Optimal Blackjack Strategies Using Monte Carlo Simulation

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Abstract

A complete framework is described for producing optimal blackjack strategies for a combination of rules and decks in play. The strategies can be produced analytically or using an efficient Monte Carlo based method. The MC based approach employs a set of algorithms capable of producing strategies with over 99.9% of moves optimally classified. This process is completed in under 3 minutes on modern consumer hardware involving over 2 billion Monte Carlo simulations. The strategies can account for 3,072 combinations of rules, including any number of decks and an arbitrary return for obtaining blackjack. The functionality is provided in a fully documented R package which can be used on any machine capable of running R.

1 Introduction

1.1 Background

The goal of blackjack is to obtain as high a score as possible without exceeding 21. This led to the game initially taking the name "twenty-one". It is referred to as such in the first evidence of the game which came in a novel by 16^{th} Century Spanish novelist Miguel de Cervante[1]. The version described in the novel is reminiscent of the basic structure of the game today. Despite this, the game "vingt-et-un" is commonly associated with 18^{th} Century France[2]. It is often claimed that the name "blackjack" originated in 19^{th} Century New Orleans due to a 10:1 return for drawing an ace of spades and one of the black jack cards. There is no evidence to suggest that this claim is anything but rumour. A more substantial claim is that the name comes from the Klondike Gold Rush where a precursor to gold called Sphalerite was referred to as "blackjack" from which the miners coined the name[3].

The game would go on to become a cornerstone in casinos all over the world with its strategic depth masquerading behind apparent simplicity. This veil was removed in 1956 when a group of Maryland mathematicians derived a mathematical framework that contradicted traditional blackjack strategy [4]. Though the work used approximations to true probabilities, it began an iterative process of creating optimal strategies for every possible ruleset. As computer performance began to increase, fewer approximations were needed in strategy production. A leader in this area is Edward Thorp who used Fortran to develop card counting strategies which had not been fully understood before this[5]. Card counting accounts for the cards removed from the deck since the last reshuffle giving the player a potentially profitable strategy with an appropriate ruleset. When these findings were synopsised in his 1964 book *Beat the Dealer: A Winning Strategy for the Game of Twenty-One*, it began a perpetual cycle of players creating new strategies and casinos mitigating their effectiveness[5].

The thousands of possible hands, the differing number of decks and endless other additives create an enumerable amount of possible blackjack configurations. An optimal strategy is only optimal for the rules on which it is based as well as the number of decks used. Any deviation in rules or decks used will render the previously calculated strategy sub-optimal which justified the study of computational strategy generation. Using modern computer hardware, strategies for individual casino rulesets can be constructed in a short time, which is a goal of this work.

The gambling industry recently begun to use personal data to target individuals with low income and those recovering from gambling addiction[6]. As casino profit is directly linked with maximizing customer loss, there is significant potential danger in these practices. The increasing prevalence of roulette based "loot-boxes" in games made for children exemplifies this[7]. There is evidence that even when these boxes are free to acquire, that they act as a gateway to payed gambling[8]. The gambling industry has also begun to exploit this phenomenon known as "social gambling" by targeting the same young adults[9]. This project provides a set of tools to those interested in gambling to quantify the game of blackjack in an accessible and efficient way. Expected returns can be illustrated to show how much one can expect to lose per play session. In doing so, it represents a small contribution to tackling the predatory practices of the gambling industry against vulnerable people.

1.2 Rules

Blackjack is played between two distinct types of players which will be referred to as the "player" and the "dealer". There can be many players in a single game but there is always one dealer only. To determine the score of a hand, the values of the individual cards in the hand are summed. The numeric cards take their numeric value. The face cards are equivalent to 10 so there are effectively 16 unique 10 cards per deck. The ace can be 11 or 1 which is chosen at the player's discretion. The dealer can only set the ace to a score of 1 if their score would otherwise exceed 21. It is sometimes permitted for the dealer to attribute a score of 1 to the ace if they have a score of 17 otherwise, but this is not standard[10]. At the beginning of the game, the player places a unit bet down. If the player wins, they receive their original bet back with an additional unit bet for wining. If the dealer wins, the original bet is kept. When the bet is made, the dealer shuffles up to 8 decks together. Two cards are then dealt to both the player and the dealer. The player's cards are face up while only one of the dealer's cards is visible. If the player has a score of 21 on their initial hand, their hand is called blackjack. When this occurs, the player receives an additional return on their original bet. If the dealer too receives blackjack, the hand is a draw and the bet is returned to the player[11].

Once both people receive their initial hands, the player takes control of the game. The player has several choices of how to proceed, which all must be accounted for in an optimal strategy. The possible moves are hitting, sticking, surrendering, doubling down and splitting. Splitting and surrendering can be exercised on the first move only. As the player can exercise these moves at their discretion, each rule increases the player's expected return. Different casinos have conditions on when and how often these options can be exercised. The options are described below:

Stick: The player chooses to not draw a card, ceasing their involvement in the game. All hands end in sticking or in the player going bust unless there is a surrender.

Hit: The player draws an additional card from the deck.

Surrender: The player may choose to surrender on their initial hand. This ends the hand immediately with the player receiving half of their original bet back.

Split: If the player draws a pair, they may split their hand into two independent hands with each holding one of the pair cards. To facilitate this, the player must place an additional bet. These hands are then dealt an additional card and the game continues as though there are two independent players opposing the same dealer. The split hands cannot obtain blackjack and split hands may often be re-split if another pair is drawn. The number of permitted re-splits varies depending on the betting house. Split hands are often permitted to double down but never to surrender.

Double Down: The player doubles their bet with the condition that they must draw one card and stick.

Once the player has stuck or gone bust, their involvement in the game ceases. The dealer then reveals their face down card. The dealer must then draw cards until their score exceeds 16. As mentioned, the ace attributes a score of 11 unless this busts the dealer's hand in which case the ace attributes a score of 1. Once the dealer's score exceeds 16, the player and dealer scores are compared. The winner is the person with the higher score conditional on the score not exceeding 21. If the scores are equal, the game is a tie and the player receives their bet back. If both parties bust, the dealer still receives the player's bet.

The information available to the player is the cards in their hand and the visible dealer card. This provides information on the cards still in the deck and the probability distribution of their next draw. As more decks are included, drawn cards perturb the deck probability distribution by smaller amounts. This decreases the effect of an optimal player strategy and always favors the dealer.

1.3 Nomenclature

Position : A combination of a player hand and a dealer visible card representing all information available to the player. A position can be described by a unique 12 digit number, which is stored as a floating point number due to its size. The first digit from left to right is 1 which avoids truncation of the number. The second number is the dealer's visible card. The remaining digits describe the player's hand. The third digit from the left is the number of 9 cards in the players hand. Next is the number of 8 cards etc. The final two digits are the number of 10 and ace cards, respectively. Jack, queen and king are not used in this work with each deck having 16 unique 10 cards instead. A position with the player holding a 2 and 3 with the dealer holding a 7 is described by the index 170000001100. Note: This indexing system cannot store hands with more than 9 of a single card. This can occur with 10 ace cards and 10 2 cards in games with more than 2 decks in play. The probability of these hands occurring is sufficiently small that they are neglected ($< 1 : 10^8$).

Daughter : A position which can be obtained from a parent position by drawing one specific card. The position 170000001100 is a daughter of the position 170000001000 obtained by drawing a 3.

Parent : A position which can be obtained from a daughter position by removing one specific card. The position 170000001000 is a parent of the position 170000001100 obtained by removing a 3.

Terminal Position : A position in which drawing any possible card will guarantee a score above 21. $\{10, 9, 2\}$ is an example of a terminal position.

2 Backward Induction

The main focus of this project is to obtain optimal player strategies for blackjack. This involves obtaining the move which maximizes the player's expected return for every possible position. This requires the expected return for all optional moves to be quantified at every possible position to obtain the set of optimal choices. The crucial device for obtaining these values is backward induction. Backward induction has been used in blackjack analysis before to investigate card counting in Zimram, *et al.*, (2009)[12]. Another analytical method exists for general stopping problems by Hill, *et al.*, (1990) which could be used as an alternative to backward induction but it is not explored in this work[13]. The method used in this work only produces legal strategies, unlike card counting and was derived independently. Given the expected return for sticking on every position, a backward induction algorithm returns a complete set of expected returns for hitting and doubling down at all positions. All other required expectation values can be calculated using the hit, stick and double down expected returns.

When the player hits, they may obtain 1 of up to 10 daughter hands. Backward induction defines the expected return of hitting at a position based on the optimal moves of those daughter hands. Each daughter position can hit, stick or double down. The one with the superior expected return is chosen as optimal and weighted by the probability of drawing that daughter. This is repeated for up to 10 daughters resulting in an expected hitting return for the parent position in terms of its daughter's optimal expected returns. Knowledge of the optimal choice for up to 10 daughter hands is required to obtain the numerical expected return, which is not known without knowing the daughter's expected return when hitting and doubling down. This recursive definition repeats until a *terminal hand* is reached, which is a position that guarantees a bust hand if the player draws. The existence of these hands makes the problem solvable.

The largest player hands in single-deck blackjack hold 11 cards. The largest hands are terminal by definition and therefore, sticking is the optimal move. The sticking expected return may be computed analytically or through Monte Carlo simulation. A position's double down expected return is the weighted sticking return of that position's 10 daughters. The double down expected returns must be calculated in tandem with the sticking probabilities as they affect the hitting expected returns for parent positions. When the optimal move's expected return is known for all positions with 11 cards, the expected return when hitting for the positions with 10 cards can be obtained. The 10 card positions allow for expected returns to be found for the 9 card positions and so on. This process is repeated for each level of player hand size until a full set of expectation values is obtained.

Only the ranking of the hitting and sticking expected returns must be correct to properly classify the optimal move. When a move is misclassified, this is because the most optimal move and second most optimal move have similar expected returns. Therefore, misclassified moves will result in minuscule reductions in win rate. This is why the problem is well suited to MC simulation, with relatively few simulations resulting in reasonably high classification accuracy. This remains true when surrendering and splitting are included in a strategy.

2.1 Exact Solution

The optimal strategy can be obtained provided the sticking expected return for every position is known. Determining these expectation values analytically is essential for testing the convergence of the MC simulations. Knowing the exact expectation values allows for approximation of the number of MC simulations required to properly classify most positions. By using the exact solution, the smallest difference between the hit and stick expected returns can be found which can be used as a standard error for the MC derived probabilities. The data required is the player's win rate for sticking at every possible position which may be obtained using a computer algorithm.

The algorithm takes a position where it is understood that the player has stuck, ending their involvement in the game. The cards in the position are removed from the deck. The player's score is calculated and the dealer hand is initialized with the single visible card associated with the position. The algorithm then calculates all final hands that the dealer can obtain and their associated probabilities. This is achieved by finding all of the routes to a final dealer hand and summing the probability of each route to that hand. The result is a complete set of probabilities for each possible final dealer hand. The proportion that are beaten by the position's associated player score is the player's win rate. These rates are then inputted into a backward induction algorithm resulting in an optimal strategy which may be used as a benchmark for the Monte

Carlo based approach.

2.2 Monte Carlo Simulation

A blackjack game begins with a number of decks being shuffled together into a single deck. Four cards are drawn with two given to both parties. The player then operates based on some strategy and the dealer performs their fixed role. These tasks are highly suited to simulation. The purpose of the model is to take as input a number of decks and a position on which it understood that the player has stuck. The model removes the cards in the position from the deck and initializes the dealer deck with the dealer visible card associated with the position. As the dealer has no input in how their involvement plays out, the model is only required to draw cards and check if the dealer score has exceeded 16. This is very simple computationally, allowing for rapid simulations. The model performs MC simulations a chosen number of times and outputs the player's approximate win rate as the ratio between games won and games played. The model can simulate 7×10^6 games/second from a sticking position on an Intel i5-8250u, the most ubiquitous laptop processor[14].

A second model is used to simulate games using the optimal strategy. The optimal strategy is inputted as a hash map where the keys represent a complete set of possible positions for the number of decks in play. The value associated with each key is the player's optimal decision at the position associated with the key. Stick, hit, double down, surrender and split are represented by 0,1,2,3 and 4, respectively. This means for different casino rulesets, the same function may be used where any optional move may be prohibited or allowed when producing the strategy. For example, if surrender is to be prohibited, the analysis will initialize all surrender expected returns to -2, guaranteeing them to be sub-optimal. When the strategy is used to play in this example, the optimal move will never be surrender. This model simulates 2.5×10^6 optimal games/second on the same hardware as previous.

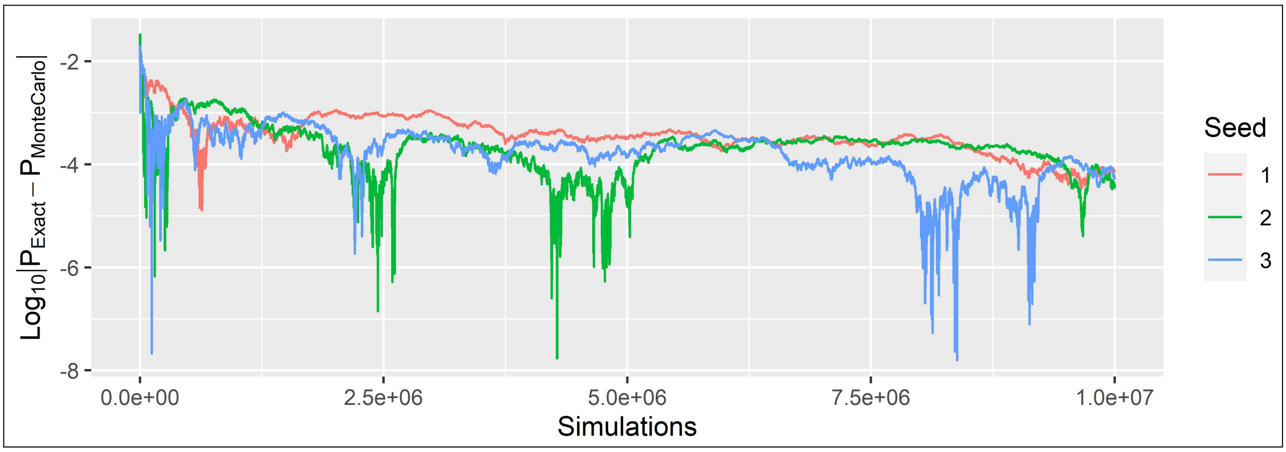

Figure 1: *The Monte Carlo derived sticking probabilities converge to the true probability to within* 10^{-3} *in* $\mathcal{O}(10^4)$ *simulations. An accuracy of* 10^{-4} *is achieved in* $\mathcal{O}(10^7)$ *simulations. This is in line with the expectation outlined in Eqn.(4.2).*

3 Fundamental Game Properties

The dealer is a proxy for the game's rules with every dealer in the world playing every position the same way. The predefined nature of the role allows for the dealer to be fully understood before investigating the player's strategy. The dealer's visible card provides significant information to the player. Different visible cards have vastly different distributions for the dealer's final score. This is investigated by simulating games and matching the dealer's final score with their first drawn card which is shown in Figure 2.

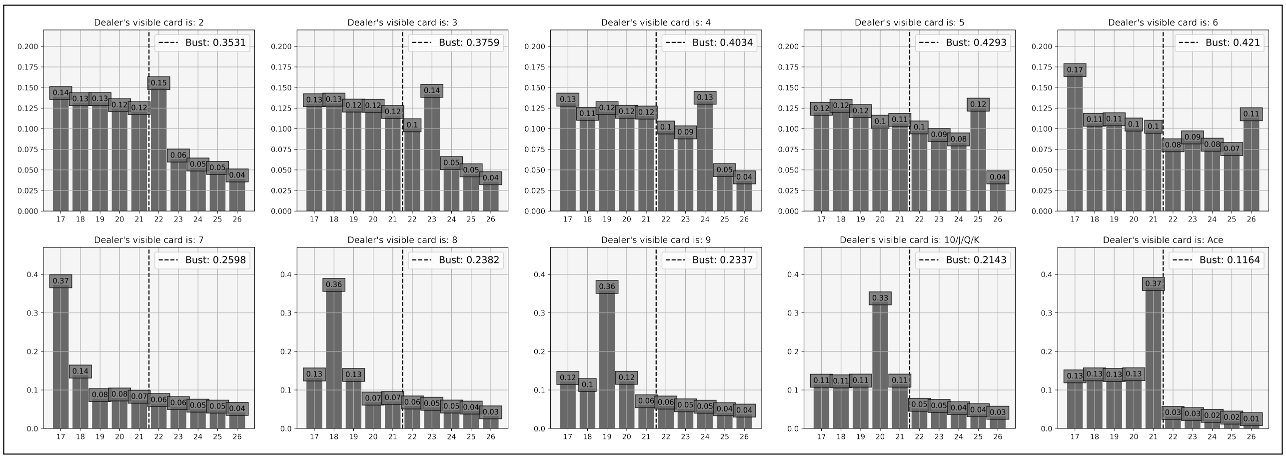

Figure 2: *The dealer's final score and bust rate depends on their visible card.* 10^8 *games were simulated resulting in the above dealer score distributions for each possible visible card.*

A simple strategy can be determined based on the dealer's visible card. Some cards result in very high bust rates such as a visible 6 card busting in 42.4% of games. In this case, the player could secure any valid hand of 17 or over. Conversely, a visible ace results in a bust hand in 11.61% of games, with 36% chance for a score of 21. In this case, the player could surrender if possible.

Another simple strategy for the player is to mimic the dealer by sticking on or above a specific number. To find out which number maximizes return, 10^8 games are played with the player sticking on or above x $\in$ [10,21]. Sticking below 10 is sub-optimal as no drawn card results in a bust hand. The dealer then sticks on 17. One of the many shortcomings of this strategy is that it treats a score with and without an ace as equivalent:

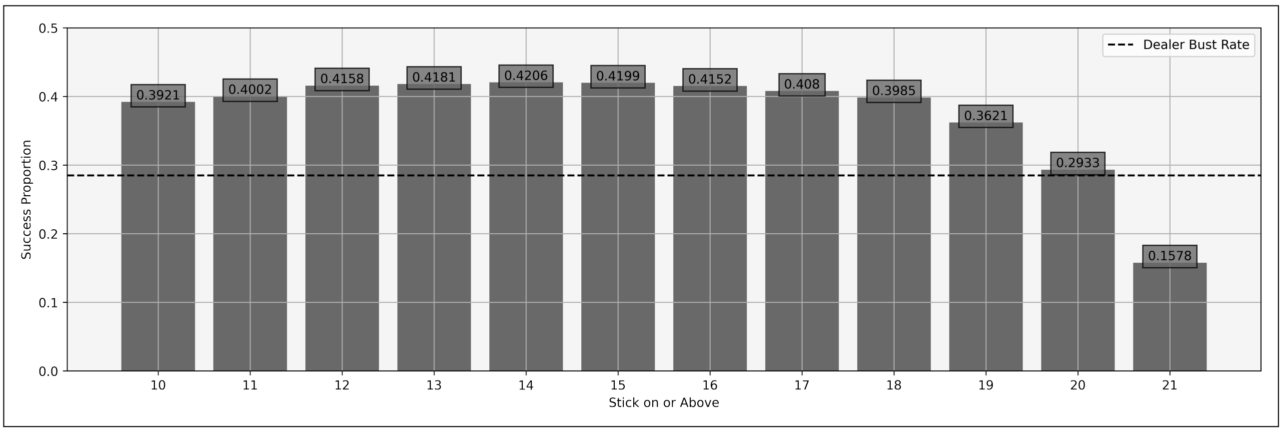

Figure 3: *Optimal player return is achieved for sticking on or above a score of 14 which results in a 42.06% win probability.*

There are 55 possible initial player hands, each having 10 possible dealer visible cards making 550 initial combinations. This number grows to 1,790 and 3,355 for player hand sizes of 3 and 4, respectively. The number of cards in the player's winning hands is an important quantity. This distribution of the number of cards in winning hands quantifies how significant the larger hand sizes are. Large hand sizes make up the bulk of simulations so if they rarely occur, omitting their simulation would dramatically improve efficiency. The model was used to simulate 10^8 games to determine how many cards and which cards were in the player's winning hands:

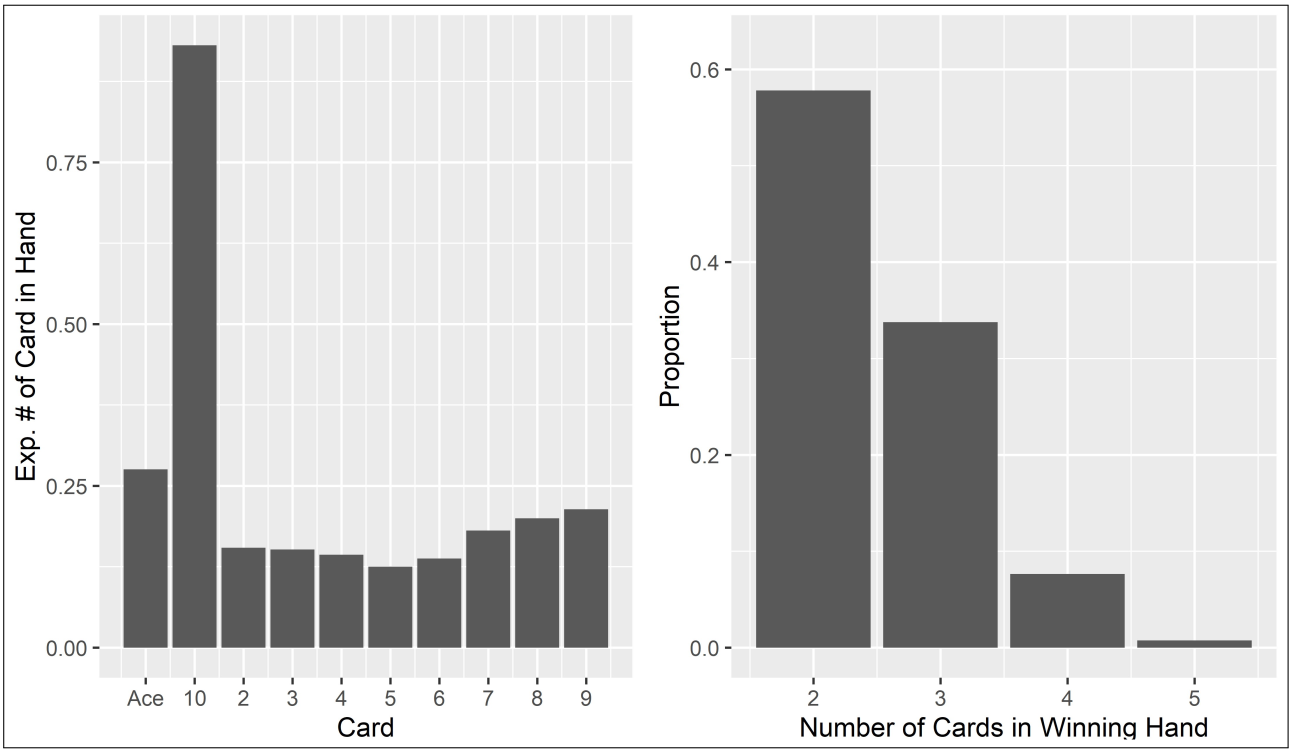

Figure 4: *(Left)The expected number of each card in a winning player hand. Ace has a proportion of roughly 0.25 implying an ace is in a quarter of player winning hands.(Right)The proportion of total cards in winning hands for the player when sticking on or above 14.*

The 10 cards represent 0.25 of the deck, hence its substantial presence in winning hands. The ace is the card most frequently in winning hands, after 10, which is expected as it allows the player an additional attempt at a high score without going bust. The right image is crucial as over 99% of games are completed in two draws or less from the initial hand. For single deck blackjack, this represents approximately 29% of the 19,620 possible positions. This proportion decreases further when using additional decks.

4 Data Analysis

The backward induction algorithm requires the stick expected return for every possible position. With these values known, the expected return for every optional move can be calculated. The required data analysis is intensive as each of stick, hit, double down, surrender and split require a unique framework to calculate their expected return. The five expectation values are compared at every position with the optimal one retained for use in the strategy. This process represents the bulk of the computation time for the Monte Carlo based strategies. The simplest of the frameworks is that of sticking, which is the win rate when sticking on a position. The approximate expected return when sticking, $\hat{ER}_{\text{stick}}$, at a position, p, is defined as

$$\hat{ER}(p)_{\text{stick}} = \frac{W - L}{W + L + T} = \frac{W - L}{N} \tag{4.1}$$

where W, L and T are the games won, lost and tied by the player, respectively. The outcome of a game follows a multinomial distribution, specifically the trinomial distribution. The variance of the player win rate is therefore, $\sigma^2(W) = p(W)(1 - p(W))$ where $p(W)$ is the probability of a player win. The variance for the win rate is maximized when the player win rate $p(W) = 0.5$

resulting in $\sigma^2(W) = 0.5(1-0.5) = 0.25$. Over N simulations, the maximum standard error of an approximated probability can be estimated as

$$\hat{S}^2(\hat{ER}(p)) \approx \frac{\sigma^2(W)}{N} \leq \frac{0.25}{N}. \tag{4.2}$$

For single deck blackjack, the smallest difference between a position's sticking, hitting and doubling down expected return is 7.5×10^{-6}. This number represents a bound on the magnitude of convergence required for the probabilities obtained using MC simulation. Substituting this value into Eq. (4.2) and solving for the number of simulations, N, yields an estimated upper bound for the magnitude of the number of simulations required for an optimal strategy of

$$N \approx \frac{\sigma^2}{S^2} \approx \frac{0.25}{(7.5 * 10^{-6})^2} \sim \mathcal{O}(10^9). \tag{4.3}$$

The expected return of hitting positions is determined by the positions that can be obtained from the hit. Each of these positions can hit, stick and double down so the choice with the largest expected return is used. The daughter position's optimal returns are then weighted by their probability of being drawn from the current position, w_i, to approximate the hitting expected return. For positions which cannot be drawn due to the required card not being in the deck, their *weight*, $w_i = 0$. This is the basis for the backward induction algorithm. With $\hat{ER}(p+i)_{\text{optimal}}$, the estimated optimal expected return for the daughter position obtained from position p by drawing card i, the estimated hitting return is

$$\hat{ER}(p)_{\text{hit}} = \sum_{i=0}^{9} w_i \hat{ER}(p+i)_{\text{optimal}}. \tag{4.4}$$

This extends easily to calculating the expected return for doubling down at a position p as

$$\hat{ER}(p)_{\text{double}} = 2\sum_{i=0}^{9} w_i \hat{ER}(p+i)_{\text{stick}}. \tag{4.5}$$

Surrendering is only valid for non-split initial hands and has an expected return $\hat{ER}(p)_{\text{surr.}} = -0.5$ for all initial hands when permitted. As initial draws make up a maximum of $\approx 3\%$ of all positions, surrender is the least frequent optimal move. However, the initial positions are drawn more frequently than any other positions so the player advantage associated with surrender is significant. This advantage is smaller than that attributed to splitting and smaller still compared to doubling down.

The final expected return is that of splitting. Splitting is only permitted for starting positions with a pair. There are 10 possible pairs each of which is matched with one of 10 possible visible dealer cards amounting to 100 positions which can split. The value of splitting for a candidate position is determined in a similar way to hitting. The relevant hitting position is the splitting position's visible dealer card and one of the pair cards in the player hand. The splitting expected return is twice the expected hit return for this position. The expected return for a sticking position, p, with parent position $p-i$ obtained by removing card i from p is

$$\hat{ER}(p)_{\text{split}} = 2\sum_{i=0}^{9} w_i \hat{ER}(p-i)_{\text{hit}}. \tag{4.6}$$

The deck is described by a discrete probability distribution. When a card is drawn and shown, the distribution is perturbed. The purpose of the optimal strategy is to choose actions based on the current state of this distribution. When hands are split, the first hand plays with the knowledge that its corresponding pair card is no longer in the deck. The second hand plays with the same knowledge as well as knowing the cards that the first hand drew. The depth of combinations possible is too extensive to simulate every splitting position. Therefore, this work assumes that the optimal strategy in the absence of splitting remains optimal when splitting has occurred. This could be avoided by implementing a pseudo card counting framework which could dynamically account for the cards in previous split hands. Another possible solution is to only simulate up to two draws and stick on anything more than this. Over 99% of moves that occur would be optimal this way while reducing the number of simulations by 71%. These options are possible additions to this work but will not be explored further.

All player hands with a valid score must be simulated, including the hands with a score of 21. The number of player hands that satisfy this when using an unlimited number of decks - equivalent to drawing from a single deck with replacement - is 3,072. Each of these hands is paired with one of ten visible dealer cards amounting to 30,720 positions. For a single deck game, there are 2,008 valid hands with 19,620 combinations of hand and visible dealer card, all of which must be simulated until sufficient levels of convergence are achieved. A classification accuracy of 99.6% is achieved with $\mathcal{O}(10^4)$ Monte Carlo simulations per position. This amounts to $\mathcal{O}(10^8)$ simulations per optimal strategy. When a change occurs to the number of decks used in the game or to any rule, the strategy must be remade entirely.

4.1 Employing Strategy

The data from the model is analysed using the framework discussed above. The result is represented in a hash map where the keys correspond to the positions using the indexing method from Section 1.3. The value associated with a specific key is the optimal move for that position. Stick, hit, double down, surrender and split are represented by 0,1,2,3 and 4, respectively. The constant lookup times of a hash map are essential for fast game simulation. Random games are played with the player's current position inputted to the hash map which outputs the optimal decision. The expected return of the strategy is the difference between games won and lost divided by the total number of games played.

5 Results

When discussing the rules for a particular strategy, the following should be assumed unless stated otherwise:

- Blackjack has a return of 3:2.
- Hands can be split up to 4 times when splitting is permitted.
- Double down is permitted on split hands.
- Split aces cannot be hit.
- Surrender permitted for all visible dealer cards, even when dealer blackjack is possible.
- The face cards and 10 cards are equivalent so non-identical pairs may be split.
- The ace scores 11 for the dealer unless they are bust, in which case the ace scores 1.
- Dealer blackjack takes the player's initial bet only.

The strategy master function can generate 3,072 possible rulesets which could reasonably be found in a gambling house. This is achieved using any combination of the following common rules where the number of configurations for each option is shown in brackets:

- (8)Arbitrary number of decks but only $\in [1, 8]$ are common.
- (8)Any of double down, surrender, split permitted or not.
- (4)Hand can be split any number of times but $\in [1, 4]$ times are common.
- (2)Surrender allowed or not with the dealer showing an ace or 10.
- (3)Arbitrary blackjack return but 2:1, 3:2 and 6:5 are common.
- (2)Hit on split aces allowed or not.

5.1 Single Deck

A strategy is generated for a set of rules using the methodology discussed in Section 4. The most significant rules are whether surrendering, splitting and doubling down are permitted. To determine how significant each one is, the rules must be implemented individually as they do not affect the game independently. A strategy is generated where each option is permitted alone and each strategy is produced using 100, 500, 1,000 and 10,000 simulations to illustrate how the strategy win rate improves as more simulations are used. This is repeated 10 times where each calculated strategy plays 10^7 games to approximate the player's expected return. The results are compared to the optimal strategy outlined in Section 2.1. This amounts to 250 individual strategies and 2.5×10^9 simulated games in total:

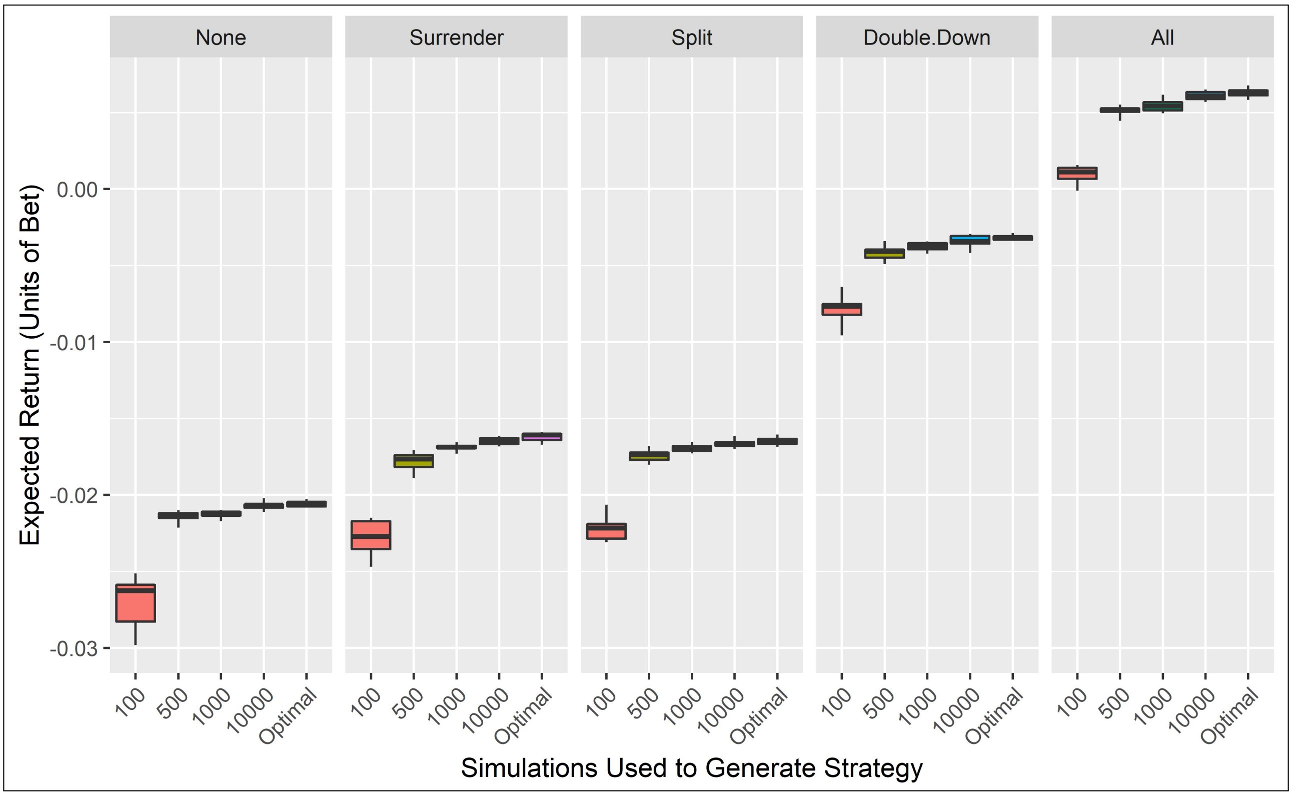

Figure 5: *The expected return per game where each box represents 10 independent strategies, each playing* 10^7 *games. The "optimal" strategy is derived in Section 2.1 using an independent method. The Monte Carlo derived strategies converge to the optimal strategies. The pane labels describe the options available to the player.*

The image illustrates that doubling down is by far the most significant option available to the player. Unrestricted doubling down increases the player expected return by 0.0175, equivalent to 1.75 cents on a €1 stake. Splitting is simulated with its common restrictions in place. Split aces are not permitted to hit which reduces the increased return when splitting is permitted by 0.002. A common restriction placed on surrendering does not allow surrender when the dealer's visible card is 10 or ace. These hands are the strongest and therefore, represent most of the hands for which surrender is optimal. This restriction reduces the expected return game from surrender by 0.003 which is a large proportion of the gain associated surrendering. This justified the omission of this restriction above. When all three options are permitted, the expected return is approximately 80% of their independent contributions, implying only slight dependence between the options.

5.2 Multiple Decks

The number of possible positions per game increases with the number of decks used. Once three decks have been included, all positions with a valid player score can be acquired. As more decks are added, the expected return for each position changes and the value of the information available to the player is diluted. The player is also less likely to obtain blackjack when more decks are used. Therefore, adding more decks always increases the house advantage. To quantify how much the house advantage increases with number of decks, strategies with varying rules are produced. The strategies are made using 10^4 simulations for each position with each strategy optimized for a specific set of rules and number of decks. The return from a real casino's ruleset can be approximated by matching it to ruleset from Table 1 that it most closely resembles. Each strategy is used to play 10^8 games to quantify the player's expected return.

Decks	None	F	S	FS	D	FD	SD	FSD
1	-0.02071	-0.01667	-0.01668	-0.01264	-0.00324	0.00123	0.00129	0.00613
2	-0.02316	-0.01909	-0.01845	-0.01427	-0.00668	-0.00230	-0.00160	0.00313
3	-0.02391	-0.01976	-0.01914	-0.01464	-0.00797	-0.00332	-0.00225	0.00208
4	-0.02416	-0.02018	-0.01929	-0.01490	-0.00826	-0.00397	-0.00270	0.00162
5	-0.02442	-0.02038	-0.01938	-0.01516	-0.00840	-0.00428	-0.00290	0.00144
6	-0.02461	-0.02044	-0.01944	-0.01528	-0.00915	-0.00447	-0.00316	0.00136
7	-0.02485	-0.02047	-0.01970	-0.01531	-0.00922	-0.00458	-0.00331	0.00118
8	-0.02505	-0.02052	-0.01992	-0.01531	-0.00927	-0.00481	-0.00350	0.00087

Table 1: *Player's expected return per unit bet when playing with a substantial spread of common rulesets. A value of 0.01 implies that a player betting a €1 stake should expect a 1 cent return per game. Standard error on all values is S = 0.00005. Key: F: Surrender permitted on initial hand for any dealer visible card. S: Splitting permitted on any hand and split aces cannot hit. D: Double down permitted on any hand including split hands.*

5.3 Practical Strategy Application

There are nearly 20,000 unique positions in the optimal single deck blackjack strategy. Many are insignificant as they describe positions that rarely occur or trivial decisions such as sticking on 21. An elite player could learn a subset of the strategy that retains most of the value in significantly fewer rules. To illustrate an idealized player's expected performance, 50 games are simulated while playing optimally, equivalent to roughly one hour of play. This is repeated 10^4 times to produce the densities shown in Figure 6.

To mimic a realistic player, the strategies produced via MC simulation are altered with proportions of the data deliberately misclassified. This is closer to what can be expected of a novice player and is shown in Figure 7.

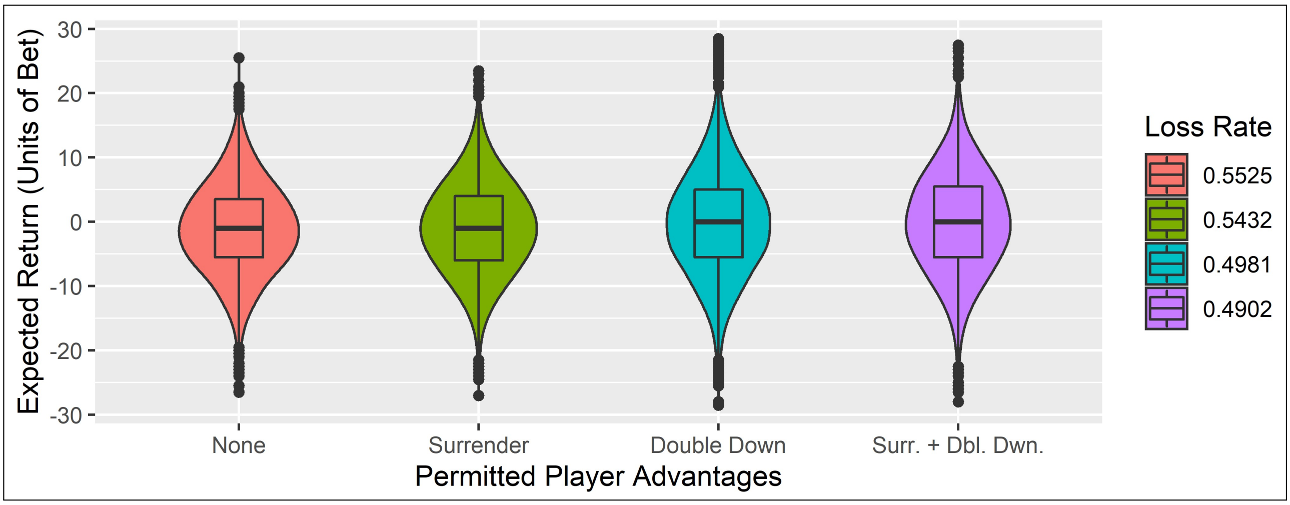

Figure 6: *The expected player return after 50 games of optimal play. The units are in the units of the player's initial stake. The loss rate is the fraction of games in which the player loses money.*

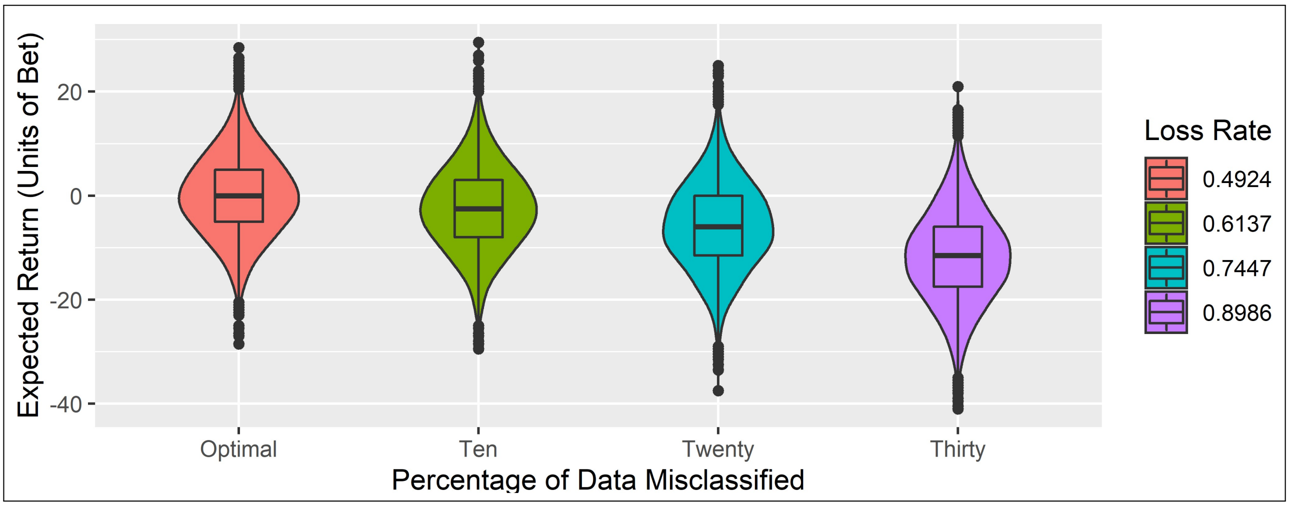

Figure 7: *The expected player return after 50 games with fractions of the optimal strategy's decisions deliberately misclassified to simulate a novice player's mistakes. The loss rate is the fraction of games in which the player loses money.*

6 Discussion

Blackjack is one of the most played games in casinos and on betting sites around the world. The confirmation bias that players feel when their decision leads to a win is something casinos have always exploited. It is often argued that this is acceptable as gambling is a form of entertainment where patrons pay through the house advantage. This naive viewpoint has been struck down by whistleblowers in the advertising industry revealing how gambling sites target those most likely gamble irresponsibly[6]. This work aims to use data to hinder these practices by demystifying the game of blackjack.

Gambling houses often have a custom set of rules for blackjack rather than adopting a general ruleset. This ensures that an optimal strategy in one casino or gambling site will almost certainly be sub-optimal everywhere else. The package accompanying this work has a set of fully documented functions to test over 3,000 possible rule combinations in a way that is accessible to anyone with access to a computer. Using the package, the rulesets of real-world gambling houses can be quantified and compared in seconds on even modest computer hardware.

Prospective players can analyse the optimal strategies in a simple data frame structure. The master function cannot generate an arbitrary ruleset but it can produce a sufficiently similar ruleset to approximate an arbitrary rulesets' house advantage.

The optimal strategy obtained analytically allows for verification of the algorithms used in this work. The win rate for strategies produced using Monte Carlo simulation converge to that of the optimal strategy using approximately 10^4 simulations per position. The classification accuracy when producing a strategy is 90.9%, 96.6%, 98.9%, 99.6% and 99.9% when using 10^1, 10^2, 10^3, 10^4 and 10^5 simulations, respectively with split, double down and surrender enabled.

A significant finding of this work is how little variability exists in the house advantage for different rulesets when played optimally. Betting houses often have small additions to rulesets such as no hitting on split aces or no surrender when the dealer can obtain blackjack. This work shows precisely how much the house advantage is affected by splitting, doubling down and surrendering. Placing limits on small subsets of these options has nearly no effect on house advantage. The reasons for these small alterations could be how they change the optimal strategy or to maximize house advantage in a more concealed way than prohibiting the more substantial rules.

7 Conclusion

When played optimally, blackjack can have one of the smallest house advantages of any common casino game. Gambling houses have increased their use of personal data in order to target people who do not play optimally. These practices encourage addiction and increase poverty. This work provides a way to quantify the approximate house advantage for blackjack at any gambling house. The framework is encapsulated in an extensively documented R package making it accessible to anyone with access to a computer. The simulation algorithms leverage parallel computing to simulate millions of complete games per second enabling the production of optimal strategies using Monte Carlo simulation. The strategies can be used to determine the expected return for a player over an arbitrary number of games. This allows prospective gamblers to understand how much they can expect to lose during their visit to a casino.

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