Properties of Convolution

1. *Time-reversed signals*. This is true for both DT and CT convolution:

If $y[n] = x[n] * h[n]$, then $y[-n] = x[-n] * h[-n]$

If $y(t) = x(t) * h(t)$, then $y(-t) = x(-t) * h(-t)$.

Prove it for the CT case.

$y(t) = x(t) * h(t)$, then $y(-t) = x(-t) \times h(-t)$

$$= \int_{-\infty}^{\infty} x(t') h(t - t') \, dt'$$

$$y(-t) = \int_{-\infty}^{\infty} x(t') h(-t - t') \, dt'$$

$$x(-t) \times h(-t) = \int_{-\infty}^{\infty} x(-t') h(-(t - t')) \, dt'$$

$$= \int_{-\infty}^{\infty} x(-t') h(-t + t') \, dt'$$

$$= \int_{-\infty}^{\infty} x(k) h(-t - k) \, dk$$

since we are integrating over infinities

$$\therefore \quad = \int_{-\infty}^{\infty} x(t') h(-t - t') \, dt' = y(-t) \text{ as required}$$

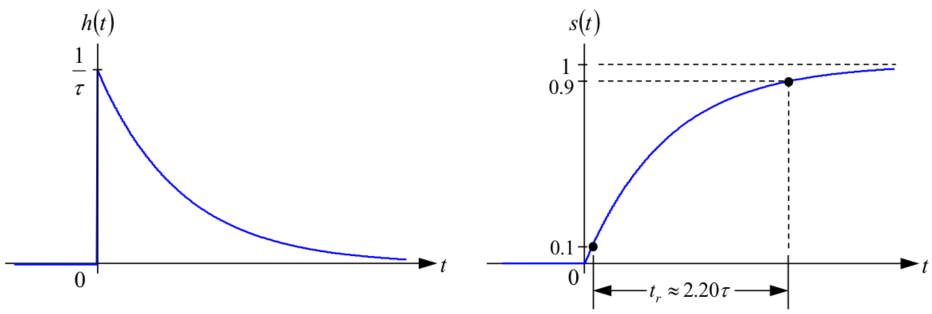
2. *First-order lowpass and highpass filters*. A *first-order lowpass filter* with input $x(t)$ and output $y(t)$ is described by a differential equation

$$\tau \frac{dy}{dt} + y(t) = x(t).$$

It has impulse and step responses

$$h(t) = \frac{1}{\tau} e^{-\frac{t}{\tau}} u(t) \text{ and } s(t) = \left(1 - e^{-\frac{t}{\tau}}\right) u(t),$$

which are shown here.



a. Show that the *rise time* required for the step response to increase from 10% to 90% of its maximum value is $t_r \approx 2.2\tau$.

a.

$$y(t) = x(t) - \tau \frac{dy}{dt}$$

$$s(t) = \left(1 - e^{-\frac{t}{\tau}}\right) u(t)$$

$$s(t_2) - s(t_1) = 0.8$$

$$0.9 - 0.1 = 0.8$$

$$s(t_2) = \left(1 - e^{-\frac{t_2}{\tau}}\right) u(t)$$

$$0.9 = 1 - e^{-\frac{t_2}{\tau}}$$

$$0.1 = e^{-\frac{t_2}{\tau}}$$

$$\ln(0.1) = -\frac{t_2}{\tau}$$

$$-\tau \ln(0.1) = t_2$$

$$s(t_1) = \left(1 - e^{-\frac{t_1}{\tau}}\right) u(t)$$

$$0.1 = 1 - e^{-\frac{t_1}{\tau}}$$

$$-\tau \ln(0.9) = t_1$$

$$\therefore t_r = t_2 - t_1 = -\tau \ln(0.1) + \tau \ln(0.9)$$

$$= 2.197\tau \text{ as required}$$

A *first-order highpass filter* is described by a differential equation

$$\frac{dy}{dt}+\frac{1}{\tau}y(t)=\frac{dx}{dt}.$$

b. In lecture, we stated its impulse response to be

$$h(t)=\delta(t)-\frac{1}{\tau}e^{-\frac{t}{\tau}}u(t).$$

Verify that this $h(t)$ satisfies the differential equation with input $x(t)=\delta(t)$, output $y(t)=h(t)$, and zero initial condition $y(t)=0$, $t<0$. Sketch $h(t)$.

$$h(t) = x(t) - \frac{1}{\tau} e^{-\frac{t}{\tau}} u(t)$$

$$y(t) = x(t) - \frac{1}{\tau} e^{-\frac{t}{\tau}}$$

$$\frac{dy}{dt} = \frac{d}{dt}\left(x(t) - \frac{1}{\tau} e^{-\frac{t}{\tau}}\right)$$

$$= \frac{dx}{dt} - \frac{d}{dt}\left(\frac{1}{\tau} e^{-\frac{t}{\tau}}\right)$$

$$= \frac{dx}{dt} - \frac{1}{\tau} \frac{d}{dt}\left(e^{-\frac{t}{\tau}}\right)$$

$$\frac{dy}{dt} = \frac{dx}{dt} - \frac{1}{\tau}\left(-\frac{1}{\tau} e^{-\frac{t}{\tau}}\right) = \frac{dx}{dt} - \frac{1}{\tau} y(t) \quad , \quad t>0$$

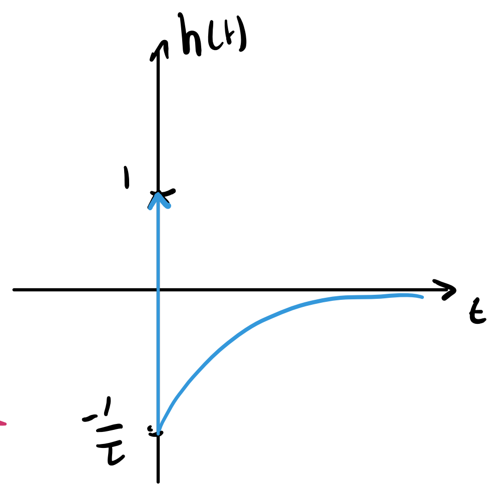
as required

at $y=0$, $y(t)=0$

$\therefore$ $y(0) = h(0) = 0$

$\therefore$ $h(0) = d(t) - \frac{1}{\tau}$

$= -\frac{1}{\tau}$ and impulse



c. Derive an expression for the step response $s(t)$. Sketch $s(t)$.

$$s(t) = \int_{-\infty}^{t} h(t')\,dt'$$

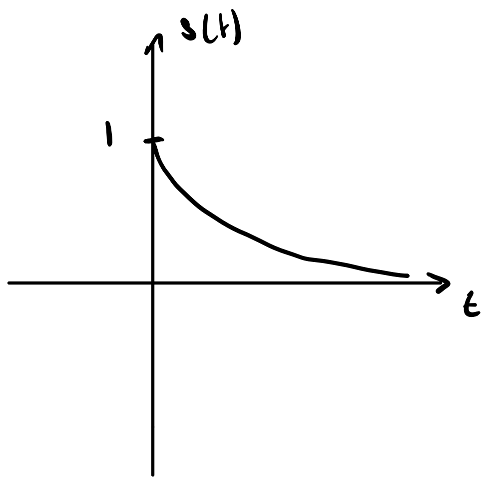
$$s(t) = \int_{-\infty}^{t} \delta(t) - \frac{1}{\tau} e^{-t/\tau} u(t)\,dt$$

$$= \int_{-\infty}^{t} \delta(t)\,dt - \int_{-\infty}^{t} \frac{1}{\tau} e^{-t/\tau} u(t)\,dt$$

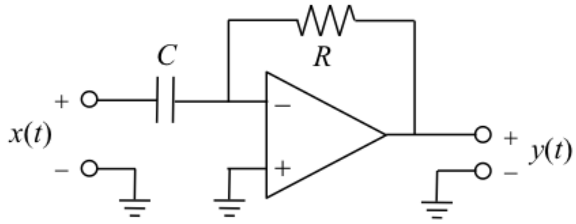
$$= u(t) - \int_{0}^{t} \frac{1}{\tau} e^{-t/\tau}\,dt$$

$$s(t) = u(t) - \left[-e^{-t/\tau}\right]_0^t$$

$$s(t) = u(t) - \left(1 - e^{-t/\tau}\right) = e^{-t/\tau}\,u(t)$$



3. *Differentiator and integrator*. If any of the following questions have been answered in the EE 102A lectures, feel free to state that and give the answer without proof. For a more detailed discussion of these systems, see *EE 102B Course Reader*, Chapter 5. A *differentiator* implemented using an operational amplifier (op amp) is shown here.



Given an input $x(t)$, ideally, the output is given by

$$y(t) = -RC\frac{dx}{dt}.$$

It is impossible to realize a perfect differentiator, as that would require the op amp to have infinite gain-bandwidth product and infinite slew rate. Here we assume ideality and ignore the factor $-RC$ so the output is given by

$$y(t) = \frac{dx}{dt}. \Rightarrow y(t) = \frac{1}{a_0} \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k}$$

a. What is the impulse response $h(t)$?

b. What is the step response $s(t)$?

c. Explain how the result of part (b) allows us to determine whether the ideal differentiator is a bounded-input bounded-output (BIBO)-stable system.

a. $h(t) = \frac{d(\delta(t))}{dt}$

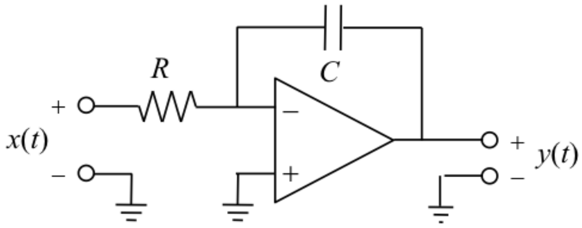
b. $s(t) = \int_{\infty}^{t} h(t')\, dt'$

$= \int_{\infty}^{t} \frac{d(\delta(t))}{dt}\, dt'$

$= \delta(t)$

c. It is not BIBO stable since $\delta(t) = \infty$, $t = 0$

An *integrator* implemented using an op amp is shown here.



Ideally, its output is given by

$$y(t) = -\frac{1}{RC}\int_{-\infty}^{t} x(t')dt' .$$

It is impossible to realize a perfect integrator, as that would require the op amp to have infinite d.c. gain and infinite output swing. Here we assume ideality and ignore the factor $-1/RC$ so the output becomes

$$y(t) = \int_{-\infty}^{t} x(t')dt' .$$

d. What is the impulse response $h(t)$?

e. What is the step response $s(t)$?

f. Explain how the result of part (e) allows us to determine whether the ideal integrator is a BIBO-stable system.

d. $h(t) = \int_{-\infty}^{t} \delta(t')\, dt' = u(t)$

$$y(t) = \frac{1}{a_0}\sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k} .$$

e. $s(t) = \int_{-\infty}^{t} h(t')dt'$

$= \int_{-\infty}^{t} u(t')\, dt'$

$= r(t) = t \times u(t)$

f. No, it is not BIBO stable as $t \to \infty$

4. *Finite integration*. A CT LTI system H with input $x(t)$ yields an output

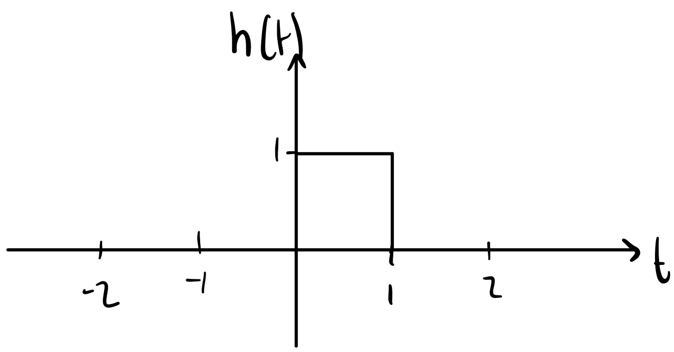
$$H[x(t)] = y(t) = \int_{t-1}^{t} x(t')dt'.$$

a. Find an impulse response $h(t)$ such that $H[x(t)] = y(t) = x(t) * h(t)$. Sketch $h(t)$. *Hint*: you can think of the finite integration as the difference between two infinite integrations with different upper limits, and express $h(t)$ as the difference between two shifted step functions.

b. Is the system causal?

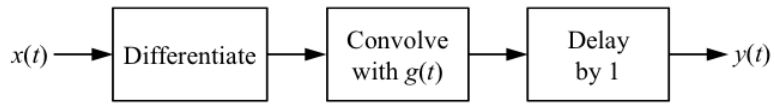
a. $y(t) = \int_{t-1}^{t} x(t')\,dt'$ $\quad\Big|\quad$ $\int_{-\infty}^{\infty} x(t')\,h(t-t')\,dt'$

$h(t) = u(t) - u(t-1)$



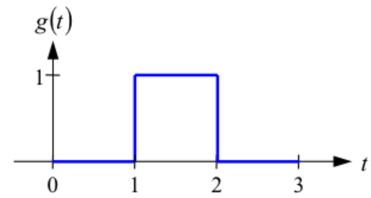
b. Yes since previous values of t are taken into account

5. A CT system H with input $x(t)$ and output $y(t)$ is the cascade of three LTI systems, so the overall system is LTI.



a. Find an impulse response $h(t)$ such that $H[x(t)] = y(t) = x(t) * h(t)$.

b. Suppose $g(t)$ is as shown. Give an expression for $h(t)$. Sketch $h(t)$.



a. Differentiator $h(t) = \frac{d\,\delta(t)}{dt}$ $y_1(t) = \frac{dx(t)}{dt}$

Convolution with $g(t)$ $\therefore$ $y_2(t) = \frac{dx(t)}{dt} \times g(t)$

delay by 1 $y_3(t) = y_2(t-1) = \frac{dx(t-1)}{dt} \times g(t-1)$

$$y(t) = x(t) \times \left(\frac{d\delta(t)}{dt} \times g(t) \times \delta(t-1) \right)$$

$$\therefore h(t) = \frac{d\delta(t)}{dt} \times g(t) \times \delta(t-1)$$

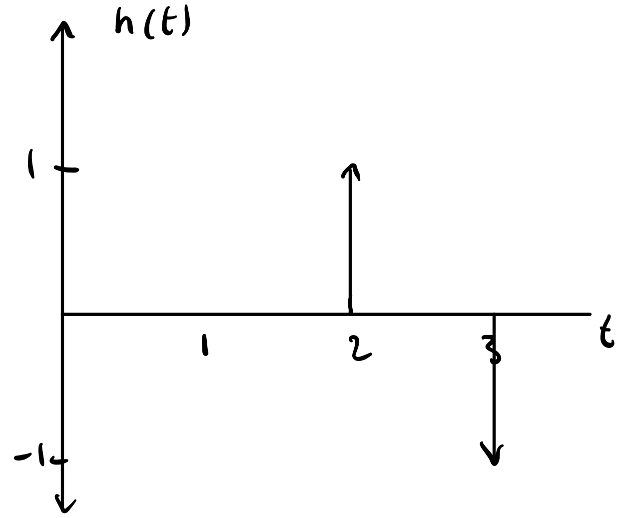
b. $h(t) = \frac{d\delta(t)}{dt} \times (u(t-1) - u(t-2)) \times \delta(t-1)$

differentiating step function is a unit impulse

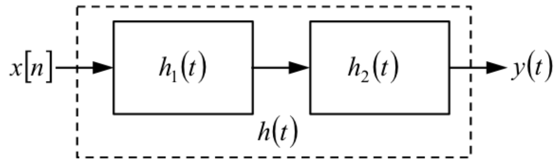
$\therefore$

$= (\delta(t-1) - \delta(t-2)) \times \delta(t-1)$

$= \delta(t-2) - \delta(t-3)$



6. LTI systems with impulse responses $h_1(t)$ and $h_2(t)$ are cascaded to form an LTI system with impulse response $h(t)$.



Both are first-order systems. Their impulse responses are

$$h_1(t) = \frac{1}{\tau_1} e^{-\frac{t}{\tau_1}} u(t)$$

$$h_2(t) = \frac{1}{\tau_2} e^{-\frac{t}{\tau_2}} u(t),$$

where τ_1 and τ_2 are real and positive and $\tau_1 \neq \tau_2$.

a. Find an expression for $h(t) = h_1(t) * h_2(t)$. Simplify your expression for $h(t)$ so it is clearly a linear combination of $h_1(t)$ and $h_2(t)$. *Hint*: you can solve this by symbolic integration, without using "flip and drag".

b. Verify that $\int_{-\infty}^{\infty} h(t)dt = \left(\int_{-\infty}^{\infty} h_1(t)dt\right)\left(\int_{-\infty}^{\infty} h_2(t)dt\right)$, as expected from Homework 2 Problem 4.

a.

$$h(t) = h_1(t) \times h_2(t)$$

$$= \int_{-\infty}^{\infty} h_1(t') h_2(t-t')\, dt'$$

$$= \int_{-\infty}^{\infty} \left(\frac{1}{\tau_1} e^{-\frac{t'}{\tau_1}} u(t')\right)\left(\frac{1}{\tau_2} e^{-\frac{t-t'}{\tau_2}} u(t')\right) dt'$$

$$= \int_0^t \left(\frac{1}{\tau_1} e^{-\frac{t'}{\tau_1}} \times \frac{1}{\tau_2} e^{-\frac{t-t'}{\tau_2}}\right) dt'$$

$$= \int_0^t \frac{1}{\tau_1 \tau_2} e^{-\frac{t'}{\tau_1} - \frac{t-t'}{\tau_2}}\, dt'$$

$$= \frac{1}{\tau_1 \tau_2} \int_0^t e^{-\frac{t'}{\tau_1} - \frac{t}{\tau_2} + \frac{t'}{\tau_2}}\, dt'$$

$$= \frac{1}{\tau_1 \tau_2} \int_0^t e^{t'\left(\frac{1}{\tau_2} - \frac{1}{\tau_1}\right) - \frac{t}{\tau_2}}\, dt'$$

$$= \frac{1}{\tau_1 \tau_2} e^{-t/\tau_2} \int_0^t e^{t'\left(\frac{1}{\tau_2} - \frac{1}{\tau_1}\right)} dt'$$

$$= \frac{1}{\tau_1 \tau_2} e^{-t/\tau_2} \left[\frac{\tau_1 \tau_2}{\tau_1 - \tau_2} e^{t'\left(\frac{1}{\tau_2} - \frac{1}{\tau_1}\right)} \right]_0^t$$

$$= \frac{1}{\tau_1 \tau_2} e^{-t/\tau_2} \left(\frac{\tau_1 \tau_2}{\tau_1 - \tau_2} e^{t\left(\frac{1}{\tau_2} - \frac{1}{\tau_1}\right)} - \frac{\tau_1 \tau_2}{\tau_1 - \tau_2} \right)$$

$$= \frac{1}{\tau_1 - \tau_2} e^{(-t/\tau_2)} \left(e^{t\left(\frac{1}{\tau_2} - \frac{1}{\tau_1}\right)} - 1 \right)$$

$$= \frac{1}{\tau_1 - \tau_2} e^{-\frac{t}{\tau_1}} - \frac{1}{\tau_1 - \tau_2} e^{-\frac{t}{\tau_2}}$$

$$= \frac{1}{\tau_1 - \tau_2} \left(\tau_1 h_1(t) - \tau_2 h_2(t) \right)$$

b. $$\int_{-\infty}^{\infty} h_1(t)\, dt = \int_0^{\infty} \frac{1}{\tau_1} e^{-t/\tau_1} dt$$

$$= \left[-e^{-t/\tau_1} \right]_0^{\infty} = 1$$

$$\int_{-\infty}^{\infty} h_2(t)\, dt = \int_0^{\infty} \frac{1}{\tau_2} e^{-t/\tau_2} dt$$

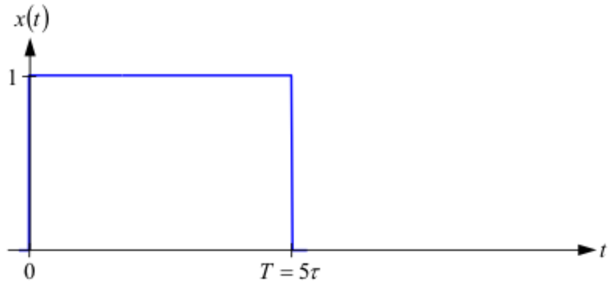
$$= \left[-e^{-t/\tau_2} \right]_0^{\infty} = 1$$

$$\int_{-\infty}^{\infty} h(t)\, dt = \frac{1}{\tau_1 - \tau_2} \int_{-\infty}^{\infty} \left(\tau_1 h_1(t) - \tau_2 h_2(t) \right) dt$$

$$= \frac{1}{\tau_1 - \tau_2} \left(\tau_1 \int_{-\infty}^{\infty} h_1(t)\, dt - \tau_2 \int_{-\infty}^{\infty} h_2(t)\, dt \right)$$

$$= \frac{1}{\tau_1 - \tau_2} \left(\tau_1 - \tau_2 \right) = 1 \text{ as required}$$

7. A rectangular pulse $x(t)$, shown below, is input to different LTI systems, each specified by an impulse response $h(t)$. For each system: (i) Find an expression for the output $y(t)$ by evaluating the convolution $x(t)*h(t)$. (ii) Make an approximate sketch of the output $y(t)$ without using a calculator or computer, assuming $T=5$, $\tau=1$. (iii) Verify that $\int_{-\infty}^{\infty} y(t)\,dt = \left(\int_{-\infty}^{\infty} x(t)\,dt\right)\left(\int_{-\infty}^{\infty} h(t)\,dt\right)$, as expected from Homework 2 Problem 4. You should compute $\int_{-\infty}^{\infty} x(t)\,dt$ and $\int_{-\infty}^{\infty} h(t)\,dt$ exactly. It is sufficient for you to estimate $\int_{-\infty}^{\infty} y(t)\,dt$ from your sketches.



a. First-order lowpass filter $h(t)=\frac{1}{\tau}e^{-\frac{t}{\tau}}u(t)$.

$$h(t-t') = \frac{1}{\tau} e^{-\frac{t-t'}{\tau}} u(t-t')$$

b. First-order highpass filter $h(t)=\delta(t)-\frac{1}{\tau}e^{-\frac{t}{\tau}}u(t)$.

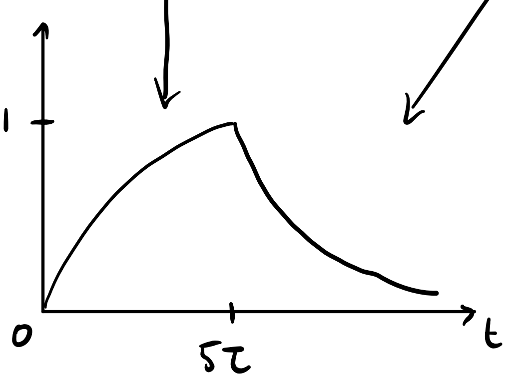
a. i $y(t) = x(t) \times h(t)$ $\qquad x(t)=0\ ,\ t>T$

for low pass filter $y(t) = 1-e^{-t/\tau}$ (course reader)

Shifted for $x(t) = u(t) - u(t-5\tau)$

$\therefore\ y(t) = \underline{(1-e^{-t/\tau})(u(t))} - \underline{(1-e^{-(t-5\tau)/\tau})(u(t-5\tau))}$

ii.



iii. $\int_{-\infty}^{\infty} u(t) - u(t-5T)\, dt = t - (t-T) = T$

$$\int_{-\infty}^{\infty} h(t) = \int_{-\infty}^{\infty} \frac{1}{\tau} e^{-\frac{t}{\tau}} u(t)\, dt$$

$$= \int_{0}^{\infty} \frac{1}{\tau} e^{-\frac{t}{\tau}}\, dt = \left[-e^{-t/\tau}\right]_0^{\infty}$$

$$= 1$$

$$\int_{-\infty}^{\infty} (1-e^{-t/\tau})(u(t)) - (1-e^{-(t-T)})(u(t-T))\, dt$$

$$= \int_{0}^{\infty} (1-e^{-t/\tau})\,dt - \int_{T}^{\infty} (1-e^{-(t-T)/\tau})\, dt$$

$$= \left[t + \frac{1}{\tau} e^{-t/\tau}\right]_0^{\infty} - \left[t + \frac{1}{\tau} e^{-(t-T)/\tau}\right]_T^{\infty}$$

$$= -\frac{1}{\tau} + T + \frac{1}{\tau} = T \text{ as required}$$

b. i $y(t) = \delta(t) - (1 - e^{-t/\tau})$ $\therefore$ opposite

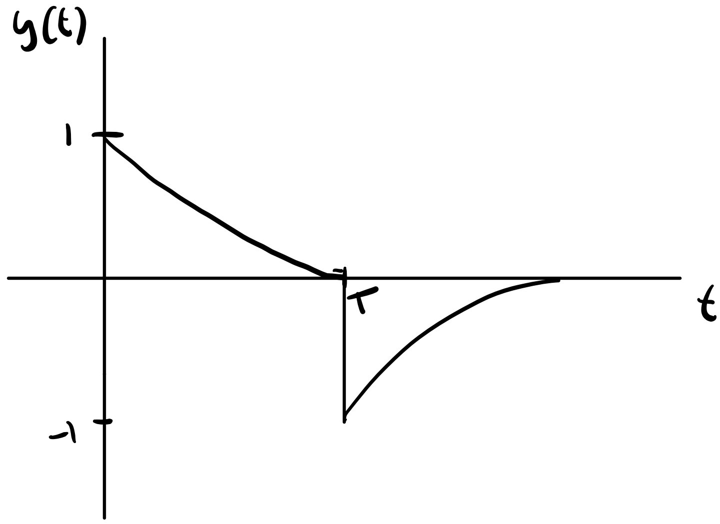
$\therefore$ Shifted for $x(t) = u(t) - u(t-T)$

$$\therefore y(t) = (\delta(t) - (1-e^{-t/\tau})) - (\delta(t-T) - (1-e^{-\frac{(t-T)}{\tau}}))$$

$t = 0 \rightarrow \delta(t) - (1-e^{-t/\tau}) = 1$

$t = T \rightarrow \delta(t) = 1$

ii.



iii. $\int_{-\infty}^{\infty} h(t)\,dt = 1$

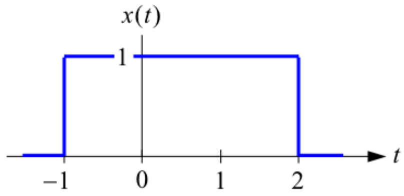
$$\int_{-\infty}^{\infty} x(t)\,dt = 5\tau = T$$

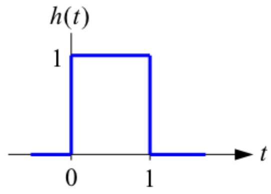
$$y(t) = \left(\delta(t) - \left(1 - e^{-t/\tau}\right)\overset{u(t)}{}\right) - \left(\delta(t-T) - \left(1 - e^{-\frac{(t-T)}{\tau}}\right)\overset{u(t-T)}{}\right)$$

$$-\int_{0}^{\infty} 1 - e^{-t/\tau}\,dt + \int_{T}^{\infty} 1 - e^{-\frac{(t-T)}{\tau}}\,dt = T$$ (from part a)

as required

8. A rectangular pulse $x(t)$ is input to a finite-duration integrator, which has an impulse response $h(t)$.





Evaluate the convolution $y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(t')h(t-t')dt'$ using the "flip and drag" method.

Hint: first sketch $x(t')$ vs. t'. Then, for relevant choices of the time *t*, sketch $h(t-t')$ vs. t' and integrate $x(t')h(t-t')$ over t' to obtain $y(t)$.

$h(t) = u(t) - u(t-1)$

$x(t) = u(t+1) - u(t-2)$

$\therefore\ h(t) \times x(t) = u(t)u(t+1) - u(t)u(t-2)$

$- u(t-1)u(t+1) + u(t-1)u(t-2)$

$u(t-1) = u(t)\, d(t-1)$

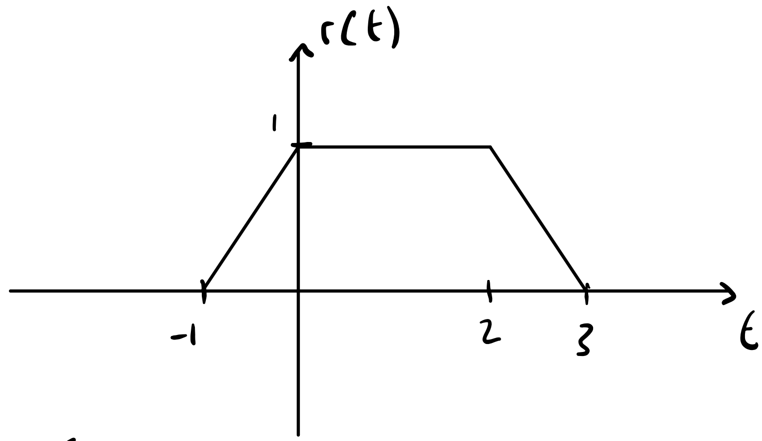
$u(t+1) = u(t)\, d(t+1)$

$r(t)\, d(t-1)\, d(t+1)$

$= r(t-(1-1)) = r(t)$

$u(t-1)u(t-2) = r(t-3)$

$= r(t+1) - r(t-2) - r(t) + r(t-3)$



Flip and Drag:

$t < -1 \Rightarrow 0$

$-1 < t < 0 \Rightarrow t+1$

$0 < t < 2 \Rightarrow 1$

$2 < t < 3 \Rightarrow 3-t$

$t > 3 \Rightarrow 0$

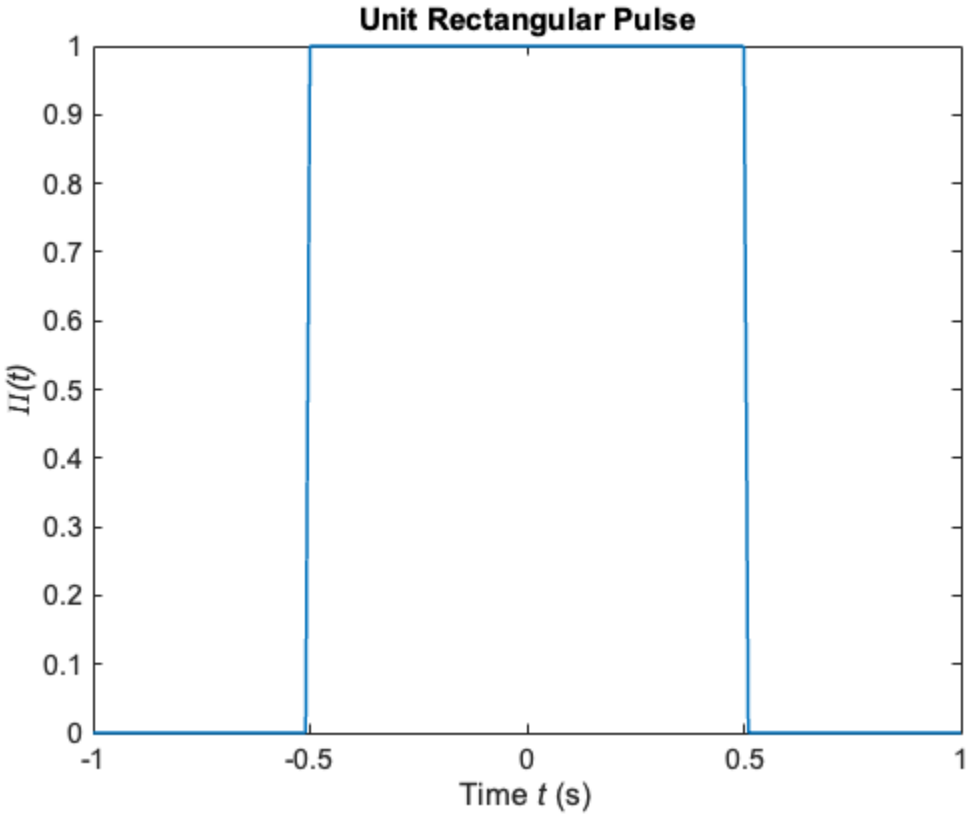
$$\therefore\ y(t) \begin{cases} 0, & t < -1 \\ t+1, & -1 < t < 0 \\ 1, & 0 < t < 2 \\ 3-t, & 2 < t < 3 \\ 0, & t > 3 \end{cases}$$

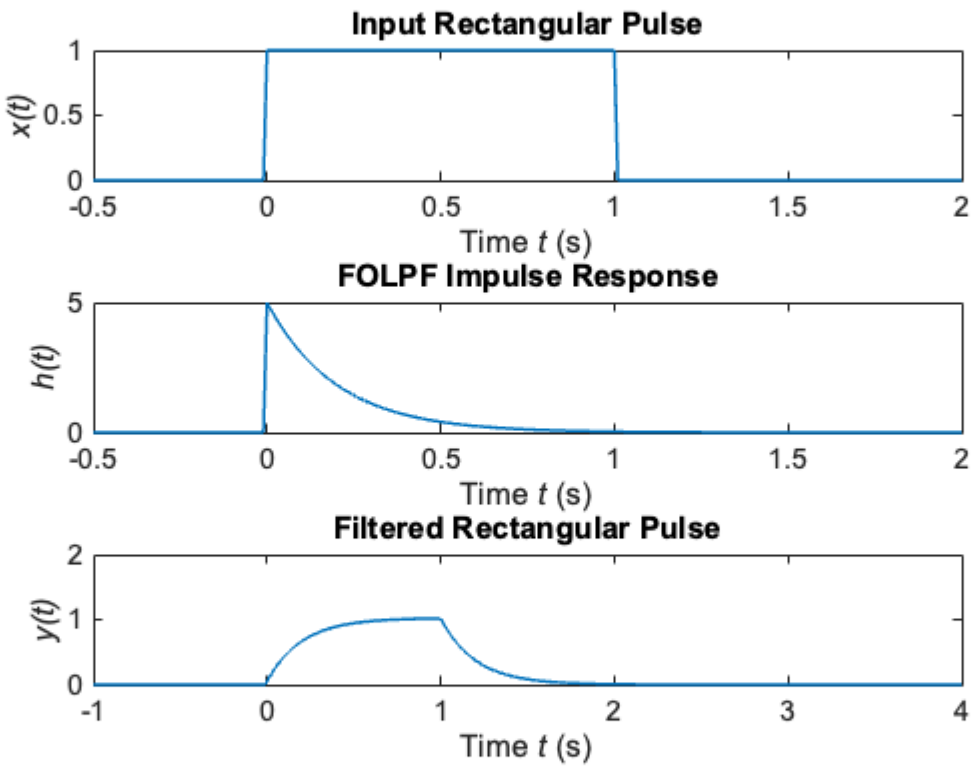
Table of Contents

Part 1

```
% A
t = -1:.01:1;
figure; plot(t,Pi(t), 'LineWidth', 1.5);
set(gca,'FontName','arial','FontSize',14);
xlabel('Time \itt\rm (s)'); ylabel('\it\Pi(t)');
title('Unit Rectangular Pulse');

% B
deltat = 0.01; % time increment
tau = 0.2; % FOLPF time constant
T = 1; % rectangular pulse width
t1 = -0.5; t2 = 10*tau;
t = t1:deltat:t2;

t1y = t1+t1; t2y = t2+t2;
ty = t1y:deltat:t2y;

x = Pi((t-T/2)/T); % x(t)
h = 1/tau * exp(-t/tau) .* double(t>=0); % h(t)

y = conv(x,h)*deltat;

% Input Rectangular Pulse
t = -0.5:.01:2;
figure; subplot(311); plot(t,x, 'LineWidth', 1.5);
set(gca,'FontName','arial','FontSize',14);
xlabel('Time \itt\rm (s)'); ylabel('\itx(t)');
title('Input Rectangular Pulse');

% FOLPF Impulse Response
t = -0.5:.01:2;
subplot(312); plot(t,h, 'LineWidth', 1.5);
set(gca,'FontName','arial','FontSize',14);
xlabel('Time \itt\rm (s)'); ylabel('\ith(t)');
title('FOLPF Impulse Response');

% Filtered Rectangular Pulse
subplot(313); plot(ty,y, 'LineWidth', 1.5);
ylim([0 2])
set(gca,'FontName','arial','FontSize',14);
```

```
xlabel('Time \itt\rm (s)'); ylabel('\ity(t)');
title('Filtered Rectangular Pulse');
```





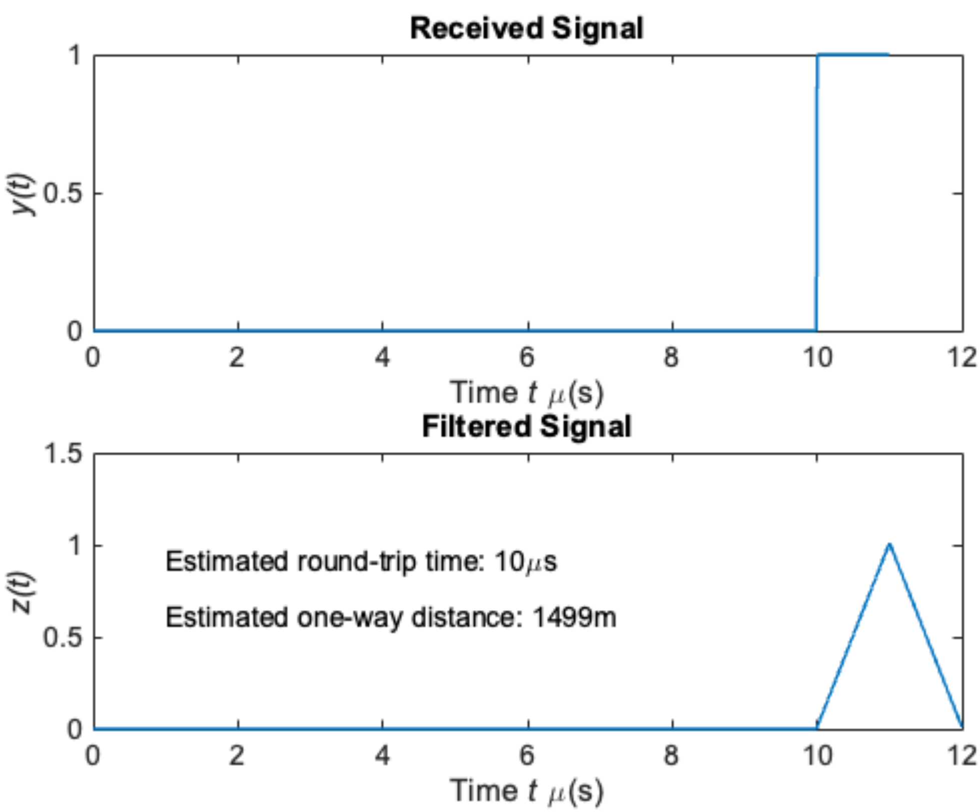
Part 2

Task 2a (noiseless)

All times are in microseconds

```
deltat = 0.01; % time increment
T = 1; % rectangular pulse width
td = 10; % round-trip time delay
c = 2.9979e2; % speed of light (m/microsecond)

% Transmitted signal x(t)
tx1 = 0; tx2 = T;
tx = tx1:deltat:tx2; % time for x
x = Pi((tx-T/2)/T); % x(t)

ty1 = tx1; ty2 = tx2 + td;
ty = ty1:deltat:ty2;
y = Pi((ty-td-T/2)/T);

% Causal matched filter h(t)= x(T-t)
th1 = tx1; th2 = tx2;
th = th1:deltat:th2; % time for h
h = fliplr(x); % h(t)
```

```
tz1 = tx1; tz2 = 2*tx2 + td;
tz = tz1:deltat:tz2;
z = conv(y, h) * deltat;

[zmax,index] = max(z); % finding the peak in z(t)
td_est = tz(index) - T; % estimated round-trip delay time
d_est = c*td_est/2; % estimated one-way propagation distance

% Received Signal
figure; subplot(211); plot(ty,y, 'LineWidth', 1.5);
set(gca,'FontName','arial','FontSize',14);
xlabel('Time \itt\rm \mu(s)'); ylabel('\ity(t)');
title('Received Signal');

% Filtered Signal
subplot(212); plot(tz,z, 'LineWidth', 1.5);
ylim([0 1.5])
set(gca,'FontName','arial','FontSize',14);
xlabel('Time \itt\rm \mu(s)'); ylabel('\itz(t)');
title('Filtered Signal');

text(1,0.9*max(z),['Estimated round-trip time: ' num2str(td_est,4) '\mus'],
'FontName','arial','FontSize',14);
text(1,0.6*max(z),['Estimated one-way distance: ' num2str(d_est,4) 'm'],
'FontName','arial','FontSize',14);
```



Task 2b (noise added)

```
Sn = 0.03; % power spectral density of noise

% Noise n(t)
sigma = sqrt(Sn/deltat); % standard deviation of noise
n = sigma*randn(size(ty)); % n(t)

% Transmitted signal x(t)
tx1 = 0; tx2 = T;
tx = tx1:deltat:tx2; % time for x
x = Pi((tx-T/2)/T); % x(t)

ty1 = tx1; ty2 = tx2 + td;
ty = ty1:deltat:ty2;
y = Pi((ty-td-T/2)/T) + n;

% Causal matched filter h(t)= x(T-t)
th1 = tx1; th2 = tx2;
th = th1:deltat:th2; % time for h
h = fliplr(x); % h(t)

tz1 = tx1; tz2 = 2*tx2 + td;
tz = tz1:deltat:tz2;
z = conv(y, h) * deltat;

[zmax,index] = max(z); % finding the peak in z(t)
td_est = tz(index) - T; % estimated round-trip delay time
d_est = c*td_est/2; % estimated one-way propagation distance


% Received Signal
figure; subplot(211); plot(ty,y, 'LineWidth', 1.5);
set(gca,'FontName','arial','FontSize',14);
xlabel('Time \itt\rm \mu(s)'); ylabel('\ity(t)');
title('Received Signal');

% Filtered Signal
subplot(212); plot(tz,z, 'LineWidth', 1.5);
ylim([0 1.5])
set(gca,'FontName','arial','FontSize',14);
xlabel('Time \itt\rm \mu(s)'); ylabel('\itz(t)');
title('Filtered Signal');

text(1,0.9*max(z),['Estimated round-trip time: ' num2str(td_est,4) '\mus'],
'FontName','arial','FontSize',14);
text(1,0.6*max(z),['Estimated one-way distance: ' num2str(d_est,4) 'm'],
'FontName','arial','FontSize',14);
```

Functions

```
function y = Pi(x)
    y = double(abs(x) <= 1/2);
end
```

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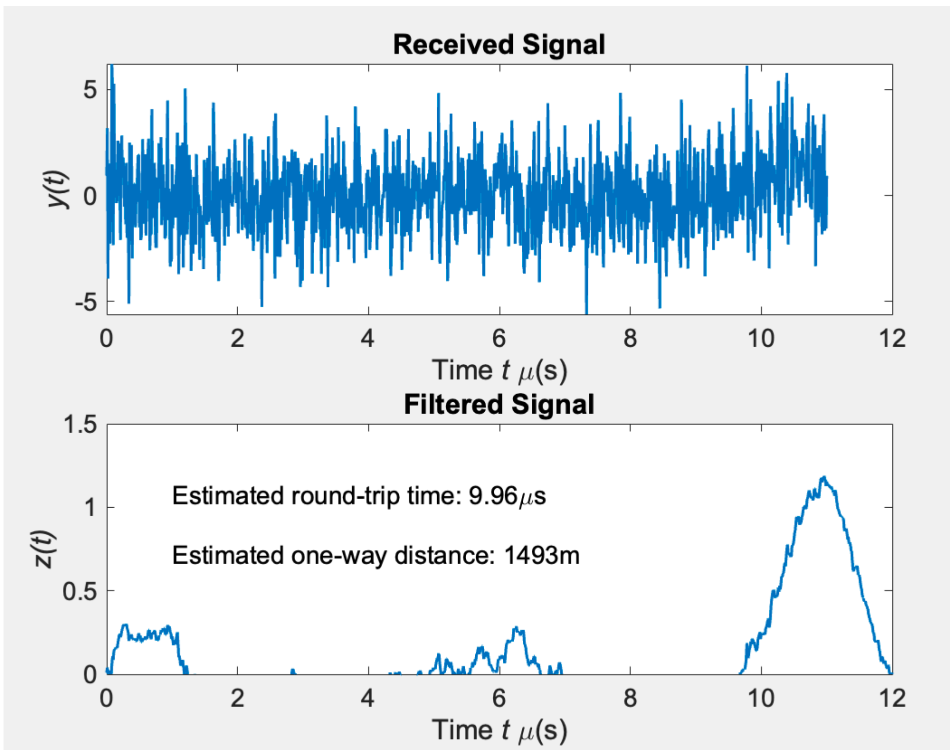

* issues with publishing on matlab so this is a screenshot