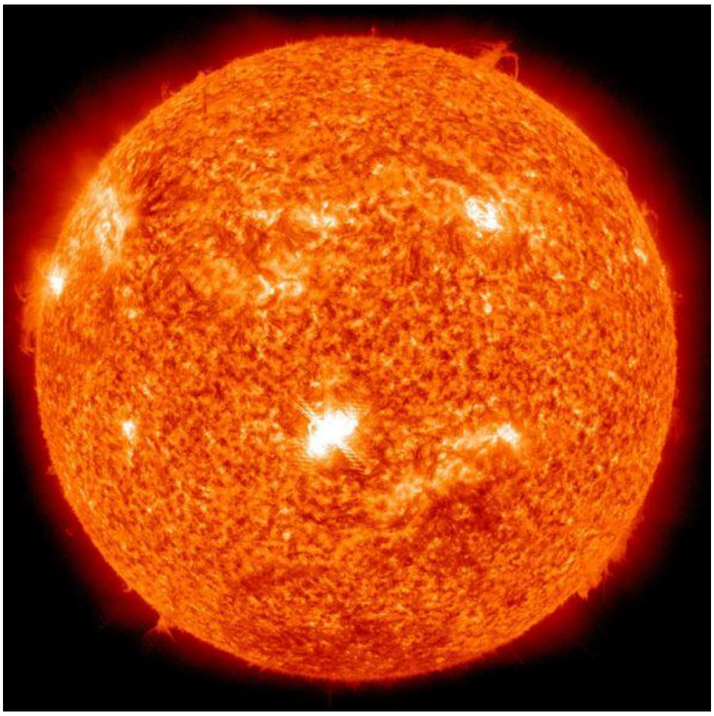
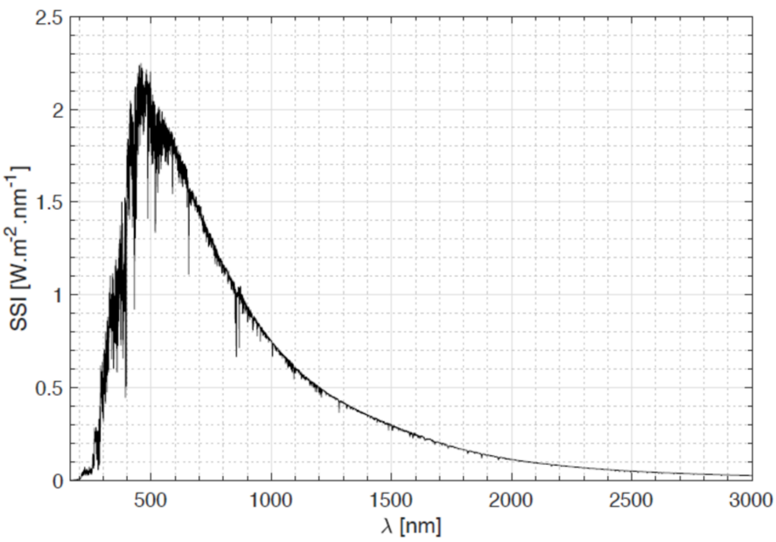
Inverse methods and parameter estimation in atmospheric physics

The Solar Spectrum

Day-3





The Minimum Length Solution

Learning goal: Better understanding of the Minimum Length Solution (the case $N < M$). Calculate and apply the averaging kernel and develop a regularization scheme using the condition number.

What do you need: working Python installation with access to the libraries numpy, netcdf4.

Exercise 1:

Read the TDID-1 HSRS solar reference spectrum F_0 from the netcdf file

hybrid_reference_spectrum_1nm_resolution_c2021-03-04_with_unc.nc

Convert the spectrum units from [W/(m2 nm)] to [number of photons/(s m2 nm)].

To perform the spectral convolution with the Instrument Spectral Response Function (ISRF) at spectral sampling points λ_i,

$$F_c = s \otimes F_o(\lambda_i)$$

$$= \int_0^{\infty} d\lambda \; s(\lambda_i, \lambda - \lambda_i) F_0(\lambda)$$

discretize the convolution integral. For the set of spectral sampling points $\{\lambda_1, \cdots, \lambda_N\}$ write the convolution in the form

$$F_c(\lambda_i) = \sum_{m=1}^{M} s(\lambda_i, \lambda_i - \lambda_m) F_o(\lambda_m)$$

and in the matrix form

$$F_c = K_{ISRF} F_o$$

with

$$F_c = [F_c(\lambda_1), \cdots, F_c(\lambda_N)]$$

$$F_0 = [F_0(\lambda_1), \cdots, F_0(\lambda_M)]$$

For the ISRF, use a Gaussian function with

$$s(\lambda_i, \lambda - \lambda_i) \propto \exp\left\{-4 \ln 2 \frac{(\lambda - \lambda_i)^2}{fwhm^2}\right\}$$

with a spectrally constant $fwhm = 0.45$. Define the sampled grid as

$$\lambda_i = \lambda_o + i \, \Delta\lambda$$

with $\lambda_o = 350\ nm$, $\lambda_M = \lambda_o + M\Delta\lambda = 405\ nm$, and $\Delta\lambda = 0.2\ nm$.

Plot the convolved solar spectrum. photons/(s m2 nm)]. Plot the solar line-by-line spectrum and the convolved spectrum as a function of wavelength.

Exercise 2:

Define a least squares function using the singular value decomposition of the kernel

$$K \ = \ USV^T$$

Suppose you use numpy.linalg.svd, check carefully the manual. V is defined differently for this function.

Calculate the gain matrix and the averaging kernel

$$G = VSU^T$$

$$A = V\,V^T$$

and return both.

Calculate in the main program the minimum length solution

$$F_{ret} = GF_{conv}$$

and check if this agrees with

$$F_{avg} = AF_o$$

Exercise 3

Determine the condition number of K_{isrf} from the SVD. Consider the truncated kernel

$$K_{trunc} = \sum_{i=1}^{N_{trunc}} u_i \sigma_i v_i^T$$

and determine the truncation index N_{trunc} such that the condition number of K_{trunc} is closest to a required value (input). Change the least squares function so that it can handle this required condition number as an input.

Plot the standard deviation of $F_{ret} - F_o$ as a function of the condition number.

Put a relative noise of 1% on F_c and repeat the simulation. Include the corresponding standard deviation in the plot and try to understand the shape of these curves.