

1 Homogeneous Notation

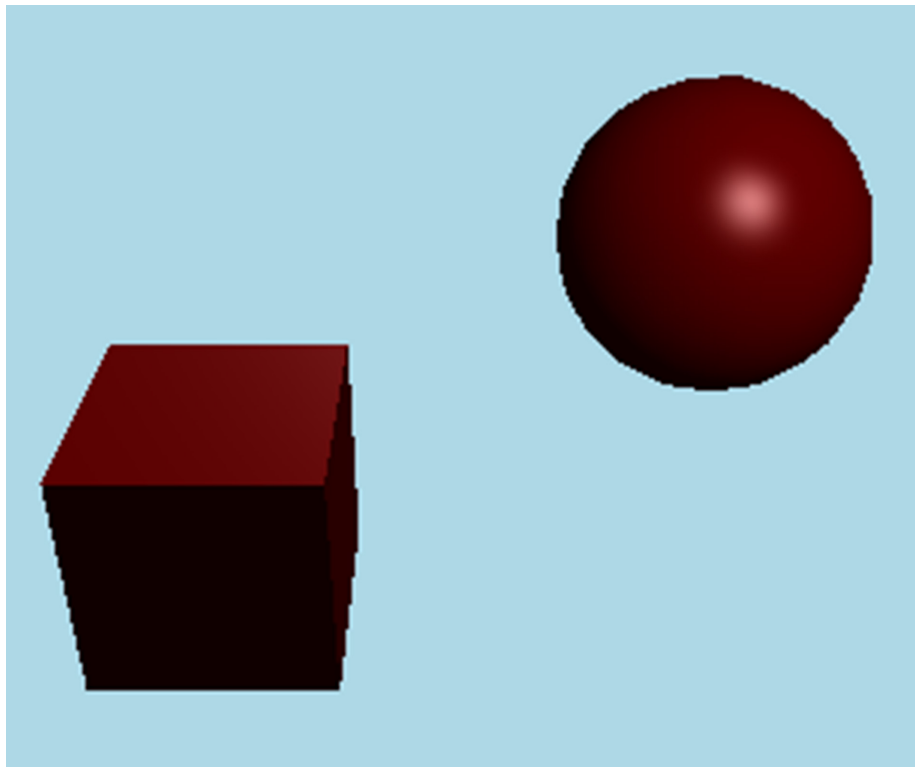


Figure 1: Different transformations are applied to cube and sphere

1.1 Homogeneous Notation for a Point

In homogeneous coordinates, a point \mathbf{p} in 3D space is represented as a 4D vector:

$$\mathbf{p} = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}.$$

1.2 Translation

To translate a point by a vector $\mathbf{t} = [t_x \ t_y \ t_z]^T$, I use the translation matrix \mathbf{T} :

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The translated point \mathbf{p}' is obtained by: $\mathbf{p}' = \mathbf{T}\mathbf{p}$.

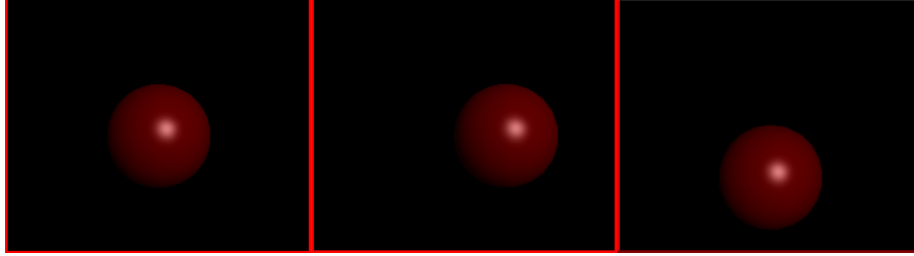


Figure 2: The sphere's position changes: in the middle, it moves to the right; on the left, it moves down.

1.3 Scaling

To scale a point by factors s_x , s_y , and s_z , I use the scaling matrix \mathbf{S} :

$$\mathbf{S} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The scaled point \mathbf{p}' is obtained by: $\mathbf{p}' = \mathbf{S}\mathbf{p}$.

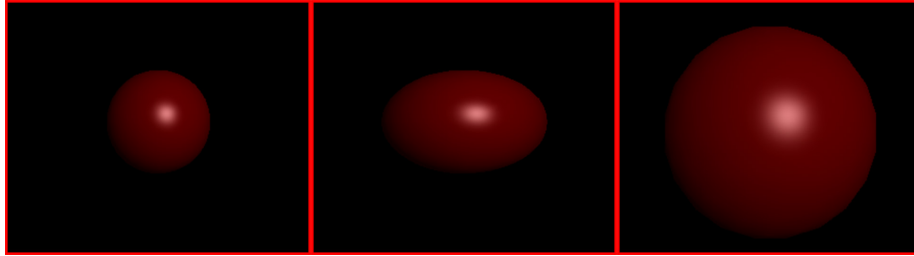


Figure 3: Sphere is scaled in x direction and is scaled by 2.

1.4 Mirroring

To mirror a point along the x , y , or z axis, I use the mirroring matrices \mathbf{M}_x , \mathbf{M}_y , and \mathbf{M}_z :

$$\mathbf{M}_x = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{M}_y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{M}_z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The mirrored point \mathbf{p}' is obtained by: $\mathbf{p}' = \mathbf{M}_x\mathbf{p}$ or $\mathbf{p}' = \mathbf{M}_y\mathbf{p}$ or $\mathbf{p}' = \mathbf{M}_z\mathbf{p}$.

1.5 Rotation

To rotate a point around the x , y , or z axis by an angle θ , I use the rotation matrices \mathbf{R}_x , \mathbf{R}_y , and \mathbf{R}_z :

$$\mathbf{R}_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta & 0 \\ 0 & \sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{R}_y = \begin{bmatrix} \cos \theta & 0 & \sin \theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \theta & 0 & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{R}_z = \begin{bmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The rotated point \mathbf{p}' is obtained by: $\mathbf{p}' = \mathbf{R}_x \mathbf{p}$ or $\mathbf{p}' = \mathbf{R}_y \mathbf{p}$ or $\mathbf{p}' = \mathbf{R}_z \mathbf{p}$.

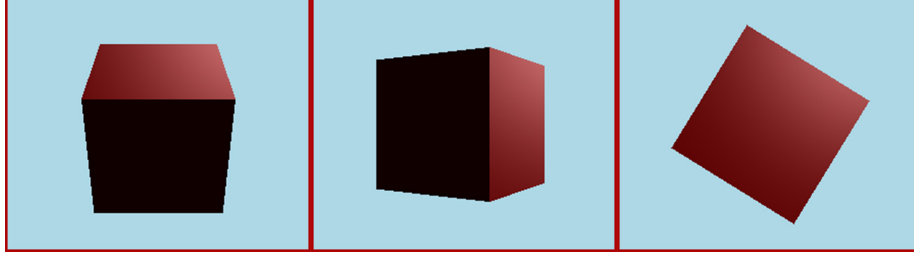


Figure 4: Cube is rotated by 45 Degrees in each direction.

1.6 Shearing

To shear a point along the x , y , or z axis, I use the shearing matrices \mathbf{H}_{xy} , \mathbf{H}_{xz} , \mathbf{H}_{yx} , \mathbf{H}_{yz} , \mathbf{H}_{zx} , and \mathbf{H}_{zy} :

$$\mathbf{H}_{xy} = \begin{bmatrix} 1 & h_{xy} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{H}_{xz} = \begin{bmatrix} 1 & 0 & h_{xz} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{H}_{yx} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ h_{yx} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{H}_{yz} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & h_{yz} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{H}_{zx} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ h_{zx} & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{H}_{zy} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & h_{zy} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The sheared point \mathbf{p}' is obtained by: $\mathbf{p}' = \mathbf{H}_{xy}\mathbf{p}$, $\mathbf{p}' = \mathbf{H}_{xz}\mathbf{p}$, $\mathbf{p}' = \mathbf{H}_{yx}\mathbf{p}$, $\mathbf{p}' = \mathbf{H}_{yz}\mathbf{p}$, $\mathbf{p}' = \mathbf{H}_{zx}\mathbf{p}$, or $\mathbf{p}' = \mathbf{H}_{zy}\mathbf{p}$.

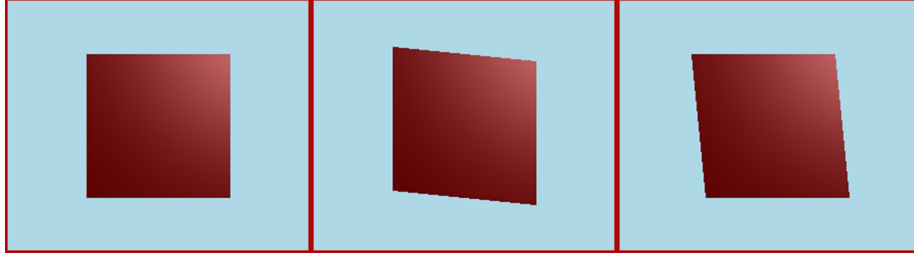


Figure 5: Cube is transformed with Shear