1. *Fourier transforms.* Sketch each signal $x(t)$. Using only tables and properties, obtain an expression for its Fourier transform $X(j\omega)$. Explain how the properties of $x(t)$ (real or imaginary, odd or even, etc.) are reflected in those of $X(j\omega)$.

a. $x(t) = \Lambda\left(\frac{t}{4}\right)\sin(2\pi t)$. Sketch the real or imaginary part of $X(j\omega)$, whichever is nonzero. *Hint*: use the known FT of the sine function and use the FT multiplication property.

a.

$$\sin\omega_0 t = x'(t) \longleftrightarrow X'(j\omega) = \frac{\pi}{j}\left[\delta(\omega-\omega_0) - \delta(\omega+\omega_0)\right]$$

$$\sin(2\pi t) \rightarrow Y(j\omega) = \frac{\pi}{j}\left[\delta(\omega-2\pi) - \delta(\omega+2\pi)\right]$$

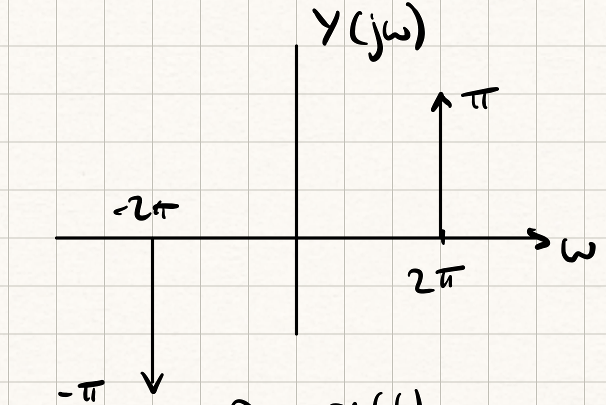
$$Z(j\omega) = 4\,\mathrm{sinc}^2\left(\frac{2\omega}{\pi}\right) \qquad \Lambda\left(\frac{t}{2T_1}\right) \longleftrightarrow 2T_1\,\mathrm{sinc}^2\left(\frac{\omega T_1}{\pi}\right)$$

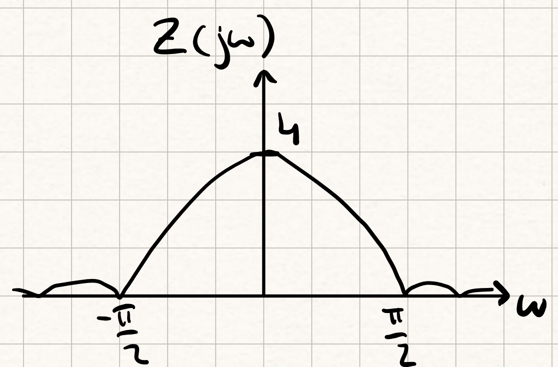
$$\therefore T_1 = 2$$

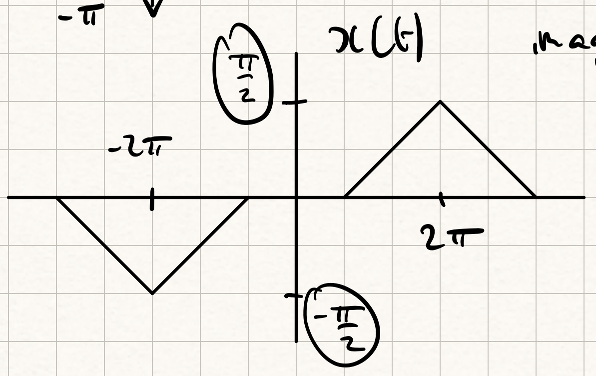
$$X(j\omega) = \frac{1}{2\pi} Z(j\omega) * Y(j\omega)$$

$$= \frac{1}{2} Z(j(\omega-2\pi)) - \frac{1}{2} Z(j(\omega+2\pi))$$

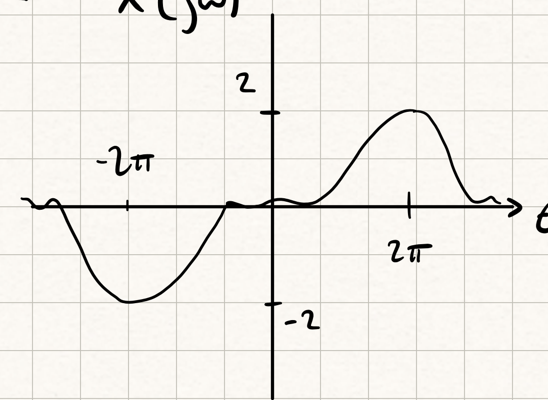
$$= 2\,\mathrm{sinc}^2\left(\frac{2\omega-4\pi}{\pi}\right) - 2\,\mathrm{sinc}^2\left(\frac{2\omega+4\pi}{\pi}\right)$$







imaginary ←



b. $x(t)=e^{-t}\left[u(t+1)-u(t-1)\right]$. You do not need to sketch $X(j\omega)$. *Hint*: write a term like $e^{-t}u(t+1)$ as $e\cdot e^{-(t+1)}u(t+1)$.

$$x(t) = e^{-t}\left[u(t+1) - u(t-1)\right]$$

$$= e^{-t}u(t+1) - e^{-t}u(t-1)$$

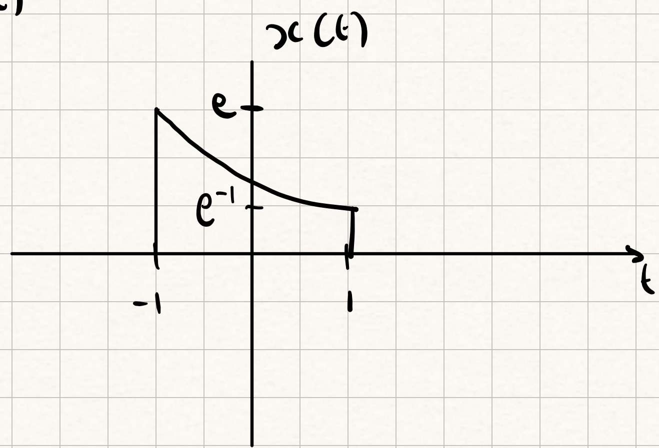
$$= \underbrace{e\cdot e^{-(t+1)}u(t+1)} - e^{-1}\cdot e^{-(t-1)}u(t-1)$$

$\hookrightarrow \quad e^{-at}u(t) \qquad X(j\omega) = \dfrac{1}{(a+j\omega)}$

$\therefore\ t = t+1, \quad a = 1$

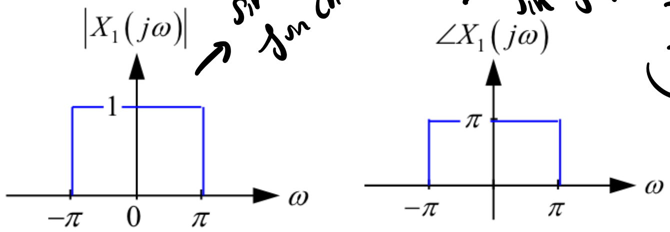
$$X(j\omega) = \frac{e^{j\omega}}{1+j\omega} - \frac{e^{-j\omega}}{(1+j\omega)} = \frac{1}{(1+j\omega)}\left(e^{j\omega} - e^{-j\omega}\right)$$

$x(t)$



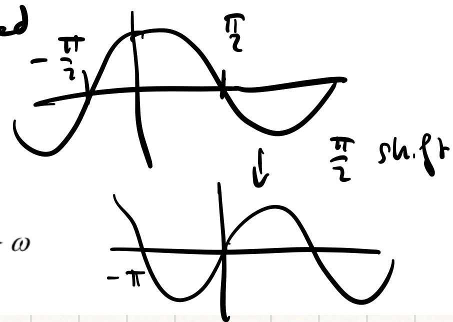
2. *Inverse Fourier transforms.* The Fourier transforms given have identical magnitudes but different phases. Obtain an expression for the inverse Fourier transform of each one (the corresponding time signal) using only tables and properties. Explain how the properties of each time signal (real or imaginary, odd or even, etc.) are reflected in those of its Fourier transform. *Hint*: express each transform in terms of one or more rectangular pulse(s), possibly shifted, and scaled by constant(s).

a.



sinc function

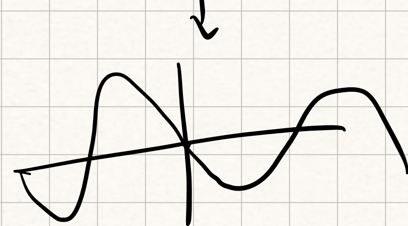
cos or sin flipped



a.

$$|X(j\omega)| + \angle X_1(j\omega) = |X(j\omega)| + e^{j\angle X_1(j\omega)}$$

$$X(j\omega) =$$



$$|X_1(j\omega)| = u(t+\pi) - u(t-\pi)$$

$$\angle X_1(j\omega) = \pi(u(t+\pi) - u(t-\pi))$$

$x(t)$ = flipped sinc function

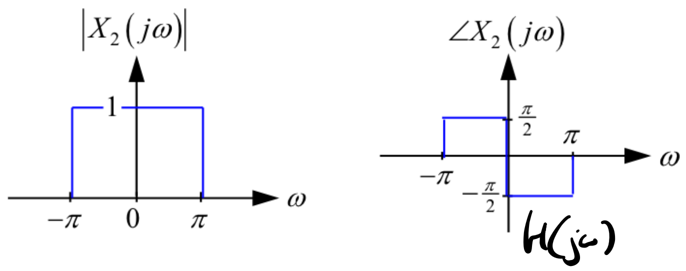
$$X(j\omega) = \underbrace{-}_{\text{phase}} \underbrace{\Pi\left(\frac{\omega}{2\pi}\right)}_{\text{magnitude}} \longleftrightarrow x(t) = -\frac{\pi}{\pi}\,\text{sinc}(t)$$

$$= -\,\text{sinc}(t)$$

Since $X(j\omega)$ is real and even

$x(t)$ is real and even

b.



$$H(j\omega) = \Pi\left(\frac{\omega}{\pi}\right) \longleftrightarrow h(t) = \frac{1}{2}\,\mathrm{sinc}\left(\frac{t}{2}\right)$$

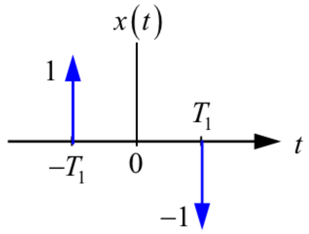
$$\therefore\ X(j\omega) = j\,H\left(j\left(\omega + \frac{\pi}{2}\right)\right) - j\,H\left(j\left(\omega - \frac{\pi}{2}\right)\right)$$

$$\therefore\ x(t) = j e^{-j\frac{\pi}{2}t} h(t) - j e^{j\frac{\pi}{2}t} h(t) \to h(t)\left(\frac{e^{j\frac{\pi}{2}t} - e^{-j\frac{\pi}{2}t}}{j}\right)$$

$$x(t) = 2\sin\left(\frac{\pi}{2}t\right) h(t) = \sin\left(\frac{\pi}{2}t\right)\mathrm{sinc}\left(\frac{t}{2}\right)$$

$X(j\omega)$ is odd and imaginary so $x(t)$ is real and odd

3. *Fourier transform integration property.* A signal $x(t)$ is a sum of two scaled, shifted impulses.



a. Obtain an expression for $X(j\omega)$, the Fourier transform of $x(t)$.

b. Now consider $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$. Sketch $y(t)$.

c. Using the Fourier transform integration property, obtain an expression for $Y(j\omega)$, the Fourier transform of $y(t)$. Put $Y(j\omega)$ in the standard form found in the table.

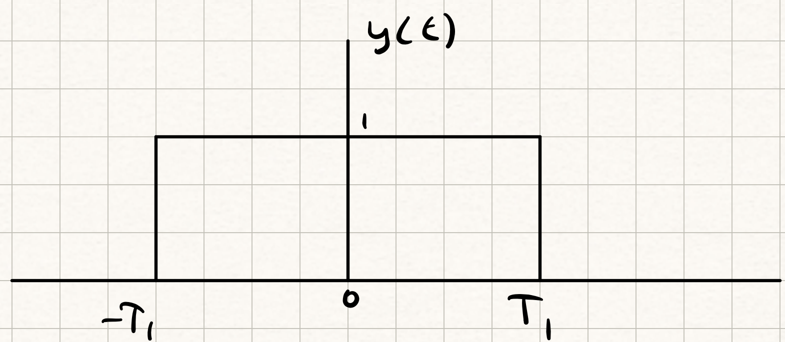
a. From appendix $\rightarrow$ $\delta(t-t_0) \rightarrow FT = e^{j\omega t_0}$

$$x(t) = \delta(t+T_1) - \delta(t-T_1)$$

$$\therefore X(j\omega) = e^{j\omega_0 T_1} - e^{-j\omega_0 T_1}$$

$$= 2j \sin(\omega_0 T_1)$$

b.



$= \Pi\left(\frac{t}{2T_1}\right)$

c. $Y(j\omega)$ is sinc function

$Y(j\omega) = 2T_1 \operatorname{sinc}\left(\frac{\omega T_1}{\pi}\right)$ (from appendix)

4. *Convolution property and relation between the Fourier series and the Fourier transform of one period.* We are given two rectangular pulses:

$$x_1(t) = \Pi\left(\frac{t}{2T_1}\right) \text{ and } x_2(t) = \Pi\left(\frac{t}{2T_2}\right).$$

Consider the convolution between them:

$$x(t) = x_1(t) * x_2(t). \rightarrow \text{triangle?}$$

a. Sketch $x(t)$, assuming $T_1 = 1$ and $T_2 = 2$.

b. Obtain an expression for $X(j\omega)$, the Fourier transform of $x(t)$, assuming general values of T_1 and T_2. Sketch $X(j\omega)$, assuming $T_1 = 1$ and $T_2 = 2$ (sketch the real or imaginary part, whichever is nonzero).

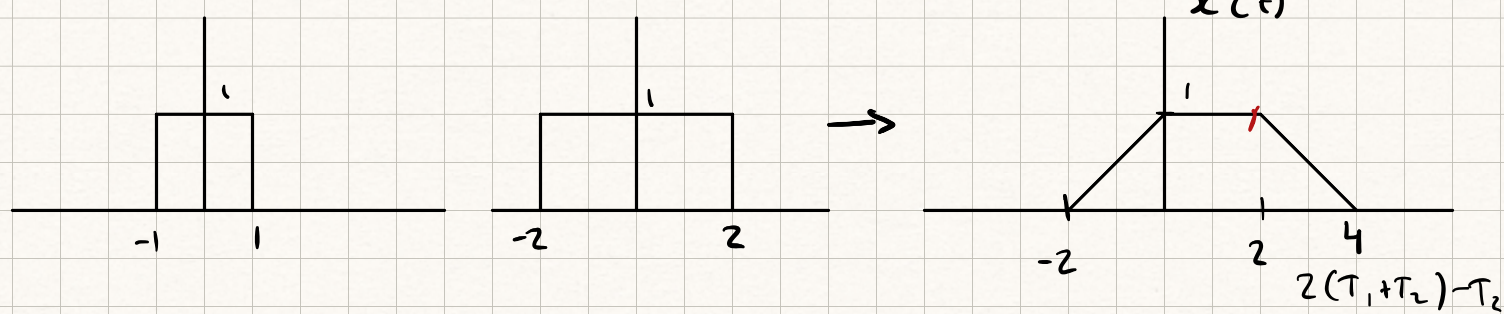
c. Now consider a periodic signal

$$\tilde{x}(t) = \sum_{n=-\infty}^{\infty} x(t - nT_0), \qquad \text{p158}$$

assuming $T_0 \geq 2(T_1 + T_2)$. Sketch two or three periods of $\tilde{x}(t)$, assuming $T_1 = 1$ $T_2 = 2$ and $T_0 = 6$.

d. By sampling $X(j\omega)$, obtain an expression for the Fourier series coefficients of $\tilde{x}(t)$, given by a_k. Sketch the a_k vs. k, assuming $T_1 = 1$ $T_2 = 2$ and $T_0 = 6$.

a.

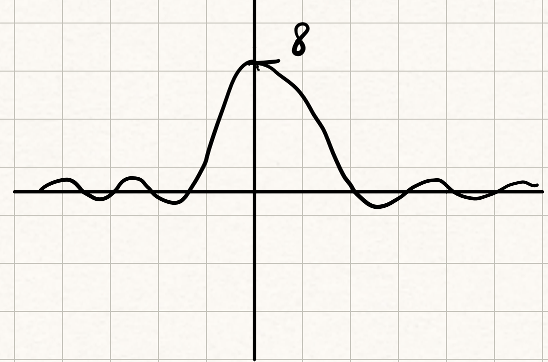


b. $X_1(j\omega) = 2T_1 \operatorname{sinc}\left(\frac{\omega T_1}{\pi}\right)$ $\qquad X_2(j\omega) = 2T_2 \operatorname{sinc}\left(\frac{\omega T_2}{\pi}\right)$

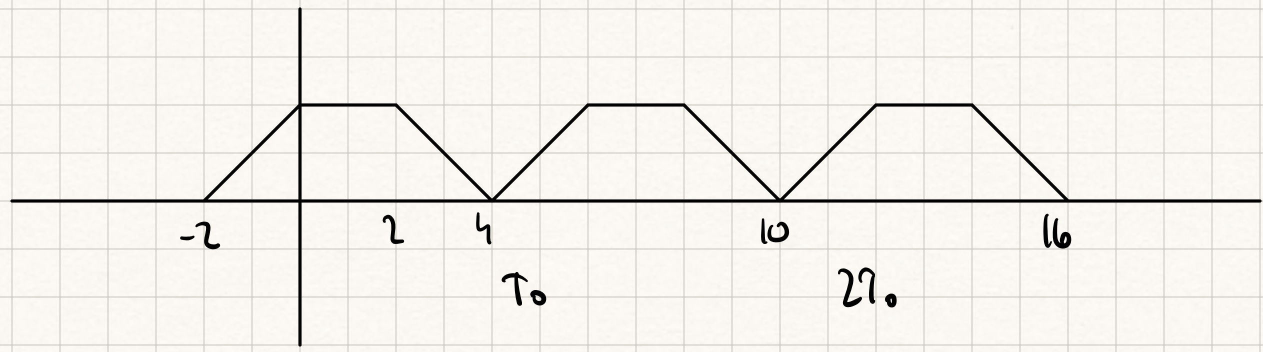
$\therefore$ Convolution property $X(j\omega) = X_1(j\omega)\, X_2(j\omega)$

$$= 4T_1T_2 \operatorname{sinc}\left(\frac{\omega T_1}{\pi}\right) \operatorname{sinc}\left(\frac{\omega T_2}{\pi}\right)$$

$X(j\omega)$ if $T_1 = 1$ and $T_2 = 2$ $= 8 \operatorname{sinc}\left(\frac{\omega}{\pi}\right) \operatorname{sinc}\left(\frac{2\omega}{\pi}\right)$



c.



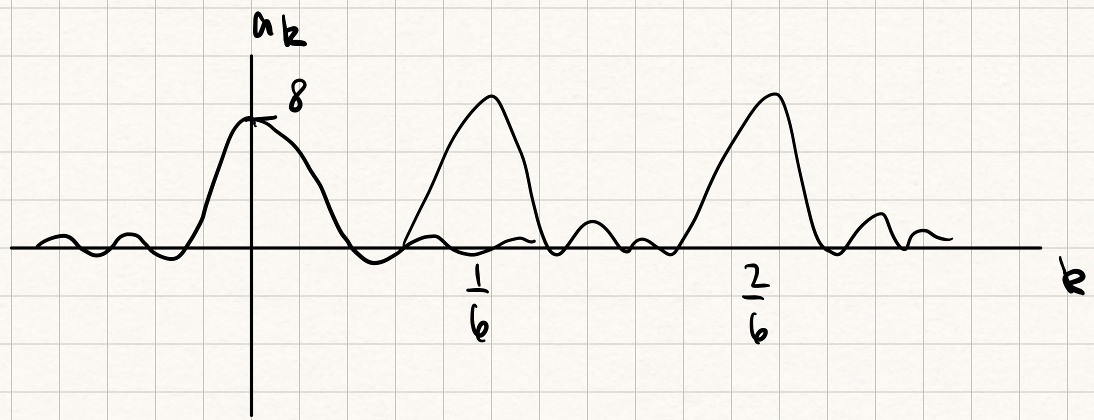
d.

d. By sampling $X(j\omega)$, obtain an expression for the Fourier series coefficients of $\tilde{x}(t)$, given by a_k. Sketch the a_k vs. k, assuming $T_1 = 1$ $T_2 = 2$ and $T_0 = 6$.

$\omega = k\omega_0$

$$8\,\mathrm{sinc}\left(\frac{\omega}{\pi}\right)\,\mathrm{sinc}\left(\frac{2\omega}{\pi}\right) = 8\,\mathrm{sinc}\left(\frac{k\omega_0}{\pi}\right)\,\mathrm{sinc}\left(\frac{2k\omega_0}{\pi}\right)$$

$$= 8\,\mathrm{sinc}\left(\frac{6k}{\pi}\right)\,\mathrm{sinc}\left(\frac{6k}{\pi}\right)$$



5. *Frequency-shift property and Parseval's Identity.* We are given two signals:

$$x_1(t) = \frac{W}{\pi}\operatorname{sinc}\left(\frac{Wt}{\pi}\right) \text{ and } x_2(t) = \frac{W}{\pi}\operatorname{sinc}\left(\frac{Wt}{\pi}\right)\underline{\sin(\omega_0 t)}, \text{ assuming } \omega_0 \geq 2W.$$

a. Sketch their Fourier transforms, $X_1(j\omega)$ and $X_2(j\omega)$, assuming $W = 2\pi$ and $\omega_0 = 4\pi$.

b. Compute the energies $E_{x_1} = \int_{-\infty}^{\infty} |x_1(t)|^2 dt$ and $E_{x_2} = \int_{-\infty}^{\infty} |x_2(t)|^2 dt$.

c. Compute the inner product $\langle x_1(t), x_2(t)\rangle = \int_{-\infty}^{\infty} x_1(t)x_2^*(t)dt$.

a. $x_1(t) = 2\operatorname{sinc}(2t) \longleftrightarrow X_1(j\omega) = \Pi\left(\frac{\omega}{2W}\right) = \Pi\left(\frac{\omega}{4\pi}\right)$

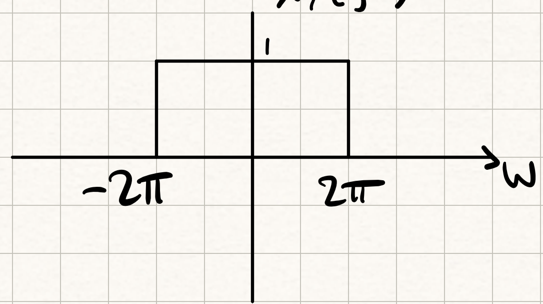
$x_2(t) = 2\operatorname{sinc}(2t)\sin(4\pi t)$

$$X_2(j\omega) = \Pi\left(\frac{\omega}{2W}\right) \times \frac{\pi}{j}\left[\delta[\omega-\omega_0] - \delta(\omega+\omega_0)\right]$$

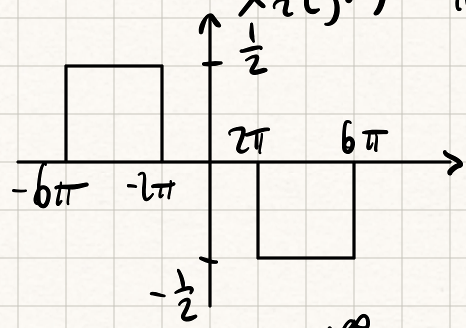
$$= \Pi\left(\frac{\omega}{4\pi}\right) \times \frac{\pi}{j}\left[\delta[\omega-4\pi] - \delta[\omega+\omega_0]\right] \times \frac{1}{2\pi}$$

$$= -\Pi\left(\frac{\omega}{4\pi}\right) \times \frac{j}{2} \times \delta[\omega-4\pi] + \Pi\left(\frac{\omega}{4\pi}\right) \times \frac{j}{2} \times \delta[\omega+4\pi]$$

$X_1(j\omega)$



$X_2(j\omega)$ imaginary



b. $$E_{x_1} = \int_{-\infty}^{\infty} |x_1(t)|^2 dt$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} |X_1(j\omega)|^2 d\omega$$

$$= \frac{1}{2\pi}\int_{-2\pi}^{2\pi} d\omega$$

$$= 2$$

$$E_{x_2} = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X_2(j\omega)|^2 d\omega$$

$$= \frac{1}{2\pi}\left(\underbrace{\int_{-6\pi}^{-2\pi} \frac{1}{2} d\omega}_{=2\pi} + \underbrace{\int_{2\pi}^{6\pi} \frac{1}{2} d\omega}_{=2\pi}\right)$$

$$= 2$$

c. $\langle x_1(t), x_2(t) \rangle = \frac{1}{2\pi} \int_{-\infty}^{\infty} X_1(j\omega)\, X_2^*(j\omega)\, d\omega$

$= 0$

6. *Ideal filter*. A filter has an impulse response

$$h(t) = \frac{W}{\pi}\operatorname{sinc}\left(\frac{W(t-t_0)}{\pi}\right)\cos\left(\omega_0(t-t_0)\right), \text{ assuming } \omega_0 \geq W .$$

a. Obtain an expression for the frequency response $H(j\omega)$. Sketch the magnitude and phase, $|H(j\omega)|$ and $\angle H(j\omega)$, assuming $W = \frac{3}{4}\pi$, $\omega_0 = \pi$ and $t_0 = 1$.

b. What kind of filter is this (lowpass, highpass, etc.)?

c. We input a signal

→ magnitude is multiplied the phase added for sine and cosine

$$x(t) = 1 + 2\sin(t) + \cos(2\pi t).$$

Obtain an expression for the output signal $y(t)$, assuming the same filter parameters as in part (a).

a. $H_1(j\omega) = \Pi\left(\frac{\omega}{2W}\right)$ $\quad$ $H_2(j\omega) = \pi\left[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right]$

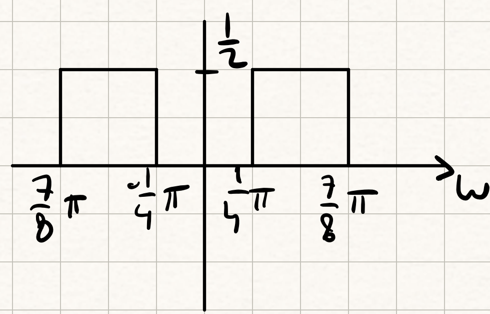
$\therefore H_3(j\omega) = \Pi\left(\frac{\omega}{2W}\right) \times \pi\left[\delta[\omega-\omega_0] + \delta[\omega+\omega_0]\right]$

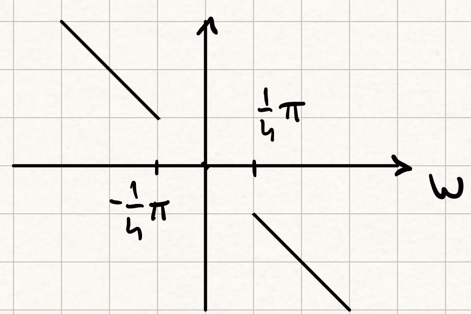
Time shift → $H(j\omega) = e^{-j\omega t_0} \times \frac{1}{2\pi}\Pi\left(\frac{\omega}{2W}\right) \times \pi\left(\delta[\omega-\omega_0] + \delta[\omega+\omega_0]\right)$

$h_1(t) * h_2(t) \leftrightarrow \frac{1}{2\pi} H_1(j\omega) * H_2(j\omega) \Big| = \frac{1}{2} e^{-j\omega t_0}\,\Pi\left(\frac{\omega}{2W}\right)\delta[\omega-\omega_0] +$

$\frac{1}{2} e^{-j\omega t_0}\,\Pi\left(\frac{\omega}{2W}\right)\delta[\omega+\omega_0] = \frac{1}{2}\Pi\left(\frac{\omega-\omega_0}{2W}\right) + \frac{1}{2}\Pi\left(\frac{\omega-\omega_0}{2W}\right)$

$|H(j\omega)|$ $\quad$ $\angle H(j\omega) = -\omega t = -\omega$





b. bandpass

$$\frac{\pi}{j} \times 2(\delta(\omega-1) - \delta(\omega+1))$$

c. $x(t) = 1 + 2\sin(t) + \cos(2\pi t) \hookrightarrow \pi(\delta(\omega-2\pi)+\delta(\omega+2\pi))$

$$Y(j\omega) = X(j\omega) * H(j\omega)$$

$$X(j\omega) = \underbrace{2\pi\,\delta(\omega)}_{0} + \frac{2\pi}{j}[\delta(\omega-1) - \delta(\omega+1)] + \pi\underbrace{(\delta(\omega-2\pi)+\delta(\omega+2\pi))}_{0}$$

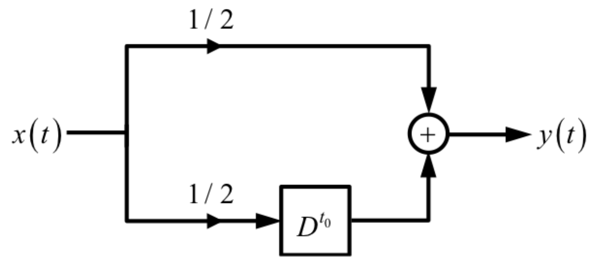
$$Y(j\omega) = X(j\omega) * H(j\omega)$$

$$= \frac{2\pi}{j} H(j)\,\delta(\omega-1) + \frac{2\pi}{j} H(-j)\,\delta(\omega+1)$$

$$= \frac{\pi}{j} e^{-jt_0}\,\delta(\omega-1) + \frac{\pi}{j} H(-j)\,\delta(\omega+1) = \sin(t-t_0)$$

$$= \sin(t-1)$$

7. *Delay-and-add system*. A system splits an input signal $x(t)$ into two copies, each scaled by 1/2, delays one copy by t_0, and combines the two copies to obtain an output signal $y(t)$. *Hint*: this is the CT analogue of a DT system we studied in lecture.



a. Obtain an expression for the impulse response $h(t)$.

b. Obtain an expression for the frequency response $H(j\omega)$. Sketch the magnitude and phase $|H(j\omega)|$ and $\angle H(j\omega)$, assuming a general value of the delay t_0.

c. An input signal is $x(t) = \sin(\omega_0 t)$. Specify all values of ω_0 such that the output is $y(t) = 0$, assuming a general value of the delay t_0.

a. $y(t) = \frac{1}{2}x(t) + \frac{1}{2}x(t-t_0)$

$y(t) = x(t) * h(t)$

$h(t) = \frac{1}{2}\delta(t) + \frac{1}{2}\delta(t-t_0)$

b. $H(j\omega) = \frac{1}{2} + \frac{1}{2}e^{-j\omega t_0} = \frac{1}{2}\left[1 + e^{-j\omega t_0}\right]$

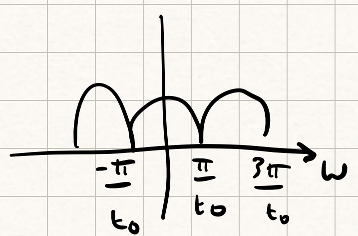
$= \frac{1}{2} e^{j\omega\frac{t_0}{2}} \cdot e^{-j\omega\frac{t_0}{2}}\left[1 + e^{-j\omega t_0}\right]$

$= \frac{1}{2} e^{-j\omega\frac{t_0}{2}}\left[e^{j\omega\frac{t_0}{2}} + e^{-j\omega\frac{t_0}{2}}\right] \rightarrow \cos\theta = \frac{e^{j\theta}+e^{-j\theta}}{2}$

$= e^{-j\omega\frac{t_0}{2}} \cdot \cos\left(\frac{\omega t_0}{2}\right)$

unit circle

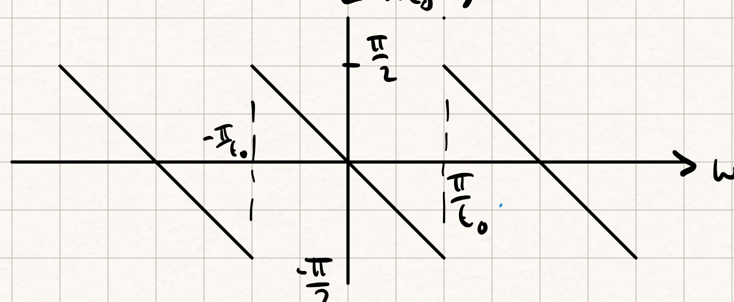
$|H(j\omega)| = 1 + \left|\cos\left(\frac{\omega t_0}{2}\right)\right|$



$\angle H(j\omega) = \angle e^{-} + \angle \cos(\;)$ → cosine phase

$= -\frac{\omega t_0}{2} + $ (π)

$\angle H(j\omega) = \begin{cases} -\frac{\omega t_0}{2} + 0 & -\frac{\pi}{t_0} < \omega < \frac{\pi}{t_0} \\ -\frac{\omega t_0}{2} + \pi & \text{other} \end{cases}$



c. $x(t) = \sin(\omega_0 t)$

$$y(t) = |H(j\omega_0)| \times \sin(\omega_0 t + \angle H(j\omega_0)) = 0$$

$$\therefore \omega_0 = \frac{\pi}{t_0} \times (2k+1)$$

$\frac{\pi}{t_0}$ or $\frac{3\pi}{t_0}$ or $\frac{5\pi}{t_0}$ ...

8. *Critically damped second-order lowpass filter*. We discuss a second-order lowpass filter in the *EE 102A Course Reader*, Chapter 4, pages 183-186. Assuming a natural frequency ω_n and a damping constant $\zeta = 1$ (critical damping), the impulse and frequency responses are given by

See pics

$$h(t) = \omega_n^2 t e^{-\omega_n t} u(t) \overset{F}{\leftrightarrow} H(j\omega) = \frac{1}{\left(1 + j\frac{\omega}{\omega_n}\right)^2} = \frac{\omega_n^2}{(j\omega)^2 + 2\omega_n (j\omega) + \omega_n^2}. \tag{1}$$

We derive the CTFT pair (1) in this problem. To facilitate using earlier results, we make a substitution $\omega_n \to 1/\tau$, so (1) becomes

$$h(t) = \frac{t}{\tau^2} e^{-\frac{t}{\tau}} u(t) \overset{F}{\leftrightarrow} H(j\omega) = \frac{1}{(1 + j\omega\tau)^2}. \tag{1'}$$

a. Use the CTFT pair

$$x(t) = \frac{1}{\tau} e^{-\frac{t}{\tau}} u(t) \overset{F}{\leftrightarrow} X(j\omega) = \frac{1}{1 + j\omega\tau} \tag{2}$$

and the CTFT differentiation-in-frequency property to prove (1').

b. In Homework 3 Problem 6, we studied the cascade of two CT first-order lowpass filters with time constants τ_1 and τ_2, $\tau_1 \neq \tau_2$, and derived the resulting impulse response

$$\frac{1}{\tau_1} e^{-\frac{t}{\tau_1}} u(t) * \frac{1}{\tau_2} e^{-\frac{t}{\tau_2}} u(t) = \frac{1}{\tau_1 - \tau_2} \left(e^{-\frac{t}{\tau_1}} - e^{-\frac{t}{\tau_2}} \right) u(t).$$

Here we study the case $\tau_1 = \tau_2 = \tau$. Use (1'), (2) and the CTFT convolution property to show that the cascade has an impulse response

$$\frac{1}{\tau} e^{-\frac{t}{\tau}} u(t) * \frac{1}{\tau} e^{-\frac{t}{\tau}} u(t) = \frac{t}{\tau^2} e^{-\frac{t}{\tau}} u(t). \tag{3}$$

This shows that a critically damped second-order lowpass filter is equivalent to a cascade of two identical first-order lowpass filters.

a. $x(t) = \frac{1}{\tau} e^{-\frac{t}{\tau}} u(t) \longleftrightarrow \frac{1}{1+j\omega\tau}$

$h(t) = \frac{t}{\tau} x(t) \quad \therefore \quad t\,x(t) \longleftrightarrow \frac{j\,dX(j\omega)}{d\omega}$

$(1+j\omega\tau)^{-1} \to -(1+j\omega\tau)^{-2}(j\tau)$

$= \frac{1}{(1+j\omega)^2}$

$$\therefore \frac{t}{\tau} x(t) = \frac{t}{\tau^2} e^{-t/\tau} u(t) \longleftrightarrow \frac{1}{\tau} \times \frac{\tau}{(1+j\omega)^2}$$

$$= \frac{1}{(1+j\omega)^2} = H(j\omega)$$

b. In Homework 3 Problem 6, we studied the cascade of two CT first-order lowpass filters with time constants τ_1 and τ_2, $\tau_1 \neq \tau_2$, and derived the resulting impulse response

$$\frac{1}{\tau_1} e^{-\frac{t}{\tau_1}} u(t) * \frac{1}{\tau_2} e^{-\frac{t}{\tau_2}} u(t) = \frac{1}{\tau_1 - \tau_2} \left(e^{-\frac{t}{\tau_1}} - e^{-\frac{t}{\tau_2}} \right) u(t).$$

Here we study the case $\tau_1 = \tau_2 = \tau$. Use (1'), (2) and the CTFT convolution property to show that the cascade has an impulse response

$$\frac{1}{\tau} e^{-\frac{t}{\tau}} u(t) * \frac{1}{\tau} e^{-\frac{t}{\tau}} u(t) = \frac{t}{\tau^2} e^{-\frac{t}{\tau}} u(t). \tag{3}$$

This shows that a critically damped second-order lowpass filter is equivalent to a cascade of two identical first-order lowpass filters.

$$H(j\omega) = \frac{1}{(1+j\omega\tau)}$$

$$\therefore h(t) * h(t) \longleftrightarrow H(j\omega)\, H(j\omega)$$

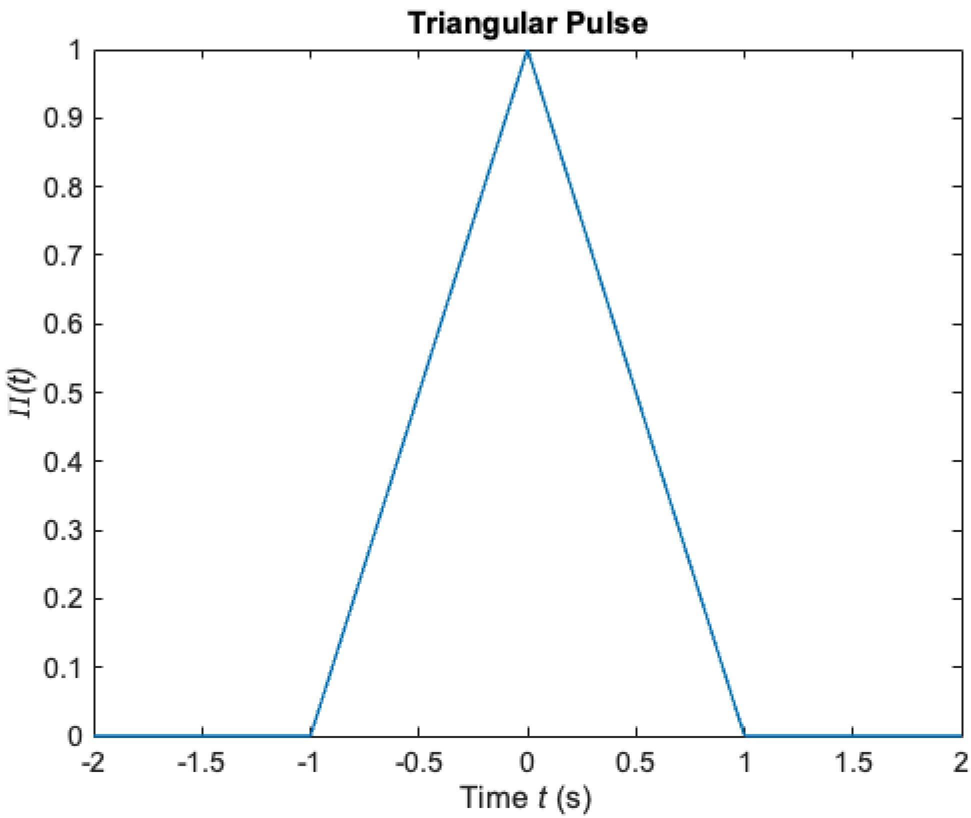
$$= \frac{1}{(1+j\omega\tau)} \times \frac{1}{(1+j\omega\tau)} = \frac{1}{(1+j\omega\tau)^2}$$

$$\therefore \frac{1}{(1+j\omega\tau)^2} \longleftrightarrow h(t) \text{ from } 1'$$

$$= \frac{t}{\tau^2} e^{-t/\tau} u(t) \quad \text{as required}$$

Table of Contents


```
t = -2:.01:2;
figure(1); plot(t,Lambda(t), 'LineWidth', 1.5);
set(gca,'FontName','arial','FontSize',14);
xlabel('Time \itt\rm (s)'); ylabel('\it\Pi(t)');
title('Triangular Pulse');
```

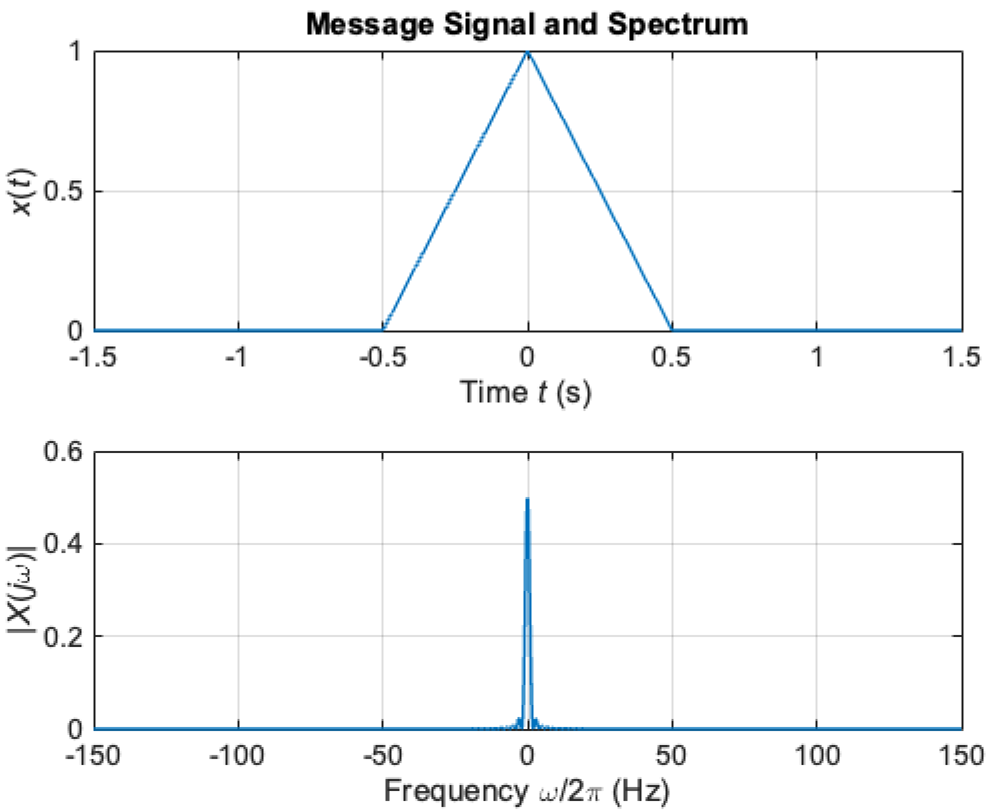


Part 2

time and frequency

```
deltat = 1/300; % discretization interval (s)
tmax = 1.5; % time vector runs from -tmax to tmax (s)
```

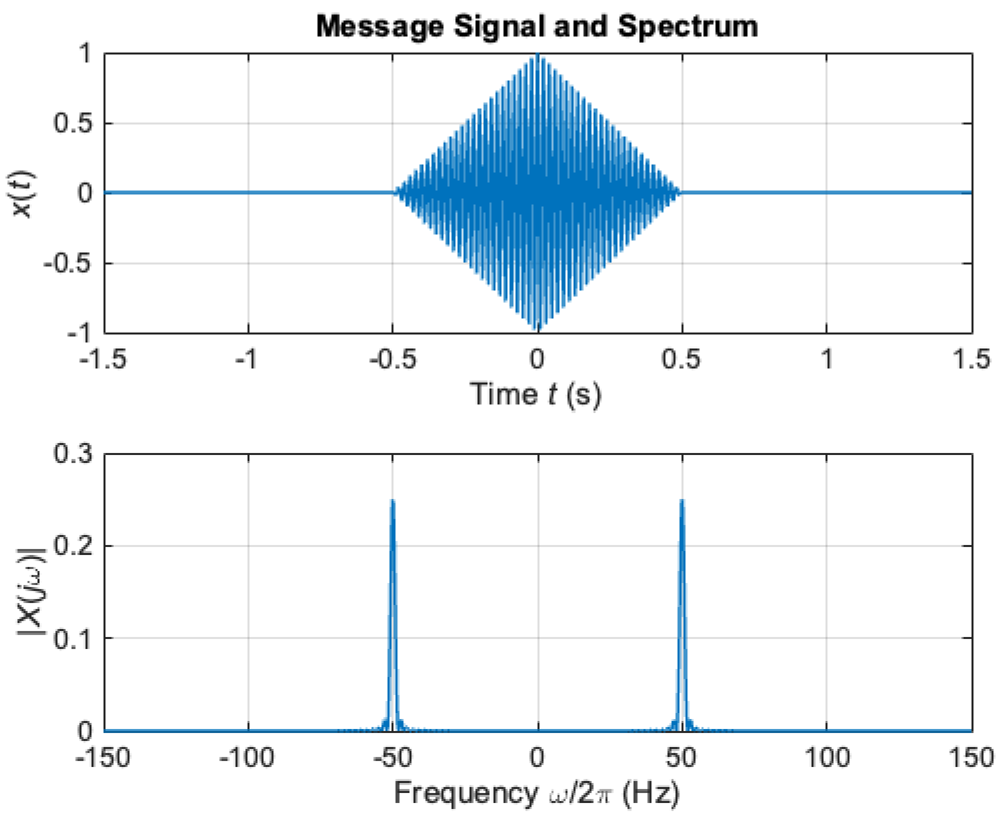
```
t = -tmax:deltat:tmax; % time vector for x(t),y(t),v(t),h(t) (s)

x = Lambda(2*t);
[X, omega] = CTFT_approx(x, t);

figure (2);
subplot(211)
plot(t,x)

l=get(gca,'children'); set(l,'linewidth',1.5)
set(gca,'FontName','arial'); set(gca,'FontSize',14);
xlabel('Time \itt\rm (s)');
ylabel('\itx\rm(\itt\rm)');
grid
title('Message Signal and Spectrum');
subplot(212)
plot(omega/(2*pi),abs(X));
l=get(gca,'children'); set(l,'linewidth',1.5);
set(gca,'FontName','arial'); set(gca,'FontSize',14);
xlabel('Frequency \omega/2\pi (Hz)');
ylabel('|\itX\rm(\itj\rm\omega)|');
grid
```



3

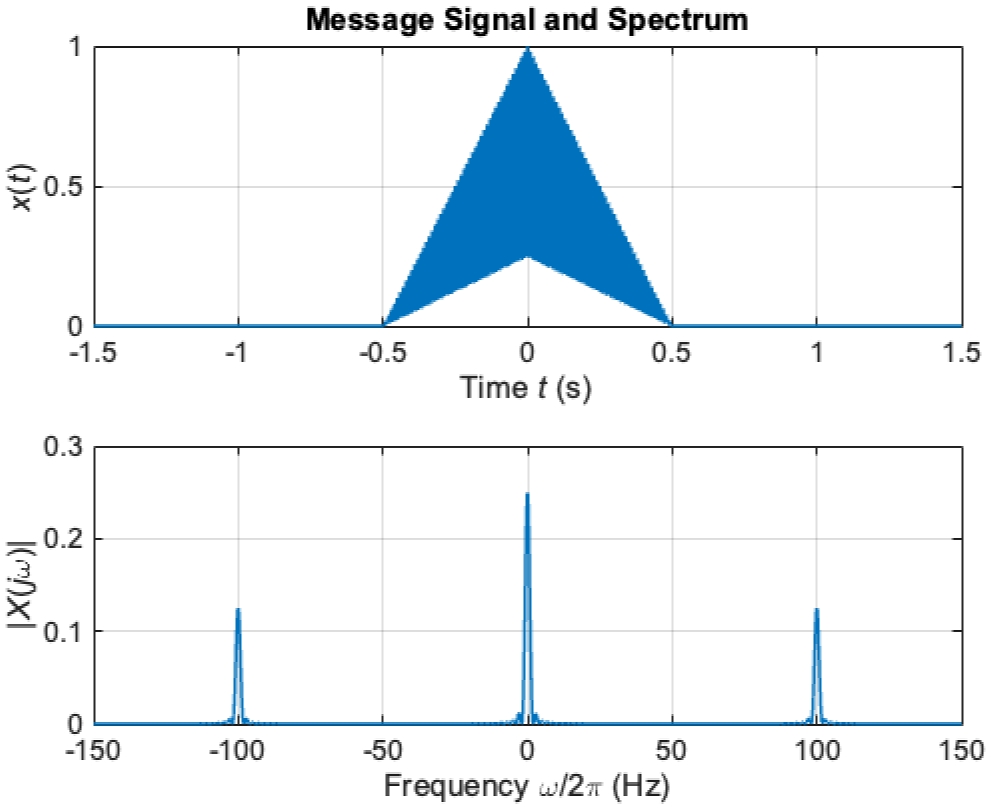
modulation

```
fc = 50; omegac = 2*pi*fc; % carrier frequency (Hz and rad/s)

y = x .* cos(omegac * t);
[Y, omega] = CTFT_approx(y, t);

figure (3);
subplot(211)
plot(t,y)

l=get(gca,'children'); set(l,'linewidth',1.5)
set(gca,'FontName','arial'); set(gca,'FontSize',14);
xlabel('Time \itt\rm (s)');
ylabel('\itx\rm(\itt\rm)');
grid
title('Message Signal and Spectrum');
subplot(212)
plot(omega/(2*pi),abs(Y));
l=get(gca,'children'); set(l,'linewidth',1.5);
set(gca,'FontName','arial'); set(gca,'FontSize',14);
xlabel('Frequency \omega/2\pi (Hz)');
ylabel('|\itX\rm(\itj\rm\omega)|');
grid
```



4

```
v = y .* cos(omegac * t);
[V, omega] = CTFT_approx(v, t);

figure (4);
subplot(211)
plot(t,v)

l=get(gca,'children'); set(l,'linewidth',1.5)
set(gca,'FontName','arial'); set(gca,'FontSize',14);
xlabel('Time \itt\rm (s)');
ylabel('\itx\rm(\itt\rm)');
grid
title('Message Signal and Spectrum');
subplot(212)
plot(omega/(2*pi),abs(V));
l=get(gca,'children'); set(l,'linewidth',1.5);
set(gca,'FontName','arial'); set(gca,'FontSize',14);
xlabel('Frequency \omega/2\pi (Hz)');
ylabel('|\itX\rm(\itj\rm\omega)|');
grid

% The first signal only represented one narrow band of frequences, the
% second filter only two, and the last filter represented from
% the first signal and also new frequences at 100Hz and -100Hz.
```

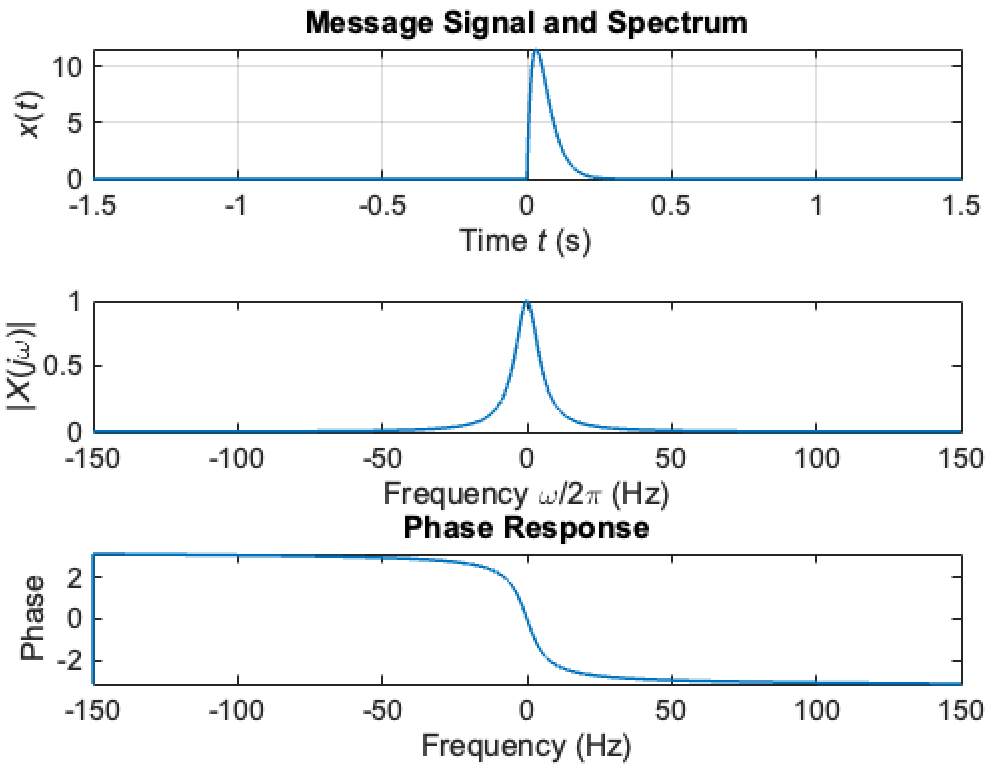


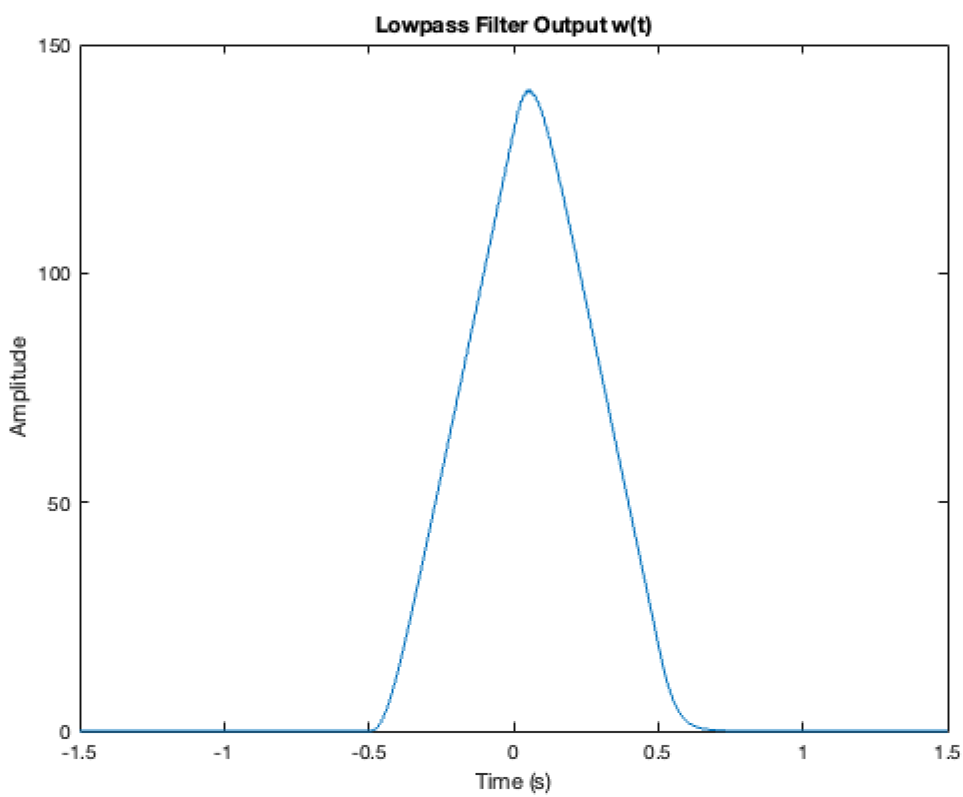
5

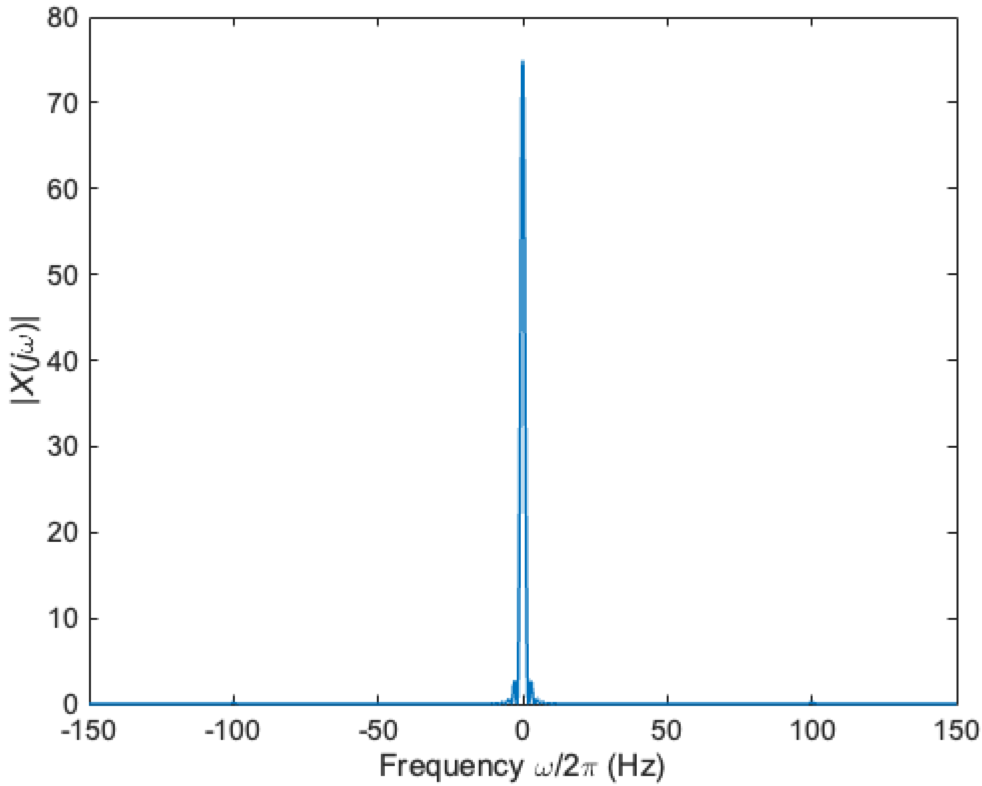
Filtering

```
fn = 5; omegan = 2*pi*fn; % LPF cutoff frequency (Hz and rad/s)

hlpf = hsolpfcd(t,omegan); % lowpass filter

[H, omega] = CTFT_approx(hlpf, t);

figure (5);
subplot(311)
plot(t,hlpf)

% Magnitude
l=get(gca,'children'); set(l,'linewidth',1.5)
set(gca,'FontName','arial'); set(gca,'FontSize',14);
xlabel('Time \itt\rm (s)');
ylabel('\itx\rm(\itt\rm)');
grid
title('Message Signal and Spectrum');
subplot(312)
plot(omega/(2*pi),abs(H));
l=get(gca,'children'); set(l,'linewidth',1.5);
set(gca,'FontName','arial'); set(gca,'FontSize',14);
```

```
xlabel('Frequency \omega/2\pi (Hz)');
ylabel('|\itX\rm(\itj\rm\omega)|');

% Phase
subplot(313);
plot(omega/(2*pi), angle(H), 'LineWidth', 1.5);
set(gca, 'FontName', 'arial', 'FontSize', 14);
xlabel('Frequency (Hz)');
ylabel('Phase');
grid on;
title('Phase Response');
grid

% W
w = conv(v, hlpf, 'same');
tw = t;
figure(6);
plot(tw, w);
title('Lowpass Filter Output w(t)');
xlabel('Time (s)');
ylabel('Amplitude');

[W, omega] = CTFT_approx(w, t);
% Magnitude

figure(7);
plot(omega/(2*pi),abs(W));
l=get(gca,'children'); set(l,'linewidth',1.5);
set(gca,'FontName','arial'); set(gca,'FontSize',14);
xlabel('Frequency \omega/2\pi (Hz)');
ylabel('|\itX\rm(\itj\rm\omega)|');

% This response narrows the potential frequences much more so that only
% lower frequencies go through. There are some bounces so it's worth
% smoothing those out
```


Message Signal and Spectrum
x(t)
Time t (s)
|X(jω)|
Frequency ω/2π (Hz)
Phase Response
Phase
Frequency (Hz)


Lowpass Filter Output w(t)
Amplitude
Time (s)



6

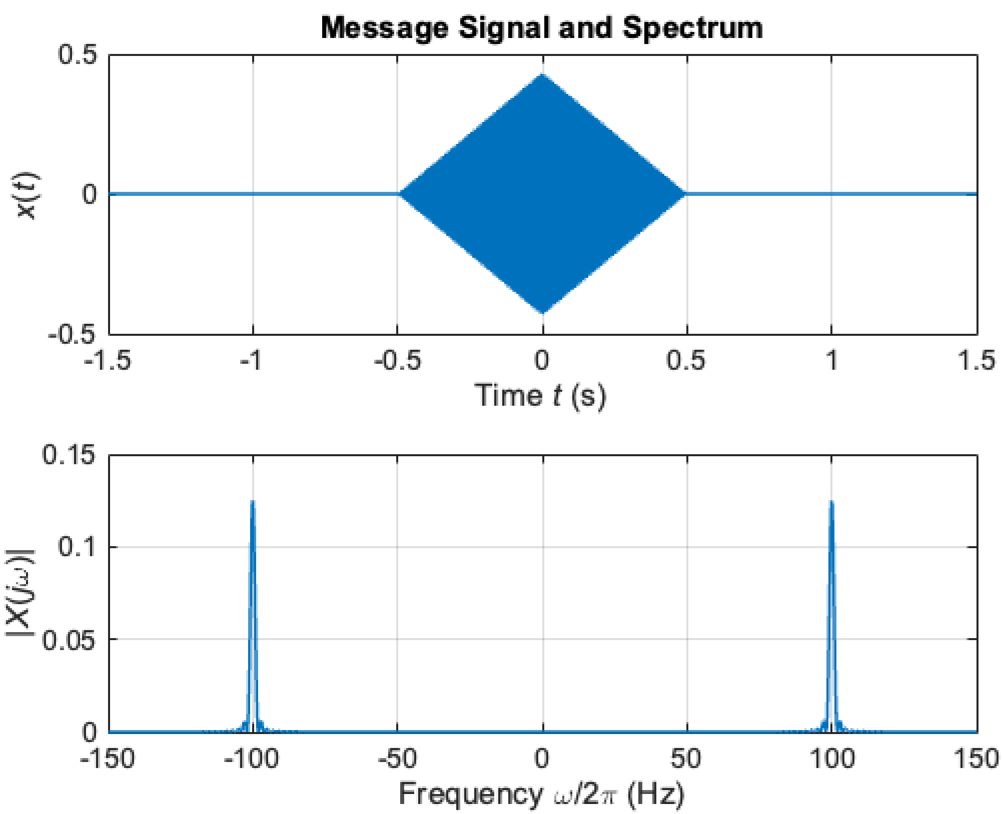
Since we convolve v with a low pass filter, the only frequencies left in w will be w(t) = (1/2)x(t)cos(phi). So if phi = 0, w(t) = (1/2)x(t) For phi = pi/2, w(t) = 0 phi = pi, w(t) = -(1/2)x(t), so phi = pi/2 is the worst one since the signal is reduced to 0

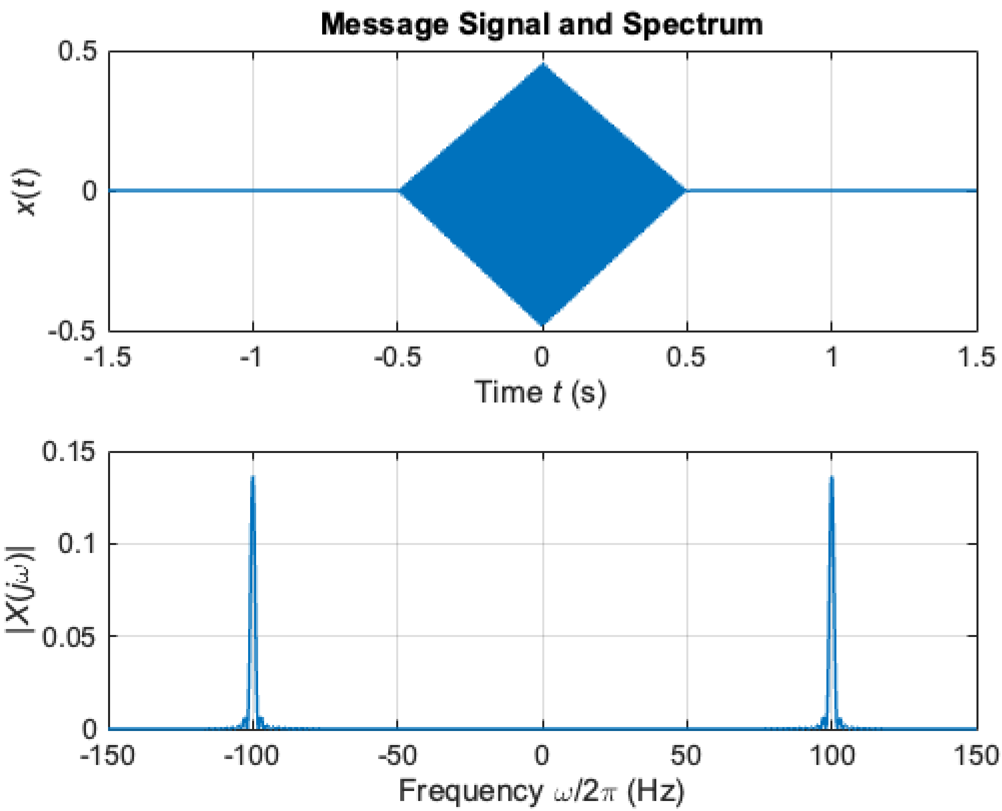
7

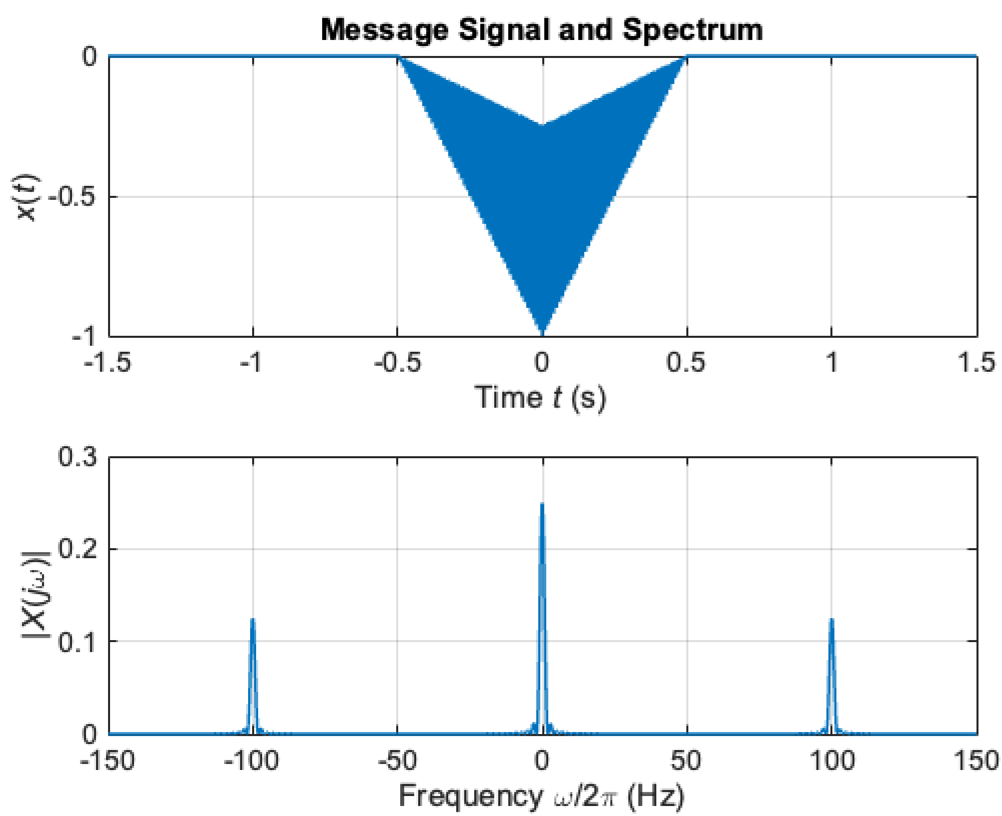
```
%V pi/2
v = y .* cos(omegac * t + pi/2);
[V, omega] = CTFT_approx(v, t);

figure (8);
subplot(211)
plot(t,v)

l=get(gca,'children'); set(l,'linewidth',1.5)
set(gca,'FontName','arial'); set(gca,'FontSize',14);
xlabel('Time \itt\rm (s)');
ylabel('\itx\rm(\itt\rm)');
grid
title('Message Signal and Spectrum');
subplot(212)
plot(omega/(2*pi),abs(V));
l=get(gca,'children'); set(l,'linewidth',1.5);
```

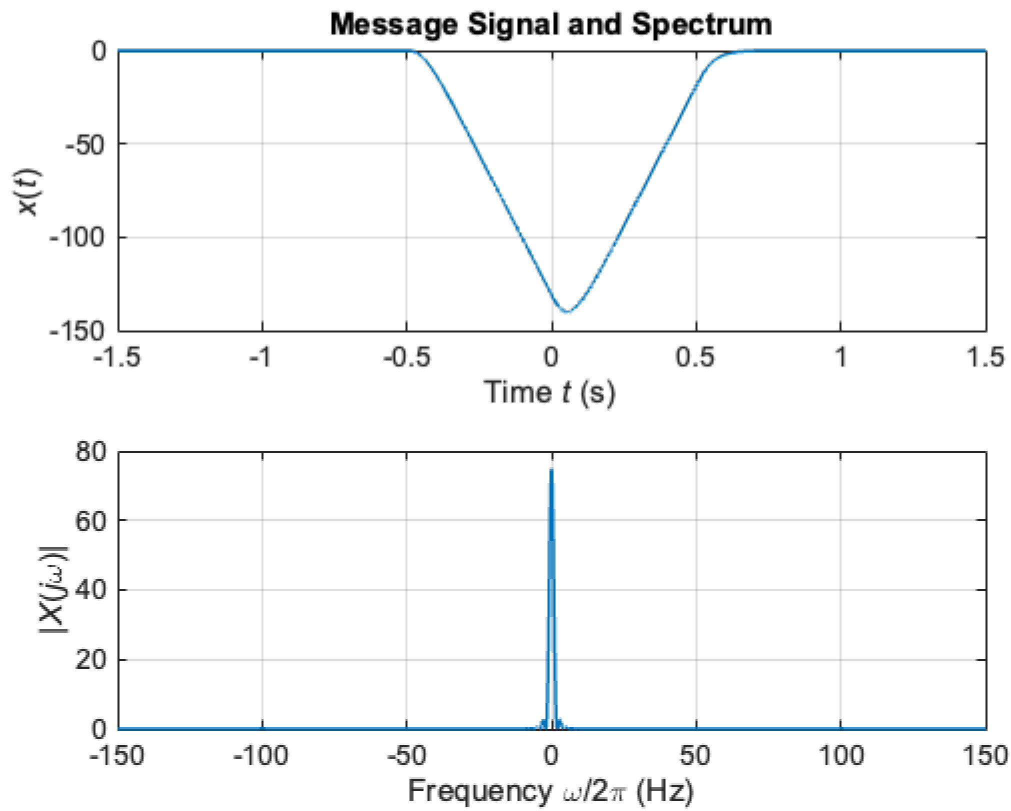
```
set(gca,'FontName','arial'); set(gca,'FontSize',14);
xlabel('Frequency \omega/2\pi (Hz)');
ylabel('|\itX\rm(\itj\rm\omega)|');
grid

%W pi/2
w = conv(v, hlpf, 'same');
[W, omega] = CTFT_approx(w, t);
tw = t;
figure (9);
subplot(211)
plot(tw,w)

l=get(gca,'children'); set(l,'linewidth',1.5)
set(gca,'FontName','arial'); set(gca,'FontSize',14);
xlabel('Time \itt\rm (s)');
ylabel('\itx\rm(\itt\rm)');
grid
title('Message Signal and Spectrum');
subplot(212)
plot(omega/(2*pi),abs(W));
l=get(gca,'children'); set(l,'linewidth',1.5);
set(gca,'FontName','arial'); set(gca,'FontSize',14);
xlabel('Frequency \omega/2\pi (Hz)');
ylabel('|\itX\rm(\itj\rm\omega)|');
grid

%V pi
v = y .* cos(omegac * t + pi);
[V, omega] = CTFT_approx(v, t); % Compute the spectrum of y

figure (10);
subplot(211)
plot(t,v)
l=get(gca,'children'); set(l,'linewidth',1.5)
set(gca,'FontName','arial'); set(gca,'FontSize',14);
xlabel('Time \itt\rm (s)');
ylabel('\itx\rm(\itt\rm)');
grid
title('Message Signal and Spectrum');
subplot(212)
plot(omega/(2*pi),abs(V));
l=get(gca,'children'); set(l,'linewidth',1.5);
set(gca,'FontName','arial'); set(gca,'FontSize',14);
xlabel('Frequency \omega/2\pi (Hz)');
ylabel('|\itX\rm(\itj\rm\omega)|');
grid

%W pi
w = conv(v, hlpf, 'same');
[W, omega] = CTFT_approx(w, t);
tw = t;
figure (11);
subplot(211)
```

```
plot(tw,w)

l=get(gca,'children'); set(l,'linewidth',1.5)
set(gca,'FontName','arial'); set(gca,'FontSize',14);
xlabel('Time \itt\rm (s)');
ylabel('\itx\rm(\itt\rm)');
grid
title('Message Signal and Spectrum');
subplot(212)
plot(omega/(2*pi),abs(W));
l=get(gca,'children'); set(l,'linewidth',1.5);
set(gca,'FontName','arial'); set(gca,'FontSize',14);
xlabel('Frequency \omega/2\pi (Hz)');
ylabel('|\itX\rm(\itj\rm\omega)|');
grid
```




Message Signal and Spectrum
0.5
0
-0.5
x(t)
-1.5 -1 -0.5 0 0.5 1 1.5
Time t (s)
0.15
0.1
0.05
0
|X(jω)|
-150 -100 -50 0 50 100 150
Frequency ω/2π (Hz)


Message Signal and Spectrum
0
-0.5
-1
x(t)
-1.5 -1 -0.5 0 0.5 1 1.5
Time t (s)
0.3
0.2
0.1
0
|X(jω)|
-150 -100 -50 0 50 100 150
Frequency ω/2π (Hz)



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