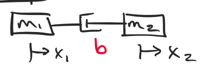
• "Cheating" to find Ξ_i:

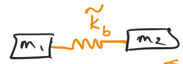
- Using the Lagrangian approach, <u>conservative</u> terms are generated automatically, e.g., from gravity or from spring forces.
- <u>Non-conservative</u> terms, in each Ξ_i's, are more challenging to find.
- A "cheating" approach is to insert a "fake spring", to see its effect in the E.O.M.'s.

→ Two cases are typical

1) <u>Damper</u>: <u>BETWEEN</u> 2 DOF's

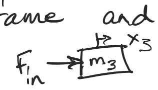


temporarily, model as:

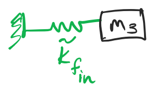


← $V = \frac{1}{2}\left(\tilde{k}_b \cdot (x_2 - x_1)^2\right)$

2) <u>External force</u>: between reference frame and a mass:



put spring on a wall...



← $V = \frac{1}{2}\left(\tilde{k}_{f_{in}} x_3^2\right)$

Case 1) $\quad \xi_1 = x_1, \quad \xi_2 = x_2$ (absolute coordinates)

$$T^* = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2$$

$\boxed{V = 0}$, then, <u>pretend</u> $\boxed{\tilde{V} = \frac{1}{2}\left(k_b \cdot (x_2 - x_1)^2\right)}$

$\mathcal{L} = T^* - V$, but pretend: $\tilde{\mathcal{L}} = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 - \tilde{V}$

pretending: $\tilde{\mathcal{L}} = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 - \frac{1}{2} k_b \left(x_1^2 - 2 x_1 x_2 + x_2^2\right)$

<u>Also</u>, $\boxed{\text{pretend } \Xi_1 = \Xi_2 = 0}$, ∴ only do "lefthand sides"

For ξ_1: $\quad \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\xi}_1}\right) - \frac{\partial \mathcal{L}}{\partial \xi_1} = 0 \rightarrow \frac{d}{dt}(m_1 \dot{x}_1) + k_b(x_1 - x_2) = 0$

Cheat! Really, this was a <u>DAMPER</u>, so...

a) replace $\tilde{k}_b$ with b

b) replace positions (x_1 & x_2) with velocities ($\dot{x}_1$ & $\dot{x}_2$)

$$m_1 \ddot{x}_1 + k_b (x_1 - x_2) = 0$$

is <u>Really</u>: $\quad \boxed{m_1 \ddot{x}_1 + b(\dot{x}_1 - \dot{x}_2) = 0}$

(<u>Truly</u>: $V = 0$ & $\Xi_1 = -b(\dot{x}_1 - \dot{x}_2)$ here...)

- Again this is cheating.

- The better approach is to use earlier notes to write:

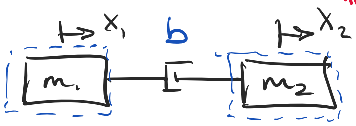
force (or torque)

velocity at this force (or torque)

$$\dot{W} = \sum_{\forall j} F_j \cdot V_j$$

then, $$\boxed{\Xi_i = \sum_{\forall j} F_j \cdot \frac{\partial v_j}{\partial \dot{\xi}_i}}.$$

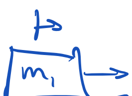
For:



, w/ $\xi_1 = x_1$ and $\xi_2 = x_2$,

Do NOT differentiate the "F_j" above!!

$$\boxed{\dot{W} = F_b \dot{x}_1 - F_b \dot{x}_2}$$



$F_b = b(\dot{x}_2 - \dot{x}_1)$

$-F_b = -b(\dot{x}_2 - \dot{x}_1)$

$F_b = b(\dot{x}_2 - \dot{x}_1)$

$$\therefore \boxed{\Xi_1 = F_b = b(\dot{x}_2 - \dot{x}_1)}$$

no spring in REAL system...

and: $$\boxed{\mathcal{L} = T^* - V = \tfrac{1}{2} m_1 \dot{x}_1^2 + \tfrac{1}{2} m_2 \dot{x}_2^2 - 0}$$

∴ E.O.M. #1 is: $$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\xi}_1}\right) - \frac{\partial \mathcal{L}}{\partial \xi_1} = \Xi_1 = b(\dot{x}_2 - \dot{x}_1)$$

$$\boxed{m_1 \ddot{x}_1 = b\dot{x}_2 - b\dot{x}_1}$$

which is, as predicted on last page: $\boxed{m_1 \ddot{x}_1 + b(\dot{x}_1 - \dot{x}_2) = 0}$