1 Homogeneous Notation

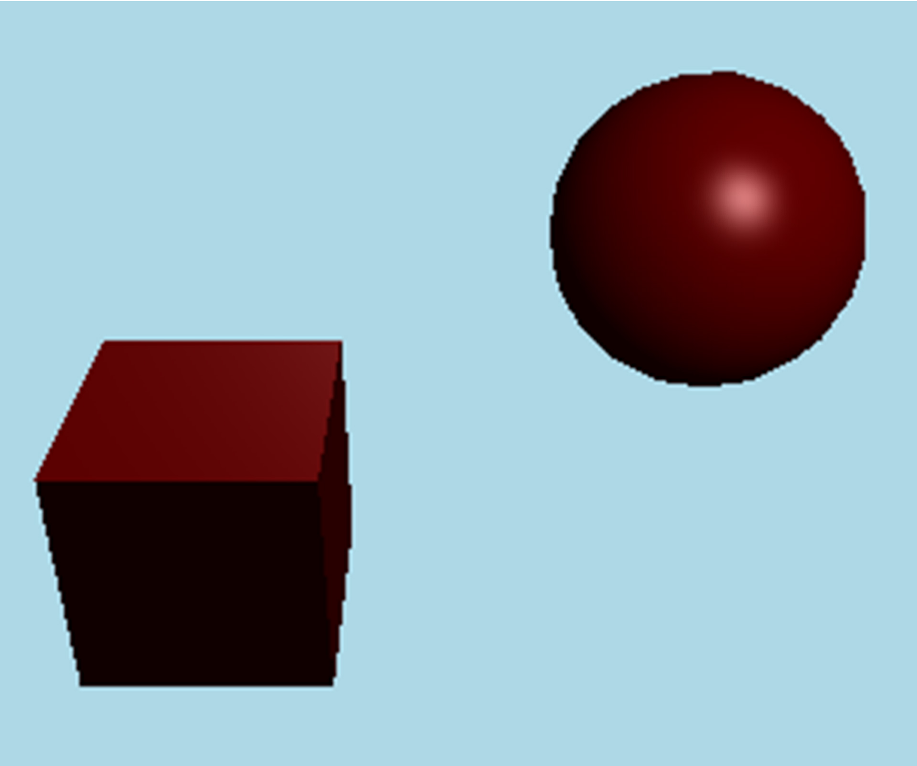

Figure 1: Different transformations are applied to cube and sphere

1.1 Homogeneous Notation for a Point

In homogeneous coordinates, a point $\boldsymbol{p}$ in 3D space is represented as a 4D vector: $\boldsymbol{p} = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$.

1.2 Translation

To translate a point by a vector $\boldsymbol{t} = \begin{bmatrix} t_x & t_y & t_z \end{bmatrix}^T$, I use the translation matrix $\boldsymbol{T}$:

$$\boldsymbol{T} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The translated point $\boldsymbol{p}'$ is obtained by: $\boldsymbol{p}' = \boldsymbol{T}\boldsymbol{p}$.

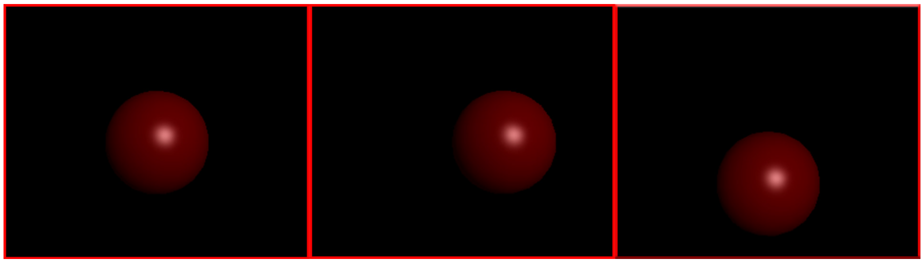

Figure 2: The sphere's position changes: in the middle, it moves to the right; on the left, it moves down.

1.3 Scaling

To scale a point by factors s_x, s_y, and s_z, I use the scaling matrix $\boldsymbol{S}$:

$$\boldsymbol{S} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The scaled point $\boldsymbol{p}'$ is obtained by: $\boldsymbol{p}' = \boldsymbol{S}\boldsymbol{p}$.

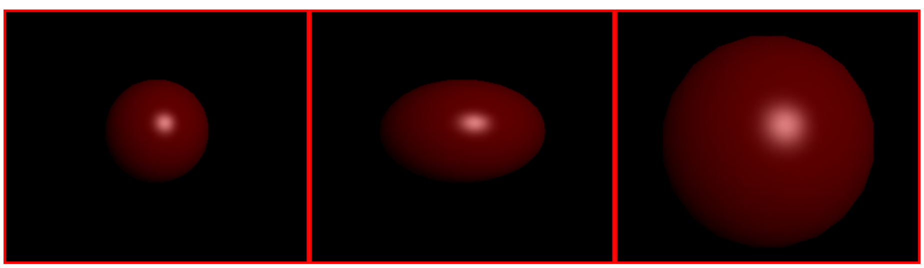

Figure 3: Sphere is scaled in x direction and is scaled by 2.

1.4 Mirroring

To mirror a point along the x, y, or z axis, I use the mirroring matrices $\boldsymbol{M}_x$, $\boldsymbol{M}_y$, and $\boldsymbol{M}_z$:

$$\boldsymbol{M}_x = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \boldsymbol{M}_y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \boldsymbol{M}_z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The mirrored point $\boldsymbol{p}'$ is obtained by: $\boldsymbol{p}' = \boldsymbol{M}_x\boldsymbol{p}$ or $\boldsymbol{p}' = \boldsymbol{M}_y\boldsymbol{p}$ or $\boldsymbol{p}' = \boldsymbol{M}_z\boldsymbol{p}$.

1.5 Rotation

To rotate a point around the x, y, or z axis by an angle θ, I use the rotation matrices $\boldsymbol{R}_x$, $\boldsymbol{R}_y$, and $\boldsymbol{R}_z$:

$$\boldsymbol{R}_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{R}_y = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{R}_z = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The rotated point $\boldsymbol{p}'$ is obtained by: $\boldsymbol{p}' = \boldsymbol{R}_x\boldsymbol{p}$ or $\boldsymbol{p}' = \boldsymbol{R}_y\boldsymbol{p}$ or $\boldsymbol{p}' = \boldsymbol{R}_z\boldsymbol{p}$.

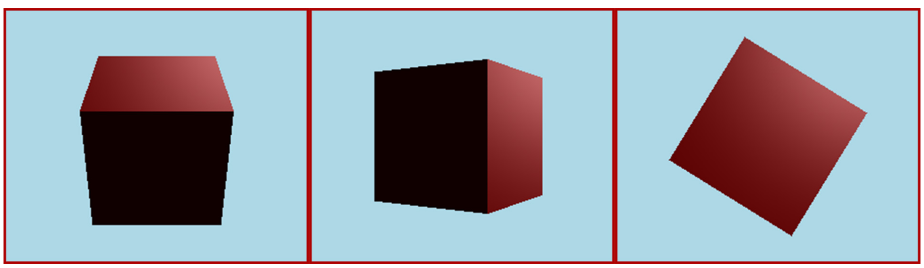

Figure 4: Cube is rotated by 45 Degrees in each direction.

1.6 Shearing

To shear a point along the x, y, or z axis, I use the shearing matrices $\boldsymbol{H}_{xy}$, $\boldsymbol{H}_{xz}$, $\boldsymbol{H}_{yx}$, $\boldsymbol{H}_{yz}$, $\boldsymbol{H}_{zx}$, and $\boldsymbol{H}_{zy}$:

$$\boldsymbol{H}_{xy} = \begin{bmatrix} 1 & h_{xy} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \boldsymbol{H}_{xz} = \begin{bmatrix} 1 & 0 & h_{xz} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \boldsymbol{H}_{yx} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ h_{yx} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{H}_{yz} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & h_{yz} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \boldsymbol{H}_{zx} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ h_{zx} & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \boldsymbol{H}_{zy} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & h_{zy} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The sheared point $\boldsymbol{p}'$ is obtained by: $\boldsymbol{p}' = \boldsymbol{H}_{xy}\boldsymbol{p}$, $\boldsymbol{p}' = \boldsymbol{H}_{xz}\boldsymbol{p}$, $\boldsymbol{p}' = \boldsymbol{H}_{yx}\boldsymbol{p}$, $\boldsymbol{p}' = \boldsymbol{H}_{yz}\boldsymbol{p}$, $\boldsymbol{p}' = \boldsymbol{H}_{zx}\boldsymbol{p}$, or $\boldsymbol{p}' = \boldsymbol{H}_{zy}\boldsymbol{p}$.

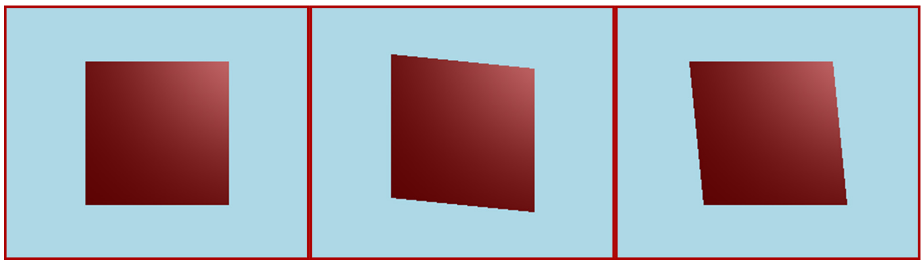

Figure 5: Cube is transformed with Shear