

OPEN Deployment of check-in nodes in complex networks

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In many real complex networks such as the city road networks and highway networks, vehicles often have to pass through some specially functioned nodes to receive check-in like services such as gas supplement at gas stations. Based on existing network structures, to guarantee every shortest path including at least a check-in node, the location selection of all check-in nodes is very essential and important to make vehicles to easily visit these check-in nodes, and it is still remains an open problem in complex network studies. In this work, we aim to find possible solutions for this problem. We first convert it into a set cover problem which is NP-complete and propose to employ the greedy algorithm to achieve an approximate result. Inspired by heuristic information of network structure, we discuss other four check-in node location deployment methods including high betweenness first (HBF), high degree first (HDF), random and low degree first (LDF). Finally, we compose extensive simulations in classical scale-free networks, random networks and real network models, and the results can well confirm the effectiveness of the greedy algorithm. This work has potential applications into many real networks.

The advent of complex network¹⁻³ theory has had a significant impact on the network and data science⁴ over the course of past 20 years. People's daily life deeply relies on kinds of artificial networks such as city road networks, highway networks, power grids, communication networks, and virtual networks such as WWW (World Wide Web), social networks, and so on. A wide range of research topics aim to solve the challenges that many real networks face. As discussed in our previous work⁵, a portion of nodes in many complex networks have special functions such gas stations in road networks and highway networks supplying for check-in like services. In air transportation, for the convenience of passengers and resource locations, e.g. maintenance crews, it is very important to locate the hub nodes of an airline⁶. In IT infrastructure, we may want to allocate specific functions to critical nodes or driver nodes⁷, for instance, the nodes that control the Internet traffic in the search for viruses. In interdependent networks (e.g. power grids and communication networks), a fraction of critical nodes may result in the collapse of whole interdependent network8, such as the largest blackout of the power gird and the outages of the Internet. In social science, for security purpose, many "inside" agents are need to intercept all communications⁹ in a network of terrorists. In food web¹⁰, the predation relation can be also considered as check-in like service, and mining the key species whose disappearance may lead to large scale species extinction is a very critical problem. These nodes with special functions can be called check-in nodes, and objects that flow in networks need to finish check-in like services at the check-in nodes. For instance, vehicles often have to pass through gas stations to get gas supplement. Then two aspects of this problem should be considered:

- (1) Efficient routing strategies. With a portion of predesigned locations (perhaps randomized ones) of gas stations, designing efficient routes for all vehicles is very essential and important to alleviate traffic congestion, save gas fuel and time consumption of drivers. Our work⁵ tried to explore a possible check-in based routing framework for this problem. Definitely, many previous routing optimization methods including the efficient routing¹¹, optimal routing¹², global dynamic routing¹³, incremental routing¹⁴ and hybrid routing¹⁵ can be referenced. For simplicity and without loss of generality, here we employ the classical shortest path routing method for path discovery.
- (2) Optimal deployment of check-in node locations. With a given number of check-in nodes, which positions are the optimal ones that can achieve the highest profits to citizens and governors? To our best knowledge, it is still an open problem in complex network research.

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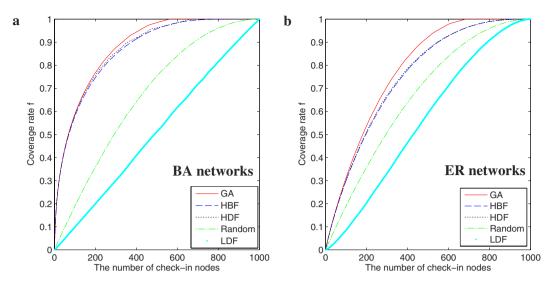


Figure 1. Evolution of cover rate f as a function of the number of check-in nodes under the five different check-in nodes selection methods. (a) BA Networks; (b) ER networks. Network parameters are N = 1000, $\langle k \rangle = 8$. Each datum is the average of 50 realizations of BA scale-free networks.

In other words, with minimum number of check-in nodes, we aim to maximize the profits of the whole network in this work. There are several aspects which need to be clarified clearly for this problem:

- (1) Clear problem definition and evaluation metric. The problem of check-in node deployment should be clear and a metric should be defined to accurately evaluate performance for check-in node deployment methods.
- (2) Efficient check-in node deployment method. Currently, to our best knowledge, the check-in node deployment method research is open, and there is a lack of deep study.
- (3) Evaluations. To verify the effectiveness of proposed methods, extensive simulations must be composed in both classical complex network models (e.g. scale-free network model and random network model) and real network models.

In the following section, we will first show the results of this work. Then we introduce the proposed algorithms and the employed network models. Finally, we close this work with a conclusion.

Results

Here we first define the check-in node deployment problem. Given a network which might be directed or undirected, assuming the shortest path routing protocol is employed, every shortest path between any pair of source and destination must include at least a check-in node to receive the check-in like services. Then the *minimum number of check-in nodes* (MNCN) which can guarantee every shortest path including at least a check-in node can be employed to evaluate the performance of a check-in node deployment method.

This problem can be converted into the set cover problem¹⁶ (see details in Methods section) and solved by employing greedy algorithm¹⁶ (GA). To compare with GA, other 4 check-in node position selection methods (see details in Methods section) including high betweenness first (HBF), high degree first (HDF), random and low degree first (LDF) are discussed.

Given a set of locations for check-in nodes J_* , the cover rate of all shortest paths f (see details in Methods section) can be employed to evaluate the effectiveness of check-in node location selection methods.

We first investigate the evolution of cover rate f as a function of the number of check-in nodes in BA¹⁷ scale-free networks and ER¹⁸ random networks in Fig. 1(a,b) respectively. One can see that the GA achieves the highest f. With the same number of check-in nodes, for instance, 150 check-in nodes in Fig. 1(a,b), f under the five methods appears to be GA > HFB > HDF > Random > LDF in both two types of networks. The HBF and HDF appear to be a bit lower than the GA, but very near. The LDF is the worst, because under the shortest path routing, paths trend to pass through the nodes with high degrees. Therefore, with the same number of check-in nodes, the number of the shortest paths that passing through check-in nodes of low degrees is very small, resulting in low f. With increasing number of check-in nodes, the f increases under the 5 methods. When the number of check-in nodes goes beyond a critical value, the f gets its maximum value of 1.0. Then the MNCN can be efficiently achieved and represented by the critical value.

In Fig. 2, we investigate the comparisons of different location selection methods in the two types of classical network models. In Fig. 2(a), based on the GA method, with the same network size and average degree, the robustness of BA¹⁷ networks appears to be better. It is related to the network structure, and in BA¹⁷ networks, most of the shortest path pass through a fraction of hub nodes. Meanwhile, the betweenness distribution of all nodes in ER¹⁸ network is much even, and more check-in nodes are needed. Similarly, under the HDF and HBF methods, the results are very similar to GA. Under the random selection, the effects are almost the same in two

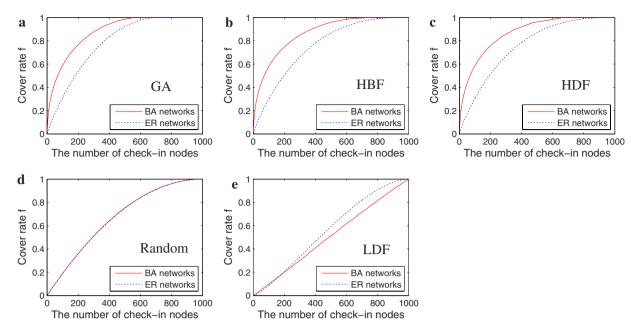


Figure 2. Comparisons of different methods in the two classical networks. (a) GA; (b) HBF; (c) HDF; (d) Random; (e) LDF. Network parameters are N = 1000, and average degree $\langle k \rangle = 8$. Each datum is the average of over 50 realizations of BA scale-free networks.

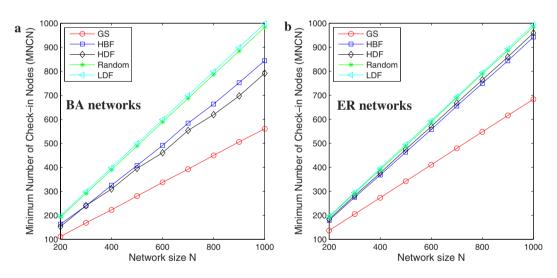


Figure 3. Evolution of MNCN as a function of the network size under the five different check-in nodes selection methods. (a) BA Networks of average degree $\langle k \rangle = 8$; (b) ER networks of average degree $\langle k \rangle = 8$. Each datum is the average of 50 realizations of BA scale-free networks.

network models. In Fig. 2(e), under the LDF method, the ER^{18} network appears to achieve better performance. It is also related to the network structure. The degree distribution of ER^{18} networks is more even than BA^{17} networks.

In Fig. 3, we investigate the evolution of minimum number of check-in nodes (MNCN) under the 5 methods in BA¹⁷ scale-free networks and ER¹⁸ random networks of different network sizes. With increasing network size, the MNCN increases. Because the larger the network size, the higher the number of the shortest paths appears, and more check-in nodes are needed. Under all network sizes, the GA method can achieve the lowest MNCN, and it can confirm the effectiveness of GA method.

In Fig. 4, we investigate the comparisons of all methods in the two network models. We can see that it is very obvious that under the GA, HBF, and HDF, the BA^{17} network models appear to have smaller MNCN, namely higher efficiency than the ER^{18} networks. The effects are almost the same under the Random and LDF methods.

So far, we can say the GA can achieve very good results when compared with all other methods. However, we may want to compare the results with the optimal solution which has been proved to be NP-hard¹⁶. Here we set network size N=20, average degree $\langle k \rangle = 4$. We run the simulations on many BA¹⁷ and ER¹⁸ networks on a PC

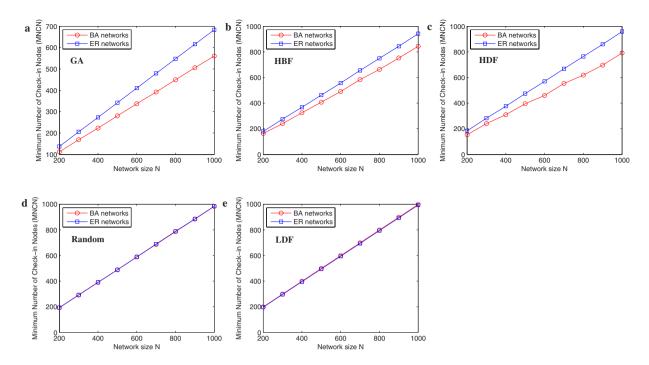


Figure 4. (a) Comparisons of different methods in the two classical networks of average degree $\langle k \rangle = 8$. (a) GA; (b) HBF; (c) HDF; (d) Random; (e) LDF. Each datum is the average of over 50 realizations of BA scale-free networks.

		GA	Optimal			
	MNCN	Computational cost (seconds)	MNCN	Computational cost (seconds		
BA networks	9	0.0136846	9	66.84760		
ER networks	11	0.015486	11	190.421226		

Table 1. The comparisons MNCN and Computational cost under GA and the Optimal methods in several small network models.

of Intel(R) Core(TM) i5-3470 CPU @3.2 GHz 3.2 GHz, RAM 4.0 GB. In Table 1, the results show that the average MNCN = 9 for both GA and optimal solution in BA¹⁷ networks, and average MNCN = 11 for both GA the optimal in ER¹⁸ networks. But the computational cost of the optimal solution is about 4800 and 12000 times more than GA in BA¹⁷ networks and ER¹⁸ networks respectively. The results are very amazing, especially for MNCN for both GA and optimal solution. For this special problem, the GA can achieve very good results.

In Table 2, we evaluate MNCN for many real networks which are widely used in previous research papers. One can see that the GA method can efficiently locate the check-in nodes than other 4 methods.

Discussion

In this work, assuming a portion of nodes were designated as check-in ones to supply check-in services for vehicles or network objects, we aimed to find efficient locations for these check-in nodes to achieve every shortest path including at least a check-in node. By carefully analyzing this problem, we transformed it into a set cover problem which has proved to be NP-complete, and proposed to use the greedy algorithm¹⁶ to find a cover. To verify the effectiveness of greedy algorithm¹⁶, we discussed other four heuristic location selection methods including high betweenness first, high degree first, random, and low degree first. To compare these methods, extensive simulations were done in BA¹⁷ scale-free networks and ER¹⁸ random networks. We investigated evolution of cover rate as functions of network sizes and average degrees, and found that with increasing network size and average degree the minimum number of check-in nodes which can guarantee every shortest path including at least a check-in node increases. Moreover, we employed these methods into many real network models. All the results can well confirm the effectiveness of the greedy algorithm for set cover problem. We compare the results of the greedy algorithm¹⁶ with the optimal results, and found that the GA method can achieve better network robustness with low computational cost. The results of this work can be employed for check-in node location selections in many potential real networks. In reality, other factors such as traffic density, source and destination distributions, and routing methods should also be comprehensively considered to efficiently solve the real challenges in complex networks. Moreover, network resilience is a very important topic in network science. In epidemic processes 19,20, it has been found that the epidemic processes are drastically affected by the first two moments of

Type	Name	N	L	GA	HBF	HDF	Random	LDF
Regulatory	TRN-Yeast-2	688	1079	123	675	521	679	678
Trust	Prison-inmate	67	182	41	58	62	61	64
	Netscience	1461	5484	899	1457	1458	1451	1459
	Leadership	32	96	19	24	24	31	31
Food Web	Grassland	88	137	33	83	54	86	86
	Seagrass	49	226	34	38	41	49	48
	Littlerock	183	2476	82	180	145	176	182
	St. Marks	49	223	33	38	41	47	48
	St. Martin	45	224	28	36	39	42	44
	Ythan	135	597	57	133	81	130	134
	E. coli-1	99	212	63	89	92	96	97
	E. coli-2	418	519	103	414	416	401	413
Biologic Network	S. cerevisiae	688	1209	145	675	521	663	678
	Ppi	990	9374	590	981	961	980	989
	Neural	297	2345	193	280	249	286	295
Electronic Circuits	S208	122	189	66	111	112	106	119
	S420	252	399	133	233	234	236	250
	S838	512	819	267	477	478	494	509
World Wide Wibe	Politicalblogs	1224	19022	563	1203	1113	1222	1221
Transposition	Airports	2939	30501	1101	2901	2915	2918	2938
Lauguage	Japanese	2704	8300	502	2623	2641	2688	2702
	SF2-1	400	797	166	201	201	386	399
	SF2-2	400	797	170	201	201	383	398
BA Model Network	SF3-1	400	1194	195	230	230	388	399
	SF3-2	400	1194	215	247	247	386	399
	SF4-1	400	1590	231	272	272	389	399
	SF4-2	400	1590	217	263	263	388	398
Persimona	Ownership	141	189	51	130	133	137	140
Business	Wtn61	218	5851	114	198	158	212	217

Table 2. The comparisons of minimum number of check-in nodes (MNCN) under different check-in node selection methods in the many real and classical networks. *L* is the number of links of the network.

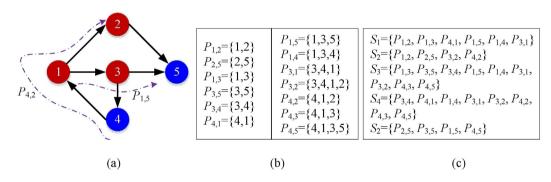


Figure 5. An example for the problem. (a) A simple directed network which employs the shortest path routing; (b) All shortest paths between all possible source and destination pairs in the network; (c) Set S_i denotes the path set in which all paths include node i.

the degree distribution²¹. Can these methods be employed into these network processes and enhance the other network resilience measures? In our future work, we will continue the research topic and share the results soon.

Methods

Algorithms. As shown in Fig. 5(a), a simple directed network with 5 nodes. The shortest path routing is employed. If many shortest paths exist between a source and destination pair, one of them is used randomly. For instance in Fig. 5(a), the shortest path from node 1 to 5 might be $P_{1,5} = \{1, 2, 5\}$ or $P_{1,5} = \{1, 3, 5\}$, and we randomly select $P_{1,5} = \{1, 3, 5\}$. Figure 5(b) shows all the shortest paths in the network.

In order to find the minimum number of check-in nodes, we first collect the shortest paths which pass through a given node. As shown in Fig. 5(c), node 1 has 6 shortest paths including this node, denoted by a set S_1 .

In fact, the MNCN problem can be described from another perspective. Given every S_i of node i in the network, find the minimum number of sets that can cover all the shortest paths in the network, namely finding a cover $J(J \subset V)$ with minimum |J| that can achieve $\bigcup (S_i \colon i \in J) = P$, where $P = \bigcup (S_i, i \in V)$ and V is the set of all nodes in the network. Then it is converted into the classical Set Cover problem²² which has been proved to be NP-complete and can be approximately solved by greedy algorithm¹⁶ described as follows.

Algorithm 1: Greedy algorithm (GA):

Step 0. Set $J = \emptyset$.

Step 1. If $S_i = 0$ for all i then stop: J is a cover. Otherwise, find a subscript J maximizing $|S_i|$ and proceed to Step 2.

Step 2. Add *j* to *J*, replace each S_i by $S_i - S_i$ and return to Step 1.

As shown in Fig. 5, by employing the greedy algorithm, the cover $J = \{3, 1, 2\}$, namely the minimum number of check-in nodes MNCN equal to |J| = 3.

The above greedy algorithm¹⁶ can find an approximate cover J. In the greedy algorithm process, at each step, we find the set which can maximize the number of included shortest paths. Finally, there is a set sequence J. However, in real check-in demands, it is not necessary to cover all paths, and we may want to only cover a large portion of them with minimum check-in nodes. Given a subset of V, denoted as J_* , here we employ a cover rate metric to evaluate the covered portion of all the shortest paths, described as

$$f(J_*) = \frac{|\bigcup_{j \in J_*} S_j|}{|\bigcup_{j \in V} S_j|}$$
 (1)

Under the greedy algorithm ¹⁶, when $J \subseteq J_*$, $f(J_*) = 1$. However, the scales of real networks are very large, and it is very difficult to emulate all shortest paths in the network and calculate the set cover. Is there any simple and heuristic algorithm to achieve an approximate cover rate f with small number of check-in nodes? Most of real networks can be modeled by the scale-free network model¹⁷, in which many nodes with the highest degrees are considered as central nodes. Moreover, the betweenness centrality²³ of a node ν is defined as the number of shortest paths passing through the node and be used to evaluate the importance of node in the network. Inspired by these heuristic information, in the following parts, we will employ several check-in node selection methods as baselines to compare with the greedy algorithm.

In general, the betweenness of a node directly represents the number of shortest path passing through the node, so the betweenness information based method can be described as follows.

Algorithm 2: High betweenness first (HBF).

Step 0. Sort the betweenness of all nodes in descend order.

Step 1. Given the number of check-in nodes, select the top $|J_*|$ nodes in the descend order.

Step 2. Calculate the $f(J_*)$.

In HBF, the betweenness of every node must be calculated first. Though the fast algorithm²⁴ can be used, it is still consuming huge computation resource especially for large scale networks. Meanwhile, getting the node degree information is relatively simple, and the degree information is also very efficient in evaluating node importance. Moreover, in complex networks, betweenness of a node is strong correlated to its degree. A node of high degree often has large betweenness²³. Therefore, here we propose a degree based check-in node deployment method

Algorithm 3: High degree first (HDF)

Step 0. Sort the degrees of all nodes in descend order.

Step 1. Given the number of check-in nodes, select the top $|I_*|$ nodes in the descend order.

Step 2. Calculate the $f(J_*)$.

In HDF, the node degree in employed. Sometimes, the network structure might not be known to us, and no heuristic information can be used. Then the random location deployment mechanism can be simply used.

Algorithm 4: Random

Step 0. Given the number of check-in nodes, randomly select the $|J_*|$ nodes in the network.

Step 1. Calculate the $f(J_*)$.

Opposite to HDF, as discussed in our previous $work^5$, if the check-in nodes are selected as the nodes of the lowest degrees, the network traffic capacity²⁵ will be remarkably reduced. Here, we assume the nodes of the lowest degrees are set as the check-in nodes, and compare the results with other methods.

Algorithm 5: Low degree first (LDF)

Step 0. Sort the degrees of all nodes in ascend order.

Step 1. Given the number of check-in nodes, select the top $|J_*|$ nodes in the ascend order.

Step 2. Calculate the $f(I_*)$.

Moreover, in order to compare the results with optimal solution, here we try to obtain optimal by emulating all possible sets of check-in nodes. Described as follows:

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Step 0. Assuming |J_{1*}|=1.
Step 1. Find all combinations C_{|J|}^{|J_{1*}|}.
Step 2. For each combination, if f(J_{1*})=1 then J_{1*} is the result, else |J_{1*}|=|J_{1*}|+1, go to Step 1.
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Network models. To verify the effectiveness of above check-in node selection methods, the network structure is the basic. In this work, the used network models include two categories: BA¹⁷ scale free networks, ER¹⁸ random networks and real network models.

The BA¹⁷ scale-free network model which is constructed by two general rules: (1) Growth; (2) Preferential attachment. Starting from m_0 fully connected nodes, a new node with m ($m \le m_0$) edges are added to the existing network, and the other end of every new edge is connected to an old node preferentially proportional to the degree of the old node.

Another classical network model is the ER¹⁸ random graph. The network generation is simple. Initially, beginning with N isolated nodes, a pair of nodes is connected by a probability p.

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Author Contributions

Z.Y.J. and J.F.M. designed the research. Z.Y.J. performed numerical simulation and analyzed data. Z.Y.J. and J.F.M. wrote the paper. All authors reviewed the paper.

Additional Information

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