Matching Students and Instructors in Higher Ed. Settings

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Abstract

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1 Introduction

Formal education institutions play a crucial role in the development of human capital. In terms of their scope, approximately XXX workers in the developed world have engaged with these institutions at some point, while XXX have done so in the developing world. Furthermore, the significance of these institutions in enhancing learning is evident in the substantial income disparities among workers who participate in them to varying degrees. Consequently, it follows that the potential gains resulting from carefully organizing the use of students and professors in these institutions can be substantial. One key component of this problem involves quantifying the extent to which learning output depend on the specific way in which students and instructors are paired within these institutions, and examining the influence of commonly used course assignment mechanisms on the resulting pairings.

The latter concern is particularly significant in the context of higher education institutions. On one hand, higher education institutions display significant diversity within the student body, encompassing individuals from various academic backgrounds and with varying aptitudes for learning. Intuitively, these differences increase the potential impact of matching effects on learning outcomes and emphasize the benefits of considering them in the design of instructor-student assignments. On the other hand, higher education institutions typically employ course enrollment mechanisms that rely on student choice. While these choice-based mechanisms may align with specific institutional goals, they raise questions about their effectiveness in terms of learning outcomes. This is especially pertinent given the emphasis on evaluating students based on their score outcomes, which might reflect, besides learning, differences in instructors' grading policies. If students' course/section choices are driven by a preference for high scores, the assignments resulting from these choice-based mechanisms might be suboptimal from a learning perspective.

This paper seeks to address these questions on empirical grounds. To be concrete, the approach involves the construction of a structural model that describes learning-related variables found in most standard academic records. Our primary focus is on modeling the creation of learning outcomes in conjunction with the factors influencing students' preferences for course sections within a given course. Regarding the former, the model allows for flexible specifications of the learning production function, which maps learning inputs to outputs. On the demand side, we explicitly model students' preferences for both learning and score outcomes, examining how these preferences affect their choices for courses within the rules defined by the university's course enrollment mechanism. We utilize our model to assess the potential improvements in learning outcomes by simulating counterfactual policies that modify the rules governing the assignment of students and professors.

To estimate the model, we utilize the academic records of INTEC, a higher education institution in Santo Domingo, Dominican Republic. These records encompass information related to the academic paths of all students who enrolled in the university between 2007.0 and 2022.0. Specifically, the dataset includes comprehensive records for each course taken by a student, among other things, encompassing the score outcomes resulting from each course enrollment instance observed. Furthermore, the records contain both pre-enrollment and post-enrollment variables associated with learning, all of which we consider in our empirical exercises.

Our approach capitalizes on the richness of the dataset to identify the learning production functions under very flexible parametric restrictions. In particular, we identify the characteristics of the learning production functions associated with each instructor in our sample using only information related to the scores achieved by students throughout their tenure in the university. The proposed approach carefully takes into consideration specific aspects of higher education settings that complicate the empirical identification of learning outcomes. For instance, it explicitly separates the confounding effects arising from differences in instructor grading policies, which can complicate the interpretation of high scores in terms of high learning outcomes. Regarding the demand model, our approach estimates student preferences for both learning and scoring concerns while dealing with the fact that not all variables describing a student's participation in the course enrollment mechanism are observed.

Two main insights can be highlighted from the estimates of our structural model. First, we find that while students' characteristics indeed play a significant role in predicting learning outcomes, these learning returns depend substantially on the instructor the student is paired with. For instance, for the average student, the worst professor in terms of learning is associated with more than a full letter score drop relative to the student's ideal professor under our definition of the learning scale. Second, we observe substantial variations in grading policies across instructors, encompassing both the marginal returns associated with learning and the baseline grading criteria used in the course. Importantly, instructors with lenient grading policies don't always coincide with high-learning instructors. As our demand estimates suggest that students value both learning and score outcomes, the latter observation implies a tension for students when choosing to request sections of a course.

In terms of the policy implications of our findings, we begin by assessing two distinct families of counterfactual scenarios. The first group, serving as a benchmark, involves a dean seeking to assign students and professors to maximize average learning outcomes without considering students' preferences for professors or sections. Within this framework, we explore different versions of the exercise, each assigning varying weights to students based on their initial ability as to capture distributional concerns in the design of a student-professor assignment. The simulations demonstrate how learning gains can be achieved across the entire distribution of student ability, with the size of these gains varying across different segments of the distribution and the dean's weighting function.

While the latter dictatorial counterfactual might serve as a benchmark, it is typically infeasible and incompatible with the university's goal of allowing students some degree of freedom when enrolling in sections within a course. For this reason, we subsequently consider counterfactual exercises that aim to change the observed assignment by modifying the course enrollment mechanism. In particular, we consider mechanisms that prioritize students in terms of allowing them to choose first, based on their past experience at the university. These prioritization rules account for student differences, including their past GPA, seniority, and whether a student is retaking a course. For each of these mechanisms, we compare the resulting assignment to that resulting from the dictatorial benchmark and attempt to assess what aspects of the learning/demand estimates lead to the resulting gaps.

Related Literature

1. Equilibrium Grade Inflation with Implications for Female Interest in STEM Majors

2 The Importance of Matching Effects for Labor Productivity: Evidence from Teacher-Student Interactions*

3. Identification of Non-Additive Fixed Effects Models: Is the Return to Teacher Quality Homogeneous?

4. The Multi-unit Assignment Problem: Theory and Evidence from Course Allocation at Harvard

2 Institutional Details and Data Sources

Our dataset encompasses the academic records of Instituto Tecnológico de Santo Domingo (INTEC), a higher education institution primarily oriented towards undergraduate STEM majors. It represents a longitudinal panel that provides insights into the academic path of all students who matriculated at INTEC between 2007.0 and 2022.0. Specifically, the primary component of this dataset comprises academic records detailing students' performance throughout their enrollment at the university. Our analysis centers on a specific subset of courses undertaken by a total of XXX students in the dataset. These students collectively comprise XXX course-section observations.

Our dataset captures a comprehensive range of information for each course in which a student enrolls. This includes unique identifiers for the course sections, instructor identifiers for each course section observed, the student's decision to either withdraw from or complete a course, and the final score achieved by the student (i.e., contingent upon not dropping the course). Additionally, we have access to an extensive set of covariate variables, which encompass learning-relevant characteristics of both students and the sections of courses in the data.

On the student side, these covariates encompass various variables, including the student's biological sex, chosen major program, age at enrollment, and individual scores on the components of the entrance exam evaluation assessing their knowledge of high school prerequisite material. The latter variable serves as our primary measure of a student's initial ability. Turning to section-related variables, we construct several key attributes for each academic period. These include the leading professor's teaching load, specific tenure within a given course, and tenure within the university overall. Moreover, for a subset of sections, we have access to additional pertinent characteristics, such as the timing of the section during the day and the highest academic degree attained by the lead instructor.

In accordance with the standard practice in Latin American higher education institutions, students at our university are mandated to declare their intended majors upon enrollment in the university. Upon making this selection, students are required to adhere to a predetermined curriculum known as the pensum. each major's pensum comprises a series of subject-related mandatory courses connected by prerequisite relationships. This system should be contrasted to the North American higher education model, where students often enter university without specifying their majors until they have accrued the necessary credit requirements. The structured nature of our system streamlines our analysis, allowing us to bypass the intricate combinatorial challenges that arise when students possess extensive flexibility in selecting courses to fulfill major requirements.

Our empirical analysis centers on the specific sequence of courses, namely Calculus

1, Calculus 2, and Calculus 3, offering several advantages from an empirical standpoint. First, the content covered in this sequence is highly standardized, enabling us to concentrate on assessing vertical rather than horizontal quality differences across instructors teaching the courses. Second, this sequence is an integral part of the curriculum for nearly all university students, affording us the opportunity to observe a substantial number of students participating in these courses. Third, students embark on this sequence in their very first academic term, ensuring a minimal time lapse between assessing their prior abilities (e.g., entrance exam scores) and their initial enrollment in the Calculus sequence. Additionally, since Calculus serves as a foundational course for most other major subjects pursued by our students, these sequence courses are typically taken in isolation, reducing potential interference bias resulting from students learning the Calculus curriculum from instructors in related courses. Finally, owing to the prevalence of STEM majors at our university, a substantial number of sections of Calculus courses are offered each term, taught by a diverse group of instructors. This diversity allows us to explore student performance and course demand across a range of teaching environments, potentially varying in pedagogical approaches and grading policies.

INTEC's grading system employs the standard 0-4.0 GPA scale, with these GPA points corresponding to letter grades, including A, B+, B, C+, C, D, F, and R (indicating a course section drop). Notably, scores are determined on an absolute basis, with each instructor having the autonomy to establish their grading policy, translating student learning into letter scores. Importantly, there is no university mandate explicitly directing professors to achieve specific score distributions within each course section. This grading approach aligns with cultural and political considerations at the university, where it is presumed that an instructor's grading conveys significance beyond mere student ranking. Moreover, it simplifies the task of comparing student grades across different cohorts, a challenge that would be formidable under a period-by-period curve grading system. A central challenge in our empirical analyses lies in disentangling the learning outcomes from the instructor's grading policy while examining the score distribution.

Table 1 serves as our initial descriptive analysis, providing statistics that depict the data's distribution of scores in each course, contingent on various characteristics we employ as controls in our empirical exercises. The data at this stage is organized at the student-course-section level. On the whole, the average score for Calculus 1 stands at approximately 2.80 out of 4.00 on the GPA scale, while for Calculus 2, it hovers around 2.75. In both instances, these averages correspond to a letter grade of B, based on the institutional letter score cutoffs. However, it's important to note that there is a considerable degree of variation surrounding these average scores. For example, in both cases, the unconditional standard deviation for students in Calculus 1 exceeds 1.00 GPA point, indicating a range that spans from a score equivalent to a letter grade of D to the highest achievable grade of A.

Table 1: Distribution of scores - Calculus 1 and 2

	Calculus 1		**Calculus 2**	
	Avg.	Std. Dev.	Avg.	Std. Dev.
All students	2.80	1.17	2.75	1.07
Ability 0% - 25%	2.41	1.21	2.45	1.09
Ability 25% - 50%	2.65	1.21	2.55	1.07
Ability 50% - 75%	2.82	1.13	2.69	1.07
Ability 75% - 100%	3.22	0.99	3.08	0.99
Female	2.94	1.10	2.87	1.01
Male	2.70	1.22	2.66	1.11
Stem	2.81	1.19	2.74	1.09
Social sciences	2.77	1.16	2.59	1.09
Health sciences	2.78	1.16	2.86	1.00
High load	2.70	1.17	2.78	1.07
Low load	2.90	1.17	2.73	1.08
High course tenure	2.96	1.10	2.85	1.04
Low course tenure	2.72	1.20	2.68	1.09
High tenure	2.92	1.11	2.85	1.05
Low tenure	2.72	1.21	2.67	1.09

Moving on to the second panel of Table 1, we delve into the same statistical measures, this time conditioned on the biological sex of each student within the dataset. In line with findings from prior research, we observe that the average scores for both Calculus 1 and Calculus 2 are higher for female students compared to their male counterparts. Intriguingly, there is also a wider dispersion in the distribution of scores among male students than among female students.

Another key variable of interest is the major choice made by each student. Intuitively, given the varying levels of mathematical emphasis across different majors, one would anticipate differences in the score distribution based on major selection. We have categorized all observed majors into four primary groups, closely aligning with the university's classification: (1) STEM majors, (2) business and social sciences majors, (3) health majors, and (4) other majors. Notably, substantial disparities in the score distribution are evident across these categories. For instance, when considering Calculus 1, the average score ranges from 2.77 for the second group to 3.07 for the last group. It's essential to bear in mind that while interpreting these averages, part of the observed differences reflects variations in course drop rates among students across different majors.

Furthermore, we investigate variations in the score distribution while controlling for instructor-related characteristics. For example, the final panel of Table 1 offers insights into the score distribution when considering an instructor's teaching load, tenure within the specific course, and overall university tenure. It's noteworthy that in the context of Calculus 1, instructors with a lighter teaching load tend to exhibit higher average scores, whereas the opposite pattern emerges in the case of Calculus 2. Additionally, notable disparities become evident when examining scores concerning an instructor's within-course tenure and general university-wide tenure. In both Calculus courses, tenured instructors, whether specifically related to the course or in terms of their general university-wide tenure, are associated with higher scores and reduced score dispersion. These instructor characteristics play a significant role in shaping the distribution of scores and will be a focal point in our subsequent analyses.

It's crucial to note that the statistics presented above cannot be directly interpreted as a reflection of the distribution of learning outcomes. This is primarily due to the fact that, in addition to differences in pedagogical abilities, professors also vary in their grading policies. Consequently, part of the disparities outlined above may stem from students enrolling in courses taught by professors with different grading policies.

The pairing of students and instructors within the courses in the sequence follows two main rules. Firstly, in their first term, students are assigned by the administration to specific sections of the courses as specified by their pensum. This assignment is conducted in a pseudo-random manner and is unrelated to variables that could potentially impact a student's performance in Calculus 1. Secondly, after their initial term, students select their courses on a first-come-first-serve basis using an electronic platform. This platform allows students to choose sections of the courses they wish to enroll in. Each section has a capacity constraint, and a student's choice set comprises all courses with available capacity at the time they access the platform. While we have access to information about each student's choice, we do not directly observe the timing of their platform entry or the specific choice set available to each student. At the heart of our demand model will be a concern for estimating demand primitives of interest in the absence of these quantities.

Table 2 complements the preceding table by offering score-related data at the course-section level. This perspective is of important as it affords a more nuanced insight into the variations in academic performance within distinct course sections. Such distinctions often factor prominently into students' decisions when choosing which course section to enroll in.

For example, Table 2 reveals that the average section size for both Calculus 1 and Calculus 2 is approximately 33 students. This is consistent with the fact that for Calculus 1, only 27% of the observed sections are operating at their full capacity, whereas this percentage increases to 40% for Calculus 2 (i.e., accreditation requirements place the

Table 2: Descriptive statistics - Course/section level

	Calculus 1		Calculus 2	
	Avg.	Std. Dev.	Avg.	Std. Dev.
Section size	33.35	9.17	32.89	10.54
Q1	32.86	9.86	33.19	11.04
Q2	30.09	11.10	32.00	10.62
Q3	34.56	7.31	28.86	11.56
Q4	32.99	10.26	34.73	9.23
% at capacity	27.00	–	40.00	–
Q1	36.00	–	45.00	–
Q2	17.00	–	34.00	–
Q3	22.00	–	22.00	–
Q4	32.00	–	47.00	–
Mean score	2.70	0.69	2.66	0.56
Pass rate	0.63	0.24	0.63	0.22
Drop rate	0.25	0.21	0.27	0.19
Load	2.77	1.36	2.54	1.40
Course tenure	9.63	8.73	10.27	9.60
General tenure	11.41	9.21	12.38	10.3

course capacity constraint at 40 slots). Moreover, disparities are evident when examining these variables in specific academic quarters. For instance, while 36% of sections meet their capacity constraint in the first quarter, this figure drops to only 17% in the second quarter. These fluctuations can be attributed to shifts in the overall demand for Calculus sections across different quarters and variations in the total supply of sections offered by the university each term. A similar pattern emerges when considering the corresponding values for Calculus 2.

Turning our attention to the second panel of Table 2, we shift our focus to the distribution of scores at the section level. In this context, we analyze metrics such as the Mean score, Pass rate, and Drop rate. For both Calculus 1 and Calculus 2, approximately 63% of students who enroll in the course successfully complete it. In Calculus 1, the average mean section score is 2.70, while in Calculus 2, it is 2.66. However, a noteworthy aspect to consider is that a substantial proportion of students, accounting for 25% in the case of Calculus 1 and 27% for Calculus 2, opt to drop the section they initially enrolled in. This choice to withdraw from a section results in these students not receiving a final score. Subsequent sections will have to deal with the identification challenge resulting from the truncation in the distributions of scores arising from this possibility.

When we delve into instructor characteristics, we observe that, on average, an instructor responsible for a Calculus 1 section is handling a load of 2.77 courses, while for

Calculus 2, this figure is 2.54. Regarding course-specific and general tenure, the average Calculus section is taught by a professor with approximately 9.63 terms of experience for Calculus 1 and 10.27 terms for Calculus 2. Additionally, when considering general university-wide tenure, the respective figures are 11.41 and 12.38 terms. These substantial variations in the latter two variables underscore the significant differences that students encounter within the pool of instructors to whom they are exposed. This diversity becomes particularly relevant if we assume that tenure may impact student learning outcomes. These insights, extracted from Table 2, play a crucial role in our understanding of the dynamics of academic performance at the section level and lay the foundation for our subsequent analyses.

3 Stylized Facts and Reduced Form Evidence

In this section, we present preliminary evidence that anticipates the discussions to be elaborated upon in subsequent sections. Our emphasis is on three key insights that collectively serve as the foundational guide for the structural model presented in the main conceptual sections of the paper.

First, we emphasize the role of student characteristics, specifically their initial ability as measured by the entrance exam scores, in explaining variations in observed learning-related academic outcomes. Second, we explore how the learning returns to ability vary across different instructors. In other words, we test for the existence of matching effects within the learning production function, to study the merit of our concerns regarding the potential benefits of reassigning students and professors. Finally, our analysis sheds light on how a student's demand for specific course sections can be predicted based on the student's individual characteristics. For instance, we argue that students of varying ability levels often opt for different course sections, a phenomenon we interpret as indicative of heterogeneity in preferences for both learning and score outcomes resulting from such choices.

Learning is predicted by student ability.

Our initial task involves collecting evidence to underscore the significance of student characteristics in elucidating disparities in learning outcomes, specifically focusing on the measure of student initial ability as gauged by the entrance exam score. To begin with, let's consider Table **??**, which illustrates the distribution of letter scores contingent upon a student's initial ability measure. In this table, the columns correspond to the scores achieved by individual course-enrollment instances for students within the sample. Meanwhile, the rows in the table delineate the segments within the distribution of initial ability where each student's entrance exam score falls.

The table reveals discernible relationship between a student's obtained score and their initial ability. For instance, let's consider the case of Calculus 1. When we focus on students in the top 20.00% of the distribution of initial abilities, they exhibit a 46.00% likelihood of attaining a score of A. In contrast, this probability decreases significantly to 11.74% when considering students at the lower end of the ability distribution. Notably, roughly 75.00% of students in the lower segment of the ability distribution ultimately receive either a C grade or a score associated with course failure or withdrawal. This pattern is mirrored in the context of Calculus 2. Here, students positioned at the top and bottom of the ability distribution exhibit odds of 34.31% and 9.27%, respectively, for obtaining a score of A. Once again, we observe a substantial and meaningful disparity in outcomes.

In simpler terms, Table **??** tells a story where our measure of initial ability reflects variations in students' knowledge of the prerequisite material for Calculus which map into actual score outcome differences. However, we should be cautious about potential complications in interpreting the table. For instance, although the assignment of first-time students to Calculus 1 is random, the small average class size could mean that significant differences in student characteristics exist across sections. It might be more accurate to use an approach that directly controls for these potential differences. This becomes even more crucial in the case of Calculus 2, where students have the freedom to choose their course, making it essential to control for observable student and professor characteristics as a top priority. Additionally, we may want to employ a reduced-form model to formally test the hypotheses of interest, particularly examining the null hypothesis that assesses the predictive power of students' initial ability measures.

With this in mind, let's try to construct a formal test capturing the following thought experiment: imagine we have two students who are identical in every aspect except for their initial abilities. After both students are assigned to the same section of Calculus 1, we compare the resulting learning-related outcomes. Under the ideal "ceteris paribus" assumption, any disparities in the outcomes can be solely attributed to the initial differences in their ability levels. In particular, we should not be concerned about differences in grading policies guiding the differences as we consider a within professor Calculus 1 professor comparison. One can formalize this experiment and test the null hypothesis that ability has no predictive power by using a reduced-form model that connects a student's Calculus 1 scores with their ability level and other relevant control covariates.

$$y_i = \mathbf{1}\left\{\gamma^0 + \sum_{j=1} \gamma_j^0 \cdot d_{i,j} + \gamma^1 \cdot a_i + \sum_{j=1} \gamma_j^1 \cdot (a_i \times d_{i,j}) + \boldsymbol{\gamma}^2 \mathbf{z}_i + \varepsilon_i \geq 0\right\}.$$

In the equation above, the variable y_i represents a binary outcome, indicating whether a specific learning event has occurred. We entertain two specific outcomes with a cutoff structure: "student i scores just above an A in Calculus 1" and"student i fails to pass Calculus 1". The model above links the likelihood of each of these events to a linear function of the students' characteristics: their ability denoted as a_i and a vector of learning-related covariates, represented by $\mathbf{z}_i$. To capture the within-professor description given above, we consider binary variables $d_{i,j}$, for whether student i is matched with professor j, which allow us to write the reduced form model in terms of professor specific intercepts and ability slopes. Assuming the distribution of the error terms ε_i to be logistical, one can essentially think of the model as being a collection of professor-specific logistical regression models. Of particular interest is the reduced-form parameter γ_j^1, which quantifies how an increase in a student's ability affects the likelihood of the event $y_i = 1$ under instructor j.

We conduct hypothesis tests for each individual parameter γ^1 to ascertain whether they are equal to zero and for the joint hypothesis that all of them are jointly equal to zero. Table 3 presents the estimates from the estimation exercise described earlier, focusing on a subset of instructors in the dataset. The first panel of the table collects the point estimates and standard errors for the ability slope parameters corresponding to the four Calculus 1 instructors with the highest number of associated students. Each column corresponds to one of the two dichotomous outcomes considered. In addition, each regressions controls for various observable student-professor characteristics, including the student's sex, major choice, the professor's workload, and both the professor's general and course-specific tenure.

Table 3: Learning increases with ability?

	A	**Fail/Retire**
Point estimates for the ability slopes (γ_j^1)		
Professor 1 ($\gamma_{j_1}^1$)	1.80	-1.12
	(0.19)	(0.18)
Professor 2 ($\gamma_{j_2}^1$)	2.41	-1.83
	(0.23)	(0.17)
Professor 3 ($\gamma_{j_3}^1$)	2.50	-2.31
	(0.19)	(0.15)
Professor 4 ($\gamma_{j_4}^1$)	2.05	-1.86
	(0.11)	(0.11)
H_0: Joint insignificance of ability slopes (γ_j^1)		
p.value	0.00	0.00

In general, the results align with those in the table above. Let's focus for example on the outcome 'A' in the first column of the table. We can interpret the coefficients in terms of the change in the odds ratio $\mathbb{P}(y_i = 1 \mid a_{i,0})/\mathbb{P}(y_i = 0 \mid a_{i,0})$ following from a unit increase in the student's ability. To illustrate this, consider the coefficient $\gamma_j^1 = 2.41$ for the second instructor instructor. Starting from a scenario where the outcomes $y_i = 1$ and $y_i = 0$ are equally likely, an increase of one unit in a student's ability level maps into a 1.08 increase in the odds ratio. In simpler terms, this represents a change of $\mathbb{P}(y_i = 1 \,|\, a_{i,0})$ from 0.5 to approximately 0.70. The coefficients for all ability slopes under the 'A' outcome are positive, indicating that higher ability is associated with a higher probability of obtaining an 'A' score. Furthermore, the null hypothesis of a zero slope is rejected for all eight instructors considered. Similar patterns show up in the case of Fail/Retire: (i) all parameters are significantly different from zero, and (ii) we reject the null hypothesis of joint insignificance.

The learning returns to ability differ across instructors.

The exercise detailed in the previous section enables us to test whether the learning returns associated with a student's ability are significantly different from zero. However, this approach doesn't address the question of whether these learning returns to student ability vary among different instructors. In other words, while the approach allows us to compare the returns to different ability levels within each instructor's production function, it doesn't allow for comparisons across different instructors.

Once again, let's entertain a simple experiment as the ideal comparison we would like in order to tackle the latter question: imagine two students with the same entrance exam score. Enroll both of them with different Calculus 1 instructors, and after completing Calculus 1, have both students enroll under a common Calculus 2 instructor. Under the "ceteris paribus" assumption, since the only distinguishing factor in the paths these students follow is their Calculus 1 instructor, any differences in their Calculus 2 performance should reflect disparities in the learning outcomes associated with their respective Calculus 1 instructors. While the experiment doesn't allow us to directly quantify the magnitude of this gap, as it is influenced by the Calculus 2 professor's own contribution to learning, it can be used to test whether the returns to learning outcomes of the two Calculus 1 instructors differ.

We initiate this comparison by examining the distribution of letter scores among students who follow a particular instructor path during their initial two academic terms. Table **??** in its entirety pertains to information regarding students who enroll with a specific Calculus 2 professor (chosen as the instructor with the highest number of Calculus 2 enrollment observations) during their second academic term. Each panel within the table conditions on the Calculus 1 instructor associated with a student in their first academic term. For example, the students represented in the first panel correspond to an instructor path (j_1, j_2), while those in the second panel correspond to a path $(j_1^{'}, j_2)$ with $j_1 \neq j_1^{'}$. Once again, we select the three Calculus 1 instructors corresponding to each panel based on their enrollment numbers.

As before, within any given panel, each cell displays the fraction of students who achieve a particular Calculus 2 score, as indicated by the columns, conditional on their initial ability measurement. We employ this table to facilitate comparisons across different Calculus 1 instructors. For instance, it allows us to assess how effective a Calculus 1 professor is at guiding students from specific ability regions to attain Calculus 2 letter scores associated with high performance. Importantly, since all students in the table share a common Calculus 2 instructor, we rule out the possibility of any disparities arising from differences in the grading policies of Calculus 2 that students encounter.

In the passage above, we use red highlighting to emphasize the cell associated with the most frequently observed letter score for each region within the ability distribution. The table reveals noteworthy disparities in the learning outcomes across the three Calculus 1 instructors under consideration. For example, observe that the instructor in the first panel is considerably more likely to yield an A score for students at the top of the ability distribution compared to the other two professors. This trend holds true for all students within the first four ability categories. Further comparisons uncover additional differences. For instance, while the structure of the second panel appears to allocate more weight to high letter scores for students in the top three ability categories, relative to the third panel instructor, the same pattern doesn't hold for the lowest category. To illustrate, in panel 2, students in the bottom 20% of the ability distribution attain a B score with a frequency of 15.56%, whereas the corresponding figure in the bottom panel is 30.0%.

As before, a more formal statistical exercise might be of value. We can frame our initial thought experiment in terms of the reduced form model described below. This mimics are previous specification except for two things: (i) we now focus on students who enroll a specific Calculus 2 instructor in their second academic period, and (ii) we take y_i as a Calculus 2 learning outcome. Notice that in this scenario, the coefficients γ differ from those considered before. Intuitively, it pools structural parameters corresponding to both the Calculus 1 and the Calculus 2 instructors. Nevertheless, as we construct the exercise by fixing the Calculus 2 instructor, we can still interpret differences in the ability slopes as resulting from differeneces in student learning across Calculus 1 instructors.

$$y_i = \mathbf{1}\left\{\gamma^0 + \sum_{j_1=1} \gamma^0_{j_1} \cdot d_{i,j_1} + \gamma^1 \cdot a_i + \sum_{j_1=1} \gamma^1_{j_1} \cdot (a_i \times d_{i,j_1}) + \varepsilon_i \geq 0\right\}.$$

The table presented here closely resembles the one in the previous subsection. It showcases the point estimates and standard errors for the interaction terms $\gamma^1_{j_1}$.

Furthermore, at the bottom of the table, we report the results of a joint test for all ability slopes being equal. We reject the null for the outcome "i obtains a score of A" at a 10% confidence level and at a 5% confidence level for the outcome "i obtains a score of Fail".

Table 4: Do learning returns differ across instructors?

	A	**Fail/Retire**
Point estimates for the ability slopes (γ^1_{j,j_2})		
Professor 1 (γ^1_{j,j_2})	1.43	-2.32
	(0.54)	(0.50)
Professor 2 (γ^1_{j,j_2})	2.09	-0.89
	(0.51)	(0.42)
Professor 3 (γ^1_{j,j_2})	1.27	-1.10
	(0.26)	(0.27)
Professor 4 (γ^1_{j,j_2})	0.39	-0.49
	(0.38)	(0.39)
H_0: Equality of the ability slopes (γ^1_j)		
p.value	0.07	0.04

The nature of instructor's grading policies

Students select into courses based on their characteristics

4 The model

This section presents a formal model for a student's academic progression through a series of mandatory subject-related courses. The model takes into account the interaction between students and instructors, recognizing the significance of both these factors in shaping learning outcomes. Furthermore, it elucidates the process through which students demand sections within a course according to the institutional rules governing our empirical setting. The model is dynamic, as it considers the evolution of a student's abilities over time based on its prior learning outcomes, and it explicitly addresses how the a student-professor match translates into the typical academic results found in higher education records.

4.1 The learning production function

Consider a higher education institution which each academic term $t \in \mathcal{T}$, faces the task of assigning students, seeking to enroll a section of a given course, to instructors, who lead the teaching in such sections. We index an arbitrary student by $i \in \mathcal{I} \equiv \{1, ..., N\}$, and an instructor by $j \in \mathcal{J} \equiv \{1, ..., J\}$. The focus is placed on a sequence of courses centered around a common subject. These courses, which we denote by $\kappa \in \mathcal{K} \equiv \{1, 2, \ldots, K\}$, might correspond for instance to Calculus 1, Calculus 2, Calculus 3, and so forth. Students are required to enroll and successfully complete all of the courses in the sequence in the order specified by the indexes in $\mathcal{K}$. For instance, a section of course $\kappa > 1$ can be enrolled only after obtaining a pass score for course $\kappa - 1$ (e.g., achieving a pass score in Calculus 1 is a prerequisite for enrolling in Calculus 2). This sequential arrangement reflects the constraints imposed by the university's curriculum.

Let $t_i \in \mathcal{T}$ be the academic period in which student i enrolls in the university. Upon enrollment, i draws an ability $a_{i,0} \in \mathbb{R}_+$ which we occasionally refer to as i's initial type. The value $a_{i,0}$ can be interpreted as i's understanding of the prerequisite material required for the courses in $\mathcal{K}$. It may correspond, for example, to the student's score in the math component of a college entrance examination designed to asses a student's understanding of high-school pre-calculus. As the student progresses along the sequence of courses, its ability measure is updated in a way that shows i's acquired knowledge (or knowledge depreciation) of the sequence curriculum given its past experience along the sequence. We denote a student's type at the end of any period $t > 0$ by $a_{i,t}$.

In any given academic term t, multiple sections of a course κ may be offered, with each section being guided by a single instructor. The pool of all instructors leading a section of course κ during academic term t is denoted by $\mathcal{J}_t^\kappa \subseteq \mathcal{J}$. Notice that this is potentially a strict subset of $\mathcal{J}$ as some instructors might not be active in certain periods for exogenous reasons. The multiplicity of instructors under a common course implies that more than one way of matching students to professors might exist in any given course/period pair. Let $\kappa_{i,t}$ stand for the course in the sequence student i seeks to enroll in period t and $j_{i,t}$ for the instructor student i is paired with. While subsequent subsections describe the process by which these assignments take place, our interest here is in describing the academic outcomes conditional on the student-instructor match.

With this goal in mind, let's entertain a student i who in period t is paired with instructor $j_{i,t} = j$ under course $\kappa_{i,t} = \kappa$. Two potential academic outcomes might arise. First, i might decide to drop the section of the course, in which case an R (i.e., the notation stands for retire) score is recorded. Such a situation is deemed as an unsuccessful attempt at completing the course and therefore requires i to enroll the same course again in a subsequent term. The dummy variable $R^{\kappa}_{i,t}$ records i's decision not to drop the section of course κ (i.e., $R^{\kappa}_{i,t} = 0$ corresponds to dropping the section). Alternatively, the student might choose to complete the course, which results in a discrete course score (i.e., analog to the A,$B+$, B,... system ubiquitous in higher education institutions). Student i's discrete score upon completion of the course is recorded by the discrete random variable $S^{\kappa}_{i,t}$. The setting described above is formally captured by the following collection of equations.

$$
\begin{aligned}
&[\,0\,] \quad && a_{i,t_i} && \sim F_a(\cdot\,),; \quad j_{i,t} = j, \quad \kappa_{i,t} = \kappa, \\
&[\,1\,] && a_{i,t} && = f_j(a_{i,t-1}, \mathbf{x}_{i,j,t}), \\
&[\,2\,] && s_{i,t} && = \beta_j \cdot a_{i,t} + c_j, \\
&[\,3\,] && S^{\kappa}_{i,t} && = \sum_l s_l \cdot \mathbf{1}\big\{ s_{l+1} > s_{i,t} + \tilde{\eta}^{\kappa}_{i,j,t} \geq s_l \big\}, \\
&[\,4\,] && R^{\kappa}_{i,t} && = \mathbf{1}\big\{ s_{i,t} + \tilde{\varepsilon}^{\kappa}_{i,j,t} \geq s_{l^*} \big\}.
\end{aligned}
$$

To fix ideas, suppose a student i enters academic term t with a type given by $a_{i,t-1}$. Equation [1] describes the learning output of such a student after being paired with instructor j. This quantity, unobserved by the researcher, is denoted by $f_j(a_{i,t-1}, \mathbf{x}_{i,j,t})$. Notice that besides the student's ability, learning outputs are affected by a vector $\mathbf{x}_{i,j,t}$ capturing learning-related covariates. That production functions are indexed by j implies the possibility of different learning outputs across instructors even conditional on the values of $a_{i,t-1}$ and $\mathbf{x}_{i,j,t}$. In turn equation [2] describes the score outcome the student obtains. In other words, the student's learning output as expressed by its instructor's grading policy.

Equations [3] and [4] describe how i's learning output maps into a course discrete score and a course dropping decision. Intuitively, we can think of $\beta_j \cdot a_{i,t} + c_j$ as the student's expected continuous score obtained in period t. To make it clear that such a score depends on the student's ability and the underlying covariates, we will occasionally use the notation $s_{i,t} \equiv s_j(a_{i,t-1}, \mathbf{x}_{i,j,t})$. Notice that this quantity differs from $S^{\kappa}_{i,t}$, the discrete score obtained by the student. While the former represents the instructor's granular assessment of the student's performance (e.g., the 100 points based raw score i obtains in j's course) the latter is a discrete variable indicating the region of the score support where $s_j(a_{i,t}, \mathbf{x}_{i,j,t})$ falls. Institutional rules fix thresholds $s_1 > s_2 > ... > s_L$ which determine the map between a student's continuous underlying score and its final discrete score. As an example, student i obtains a score of s_l if $s_j(a_{i,t}, \mathbf{x}_{i,j,t})$ (plus a random perturbation) exceeds the threshold s_l but falls short of the threshold for score s_{l+1}. The error terms $\tilde{\eta}^{\kappa}_{i,j,t}$ and $\tilde{\varepsilon}^{\kappa}_{i,j,t}$ perturb the relationship between a student's continuous and discrete course scores.

We highlight that ultimately, a student's outputs depend on the instructor's grading policy (β_j, c_j). We can interpret these as encapsulating the leniency or stringency with which a student's learning is evaluated in the course. Figure 1 depicts this by plotting the map $a_{i,t} \to \beta_j \cdot a_{i,t} + c_j$ for two different instructors who differ only in their grading policies (i.e., but whose learning production functions coincide: $f_j = f_{j'}$). For instance, the blue curve depicts an instructor who while more lenient in terms of the marginal return to learning (e.g., a higher β_j), is more stringent in terms of the level of the scoring equation (e.g., a smaller c_j). These differences map two students, with the same underlying learning output, to different scores under each of the professors. For example, while under the red curve students with low ability levels end up above the s_l threshold, the same is not true under the scoring equation corresponding to the blue curve.

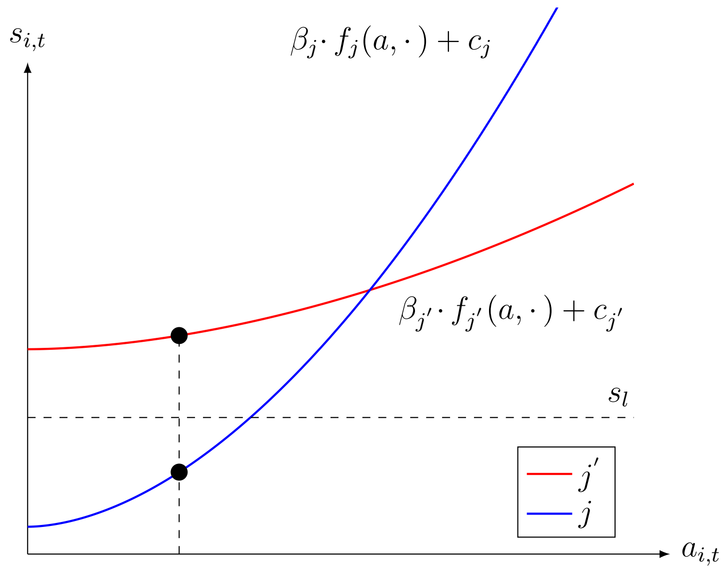


Figure 1: Grading policy differences

As explained before, student i can choose to drop instructor j's section of course κ which is explained by equation [3]. Intuitively, i chooses to drop the section whenever its underlying continuous score, $s_j(a_{i,t}, \mathbf{x}_{i,j,t})$, places him below a certain threshold s_{l^*}. As an example, one can think of a student choosing to drop the course whenever it expects to end up with a fail score. Notice that the error term $\tilde{\varepsilon}^{\kappa}_{i,j,t}$ allows for heterogeneity in the course dropping threshold. It can also capture uncertainty resulting from course dropping decisions depending on noisy signals of the true underlying score (i.e., the student's perception of its position in the grading policy after the midterm, but without the final exam signal). An error term $\tilde{\varepsilon}^{\kappa}_{i,j,t}$ with a large variance could for instance correspond to a situation in which students face a lot of uncertainty before the dropout deadline.

We conclude our description of the learning production function side of the model by highlighting the need of specifying the units under which learning output is measured. Under a setting with standardized testing, all students are tested under a common grading policy so that a natural choice is to measure $f_j(a, \mathbf{x})$ in the units of the common test. This is not the situation in our empirical context as in higher education institutions students are evaluated according to the grading policy of their matched instructor. We instead propose defining learning output in terms of the grading policy of a reference professor $\hat{j}^{\kappa}$ for each course κ. It then follows that for any course κ instructor j, we interpret $f_j(a, \mathbf{x})$ as the learning output of a student who is instructed by professor j but graded according to instructor $\hat{j}^{\kappa}$'s grading policy. We can then interpret (β_j and c_j) as deviations of instructor j's grading policy from that of the reference professor. Under this intuition, c_j corresponds to the baseline score granted by the instructor and β_j captures the marginal reward to learning under instructor j (i.e., in both cases relative to the reference professor). It is immediate that under the proposed normalization, $\beta_{\hat{j}^{\kappa}} = 1$ and $c_{\hat{j}^{\kappa}} = 0$.

Discussion

At the core of the project is the objective of quantifying differences in instructors' pedagogical abilities. The fact that this is a prevalent theme in many Education Economics studies prompts us to compare our framework with other models commonly employed in the literature. For instance, consider the following model for the creation of learning outputs presented below. It abstracts from many of the modeling features of our model by assuming that continuous scores are directly observed by the researcher, student ability is the only learning relevant characteristic on the student side, that students cannot drop sections of a course, among others.

$$
\begin{aligned}
&[\,0\,] \qquad a_{i,t} \sim F_a(\cdot\,),\\
&[\,1\,] \qquad s_{i,t} = f_j(a_{i,t}) + \tilde{\eta}_{i,j,t}.
\end{aligned}
$$

This simple model encapsulates (in essence) a considerable body of work in measuring learning outcomes. For example, the restriction of the production function above to an additively separable form in student and professor attributes is common in many education papers. As another example, the fact that scores are expressed in the same units as the learning production function corresponds to situations in which students are evaluated through a standardized test. That such a model is commonly found in the literature to a great extent reflects the fact that from an econometric standpoint, the model is highly tractable as it allows for the inference of learning returns associated with each instructor through a direct examination of within-professor score distributions. For example, in the above, the average score for students enrolled under instructor j conditional on the ability type $a_{i,t}$, constitutes a consistent estimator for the image $f_j(a_{i,t})$.

However, as is clear from our previous discussions, the model is a poor description of higher education environments, which is why we choose to deviate from it. We expand on this discussion by showing how each deviation in the direction of our model of choice involves empirical challenges in the identification of learning returns which we will have to solve when thinking about the identification of the model. To be concrete, consider a slight modification of the previous model as to account for differences in instructors' grading policies while keeping other aspects the same,

$$\begin{aligned} &[\,0\,] \qquad a_{i,t} \sim F_a(\cdot\,), \\ &[\,1\,] \qquad s_{i,t} = \beta_j \cdot f_j(a_{i,t}) + c_j + \tilde{\eta}_{i,j,t}. \end{aligned}$$

Even without considering the other elements in our framework, it is evident that the within-professor conditional average approach discussed earlier is no longer useful in identifying the learning production functions. For example, the average scores of students under instructor j conditional on the ability type $a_{i,t} = a_0$, is now consistent for a quantity that conflates both learning returns and grading policies, $\frac{1}{n}\sum_i s_{i,t} \quad \rightarrow_p \quad \beta_j \cdot f_j(a_0) + c_j$. Put simply, observing high average scores may indicate either a high learning return under professor j, a choice of a very lenient grading policy, or both. Clearly, the latter is unsatisfactory if the aim is to deduce the nature of an instructor's production function.

As a second example, consider the following alternative deviation from the model in the direction of our framework. Specifically, let's modify the model by allowing students to withdraw from previously enrolled courses/sections. Following our formulation, an example of this corresponds to the following,

$$\begin{aligned} &[\,0\,] \qquad a_{i,t} \sim F_a(\cdot\,), \\ &[\,1\,] \qquad s_{i,t} = f_j(a_{i,t}) + \tilde{\eta}_{i,j,t}, \\ &[\,2\,] \qquad R_{i,t} = \mathbf{1}\{f_j(a_{i,t}) + \tilde{\varepsilon}_{i,j,t} \geq s_{l^*}\}. \end{aligned}$$

As before, consider a similar approach to identifying the learning contribution of j by observing the distribution of scores of its students. Since only the scores of students who choose not to withdraw from a course can be observed in the academic records, the approach based on the average scores of students conditional on ability must also condition on the students not choosing to withdraw from the section of the course. In this case, such an average is again consistent for a quantity that differs from the learning production function, $\frac{1}{n}\sum_i s_{i,t} \quad \to_p \quad f_j(a_0) + \mathbb{E}(\varepsilon_{i,j} \mid R_{i,j} = 1)$. Since under the proposed model, students with a small value of the draw $\varepsilon_{i,j}$ are more likely to withdraw from the course, we should anticipate the expectation of such an error term conditional on not withdrawing from the course to be positive. The implication is that, after observing high average scores for the conditioning set, the researcher is unable to determine whether these scores reflect a high learning return or merely the fact that the average is computed for students with high $\varepsilon_{i,j,t}$ draws.

The concerns listed above, while not exhaustive, clarify the point. The modeling realism gains resulting from our framework come at the cost of a need for additional work in order to estimate the primitives of interest. Indeed, in subsequent sections, we devote a significant amount of effort towards establishing identification results under which the learning production functions and the related primitives can be inferred from the data.

4.2 The demand for sections within a course

The preceding subsection offers a model for how learning and related academic outcomes are determined while conditioning on the student-instructor match observed. We now describe how these matches emerge given the institutional rules governing the demand for sections within a course in our empirical setting.

In Intec, the demand for sections is described by two rules. First, first time students are randomly assigned to instructors of Calculus 1. Second, all other students demand sections of a course by participating of a first-come-first-serve mechanism wherein every academic term t a course enrollment platform opens enabling students to request a section of a course $\kappa \in \mathcal{K}$. Since multiple sections can be associated with the same instructor, we must introduce additional notation that distinguishes two sections under the same instructor. In particular, consider denoting a particular section by $s \in Sec_t^{\kappa}$, where Sec_t^{κ} represents the collection of all sections of course κ active in period t. Of course, each of these sections must be under an instructor j in $\mathcal{J}_t^{\kappa}$. Whenever it is not obviously the case from the context, we explicitly keep track of the professor associated to a section s by using the notation j_s.

The course-enrollment mechanism implies a student trying to enroll a course section faces two sequential decisions. First, student i must opt for an entry time, $\tau > 0$, to access the platform. Subsequently, i must select, from the available sections, which to enroll. These two problems are intertwined as sections are subject to capacity constraints (e.g., limited slots available for each section due to institutional constraints), implying that students may need to access the platform early to secure enrollment in highly demanded sections. Formally, we frame the decision problem of a student i demanding a section of course κ in terms of the following two-stages optimization problem.

$$\max_{\tau \geq 0} \left[\max_{s} \left\{ U_{i,s,t} \text{ s.t. } s \in \mathcal{C}_t(\tau) \right\} + \phi(\tau) \right];$$
$$\mathcal{C}_t(\tau) \equiv \{ s \in Sect_t^{\kappa} : \tau \leq \tau_{s,t}^{eq} \}.$$

The inner maximization problem corresponds to a standard discrete choice problem, where students choose the section of the course that maximizes their utility, denoted by $U_{i,s,t}$. Importantly, students can only select sections from the set $\mathcal{C}_t(\tau)$, which includes all active sections whose capacity constraint is not binding at the entry time τ. In other words, i's choice set at period t is potentially smaller than $Sect_t^{\kappa}$, the set of all active sections in t. As the availability of a slot in a given section depends on the demand decisions of other students, we must treat the choice set faced by i as an equilibrium object. The term $\tau_{s,t}^{eq} > 0$ denotes the equilibrium time at which the capacity constraint of section s becomes binding. For instance, very popular sections will be associated to small values of $\tau_{s,t}^{eq}$ while the opposite holds for unpopular sections. In turn, the outer maximization problem pertains to the decision of when to enter the platform, recognizing that this choice influences the set of options the student will ultimately face. The formulation above assumes that students face a cost from participating in the course enrollment mechanism, $\phi(\tau)$, and that such a cost is a function of their platform entry time decision. This cost rationalizes the fact that not all students choose to enter the platform at the same time and can be interpreted as a reluctance towards early enrollment or more generally of participating in the mechanism.

We adopt a random utility model approach for the inner maximization problem by treating $U_{i,s,t}$ as a random variable. This allows us to model preferences in terms of a systematic component, shared by all students with common characteristics, as well as an idiosyncratic component capturing unobserved heterogeneity in students' preferences. In particular, as seems reasonable from our descriptive evidence exercises, we assume student i's utility for section s under an instructor j takes the following form

$$U_{i,s,t} = U_{s,t}(s_j(a_{i,t}, \mathbf{x}_{i,j,t}) \, , \, f_j(a_{i,t-1}, \mathbf{x}_{i,j,t})) + \nu_{i,s,t}.$$

Intuitively, our model for section preferences postulates that students derive utility not only from the score they expect to obtain under instructor j but also from the actual learning derived from the match. Different forms for the systematic utility term $U_{s,t}(\cdot)$ can be used to capture different valuations for these two components in the student population. For instance, at the extremes students might have no preferences over either scores or learning outputs. As suggested by the indexing of the systematic utility, students might also have preferences related to other aspects of the section being demanded, such as the schedule of the course or characteristics of the instructor, not directly related to the learning or scoring outcomes expected by the student. These can be incorporated in the formulation above by the use of (for example) preference fixed effects as part of the systematic utility $U_{s,t}$ specification. The term $\nu_{i,s,t}$ is an error term reflecting the idiosyncratic component of utility.

While the above formulation clearly outlines the two steps involved in the course demand problem faced by students, it is also possible (and potentially advantageous from an empirical point of view) to express the demand problem from a different perspective. Namely, one can think of students first choosing a section s from the full set of active sections $Sect_t^{\kappa}$, and subsequently choosing a platform entry time that maximizes utility while securing a slot at such a section choice. We can derive this alternative formulation by manipulating the expression for the demand model as in the following.

$$\max_{\tau}\left[\max_{s}\left\{U_{i,s,t} \text{ s.t. } s \in \mathcal{C}_t(\tau)\right\} + \phi(\tau)\right] = \max_{s}\left[U_{i,s,t} + \max_{\tau}\left\{\phi(\tau); \text{ s.t. } s \in \mathcal{C}_t(\tau)\right\}\right]$$

One can think of this formulation as the decision of student i from an ex-ante perspective. Before the platform opens students face no constraints on their choice set, as they can always choose to enter the platform sufficiently early (i.e., which requires accepting the cost of such decision) in a way that ensures the availability of the section being demanded.

Discussion

The story behind the project is one of the potential emergence of undesirable student-instructor assignments due to institutional policies governing the way in which the matches are constructed. Having introduced our demand model for course/sections, it is of value to illustrate, at a conceptual level, how such undesirable allocations might arise in our setting. One channel is the obvious one: in the presence of matching effects, the random assignment of first-term students to sections is unlikely to result in a learning optimal assignment. In what follows we emphasize a second channel: students, while concerned about learning outcomes in their enrollment decisions, also consider the scores they anticipate achieving under their chosen section. Given that instructors leading to high learning returns may differ from those associated with lenient grading policies, certain students on the margin might end up in suboptimal learning matches. A straightforward example from the model illuminates the mechanisms driving this argument.

For instance, consider a scenario where the learning production functions assume the following multiplicatively separable form, $f_j(a_i) = \delta_j \cdot a_i$. Let's think about the problem of assigning two students, i_H and i_L, to two instructors, j_H and j_L (each under a single slot section). We assume that students initial abilities are such that $a_{i_H} > a_{i_L}$, and that

$\delta_{j_H} > \delta_{j_L}$ so that both students are more productive under j_H relative to j_L. For the sake of simplicity, let's also set aside the heterogeneity and uncertainty in the learning and scoring equations by assuming the $\tilde{\eta}$ and $\tilde{\varepsilon}$ terms to be exactly equal to zero. When judging the nature of an assignment, one can think about different reasonable ranking criteria. Let's entertain a simple rule under which assignments are ranked in terms of the average learning output they induce.

In our example simple toy example two assignments are possible: either (j_H, i_H) or (j_H, i_L). It is clear that under our assumptions the former assignment leads to a higher average learning output than the latter. One can see this from the increasing differences nature of the production function which implies that in moving from the latter assignment to the former one, the net loss resulting from dissolving the match (j_H, i_L) (i.e., $a_{i_L} \cdot (\delta_{j_H} - \delta_{j_L})$) is more than compensated by the net gain resulting from pairing student i_H with the high return instructor j_H (i.e., $a_{i_H} \cdot (\delta_{j_H} - \delta_{j_L})$). For our purposes, however, the relevant question is whether or not such assignment would arise under the observed course enrollment mechanism. It is useful to think about the question under the extreme scenarii of students who care only about scores or only about learning.

For instance, consider the latter. In this case it is clear that a course enrollment mechanism as the one in our setting must lead to the ideal assignment. To see this, suppose that this is not the case so that we observe the match (i_L, j_H) in the data. Since in our model students freely demand course sections at the equilibrium costs $\phi_j \equiv \phi(\tau^{eq}_{\kappa,j})$, our observation implies that $f_{j_H}(a_{i_L}) + \phi_{j_H} > f_{j_L}(a_{i_L}) + \phi_{j_L}$ or equivalently $f_{j_H}(a_{i_L}) - f_{j_L}(a_{i_L}) > +\phi_{j_L} - \phi_{j_H}$. In other words, i_L is willing to assume the equilibrium cost associated to demanding j_H's section. Notice however that given the nature of the learning production function, it must also be the case that i_H finds it optimal to demand such section since (i.e., $f_{j_H}(a_{i_H}) - f_{j_L}(a_{i_H}) > f_{j_H}(a_{i_L}) - f_{j_L}(a_{i_L})$). Since at equilibrium this cannot be the case, one should expect equilibrium costs to adjust until student i_L is discouraged from demanding j_H

In the other hand, the situation can be quite different when students are concerned instead about the score they obtain under each section: $\beta_j f_j(\cdot) + c_j$. In such a situation it is immediate that under certain grading policy configurations, the demand patterns described above might be reversed. For instance, if $\beta_{j_H} \cdot \delta_{j_H} < \beta_{j_L} \cdot \delta_{j_L}$ it is easy to see that while both students prefer instructor j_L, student i_H will end up matched to such an instructor as it can outbid i_L under the current assignment mechanism.

This example is of value in that it illustrates the main tension in simple terms. Our model can however accomodate much more interesting matching situations. To start, the generality of our learning production function model can result in situations under which even while $f_{j_H}(\cdot)$ is pointwise above f_{j_L}, the efficient assignment involves the match (i_L, j_H). This could arise, for instance, when the learning outcome differences across professors are large for low ability students but small for high ability students due to non-linearities of the production functions. Second, the fact that we allow for the production of learning to be influenced by other student/professor covariates again implies the possibility of much richer tensions than the one described in the example above. Finally, while the goal of maximizing average learning outputs might be important, other criteria might be relevant for the university authorities such as equity in the access to high quality instruction.

The discussion also makes it clear that the outcome assignment depends crucially on the course enrollment mechanism used by the university. For instance, in our example a priority mechanisms that allows certain students to enroll sections first might lead to an either desirable or undesirable assignment depending on the context considered, by reducing the competition faced by certain students when demanding slots in a course. The complexity associated to studying any of these situations theoretically suggests an empirical approach is better suited for such purposes. In subsequent sections we follow this approach by estimating the primitives of the model and conducting counterfactual exercises reflecting different assignment mechanisms a university's administration might entertain.

5 Empirical model and Identification

In this section, we consider the econometric specification of the conceptual model introduced earlier. Additionally, we present arguments for identifying the quantities of interest in the model. Following the structure of the previous section, the analysis is divided in terms of identification arguments for the learning production functions, followed by a discussion of the identification of the course/section demand model.

5.1 Identification of the Learning Production Function

Consider the problem of inferring the shape of the learning production function associated to a given instructor in course $\kappa \in \mathcal{K}$. As explained before, the main challenge this poses lies in disentangling the contributions of grading policies and actual learning outputs from the observed distribution of scores. To address this, we propose exploiting the sequential enrollment of students into courses in the sequence $\mathcal{K}$, and the fact that while learning in course κ impacts outcomes in course $\kappa + 1$, the same is not true about the grading policies used by κ instructors. The starting point is a set of assumptions regarding the distribution of the error terms in our production function model.

Assumption 1. *The following assumptions are assumed to hold,*

1. *Random variables* $\eta^{\kappa}_{i,j,t}$ *and* $\varepsilon^{\kappa}_{i,j,t}$ *exists such that* $\tilde{\varepsilon}^{\kappa}_{i,j,t} = \sigma^{\kappa}_{\varepsilon} \cdot \varepsilon^{\kappa}_{i,j,t}$ *and* $\tilde{\eta}^{\kappa}_{i,j,t} = \sigma^{\kappa}_{\eta} \cdot \eta^{\kappa}_{i,j,t} + \sigma^{\kappa}_{\varepsilon} \cdot \varepsilon^{\kappa}_{i,j,t}$ *for the scalars* $\sigma^{\kappa}_{\varepsilon}, \sigma^{\kappa}_{\eta}$.

2. *The sequences* $\{\eta^{\kappa}_{i,j,t}\}_{i,j,t}$ *and* $\{\varepsilon^{\kappa}_{i,j,t}\}_{i,j,t}$ *are mean zero and i.i.d.. Their distributions, denoted by* F_{η} *and* F_{ε}*, are known by the researcher.*

3. *The random variable* $\nu_{i,s,t}$ *is independent of* $\eta^{\kappa}_{i,j,t}$*,* $\varepsilon^{\kappa}_{i,j,t}$.

The first two parts of the assumption are technical and it is used to facilitate inversion arguments used in identifying the learning production functions. In essence it parameterizes the error terms in both the scoring and the dropping equations. Collectively they can be understood in terms of the distribution of the error terms in the scoring/dropping equation belonging to a parameteric family known to the researcher up to their variance. Notice that correlations between the unobserved components influencing students' dropping decisions and their final scores are embedded into the model through the addition of the $\eta^{\kappa}_{i,j,t}$ and $\varepsilon^{\kappa}_{i,j,t}$ error terms in the scoring equation (i.e., $\sigma^{\kappa}_{\eta} \cdot \eta^{\kappa}_{i,j,t} + \sigma^{\kappa}_{\varepsilon} \cdot \varepsilon^{\kappa}_{i,j,t}$). The more restrictive assumption is 3. which in simple terms assumes that conditional on the score and the learning outputs a student expects when demanding a given section of a course, any utility differences across students are just preference noise. In particular this noise is unrelated to the unobserved components that might affect learning outcomes. The assumption resembles a selection on observables assumption with the exception that neither the score or the learning outputs a student selects on are directly observed by the econometrician.

Main identification arguments

Let's now construct an argument for the identification of the learning production function given Assumption 1. For expositional reasons, we present results for a simplified version of our model and leave a treatment of the fully fledged framework for the appendix section. In particular, we consider a version of our model in which students don't have the option of dropping a section of a course. This allows us to bypass some technical details which are not a the core of the results. Second, the focus here is on the identification of the production function primitives associated to instructors in the first course of the sequence, $\kappa = 1$. Constructing arguments for other courses will be a simple matter of adapting the notation in what follows. In addition, since our arguments will not depend on the specific time period in which a student enrolls a course/section but instead just require keeping track of whether a student is in its first or second academic term in the university, we simplify the notation by omitting the time indices. It will be clear from the context whether an argument is based on first or second term students.

With this in mind, consider the problem of identifying the learning production function of a $\kappa = 1$ instructor j^1. Our analysis focuses on the collection of all students who in their first enrollment instance of course $\kappa = 1$ obtain a score of s_l or higher conditional on enrolling a section under j^1. Furthermore, we condition on students of an initial type $a_{i,t_i} = a_0$ and who enroll j^1's section under a vector of covariates $\mathbf{x}_1$. Our structural model offers an expression for the conditional probability described above.

$$\begin{aligned}
\mathbb{P}\big(S^1_{i,j^1} \geq s_l \mid a_0,\ \mathbf{x}_1,\ j^1\big) &= \int_\eta \mathbf{1}\big\{\beta_{j^1}\cdot f_{j^1}(a_0, \mathbf{x}_1) + c_{j^1} + \sigma^1_\eta \cdot \eta \geq s_l\big\}\ f_\eta(\eta)\ d\eta, \\
&= \int_\eta \mathbf{1}\left\{\eta \geq \frac{s_l - \beta_{j^1}\cdot f_{j^1}(a_0, \mathbf{x}_1) - c_{j^1}}{\sigma^1_\eta}\right\} f_\eta(\eta)\ d\eta, \\
&= \left[1 - F_\eta\left(\frac{s_l - \beta_{j^1}\cdot f_{j^1}(a_0, \mathbf{x}_1) - c_{j^1}}{\sigma^1_\eta}\right)\right].
\end{aligned}$$

In words, student i achieves a score above s_l whenever $\beta_{j^1}\cdot f_{j^1}(a_0, \mathbf{x}_1) + c_{j^1} + \sigma^1_\eta \cdot \eta^1_{i,j^1}$ (i.e., the students expected continuous score) falls weakly above the threshold s_l. The expression above just establishes a relationship between the observed mass of students satisfying the event $S^1_{i,j^1} \geq s_l$ (within the conditioning set), and a function of the primitives of the model. Under Assumption 1, we can invert the relationship to obtain an equivalent expression as is given below,

$$\frac{s_l - \beta_{j^1} \cdot f_{j^1}(a_0, \mathbf{x}_1) - c_{j^1}}{\sigma^1_\eta} = \underbrace{F^{-1}_\eta\Big[\mathbb{P}\big(S^1_{i,j^1} \geq s_l \mid a_0,\ \mathbf{x}_1,\ j^1\big)\Big]}_{\equiv \theta(s_l \,\mid\, a_0, \mathbf{x}_1, j^1)}.$$

The latter lends itself to an intuitive interpretation. Consider the marginal student, whose η^1_{i,j^1} draw places him precisely at the boundary between scores s_l and s_{l-1}. Within the conditioning set, such marginal student's η draw defines the entire mass of students who ultimately receive a score above s_l within the conditioning set. For example, those with a higher η^1_{i,j^1} will achieve scores weakly above s_l, while those with smaller draws obtain a strictly smaller score. The expression above identifies the marginal student's draw by finding the precise η^1 value such that the mass to its right under $F_\eta(\cdot)$ corresponds exactly to the observed share of students who obtain a score above s_l, $\mathbb{P}\big(S^1_{i,j^1} \geq s_l \mid a_0,\ \mathbf{x}_1,\ j^1\big)$. Importantly, we can also identify the marginal student as that with a draw satisfying $(s_l - \beta_{j^1}\cdot f_{j^1}(a_0, \mathbf{x}_1) + c_{j^1} + \sigma^1_\eta)/\sigma^1_\eta = \eta$. The equality derived merely states that these two expressions identifying the marginal student must agree. The notation $\theta(s_l \mid a_0, \mathbf{x}_1, j^1)$ makes it clear that any change in the conditioning arguments leads to a change in the marginal student's identity.

By itself $\theta(s_l \mid a_0, \mathbf{x}_1, j^1)$ does not offer much insight into the underlying model as it pools multiple primitives into a single expression. However, as stated in the following proposition, when considered for two different score cutoffs, s_l and $s_{l'}$, it is possible to start gaining some understanding of some parameters of interest.

Proposition 1. *The images $f_{\hat{j}^1}(a_0, \boldsymbol{x}_1)$ (i.e., $\kappa = 1$'s reference learning output) and the* *variance parameter σ^1_η are point identified.*

Proof. Fixing the conditioning quantities $a_0, \mathbf{x}_1$, we can identify the marginal students associate to the letter scores s_l and $s_{l'}$,

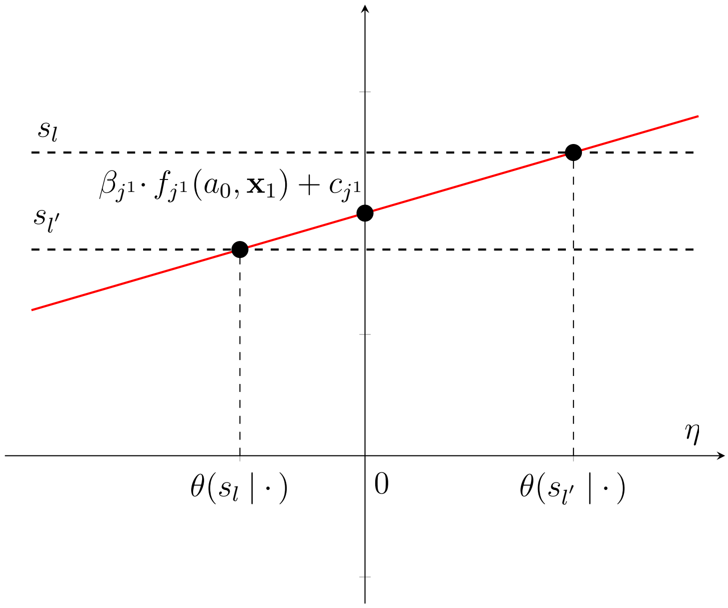
$$\theta(s_l|a_0, \mathbf{x}_1, j^1) = \frac{s_l - \beta_{j^1} f_{j^1}(a_0, \mathbf{x}_1) - c_{j^1}}{\sigma^1_\eta} \text{ and } \theta(s_{l'}|a_0, \mathbf{x}_1, j^1) = \frac{s_{l'} - \beta_{j^1} f_{j^1}(a_0, \mathbf{x}_1) - c_{j^1}}{\sigma^1_\eta}.$$

When $l \neq l'$, the latter defines a system of two equations on the unknowns σ^1_η and $\beta_{j^1} f_{j^1}(a_0, \mathbf{x}_1) + c_{j^1}$. Solving for the unique solution to the system leads to the following expression,

$$\sigma^1_\eta = \frac{s_l - s_{l'}}{\theta(s_l \mid a_0, \mathbf{x}_1, j^1) - \theta(s_{l'} \mid a_0, \mathbf{x}_1, j^1)},$$
$$\beta_{j^1} f_{j^1}(a_0) + c_{j^1} = s_l - \frac{s_l - s_{l'}}{\theta(s_l \mid a_0, \mathbf{x}_1, j^1) - \theta(s_{l'} \mid a_0, \mathbf{x}_1, j^1)} \cdot \theta(s_l \mid a_0, \mathbf{x}_1, j^1).$$

The identification of the image $f_{\hat{j}^1}(a_0, \mathbf{x}_1)$ follows from considering the second equation above for $j^1 = \hat{j}^1$ and recalling that for the reference instructor $\beta_{\hat{j}^1} = 1$ and $c_{\hat{j}^1} = 0$. ■

A graph serves as a visual representation of the argument behind the proof. Within the conditioning set, we can think of a student i's score as a linear function of its unobserved draw η^1_{i,j^1}, with an intercept of $\beta_{j^1} \cdot f_{j^1}(a_0, \mathbf{x}_1) + c_{j^1}$ and a slope of σ^1_η. Our identification of the marginal student associated to each letter score corresponds to identifying a point in the score equation. In the graph, for instance, our arguments allow us to identify the points $(\theta(s_l \mid \cdot), s_l)$ and $(\theta(s_{l'} \mid \cdot), s_{l'})$. The fact that a linear equation is pinned down by two of its points allows us to identify both the slope and the intercept of the curve. In more intuitive terms, the result states that we can always identify the learning output of an instructor j^1, in terms of its own grading policy, by directly inspecting the distribution of scores such instructor induces.



It must be emphasized that in the absence of variations in grading policies among professors, the aforementioned arguments would allow us to fully identify the learning production functions associated to each instructor. The situation resembles a scenario under standardized tests where observed scores directly reflect disparities in teaching abilities. In our context, the presence of grading policies necessitates additional efforts to separate the effects of learning returns from grading policies. With this purpose in mind let's consider the performance of students in our conditioning set in the subsequent course, $\kappa = 2$. To be precise, we are interested in the fraction of students (within our conditioning set) who after successfully completing $\kappa = 1$, enroll a section of $\kappa = 2$ under instructor j^2 and obtain a score above s_l. In addition, we focus our attention of the subpopulation of students who enroll j^2's section under a vector of covariates $\mathbf{x}_2$. As before, our model implies a concrete expression for the conditional probability of the event described above,

$$
\begin{aligned}
&\mathbb{P}(S^2_{i,j^2} \geq s_l, \ \mid a_0, \ \mathbf{x}_1, \ \mathbf{x}_2, \ j^1, \ j^2, \ S^1_{i,j^1} \geq s_{l^*}), \\
&\quad = \left[1 - F_\eta\left(\frac{s_{l^*} - \beta_{j^1}\cdot f_{j^1}(a_0,\mathbf{x}_1) - c_{j^1}}{\sigma^1_\eta}\right)\right]\left[1 - F_\eta\left(\frac{s_l - \beta_{j^2}\cdot f_{j^2}(f_{j^1}(a_0,\mathbf{x}_1),\mathbf{x}_2) - c_{j^2}}{\sigma^2_\eta}\right)\right], \\
&\quad = \left[1 - F_\eta\left(\theta(s_{l^*} \mid a_0,\mathbf{x}_1,j^1)\right)\right]\left[1 - F_\eta\left(\frac{s_l - \beta_{j^2}\cdot f_{j^2}(f_{j^1}(a_0,\mathbf{x}_1),\mathbf{x}_2) - c_{j^2}}{\sigma^2_\eta}\right)\right].
\end{aligned}
$$

This expression bears a close resemblance to the one discussed earlier for the identification of $\theta(s_l \mid a_0, \mathbf{x}_1, j^1)$. The difference lies in the consideration of students who not only achieve a score of s_l or above in course $\kappa = 2$, but also those who are assigned to a specific instructor j^1 in course $\kappa = 1$ and successfully complete the course under such professor. Just as before, this ratio represents the minimum value of η^2 consistent with a student obtaining a score weakly above s_l in course $\kappa = 2$ (given the conditioning set of students). Inverting the relationship we obtain the following identity,

$$\frac{s_l - \beta_{j_2} \cdot f_{j_2}(f_{j_1}(a_0, \mathbf{x}_1), \mathbf{x}_2) - c_{j_2}}{\sigma_\eta^2} = \underbrace{F_\eta^{-1}\left[\frac{\mathbb{P}\big(S^2_{i,j^2} \geq s_l \mid a_0,\ \mathbf{x}_1,\ \mathbf{x}_2,\ j^1,\ j^2,\ S^1_{i,j^1} \geq s_{l^*}\big)}{(1 - F_\eta(\theta(s_{l^*} \mid a_0, \mathbf{x}_1, j^1))}\right)}_{\theta(s_l \mid a_0, \mathbf{x}_1, \mathbf{x}_2, j^1, j^2)}.$$

We are now in a position that allows us to state the main result of this section. The result establishes the identification of the $\kappa = 1$ production functions under an injectivity assumption. We state and prove the result before considering an intuitive discussion of the content behind the Proposition.

Proposition 2. *The following identification results hold,*

1. *The image of the composition $\beta_{j^2} \cdot f_{j^2}(f_{j^1}(a_0, \boldsymbol{x}_1), \boldsymbol{x}_2) + c_{j^2}$ and the variance term σ_η^2 are point identified,*

3. *Suppose that $f_{j^2}(\cdot\,, \boldsymbol{x}_2)$ is injective for $\boldsymbol{x}_2$ fixed. Then the image $f_{j^1}(a_0, \boldsymbol{x}_1)$ is point identified provided the existence of $\tilde{a}_0$ such that $f_{j^2}(f_{j^1}(a_0, \boldsymbol{x}_1), \boldsymbol{x}_2) = f_{j^2}(f_{\hat{j}^1}(\tilde{a}_0, \boldsymbol{x}_1), \boldsymbol{x}_2)$.*

Proof. We start by following the same reasoning as in the previous proposition. In particular, fixing the conditioning variables $(a_0, \mathbf{x}_1, \mathbf{x}_2)$ consider the following system of equations on the unknowns σ_η^2 and $\beta_{j^2} \cdot f_{j^2}(f_{j^1}(a_0, \mathbf{x}_1), \mathbf{x}_2) + c_{j^2}$,

$$\frac{s_l - \beta_{j^2} \cdot f_{j^2}(f_{j^1}(a_0, \mathbf{x}_1), \mathbf{x}_2) - c_{j^2}}{\sigma_\eta^2} = \theta(s_l \mid a_0, \mathbf{x}_1, \mathbf{x}_2, j^1, j^2),$$
$$\frac{s_{l'} - \beta_{j^2} \cdot f_{j^2}(f_{j^1}(a_0, \mathbf{x}_1), \mathbf{x}_2) - c_{j^2}}{\sigma_\eta^2} = \theta(s_{l'} \mid a_0, \mathbf{x}_1, \mathbf{x}_2, j^1, j^2).$$

The first claim follows from noticing that when considering $l \neq l'$, the equations above define a system with a unique solution identifying both σ_η^2 and $\beta_{j^2} \cdot f_{j^2}(f_{j^1}(a_0, \mathbf{x}_1), \mathbf{x}_2) + c_{j^2}$.

Consider now the final claim in the proposition. Under the premise, we can find an ability level $\tilde{a}_0$ such that $\beta_{j^2} \cdot f_{j^2}(f_{j^1}(a_0, \mathbf{x}_1), \mathbf{x}_2) + c_{j^2} = \beta_{j^2} \cdot f_{j^2}(f_{\hat{j}^1}(\tilde{a}_0, \mathbf{x}_1), \mathbf{x}_2) + c_{j^2}$. Moreover, given the validity of the first part of the claim in the proposition, whose truth we have already asserted, we can find $\tilde{a}_0$ by directly inspecting the observed data. It follows from the injectivity of $f_{\hat{j}^2}(\cdot)$ that $f_{j^1}(a_0, \mathbf{x}_1) = f_{\hat{j}^1}(\tilde{a}_0, \mathbf{x}_1)$. However, since Proposition 1 has already established the identification of $\hat{j}_1$'s production function, the latter equality implies we can directly infer the image of j^1's production function at the argument $(a_0, \mathbf{x}_1)$. ■

Proposition 1, our main identification argument, can be understood in terms of a simple though experiment. Suppose we observe two students with the same initial ability level a_0 but assigned to different $\kappa = 1$ instructors: student one with instructor j^1 and student two with $\tilde{j}^1$. As discussed, comparing their $\kappa = 1$ scores directly is uninformative for discerning potential gaps in their learning outputs. The disparity in the scores could be due to either instructor quality differences or differences in the grading policies used by these instructors.

A potential solution to this issue consists in comparing these two students, not in terms of their $\kappa = 1$ scores, but in terms of some other future signal related to $\kappa = 1$'s learning returns but not $\kappa = 1$'s grading policies. Carefully choosing such signals then becomes very important. For instance, one concern is that as we increase the time distance between the enrollment of $\kappa = 1$ and the measurement of the signal, the noise in the latter might increase, making it difficult to detect differences in instruction quality empirically. Moreover, one might be concerned about differences in the academic path followed by the students after $\kappa = 1$ enrollment, and prior to the measurement of the signal, which would invalidate the ideal type of ceteris paribus exercise we would like to approximate.

Given these concerns, a natural choice is to consider the scores of these students in the immediately subsequent course in the sequence. Some care is required in implementing the approach. For instance, it is reasonable to limit the comparison to students who share a common $\kappa = 2$ professor to avoid confounding effects from different $\kappa = 2$ instructors. But even then, one might be concerned about separating the contribution of this common $\kappa = 2$ professor in the observed score differences across the students. Proposition 2 states that this last point is not an issue as the contribution of the $\kappa = 2$ instructor can be filtered out from the accounting under the injectivity of its production function. Figure 2 captures this intuition graphically (while omitting from the notation $\mathbf{x}_1$ and $\mathbf{x}_2$).

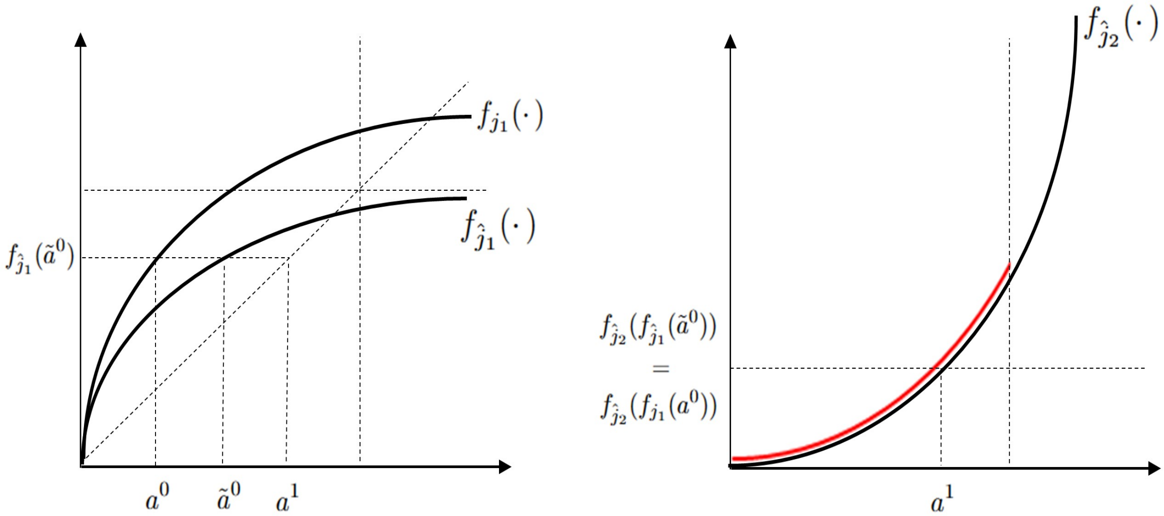


Figure 2: Identification Argument for $f_j(a, \mathbf{x})$

Notice that the same arguments in Proposition 6 lend themselves to a partial identification argument in the absence of a type $\tilde{a}_0$ as required by the premise. For example, let's entertain a situation in which $f_{\hat{j}_2}(f_{j_1}(a_0, \mathbf{x}_1), \mathbf{x}_2) > f_{\hat{j}_2}(f_{\hat{j}_1}(\tilde{a}_0, \mathbf{x}_1), \mathbf{x}_2)$ for all types $\tilde{a}_0$ whose validity can be directly observed from the data. The logic of the proof above suggests there is still a lot of information we can extract from this inequality. For instance, assuming $f_{\hat{j}_2}(\cdot\,, \mathbf{x}_2)$ is monotone increasing for a fixed $\mathbf{x}_2$, we can conclude that $f_{j_1}(a_0, \mathbf{x}_1)$ must exceed the learning return induced by instructor $\hat{j}_1$ under any student $\tilde{a}_0$ in record. In other words, we can construct a lower bound for the unknown $f_{j_1}(a_0, \mathbf{x}_1)$. Similar situations can be treated in an analog way.

We conclude the subsection by highlighting that once the images $f_{j_1}(a_0, \mathbf{x}_1)$ are identified, we can identify the grading policies associated to each instructor by going back to our results on the marginal student associated to each score cutoff for $\kappa = 1$. Proposition 5.1 formally states the latter.

Proposition 3. *The grading policy of instructor j^1 (i.e., β_{j^1}, c_{j^1}) is point identified provided that $f_{j^1}(a_0, \boldsymbol{x}_1)$ is known for some $(a_0, \boldsymbol{x}_1)$.*

Proof. Consider the expressions for $\theta(s_l \mid a_0, \mathbf{x}_1, j^1)$ and $\theta(s_l \mid \tilde{a}_0, \mathbf{x}_1, j^1)$ for two different student types such that $f_{j^1}(a_0, \mathbf{x}_1) \neq f_{j^1}(\tilde{a}_0, \mathbf{x}_1)$

$$\begin{aligned}
\theta(s_l \mid a_0, \mathbf{x}_1, j^1) &= \frac{s_l - \beta_{j^1} \cdot f_{j^1}(a_0, \mathbf{x}_1) + c_{j^1}}{\sigma_\eta^1}, \\
\theta(s_l \mid \tilde{a}_0, \mathbf{x}_1, j^1) &= \frac{s_l - \beta_{j^1} \cdot f_{j^1}(\tilde{a}_0, \mathbf{x}_1) + c_{j^1}}{\sigma_\eta^1}.
\end{aligned}$$

It is easy to see that given the identification of the images $f_{j_1}(a_0, \mathbf{x}_1)$ and $f_{j_1}(\tilde{a}_0, \mathbf{x}_1)$, the two equations above define a system of equations on the unknowns β_{j^1} and c_{j^1}. The unique solution associated to the system identifies j^1's grading policy. ■

Special Cases

1. A simple example resulting from a concrete parametric form for the production function.

1. Identification under covariates.

1. Course dropping decisions.

2. More general grading policies.

3. Identification via chains.

5.2 Identification of the Course/Section Demand

The preceding section introduces arguments regarding the identification of the production function model. We now turn our attention into the identification of the underlying primitives within the model for the demand of course/sections. We start the discussion the following assumptions regarding the nature of the demand model error terms.

Assumption 2. *The following assumptions are assumed to be satisfied,*

1. *$\{\nu_{i,s,t}\}_{i,s,t}$ is a collection of mean zero i.i.d. random variables whose distribution, denoted by F_ν, is known to the researcher.*
2. *The distribution F_ν is continuous and of full support.*

The first part of the assumption is a standard independence assumption for the demand model error terms. Assuming a common distribution for $\nu_{i,s,t}$ across all indices, is also a standard assumption that allows us map the demand model objects to the observed instructor market shares via simple conditional choice probabilities. The second assumption is technical and simply allows us to borrow some identification results from the discrete choice literature.

Given these assumptions, our identification argument can be frame in terms of two main observations. First, we emphasize the implicit assumption that the cost associated with participating in the course enrollment mechanism (i.e., $\phi(\tau)$) remains uniform across all students, thereby rendering it independent of the student index i. Consequently, we can conceptualize $\max_{\tau}\big\{\phi(\tau);\ \text{s.t. } s \in \mathcal{C}(\tau)\big\}$ as a professor-period specific fixed cost incurred by a student when expressing their preference for instructor j during academic period t. This characteristic proves crucial as it enables us to formulate the course demand problem as a standard discrete choice problem with utilities under alternative-period fixed effects. Denoting the sum term $\lambda_{s,t} + \max_{\tau}\big\{\phi(\tau);\ \text{s.t. } s \in \mathcal{C}(\tau)\big\}$ by $\Phi_{s,t}$, the ex-ante formulation of the demand model for a student enrolling course κ in period t can be written as follows,

$$\max_{s \in Sect_t^\kappa} \Big[\Phi_{s,t} + \alpha_0 \cdot s_j(a_{i,t-1}, \mathbf{x}_{i,j,t}) + \alpha_1 \cdot f_j(a_{i,t-1}, \mathbf{x}_{i,j,t}) + \nu_{i,j,t}\Big]$$

Second, we draw attention to the nature of the identification arguments put forth in the preceding section, which establish the identification of both the learning production functions and the grading policies associated to each professor. Consequently, when considering the identification of the course demand primitives, it becomes possible to regard $f_j(a, \mathbf{x})$ and $s_j(a, \mathbf{x})$ as observed quantities. These two observations significantly simplify the identification problem of the demand model, reducing it to the identification of a simple Random Utility Model (RUM). The conditions under which the primitives of the model are identified are known. We state the result in the following proposition.

Proposition 4. *Suppose that ν_{ijt} are iid error terms across i, jt. In addition suppose that ν_{ijt} is continuously distributed and full support. It follows that α_0, α_1, and Φ_{jt} are point identified.*

Proof. *** Cite Alfred's result. ■

One limitation of the approach described above is that while we can identify the structural terms α_0 and α_1, the term $\Phi_{j,t}$ bundles both the preference fixed effect and the cost fixed effect. Moreover, the latter is an equilibrium object so that one cannot assume that its values will remain constant while conducting counterfactual exercises that affect the demand of students for different sections. We discuss in subsequent sections ways in which we can address this issue for the purpose of conducting meaningful counterfactual exercises.

6 Estimation and Results

6.1 Some Nonparametric Estimates of the Model

1. Quang thinks I should nonparametrically estimate the shape of the learning production functions for some instructors (even if these are not the estimates I ultimately use for the counterfactuals). The idea is that this would (i) allow me to justify then choice of parameterization for the production function to be introduced later on, (ii) allow me to show some muscle by directly implementing the identification argument.

6.2 Parameterizing the model

As discussed above, in principle we could completely estimate the model by implementing a nonparametric estimator based on our identification arguments. While this approach offers certain desirable features, including the ability to refrain from imposing parametric assumptions on key elements such as the learning production functions, practical considerations make the idea of a more restrictive parametric approach attractive. For example, on the side of learning production, our arguments rely on the possibility of matching empirical and theoretical moments for subpopulations of students who share common academic paths. Given the relatively modest class sizes in our context, the observed student count within these subpopulations might be insufficient for empirical moments to closely resemble their theoretical counterparts. Analogous concerns may arise within the demand model.

For this reason, we consider here adopting a fully parametric approach for the estimation exercise. This adjustment not only alleviates data limitation constraints but also affords us the opportunity to specify certain parameters of interest as being common to all instructors which further reduces the data demands of the model. In what follows, we delve into the specifics of these empirical model restrictions and explain how we estimate the resulting model. The final subsection presents the estimates resulting from the approach.

Parameterizing the Learning Production Function

The specification of our empirical model commences with the parameterization of the learning production function associated with each instructor. This is guided by two principal considerations. Firstly, the functional form must exhibit enough flexibility as to accommodate a wide array of learning production shapes. Secondly, the model should be able to capture non-trivial matching effects in the learning production process. Specifically, we aim to capture interactions between instructor characteristics and our measure of student ability.

These two concerns respond to the need to allowing for flexibility at the estimation stage so that the model is capable of capturing the true shape of the learning production functions. For example, consider the additively separable specification common in empirical work, $f_j(a_0, \mathbf{x}) = \delta_j + g(a_0)$. The parameterization would be inadequate for our purposes as it would eliminate, at the modeling stage, the possibility of matching effects in the production of learning. As a second example, the multiplicatively separable parameterization $f_j(a_0, \mathbf{x}) = \delta_j \cdot a_0)$, common in theoretical settings, address the previous concern but implies simplistic positive/negative assortative matching as the only plausible learning ideal scenarios. To address these concerns, our proposal is the following: $f_j(a, \mathbf{x}) = \tilde{\delta}_j^0(\mathbf{x}) + \tilde{\delta}_j^1(\mathbf{x}) \cdot a^{\tilde{\delta}_j^2(\mathbf{x})}$. This formulation accounts for differences in the level of the learning production function, the size of the marginal returns to ability, and the nature of the returns to scale to ability. Additionally, it allows each of these coefficients to vary across instructors both in terms of observed and unobserved attributes.

To be concrete consider partitioning the covariate vector as $\mathbf{x}_{i,j,t} = (\mathbf{x}_{1,i}, \mathbf{x}_{2,j,t})$. The first component encapsulates time-invariant characteristics of students. In our estimation exercise we consider $\mathbf{x}_{1,i} = (sex_i, \{maj_{i,d}\}_{d=1}^4)$ where sex_i is a male dummy variable and $major_{i,d}$ is a dummy indicating whether student i's major choice is part of department d (i.e., we partition the set of all majors in terms of four major departments, closely following the organizational division within the university) one of four major departments in the university. Meanwhile, $\mathbf{x}_{2,j,t} = (load_{j,t}, ten_{1,j,t}, ten_{2,j,t})$. The variable $load_{j,t}$ is a binary variable specifying whether the total number of sections taught by instructor j in the academic period t exceeds a certain threshold. This allows us to account for either positive returns (learning from teaching multiple sections) or negative returns (potentially due to fatigue) associated with an instructor's teaching load in a given term. Furthermore, $ten_{1,j,t}$ and $ten_{2,j,t}$ are binary variables denoting whether or not the instructor's tenure at period t exceeds certain thresholds. The former, $ten_{1,j,t}$, reflects the number of terms of the course sequence that instructor t has taught by the beginning of period t, while $ten_{2,j,t}$ similarly measures tenure across all courses the instructor has taught in the university.

Given these considerations, we parameterize the production function coefficients in terms of Certainly! Here are the three equations aligned:

$$\begin{aligned}
\tilde{\delta}_j^0(\mathbf{x}) &= \delta_j^0 + \boldsymbol{\mu}_0^{'}\mathbf{x}_{2,j,t} + \boldsymbol{\gamma}^{'}\mathbf{x}_{1,i}, \\
\tilde{\delta}_j^1(\mathbf{x}) &= \delta_j^0 + \boldsymbol{\mu}_1^{'}\mathbf{x}_{2,j,t}, \\
\tilde{\delta}_j^2(\mathbf{x}) &= \delta_j^2 + \boldsymbol{\mu}_2^{'}\mathbf{x}_{2,j,t}.
\end{aligned}$$

. Here, $\delta_j^l;\;\; l \in \{0,1,2\}$ are production fixed effects capturing unobserved ways in which specific instructors influence production output. In turn the terms $\mu_{\cdot}^{'}\mathbf{z}_{j,t}$ capture productivity differences arising from observed heterogeneity reflected in $\mathbf{z}_{j,t}$. Importantly, notice the t index in the latter suggesting these variables change over time, thus allowing them to be separately identified from the instructor fixed effects parameters. Finally, $\gamma^{'}\mathbf{x}_{1,i}$ allows for differences in student characteristics other than ability to affect the level of the learning production function.

To complete the description of the learning output empirical model, we must specify the distribution of the error terms in the learning production model. Given that we interpret these as perturbations of the scoring and course dropping equations, a reasonable distributional assumption is $\eta_{i,j,t}^{\kappa} \sim \mathcal{N}(0, \sigma_\eta^\kappa)$ and $\varepsilon_{i,j,t}^{\kappa} \sim \mathcal{N}(0, \sigma_\varepsilon^\kappa)$.

Under the aforementioned parametric assumptions, we derive expressions for the conditional probabilities to be employed in our Maximum Likelihood Estimation (MLE) exercises: (i) the probability of observing a student dropping a course given its type, instructor, and learning covariates, and (ii) the probability of observing a particular score for a student in course κ given its type, instructor, and learning covariates. Denoting the standard normal density by $\phi(\cdot\,)$, these probabilities are given by the following expressions.

$$\mathbb{P}\big(S_{i,j^\kappa}^{\kappa} \geq s_l | a_0, \mathbf{x}, j^\kappa\big) = \int_\varepsilon \int_\eta \mathbf{1}\big\{R_{i,j}^\kappa = 1\big\} \cdot \mathbf{1}\big\{s_{l+1} > s_j(a_0,\mathbf{x}) + \sigma_\varepsilon^\kappa \cdot \varepsilon + \sigma_\eta^\kappa \cdot \eta \geq s_l\big\}\phi(\eta)\phi(\varepsilon) d\eta d\varepsilon$$

$$\mathbb{P}\big(R_{i,j}^\kappa = 1 \mid a_0,\, \mathbf{x},\, j\big) = \int_\varepsilon \mathbf{1}\big\{s_j(a_0,\mathbf{x}) \;+\; \sigma_\varepsilon^\kappa \cdot \varepsilon \geq s_{l^*}\big\}\phi(\varepsilon)\, d\eta$$

Parameterizing the Course/Section Demand model.

For our parameterization of the demand model we choose a simple linear specification of student i's utility in terms of $f_j(a_0, \mathbf{x})$, the learning output resulting from the match, and $s_j(a_0, \mathbf{x})$ the expected continuous score outcome expected by the student upon the match. In particular i's systematic utility for section s guided by instructor j is given by the following,

$$U_{s,t} = \alpha_0 \cdot s_j(a_{i,0}, \mathbf{x}_{i,j,t}) + \alpha_1 \cdot f_j(a_{i,0}, \mathbf{x}_{i,j,t}).$$

Once this is considered all that remains is to specify a concrete distributional form for the error terms $\nu_{i,j,t}$ capturing heterogeneity in taste. We assume these distribute $\nu_{i,j,t} \sim TIEV$. The resulting conditional choice probabilities are of the standard logit form as considered below for a student who demands a section s under instructor j in academic term t.

$$\mathbb{P}(s \mid a_0, \mathbf{x}, t) = \frac{\exp\big(\Phi_{s,t} + \alpha_0 \cdot s_j(a_{i,0}, \mathbf{x}_{i,j,t}) + \alpha_1 \cdot f_j(a_{i,0}, \mathbf{x}_{i,j,t})\big)}{\sum_{s'} \exp\big(\Phi_{s',t} + \alpha_0 \cdot s_{j'}(a_{i,0}, \mathbf{x}_{i,j',t}) + \alpha_1 \cdot f_{j'}(a_{i,0}, \mathbf{x}_{i,j',t})\big)}.$$

6.3 Estimation via Maximum Likelihood

Sequential Maximum Likelihood Approach

Our parametrization of the model and the distributional assumptions over the error terms suggest a simple estimation approach via Maximum Likelihood (ML). Under this approach we could proceed by maximizing the log of the likelihood associated to our observed data as specified in the following expression,

$$\mathcal{L}(\theta) = \sum_{i=1}^{N} \sum_{t=t_{i,0}}^{T_i} log\Bigg[\mathbb{P}\Bigg(j_t \ , \ S_{i,j_{i,t}}^{\kappa_{i,t}} \ \Bigg| \ j_{i,\tau}, ..., j_{i,t_{i,0}}, a_{i,t_{i,0}}, \mathbf{x}_{i,j_{i,t},t} \ ; \ \theta \Bigg)\Bigg].$$

where θ denotes the vector of all parameters associated to each of the courses in the sequence $\mathcal{K}$ considered. In practice however, this approach faces some problems. First, considering the estimation of parameters for all courses simultaneously might pose numerical complications simply due to the number of these parameters. Second, notice that since we don't observe a student's type except for the initial ability $a_{i,t_{i,0}}$ as measured by the entrance exam record, a student's type increases across time. For example, a student's type in its second academic period can be thought as a pair $(a_{i,t_{i,0}}, \mathbf{x}_{i,j_{i,t_{i,0}}}, j_{i,t_{i,0}+1})$, describing the student's initial ability, the instructor the student is paired in its initial period $t-_{i,0}$, and the vector of covariates under which such a match takes place. Even for a modest number of courses in the sequence, the resulting computations required to code the gradient, as required for the implementation of an optimization routine, can be very cumbersome.

Faced with these considerations, we we opt for a sequential ML approach that involves iterating over the different courses in $\mathcal{K}$. At the κ-th iteration of the approach er estimate the primitives θ^{κ} associated to the κ-th course in the sequence. We then use these estimates, in particular those pertaining course κ's production functions, to update each student's ability as implied by the model. The latter estimates then serve as the basis for the $\kappa+1$-th iteration of the algorithm where they are taken as observed data.

To formally illustrate this approach, let θ^{κ} denote the set of parameters associated with the κ-th course of the sequence $\mathcal{K}$. This includes both primitives of the course/section demand model and primitives of the learning production function model. We begin by constructing the log-likelihood based on the observed data for each student's first two consecutive academic periods upon enrolling in the university. For instance, if student i enrolls in course $\kappa = 1$ for the first time in academic period t, we utilize the data corresponding to academic terms t and $t+1$ for this student. Based on this, the data explained by the model for a student i with $t_{i,0} = t$ is given by: (i) $a_{i,t}$, the student's initial ability, (ii) $j_{i,t}$,$\mathbf{x}_{i,j_{i,t},t}$, the student's instructor match and vector of covariates for the first course enrollment instance, and (iii) $j_{i,t+1}$,$\mathbf{x}_{i,j_{i,t+1},t+1}$, the student's instructor match and vector of covariates for its second academic period. The average loglikelihood function for this data can be expressed in terms of the following,

$$\mathcal{L}(\theta^1, \theta^2) = \sum_{i=1}^{N}\sum_{t=1}^{2} \, log\Big[\mathbb{P}\big(j_{i,t} \mid a_{i,t-1}, \mathbf{x}_{i,j_{i,t},t} \, ; \, \theta^t\big) \cdot \mathbb{P}\big(S_{i,j_t}^{\kappa_{i,t}}, R_{i,j_t}^{\kappa_{i,t}} \mid j_{i,t}, a_{i,t}, \mathbf{x}; \theta^t\big)\Big].$$

where with a slight abuse of notation, we use $t \in \{0, 1\}$ to refer to the student i's first and second academic term as opposed to the actual academic period corresponding to these two enrollment instances. The first term inside the logarithm corresponds to the likelihood of observing the student's demand for the section they enroll in during academic term t. The second factor represents the likelihood of the observed course outcomes achieved by the student upon enrolling with a particular instructor in that term. Given that we only consider the first two academic terms of each student, our data contains observations solely for the first two courses in the sequence. Consequently, the log-likelihood function provided above depends exclusively on the parameters associated with these initial two courses: θ^1 and θ^2.

Implementing the sequential ML estimator for θ^1, θ^2 requires addressing a small identification concern. In essence, recall that in our identification results, disentangling the grading policy slopes (i.e., β_j) and the production function images (i.e., $f_j(a_{i,t}, \mathbf{x}_{i,j,t})$) for instructor in $\kappa = 2$ requires information on the student's performance on course $\kappa = 3$. However, the proposed sequential ML approach, the log-likelihood described above does not consider such information. This limitation arises because students can potentially reach course $\kappa = 3$ only in their third academic period, assuming they do not fail the first two courses in the sequence. This issue persists even if we were to estimate all parameters simultaneously, disregarding the sequential ML estimator, as it would still require the decoupling of grading policies and learning outcomes for the last course in the sequence considered.

To address this issue, we reparameterize the model for course $\kappa = 2$ in a way that renders an identified model while keeping θ^1 unchanged. Specifically, let's impose the restriction of $\beta_{j_2} = 1$ and c_{j^2} for all $\kappa = 2$ instructors. This accounts to defining the model for the second period in terms of production functions $\mathring{f}_{j^2}(a, \mathbf{x}) = \beta_{j^2} \cdot f_{j^2}(a, \mathbf{x}) + c_{j^2}$ that pool both learning output and grading policies into a single object. We denote the resulting vector of parameters for $\kappa = 2$ by $\mathring{\theta}^2$. Importantly, notice that the resulting model for student's first two academic terms can be used to identify the parameters θ^1 as our identification of the $\kappa = 1$ parameters doesn't require distinguishing $\kappa = 2$'s grading policies from production function images. We can thus optimize the log-likelihood function under the proposed parameterization and obtain consistent estimates for θ^1, $\hat{\theta}^1$.

Once the latter is achieved, we can transition to the second stage of the sequential ML approach where we estimate the parameters of the second course in the sequence. To accomplish this we use the estimates $\hat{\theta}^1$ for the learning production functions associated to any instructor j^1. These allow us to compute estimates for the implied learning outputs associated to each student's first academic period match. For example, after enrolling professor j^1's section for $\kappa = 1$, a student with an initial ability measurement of $a_{i,0}$ ends up with a new ability given by $a_{i,1} = f_{j^1}(a_{i,0}, \mathbf{x}_{i,j^1,1}\,; \hat{\theta}^1)$ where the notation makes it clear that we use the first step estimates in order to construct these ability estimates. At this point we can treat $a_{i,1}$, the ability estimates from the previous stage as observed data and use them to estimate θ^2 in the current stage of the algorithm. The process follows the same steps as before: (i) constructing the log-likelihood using data from two consecutive periods for all students upon their enrollment in a course $\kappa = 2$, and (ii) reparameterizing the model for θ^3 to account for the lack of identification of grading policies/ production functions for $\kappa = 3$.

Inner Loop for the Demand fixed effects $\Phi_{j,t}$

6.4 Estimation Results

We present the primary findings resulting from our structural model estimation. Our discussion is organized into three main components: (i) estimates for the average learning production function, (ii) estimates for the distribution of learning outputs across professors, and (iii) estimates for the demand model primitives. In all three cases, we emphasize the implications of these estimated primitives on the observed student-professor assignments in the data, as well as the potential enhancements through counterfactual policies to be explored in subsequent sections.

Average learning production function estimates

We begin by documenting the average learning production function across instructors teaching Calculus 1. This provides a compact way of understanding, on average, the relationship between the learning outcomes in our setting and the inputs that go into the learning production functions. Figure 3 depicts these average functions, with each panel corresponding to different fixed values of the vector of covariates $\mathbf{x}$. For example, the second panel describes the subpopulation of male students majoring in STEM fields who enroll in Calculus 1 under an instructor characterized by a high teaching load and both a course-specific and general tenure. On the x-axis of each panel, we display a student's ability level. The blue curve's value at any given ability level represents the average of the estimates for the professor specific learning production functions, $\hat{f}_j(a, \mathbf{x})$. In turn the shaded region corresponds to the associated 95% confidence interval.

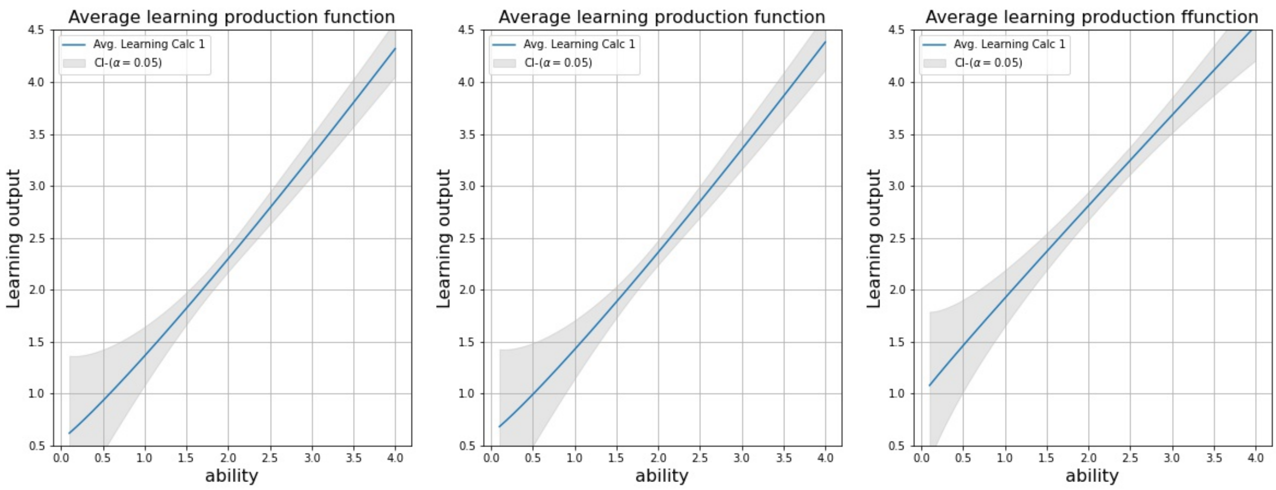


Figure 3: Average Learning Production Function

Notes: Panel 1 considers male students in the excluded major category under high teaching load, high course tenure, and high general tenure. Panel 2 coincides with the latter except that it focuses on STEM students. Panel 3 pairs female students in the excluded major category with professors having low teaching load, low course-specific tenure, and low general tenure.

Two key observations emerge from these plots. First, each panel illustrates significant disparities in learning outcomes among students of varying ability levels. To illustrate, in the first panel, consider a student with an ability level of $a_i = 1.0$, which is at the lower end of the ability spectrum. When randomly assigned to one of the Calculus 1 instructors in the sample, such a student can anticipate an average learning outcome of approximately 1.3 GPA points under the reference grading policy, or equivalently, a D grade. In contrast, a student of an ability of $a_i = 4.0$, positioned at the top of the distribution, achieves a learning outcome of approximately 3.8, equivalent to a B+ grade. Overall, the learning production functions exhibit an upward trend with a student's ability level, covering a wide range of distinct learning outcomes. This is robust to changes in the vector of covariates as suggested by looking at the remaining panels.

Second, factors beyond a student's ability type influence the shape of the average production function. For instance, while the first and second panels exhibit slightly convex relationships, the last panel portrays a scenario characterized by diminishing returns to a student's ability, as evident from the modest concavity of the average production function. In addition, the overall height of the production function varies across these panels. Consider for example a student with an ability of $a_i = 2.0$. In the first panel, this student achieves an approximate learning output of 2.0, while in the second panel, its average learning output increases to approximately 2.5. This students output is even higher under the third conditioning set, reaching approximately 3.5 GPA points under the reference grading policy. In the appendix we show that these observations correspond to our estimates of the slope parameters for the covariate variables for which we report both the point estimates and their standard errors.

The distribution of learning outcomes across instructors

Gains from our reassignment counterfactual exercises depend on the existence of pedagogical differences across instructors. To understand these differences we need to look not at the average learning outcomes, but at the dispersion in the distribution of learning outputs. The left panel of Figure 4 accomplishes this by plotting various percentiles in the distribution of learning outcomes. As before, the x-axis corresponds to the student's ability level. For any given ability, the images of the curves being depicted represent percentiles in the distribution of learning outputs for a student of such ability. While the Figure corresponds to a fixed vector of covariates, the Appendix section shows the same distribution for other conditioning vectors as to illustrate how the main conclusions remain the same.

As indicated by the stylized facts, significant variations exist in the learning outcomes a student can anticipate when randomly assigned to a Calculus 1 instructor. For instance, a student with an ability level of $a_i = 2.0$ can, on average, expect a learning outcome of 2.1 GPA points. However, the range of possible learning outcomes extends from 2.0 GPA points at the 10th percentile to 2.5 GPA points at the 90th percentile. This represents a substantial difference, equivalent to transitioning from a D to a C grade under the reference instructor's grading. The dispersion in learning outcomes becomes even more pronounced when considering students at the higher end of the ability distribution. For example, the same percentiles for a student with an ability of $a_i = 3.0$ correspond to a two-letter grade jump, ranging from 2.7 GPA points to 3.5 GPA points, or equivalently from C+ to B+.

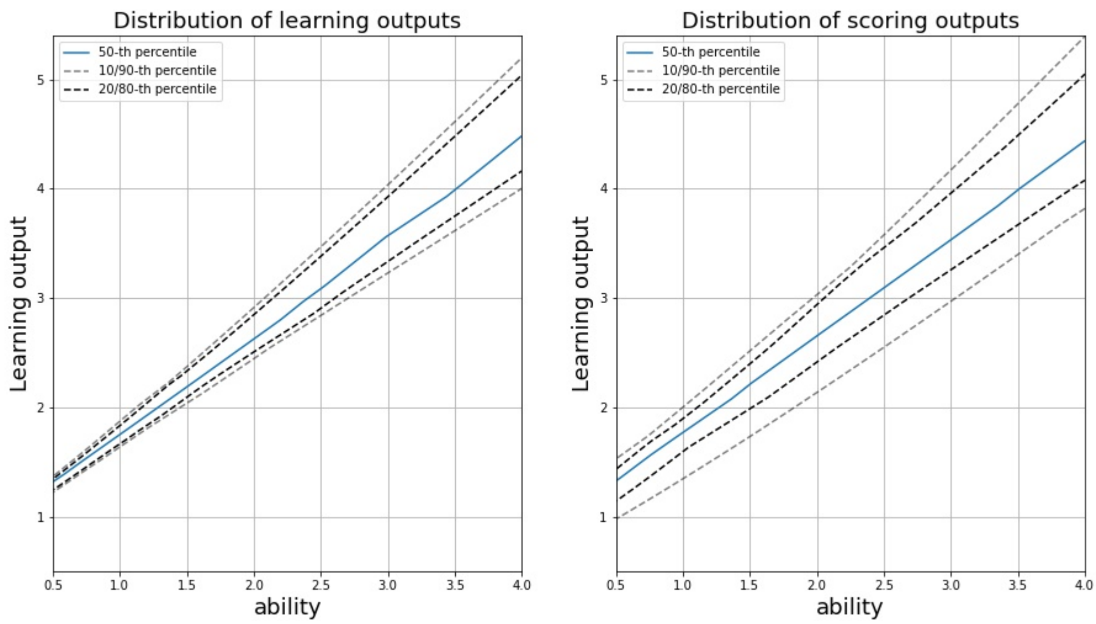


Figure 4: Learning Output Distribution

Notes: Both panels correspond to a female student in a STEM major, under a high teaching load, high course tenure, and higher general tenure instructor.

Furthermore, the second panel of Figure 4 displays the distribution of scoring outcomes that our randomly assigned student can anticipate. This provides an intuitive way of incorporating differences in instructors' grading policies into the presentation of our estimates. To start, notice that the average score output across instructors closely mirrors the average learning output curve in the first panel. This is consistent with the grading policy estimates which suggest the average instructor's grading policy coincides with that of the reference professor. However, it's worth highlighting that the spread in the scoring distribution surpasses that of the distribution for learning outcomes. This holds true for all student ability levels and remains robust across changes in the conditioning vector of covariates, as illustrated in the appendix. For instance, for the student with an ability of $a_{i,0} = 2.0$, as considered previously, shifting from the 10th percentile to the 90th percentile in the distribution of scores corresponds to a leap from approximately 1.6"to2.6" GPA points. Notably, this is a more substantial jump compared to the learning improvement discussed earlier

Collectively, the estimates in Figure 4 indicate a potential source of tension for students concerning their preferences for both learning and scoring outcomes. In other words, if instructors associated with high learning outcomes differ from those with high scoring outcomes, students will encounter a trade-off between learning and scoring when selecting sections of Calculus 1. The ultimate choice a student makes will depend on the weight its preferences place on each of these aspects. Before delving into this further, let's quantitatively analyze this trade-off by just documenting the extent to which the optimal teaching and scoring instructors for a student differ, as well as the average magnitude of these differences

For example, consider computing the score and learning outcomes for each course enrollment instance for Calculus 1 in our sample. We can then determine for such instance the number of instructors associated with a higher scoring output than that of the student's optimal professor in terms of learning output. Table 5 records this information. Each column in the table corresponds to a different integer value representing the number of professors who enhances a student's score, relative to the learning optimal instructor. The data in the k-th column for a given row reflects the proportion of students within the subpopulation represented by the row who have k score-improving instructors. To highlight variations in these score-improving opportunities across students of different abilities, we present this information while conditioning on different segments of the student ability distribution.

We can see that a large fraction of all Calculus 1 course-enrollment instances are such that the best instructors in terms of learning and scoring don't coincide. For instance, across all the course enrollment instances on average around 85% of the observations are associated to at least one score-improving instructor. Significant differences result from considering students of different ability levels, with the highest number of score-improving professors showing up among students at the top of the ability distribution. To some extent, the latter reflects the higher dispersion in the scoring and learning distributions according to our estimates.

Table 5: # of score-improving professors relative to the learning optimal professor.

	# of score-improving professors				
	1	2	3	4	≥5
All students	11.00	23.18	9.78	14.57	28.01
Ability 0% - 25%	16.08	22.95	11.42	16.49	8.21
Ability 25% - 50%	8.06	30.00	11.59	17.96	14.24
Ability 50% - 75%	9.78	23.26	8.39	12.52	38.49
Ability 75% - 100%	10.11	16.42	7.69	11.24	51.34

Notes: The k-th column cell for a given row reports the fraction of students in the subpopulation described by the row with exactly k score-improving instructors relative to the learning optimal one.

It is also of value to think about the size of these scoring deviation opportunities. Intuitively, the larger the score gap corresponding to the deviation the higher the tension faced by the student. Table 6 illustrates this by considering two differences in quantities related to the learning-optimal and scoring-optimal professors for each course-enrollment instance. The first column considers the differences in terms of their learning output, referred to as the learning gap. The second column constructs the difference between their scoring outcomes, denoted as the scoring gap. Here, the results are revealing, as in all cases the scoring gap substantially exceeds the learning gap. To put this in context, let's entertain a repeating student of average ability. While opting for the best instructor in terms of learning involves a premium of approximately 0.15 GPA learning points, opting instead for the best scoring professor involves a premium of 0.38 in terms of scoring. This implies that, if given the choice, and if the value students place on scores is as big as that placed on learning our student would find it attractive to choose the scoring optimal instructor.

Table 6: Learning/Scoring gap between the learning and scoring optimal professors.

	Outcome gaps	
	Learning gap	Score gap
New students	0.32	0.91
Ability 0% - 25%	0.17	0.62
Ability 25% - 50%	0.29	0.76
Ability 50% - 75%	0.35	1.01
Ability 75% - 100%	0.42	1.16
Old students	0.17	0.76
Ability 0% - 25%	0.10	0.43
Ability 25% - 50%	0.15	0.62
Ability 50% - 75%	0.20	0.98
Ability 75% - 100%	0.29	1.28

Notes: The learning gap is defined as the average difference between the learning output a student obtains under the learning optimal instructor and the scoring optimal instructor for the academic term in which the course enrollment instance takes place. The scoring gap is the average difference between the score output a student obtains under the scoring optimal instructor and the learning optimal instructor for the academic period of the course enrollment instance considered.

We can supplement the discussion above with estimates regarding the proportion of students, as observed in the data, who choose the learning-optimal instructor. For new students, this proportion is approximately 12.17%. Notably, this aligns with the fact that first-time students are assigned to sections within Calculus 1 randomly, and the average number of sections in the term when these students enroll in a course is 8.7. This corresponds to an 11.46% probability of being randomly matched with the learning-optimal instructor, very close to our estimate. Regarding students repeating the course, the analogous fraction resulting from our estimates is approximately 16.20%. This finding is significant as it suggests that allowing students to select sections within a course results in a higher rate of being matched with their learning-optimal instructor.

Course/Section Demand Parameters

The key parameters of interest in our demand model are α_0, representing the marginal utility of expected scores, and α_1, indicating the marginal utility of expected learning outcomes. Table 7 reports the point estimates and standard errors for both parameters under Calculus 1 and Calculus 2 courses. For Calculus 1, both α_0 and α_1 exhibit positive values, signifying that students assign positive weight to both learning and scores when making course/section selections. Notably, the magnitude associated with α_0 is slightly above that of α_1 suggesting students place a higher weight on the students they expect to obtain above their pure learning outcomes.

Table 7: Estimation Results - Course/Section Demand Parameters

	Calculus 1	
Parameter	Estimate	Std. Error
α_0	1.19	0.05
α_1	1.17	0.10

We also want to highlight that these estimates indicate a substantial portion of the variation in demand decisions can be attributed to both the scoring and learning dimensions. For instance, when considering the other utility component, $\Phi_{s,t}$, the average value for these quantities among observed (s, t) pairs is -0.35, with a standard deviation of 1.78. These values are of the same order of magnitude to the score and learning outputs for most students in the sample. An important corresponds to low-ability students for whom learning and scoring outputs are small in terms of their magnitude. This suggests that their decisions are relatively more influenced by the fixed effects terms $\Phi_{s,t}$, a possibility we explore when interpreting out counterfactual exercises.

7 Counterfactual Exercises

This section employs the estimated model to conduct simulations under various counterfactual policies influencing the assignment of students to professors in $\mathcal{K}$ courses. Our attention is directed towards two key facets of these counterfactual exercises. Firstly, we aim to comprehend the extent to which different learning-related outcomes are influenced by the implementation of the policy. An attempt is made at intuitively explaining structural rationales underlying the observed gains or losses from the policy implementations. Secondly, we meticulously analyze the policy's distributional effects, framing the gains and losses in relation to student characteristics. In essence, we present a comprehensive account of how different segments within the student type distribution experience gains or losses post-policy implementation.

We must emphasize that our approach in these exercises aligns with the perspective of the university's administration, which very often prioritizes learning over other outcomes. While we might report the consequences of our policies over other objectives of significance to students (i.e., scores as reflected in transcripts) we rank the policy interventions mainly in terms of learning outcomes.

In what follow the discussion is framed in terms of two classes of exercises: enrollment mechanisms that are independent of student choice, and mechanisms that are contingent upon student choice within specific constraints.

7.1 Counterfactual Dictatorial Assignment

Describing the Counterfactual Policy

We commence by examining the problem of assigning students and instructors under dictatorial assignment. In this context, we envision a university administration tasked with allocating students and instructors to courses, devoid of any consideration for students' preferences. As an objective the administration seeks to construct the assignment as to maximize the average learning outcomes across the entire population of students. Notice that the additional degrees of freedom resulting from the exclusion of preferences implies the average learning outcomes from this exercise will exceed that of policies under preference-based enrollment mechanisms. For this reasons, we think of the outcome of the dictatorial assignment problem as benchmark against which we can evaluate other policies.

In principle, we could envision a planner grappling with the dynamic problem wherein assignment decisions made in period t have repercussions on the number of students seeking enrollment in a given section during period $t+1$. However, for the present discussion, we confine our attention to the simplified myopic version of the problem. In this model, the planner, at the onset of each academic term, maximizes the current period learning returns while taking the pool of students and instructors as a given.

For concreteness, let's delve into the specific scenario faced by the university's administration for the first course in the sequence, Calculus, 1 during academic period t. Denoting the pool of students seeking enrollment in the course by $\mathcal{I}_t$, the university's objective is to determine the variables $\mu_{i,s} \in \{0,1\}$, where $\mu_{i,s} = 1$ dictates that student i is assigned to section s, while adhering to exogenous capacity constraints. Denoting by Con_s the capacity constraint linked to section s, and j_s the instructor associated with the section, we can write the problem as follows,

$$\max_{\mu_{i,s} \in \{0,1\}} \sum_{i \in \mathcal{I}} \sum_{s \in Sect_t} f_{j_s}(a_{i,t-1}, \mathbf{x}_{i,j_s,t}) \cdot \mu_{i,s},$$

subject to the constraints:

$$\sum_i \mu_{i,s} \leq C_s; \quad \forall\, s \in Sect_t,$$
$$\sum_s \mu_{i,s} \leq 1; \quad \forall\, i \in \mathcal{I}_t.$$

For each period t, considering the supply of sections and their respective head instructors as exogenously given (i.e., consistent with the observed data), we can calculate the learning outputs $f_{j_s}(a_{i,t-1}, \mathbf{x}_{i,j_s,t})$ before solving the optimization problem. This implies we can treat the program above as an assignment problem that can be solved under standard linear programming techniques[1]. After the planner's choice of μ, learning takes place according to the estimated production functions. In addition, student's scores and retirement decisions are simulated by drawing from the estimated distributions for the error terms. The latter quantities, together with the number of first time students in $t+1$, determines the total amount of students seeking to enroll a section of Calculus in the following period, $\mathcal{I}_{t+1}$.

[1] The existence of an integer solution is assured for assignment problems of this nature

1. Talk about how you draw missing ability values from the implied conditional distribution given the estimates. Also, hoy you plan to construct confidence intervals for the counterfactual graphs.

Simulating the Counterfactual Exercise

Figure 5 offers a visual representation of the outcomes derived from our counterfactual exercise. The first panel presents the average learning outputs, for each student ability level, for both the observed assignment (depicted by dashed line) and the dictatorial counterfactual match (illustrated by the solid line). Additionally, the second panel panel showcases the disparity between these two curves by plotting their difference. This provides a clearer insight into how various ability types experience gains or losses as a result of the counterfactual policy's implementation. From this exercise, two notable observations come to light.

Primarily, it is evident that reassignment leads to higher learning outputs for students across all ability levels. At first glance, this might appear surprising, suggesting that the university is capable of enhancing learning across the entire spectrum without incurring the typical distributional costs associated with prioritizing one subgroup's learning over another's. However, this outcome becomes more reasonable upon considering that the majority of Calculus 1 students in the observed data are randomly assigned to sections (i.e., all first-time students). The plot indicates that these gains can be particularly substantial for specific segments of the ability distribution. For instance, for students at the lower end of the distribution, for example $a_{i,t-1} = 1.0$, transitioning to the optimal assignment results in a change in the learning output from approximately 1.8 to 2.2 GPA points on the reference instructor's grading policy. This shift signifies that while the average student (here average refers to the error term perturbations η and ε) at the lower end of the distribution receives a Fail score of D under the observed assignment, the average student under the counterfactual scenario obtains a pass score of C under the reference instructor's grading policy. The same is true for students at the top of the distribution where, for example, a student of ability $a_{i,t-1} = 4.0$ moves from an average score of "B" to a "B+" score on average.

Second, it is crucial to note that while gains are universally positive across the entire distribution of ability, the magnitude of these changes exhibits significant variation depending on the considered ability level. This contrast is most stark when comparing gains at the extremes of the distribution, as described earlier, with those of students positioned in the middle of the ability spectrum. In an extreme case, students with an ability level of $a_{i,t-1} = 2.5$ experience gains of less than 0.25 reference instructor GPA points when transitioning to the counterfactual scenario.

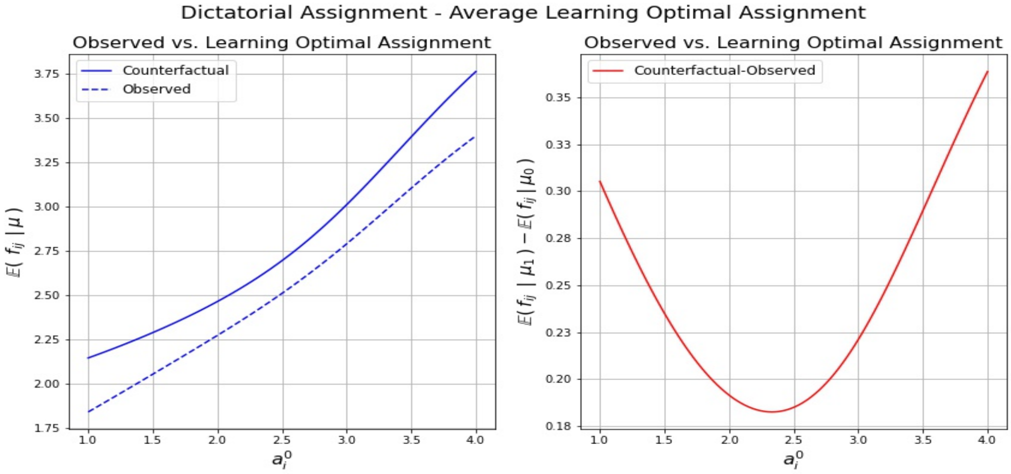
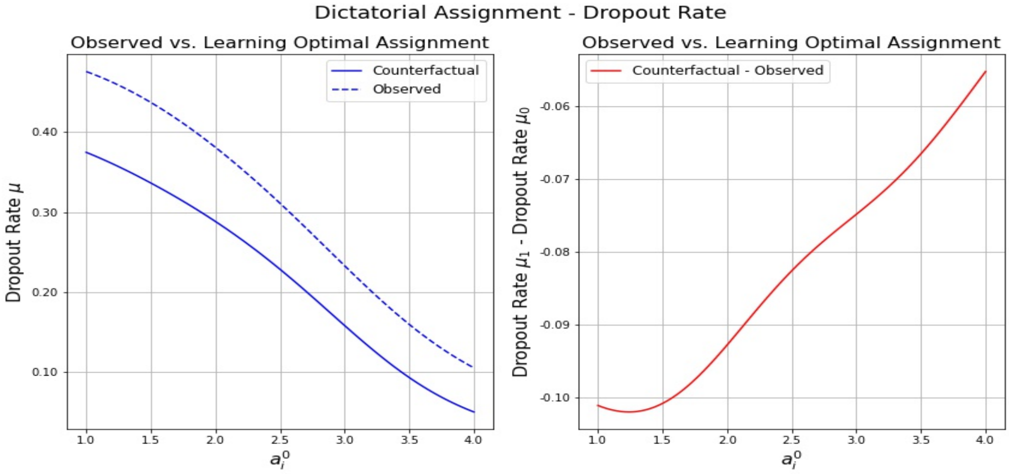
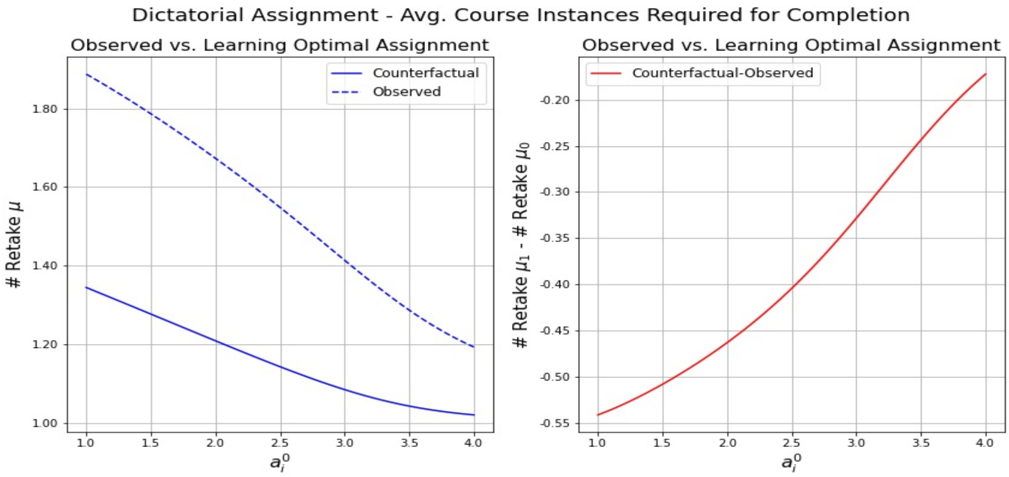


Figure 5: Dictatorial Counterfactual - Avg. Learning Output

It is also value to illustrate how quantities that don't exactly reflect learning but that are of interest to the students, differ between the observed and the counterfactual assignments. This is relevant as it might suggest whether or not the goal of maximizing average learning outputs is in tension with other objects valued by the students. Figures [...] serve as the analog to Figure 5, while focusing on the probability of a student dropping the course conditional on its ability level. Figure [...] in turn considers differences in the average number of course enrollment instances required for the successful completion of the course under both assignments.





7.2 Counterfactual Course Enrollment Mechanism

8 Endogenous Grading Policies

9 Conclusion

10 Appendix

Identification under course dropping

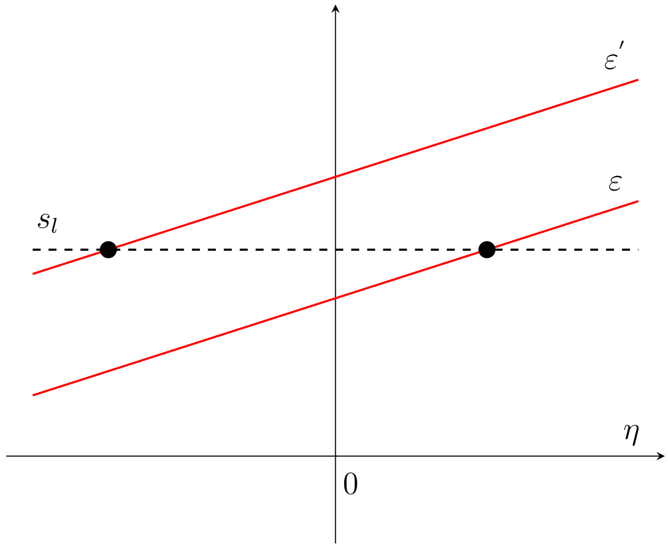
Thus far we have only provided arguments for the identification of our empirical model in a setting where students are unable to drop sections previously enrolled. In situations in which a non-negligible number of students choose to drop a section, ignoring this might be problematic. Intuitively, the issue arises from the fact that all our identification results are based on our observations of the group of students who achieve scores above a certain threshold, denoted as s_l. Yet, when students have the option to drop a course, the researcher can only observe the fraction of students who score above s_l conditional upon not dropping the course.

Failure to account for this distinction can lead to upward bias in our estimators, where estimates suggest that the learning returns of professors are higher than they truly are. Moreover, depending on the relationship between grading policies and professor productivity, this bias can result in erroneous conclusions regarding disparities in learning returns across instructors. For instance, if students are more likely to drop a course under the instruction of subpar professors, we run the risk of underestimating the gap in instructional quality between a high return and a low return instructor.

We now try to extend the arguments in the previous subsection in a way that accommodates for the truncation issue resulting when students can drop a course. Following the same arguments used before, consider the mass of students of initial type $a_{i,0} = a_0$ who in their first academic term obtain a score of at least s_l after enrolling instructor j^1's section under the covariate vector $\mathbf{x}_1$. Following the discussion above, we also condition on the subgroup of students who choose not to drop j^1's section as otherwise we wouldn't observe a score record for the student. Under our model, the latter conditional probability can be written as follows,

$$
\begin{aligned}
&\mathbb{P}\big(S^1_{i,j^1} \geq s_l \mid a_0,\ \mathbf{x}_1,\ j^1,\ R^1_{i,j^1} = 0\big) \\
&= \int_{\varepsilon^1} \int_{\eta^1} \mathbf{1}\{R^1_{i,j^1} = 0\} \mathbf{1}\{S_{i,j^1} \geq s_l\}\ f_\eta(\eta^1) f_\varepsilon(\varepsilon^1)\, d\eta^1\, d\varepsilon^1 \\
&= \int_{\varepsilon^1} \mathbf{1}\left\{\varepsilon^1 \geq \frac{s_{l^*} - \beta_{j^1} f_{j^1}(a_0, \mathbf{x}_1) - c_{j^1}}{\sigma^1_\varepsilon}\right\} \left[1 - F_\eta\left[\frac{s_l - \beta_{j^1} \cdot f_{j^1}(a^0, \mathbf{x}_1) - c_{j^1} - \sigma^1_\varepsilon \cdot \varepsilon^1}{\sigma^1_\eta}\right]\right] f_\varepsilon(\varepsilon^1) d\varepsilon^1.
\end{aligned} \tag{1}
$$

This expression closely resembles the one derived in our previous identification when trying to identify the marginal student for score s_l. It however differs in two crucial aspects that complicate this interpretation. First, students (within the conditioning set) now differ in two unobserved ways: their η^1 and ε^1 draws. Given that conditional on not dropping the course, ε^1 has an impact on the final score received by a student, this means we now should think not of a single marginal student for score s_l but of a marginal student for s_l under each potential draw of ε^1. Our observation of the mass of student scoring above s_l now corresponds to adding up the mass of students who score above s_l across all this different sub populations based on ε^1. Figure ..., the analog to Figure ... in the main subsection illustrates this by showing how a change in ε^1 shifts the linear function defining the marginal student. Second, the integral on the right hand side must now reflect the fact that we only consider students who choose not to drop the course. This is captured by the indicator term inside the integral $\mathbf{1}\{R^1_{i,j^1} = 0\}$. In terms of the graph below, this amounts to only counting the students who score above s_l for some of the ε^1 sub populations, namely, those who choose not to drop the course.



In practical terms, the implication is that we cannot directly invert the previous equation to learn about σ^1_η and $f_{\hat{j}^1}(a_0, \mathbf{x}_1)$ as considered in our previous arguments. Some additional work is required in order to accomplish this. Nevertheless, what we can do is to identify the marginal student, not in terms of obtaining a score s_l, but in terms of choosing to drop the course. To see this, consider an expression for the mass of students who choose not to drop j^1's section under our conditioning set,

$$\begin{aligned}\mathbb{P}\big(R^1_{i,j^1} = 0, \mid a_0,\ \mathbf{x}_1,\ j^1\big) &= \int_\varepsilon \mathbf{1}\left\{\varepsilon \geq \frac{s_{l^*} - \beta_{j^1}\cdot f_{j^1}(a_0, \mathbf{x}_1) - c_{j^1}}{\sigma^1_\varepsilon}\right\} f_\varepsilon(\varepsilon) d\varepsilon \\ &= 1 - F_\varepsilon\left[\frac{s_{l^*} - \beta_{j^1}\cdot f_{j^1}(a_0, \mathbf{x}_1) - c_{j^1}}{\sigma^1_\varepsilon}\right]\end{aligned}$$

Inverting the expression above delivers an expression for the ε^1 corresponding to the student who just marginally chooses not to drop j^1's section of course $\kappa = 1$.

$$\frac{s_{l^*} - \beta_{j^1}\cdot f_{j_1}(a_0, \mathbf{x}_1) - c_{j_1}}{\sigma^1_\varepsilon} = F^{-1}_\varepsilon\left[1 - \mathbb{P}\big(R^1_{i,j^1} = 1, \mid a_0,\ \mathbf{x}_1,\ j^1\big)\right]. \tag{2}$$

We can now make some progress by combining the identified expressions just derived. In particular we can use the latter to pin down the identity of the average marginal student relative to obtaining a score weakly above s_l. Once this is achieved, the same steps followed in the previous section can be used to infer the the variance parameters and the production function of at least one $\kappa = 1$ professor, namely the reference professor $\hat{j}_1$. Proposition 5 formally states and proves this claim.

Proposition 5. *The image $f_{\hat{j}_1}(a_0, \boldsymbol{x}_1)$ and the variance parameters $\sigma^1_\eta, \sigma^1_\varepsilon$ are point identified.*

Proof. Consider equation 1 describing the fraction of students (within the conditioning set) that obtains a score weakly above s_l in j^1's $\kappa = 1$ section. In particular, we consider the case for $l = l^*$, the cutoff above which students obtain a pass score. By algebraically manipulating this expression we obtain what follows,

$$\begin{aligned}&\mathbb{P}\big(S^1_{i,j_1} \geq s_{l^*} \mid a^0,\ \mathbf{x}_1,\ j^1,\ R^1_{i,j^1} = 0\big)\\ &= \int_{\varepsilon^1} \mathbf{1}\Big\{\varepsilon^1 \geq \underbrace{\frac{s_{l^*} - \beta_{j^1} \cdot f_{j^1}(a_0, \mathbf{x}_1) - c_{j^1}}{\sigma^1_\varepsilon}}_{I}\Big\}\Big[1 - F_\eta\Big(\frac{\sigma^1_\varepsilon}{\sigma^1_\eta} \cdot \Big\{\underbrace{\frac{s_{l^*} - \beta_j \cdot f_{j^1}(a_0, \mathbf{x}_1) - c_{j^1}}{\sigma^1_\varepsilon}}_{II} - \varepsilon^1\Big\}\Big)\Big] f_\varepsilon(\varepsilon^1) d\varepsilon^1.\end{aligned}$$

Notice that terms I and II (both of which coincide) are quantities we have previously identified in equation 2. We can treat them as known quantities in the equation above. Since the left hand side is also an observed quantity (i.e., crucially, this is true because the researcher is capable of observing scores for students who don't drop the course), we can treat the identity above as just a function of the quotient $\sigma^1_\varepsilon/\sigma^1_\eta$. Furthermore, it is easy to see that under the region of integration considered, term II is always below ε^1 which implies that an increasing of the quotient $\sigma^1_\varepsilon/\sigma^1_\eta$ corresponds to a pointwise decrease of the integrand considered in the right hand side. It follows from standard inversion arguments that we can use the identity above to identify the true value of the quotient $\sigma^1_\varepsilon/\sigma^1_\eta$.

Let's now consider the analog to the previous expression for an arbitrary scores threshold s_l. This is given by the equation below,

$$\begin{aligned}&\mathbb{P}\big(S^1_{i,j^1} \geq s_l \mid a_0,\ \mathbf{x}_1,\ j^1,\ R^1_{i,j^1} = 0\big),\\ &= \int_{\varepsilon^1} \mathbf{1}\Big\{\varepsilon^1 \geq \frac{s_{l^*} - \beta_{j^1} \cdot f_{j^1}(a_0, \mathbf{x}_1) - c_{j^1}}{\sigma^1_\varepsilon}\Big\}\Big[1 - F_\eta\Big(\frac{\sigma^1_\varepsilon}{\sigma^1_\eta} \cdot \Big\{\underbrace{\frac{s_l - \beta_{j^1} \cdot f_{j^1}(a_0, \mathbf{x}_1) - c_{j^1}}{\sigma^1_\varepsilon}}_{\theta(s_l \mid a_0, \mathbf{x}_1, j^1)} - \varepsilon^1\Big\}\Big)\Big] f_\varepsilon(\varepsilon^1) d\varepsilon^1.\end{aligned}$$

It is clear from the preceding discussion that both terms $(s_{l^*} - \beta_j f_{j_1}(a^0, \mathbf{x}) - c_j)/\sigma^1_\varepsilon$ and $\sigma^1_\varepsilon/\sigma^1_\eta$ inside the integral term can be treated as known quantities. The key observation is then that we can treat the right hand side as a monotone function of the quotient $(s_l - \beta_j f_{j_1}(a^0, \mathbf{x}) - c_j)/\sigma^1_\varepsilon$ and $\sigma^1_\varepsilon/\sigma^1_\eta$ for any score cutoff s_l we entertain. We can then follow the same arguments as in section 5 by considering the system of equations defined by,

$$\theta(s_l|a_0,\mathbf{x}_1,j^1) = \frac{s_l - \beta_{j^1} f_{j^1}(a_0,\mathbf{x}_1) - c_{j^1}}{\sigma^1_\eta} \text{ and } \theta(s_{l'}|a_0,\mathbf{x}_1,j^1) = \frac{s_{l'} - \beta_{j^1} f_{j^1}(a_0,\mathbf{x}_1) - c_{j^1}}{\sigma^1_\eta}.$$

As before, it is easy to see that by considering $s_l \neq s_{l'}$, the system above has a unique solution in terms of quantities σ^1_ε and $\beta_{j^1} \cdot f_{j^1}(a_0,\mathbf{x}_1) + c_{j^1}$. The former, together with our previous identification of the quotient $\sigma^1_\varepsilon/\sigma^1_\eta$, allows us to recover the variance term σ^1_η. In turn the result for $\kappa = 1$'s reference professor follows from our grading policy normalization of $(\beta_{\hat{j}^1}, c_{\hat{j}^1}) = (1,0)$ ■

We can mimic the marginal logic student logic followed in Section 5 when trying to understand the content of Proposition 5. In doing so, it is useful to recall the main challenges arising from the possibility of student's dropping a course: (i) we don't observe the score of students who drop the course, (ii) the unobserved draw affects a student's incentive to drop the course. The first part of Proposition 5 shows we can easily correct for the first issue by using our observations of who students are dropping. In other words, we can infer who the marginal student dropping the course is and use this observation when constructing an identity describing the marginal student obtaining a score of s_l. The second part shows that even when ε draws affect the scoring equation, for our purposes we can focus in identifying the marginal student for a ε draw of zero.

The remainder of the argument tracks closely our previous work on Section 5 in that we used data for the student's performance on the second course of the sequence to disentangle $\kappa = 1$'s grading policies and production functions. For instance, consider all students in the conditioning set who after obtaining a pass score for $\kappa = 1$, enroll j^2's of $\kappa = 2$ under covariates $\mathbf{x}_2$. We are interested in an expression for the fraction of these students who obtain a score weakly above s_l. Our model implies the following expression,

$$\begin{aligned}
&\mathbb{P}\big(S^2_{i,j^1} \geq s_l \mid a_0,\ \mathbf{x}_1,\ \mathbf{x}_2,\ j^1,\ j^2,\ R^2_{i,j^2} = 0\ S^1_{i,j^1} \geq s_{l^*}\big) \\
&\qquad = \int_{\varepsilon^2} \mathbf{1}\left\{\varepsilon^2 \geq \frac{s_{l^*} - \beta_{j^2} \cdot f_{j^2}(f_{j^1}(a_0,\mathbf{x}_1),\mathbf{x}_2) - c_{j^2}}{\sigma^2_\varepsilon}\right\} \\
&\qquad\qquad \times \left[1 - F_\eta\left(\frac{s_l - \beta_{j^2} \cdot f_{j^2}(f_{j^1}(a_0,\mathbf{x}_1),\mathbf{x}_2) - c_{j^2} - \sigma_\varepsilon \cdot \varepsilon^2}{\sigma^1_\eta}\right)\right] f_\varepsilon(\varepsilon^2) d\varepsilon^2.
\end{aligned}$$

Exactly the same arguments as in the preceding discussion can be applied to the $\kappa = 2$ problem. While direct inversion of the expression above is not possible, we can infer the primitives of interest by using our observations for how many students choose to drop

j^2's section. After achieving this the results are just as those considered in Section 5 in that we can identify $\kappa = 1$ production functions given injectivity of the $\kappa = 2$ instructor's production function. Below we state the result without a proof as it is identical to the arguments already outlined.

Proposition 6. *The following identification results hold,*

1. *The image of the composition* $\beta_{j^2} \cdot f_{j^2}(f_{j^1}(a_0, \boldsymbol{x}_1), \boldsymbol{x}_2) + c_{j^2}$ *and the variance term* $\sigma_\eta^2, \sigma_\varepsilon^2$ *are point identified,*

3. *Suppose that the learning production function* $f_{j^2}(\cdot, \boldsymbol{x}_2)$ *is injective. Then the image* $f_{j^1}(a_0, \boldsymbol{x}_1)$ *is point identified provided the existence of* $\tilde{a}_0$ *such that* $f_{j^2}(f_{j^1}(a_0, \boldsymbol{x}_1), \boldsymbol{x}_2) = f_{j^2}(f_{\hat{j}^1}(\tilde{a}_0, \boldsymbol{x}_1), \boldsymbol{x}_2)$.

As these parameters are not specific to individual professors, we provide detailed point estimates and standard errors for them in Table 8. The interpretation of these parameters requires the consideration of the parameterization choice for the production function model. For instance, a negative value for $\mu_{0,l}$ or γ_l implies a decrease in the level of the learning production function, while a negative values for $\mu_{1,l}$ imply diminishing returns to increases in a student's ability.

Estimates for Covariates

Table 8: Estimation Results - Learning Production Function

	Calculus 1	
Parameter	Estimate	Std. Error
Instructor Covariate Parameters		
$\mu_{0,load}$	-0.24	0.04
μ_{0,ten_1}	0.23	0.05
μ_{0,ten_2}	0.38	0.26
$\mu_{1,load}$	0.02	0.01
μ_{2,ten_1}	-0.05	0.02
μ_{1,ten_2}	-0.28	0.17
$\mu_{2,load}$	0.03	0.03
μ_{2,ten_1}	0.07	0.14
μ_{2,ten_2}	0.51	0.16
Student Covariate Parameters		
γ_{sex}	-0.21	0.01
γ_{maj_1}	0.26	0.03
γ_{maj_2}	0.11	0.03
γ_{maj_3}	0.10	0.03