 i ω → M2,
  2 τ + i ω → N1, 3 τ + i ω → N2, -2 i τ + ω → K1, -3 i τ + ω → K2}
```

Out[35]=
$$\frac{1}{16\,(-\alpha\,\text{Cosh}[M\,x_0]+M\,\text{Sinh}[M])}$$
$$c\left(-\frac{1}{M1\,N1\,\tau\,\omega}3\,i\,\text{Abs}[c\,z[s]]^2\,\left(\alpha\,\left(2\,\left(\tau^2+4\,i\,\tau\,\omega-\omega^2\right)\text{Cosh}[M\,x_0]+\right.\right.\right.$$
$$N1\,\tau\,\text{Cosh}[M2\,x_0]+i\,M1\,\omega\,\text{Cosh}[N2\,x_0]-2\,M1\,N1\,\text{Cosh}[\text{Conjugate}[M]\,x_0]\Big)+$$
$$\text{Cosh}[\tau]\,\left(-6\,i\,\tau^3+13\,\tau^2\,\omega-3\,i\,\tau\,\omega^2+4\,\omega^3+2\,M1\,N2\,\omega\,\text{Cosh}[2\,\tau]\right)\text{Sin}[\omega]+$$
$$K1\,M2\,\tau\,\text{Cosh}[\tau]\,\text{Sin}[3\,\omega]-M2\,N1\,\tau\,\text{Cos}[3\,\omega]\,\text{Sinh}[\tau]+$$
$$\left.\text{Cos}[\omega]\,\left(2\,\tau^3-7\,i\,\tau^2\,\omega+23\,\tau\,\omega^2+8\,i\,\omega^3+2\,K2\,M1\,\omega\,\text{Cosh}[2\,\tau]\right)\text{Sinh}[\tau]\right)z[s]-$$
$$\left.\frac{1}{M}8\,(M\,(\text{Cosh}[M]-2\,\text{Cosh}[M\,x_0])+\text{Sinh}[M]-\alpha\,\text{Sinh}[M\,x_0]\,x_0)\,z'[s]\right)$$

In[41]:=
```
μ1 := τ + 2 i ω
μ2 := τ + 3 i ω
η1 := 2 τ + i ω
η2 := 3 τ + i ω

Q1 := c / (16 (-α Cosh[μ x0] + μ Sinh[μ]))
  (-(3 i / (μ1 η1 τ ω)) Abs[c z[s]]^2 (α (2 (τ^2 + 4 i τ ω - ω^2) Cosh[μ x0] +
        η1 τ Cosh[μ2 x0] + i μ1 ω Cosh[η2 x0] - 2 μ1 η1 Cosh[Conjugate[μ] x0]) +
      Cosh[τ] (-6 i τ^3 + 13 τ^2 ω - 3 i τ ω^2 + 4 ω^3 + 2 μ1 η2 ω Cosh[2 τ]) Sin[ω] -
      i η1 μ2 τ Cosh[τ] Sin[3 ω] - μ2 η1 τ Cos[3 ω] Sinh[τ] +
      Cos[ω] (2 τ^3 - 7 i τ^2 ω + 23 τ ω^2 + 8 i ω^3 - 2 i η2 μ1 ω Cosh[2 τ]) Sinh[τ]) z[s] -
    (1/μ) 8 (μ (Cosh[μ] - 2 Cosh[μ x0]) + Sinh[μ] - α Sinh[μ x0] x0) z'[s])
```