School of
Computing

UNIVERSITI TEKNOLOGI MALAYSIA
FINAL EXAMINATION SEMESTER I 2018/2019

SUBJECT CODE	:	**SCSI 1013**
SUBJECT NAME	:	**DISCRETE STRUCTURE**
SECTION	:	
TIME	:	
DATE/DAY	:	
VENUES	:	

INSTRUCTIONS:

ANSWER ALL QUESTIONS IN THE ANSWER BOOKLET

(Please Write Your Lecture Name And Section In Your Answer Booklet)

Name	
I/C No.	
Year / Course	
Section	
Lecturer Name	

This questions paper consists of **SIX (6)** printed pages excluding this page.

Question 1 [17 Marks]

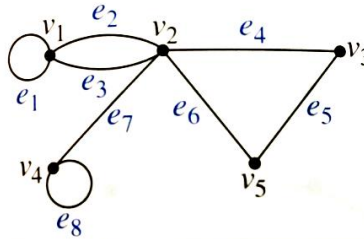a) Graph G in Figure 1 is not a simple graph.

Figure 1: Graph G

    i) Find the incidence matrix of the graph.

(5 marks)

    ii) Find a set of edges to remove to make it a simple graph.

(3 marks)

b) H_1 and H_2 are adjacency matrices of two different graphs. Check whether these graphs are isomorphic? Justify your answer.

(3 marks)

$$H_1 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}; \quad H_2 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

c) Determine whether the given graphs in Figure 2 have an Euler circuit. Construct such circuit if exists. Otherwise, determine whether the graph has an Euler path and construct such a path if one exists.
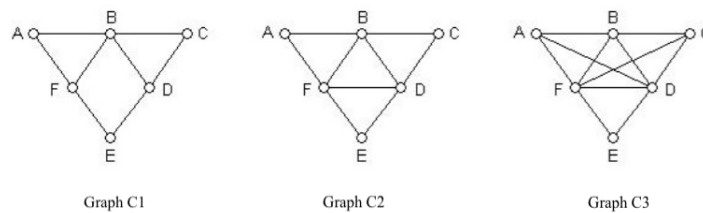
(4 marks)

Graph C1   Graph C2   Graph C3

Figure 2: Three simple graphs

d) Which of the simple graphs given in Figure 3 have a Hamilton circuit and find such a circuit?
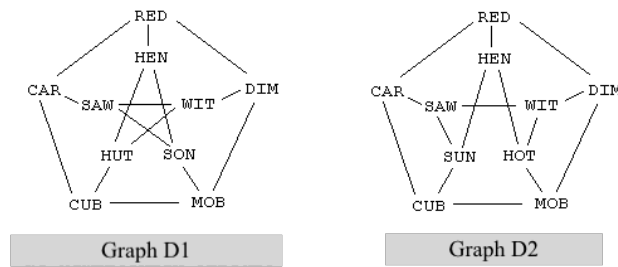
(2 marks)

Graph D1   Graph D2

Figure 3: Two simple graphs

Question 2 [15 Marks]

a) Based on the rooted tree shown in Figure 4, answer the following question.

Figure 4: Rooted tree, T

    i)   List the children of vertex d.   (1 mark)
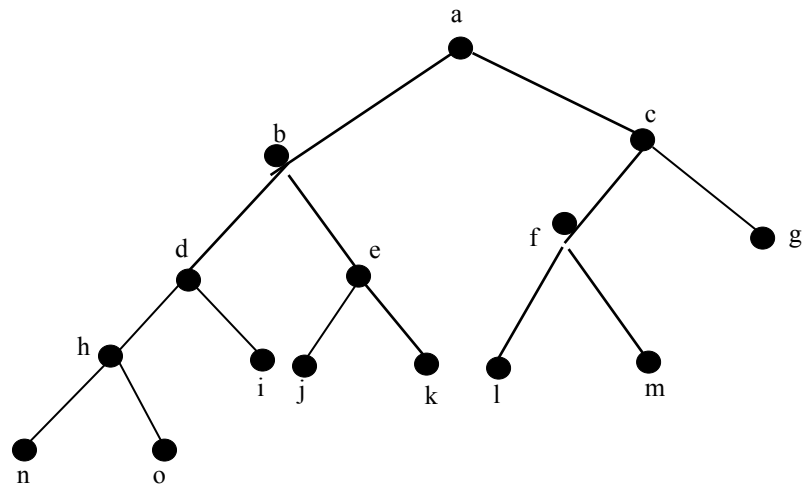   ii) List the sibling of vertex f.   (1 mark)
    iii) Find the height of this rooted tree.   (1 mark)
    iv) According to the universal address system, what is the address of vertex j?   (1 mark)
    v)   Give the order of vertices produced by preorder, inorder and postorder traversal.   (6 marks)

b) Either draw a full binary tree, 8 internal vertices(k) and 7 leaves, or show that no such tree exist.

(2 marks)

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c) A chain letter starts when a person sends a letter to 7 others. Each person receiving the letter either sends it to 7 others who have never received the letter before or does not send it to anyone. Suppose there are 6986 people received letter. How many people do not send the letter and how many people send out the letter? (3 marks)

Question 3 [10 Marks]

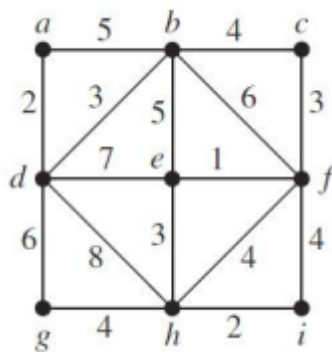a) Use Kruskal's algorithm to find a minimum spanning tree for the graph given in Figure 5.

(7 marks)

Figure 5: weighted graph

b) Draw the spanning tree obtained from (a). (3 marks)

Question 4 [20 Marks]

a) Let $M_1=(S, I, q_0, f_s, F)$ be the DFA such that $S=\{q_0, q_1, q_2, q_3, q_4, q_5\}$, $I=\{x, y, z\}$, $F=\{q_3, q_4\}$, q_0 =initial state, and f_s is defined as follows:

$f_s(q_0,x)= q_1$, $f_s(q_1,x)= q_1$, $f_s(q_2,x)= q_4$, $f_s(q_3,x)= q_1$, $f_s(q_4,x)= q_4$, $f_s(q_5,x)= q_5$,

$f_s(q_0,y)= q_2$, $f_s(q_1,y)= q_3$, $f_s(q_2,y)= q_2$, $f_s(q_3,y)= q_3$, $f_s(q_4,y)= q_2$, $f_s(q_5,x)= q_5$,

$f_s(q_0,z)= q_5$, $f_s(q_1,z)= q_1$, $f_s(q_2,z)= q_2$, $f_s(q_3,z)= q_1$, $f_s(q_4,z)= q_2$, $f_s(q_5,x)= q_5$

i) Draw the state diagram of M_1. (5 marks)
ii) Describe the set of strings that accepted by M_1. (2 marks)

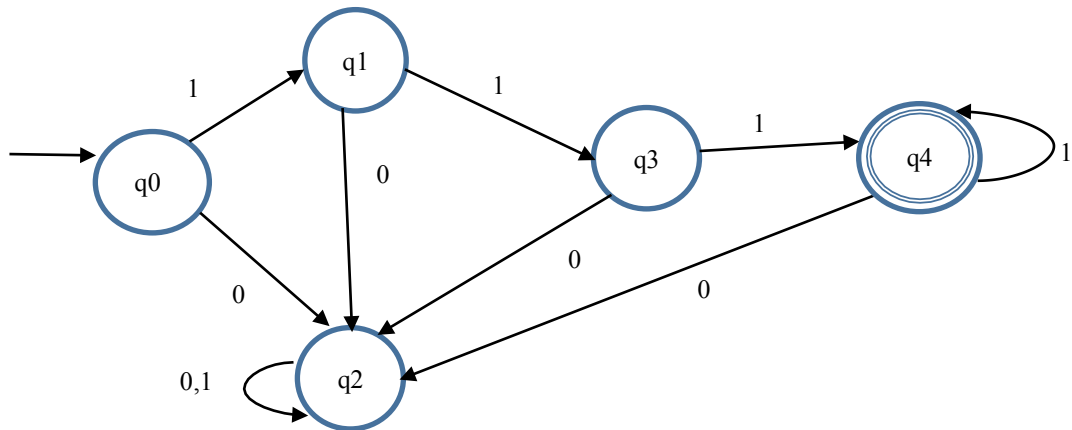b) Given the transition diagram of M_2 as in Figure 6.

Figure 6: M_2

i) What are the states of M_2? (1 mark)

ii) Write the set of input symbols. (1 mark)

iii) Construct the transition table of M_2. (3 marks)

iv) Which of the strings 11, 0011, 1110, 111 are accepted by M_2? (2 marks)

c) Construct a state transition diagram of a DFA that accepts all string over {**a,b, … , z, 0, 1, …, 9**} that start with a letter (**a-z**) and end with a digit (**0-9**). Examples of accepted strings are **a9**, **xy12**, **a1b2c3** etc. (6 marks)

Question 5 [15 marks]

In a standard washing machine operation, there are four phases which start with Idle/Stop, Wash, Rinse and Spin. When the start/stop button is pressed, the door will be automatically locked, timer will start and the washing machine will begin to wash. After the timer end, the washing machine starts the rinse phase and timer for rinsing phase will begin. After the timer is end, the spin phase will begin. At this point, the timer will start again and after it end, operation of the washing machine is finish and it returns to Idle/Stop condition. At any time during the operation, if the start/stop button is pressed again, the washing machine will stop the operation and return to Idle/Stop condition. The door will always remain locked during the operation unless it is in Idle/Stop condition.

Based on the above washing machine operation,

a) define all the states, inputs and outputs. (5 marks)

b) construct a transition diagram. (10 marks)

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Question 6 [23 Marks]

a) Which of these bytes have been transmitted correctly? Justify the answer.　　　　　(2 Marks)

   i)　　11010000

   ii)　　01100110

b) Let the parity check matrix, H is given by ,　$H = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Construct the codewords for the following message sequence and determine the weight of the codewords.

   i) 0011　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　(3 Marks)

   ii) 1010　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　(3 Marks)

c) Consider the encoding function $e : B^2 \rightarrow B^5$

$$e(00)=00000 \quad e(01)=01011 \quad e(10)=10101 \quad e(11)=11110$$

  i) What is the minimum distance of e?　　　　　　　　　　　　　　　(5 Marks)

  ii) How many transmission error can this code detected?　　　　　　　(2 marks)

d) Consider the (3,5) group encoding function $f_n : B^3 \rightarrow B^5$ defined by

$$
\begin{aligned}
f(000) &= 00000 & f(100) &= 01010 \\
f(001) &= 11110 & f(101) &= 10100 \\
f(010) &= 01101 & f(110) &= 00111 \\
f(011) &= 10011 & f(111) &= 11001
\end{aligned}
$$

Decode the following words relative to a maximum likelihood decoding function

   i)　　01011

   ii)　　10110　　　　　　　　　　　　　　　　　　　　　　　　　　(8 marks)

Formulas

i) n vertices has i = (n-1)/m internal vertices and l = [(m-1)n+1]/m leaves
ii) i internal vertices has n =mi+1 vertices and l = (m-1)i + 1 leaves
iii)l leaves has n =(ml-1)/(m-1) vertices and i =(l-1)/(m-1) internal vertices