SULIT

SCHOOL OF COMPUTING
Faculty of Engineering

# UNIVERSITI TEKNOLOGI MALAYSIA
# TEST 1 SEMESTER I, 2018 /2019

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|---|---|---|
| **SUBJECT CODE** | **:** | **SCSI1013** |
| **SUBJECT NAME** | **:** | **DISCRETE STRUCTURE** |
| **SECTION** | **:** | **01/02/03/05/05/06/07/08/09/10/11/12** |
| **TIME** | **:** | |
| **DATE/DAY** | **:** | **19 OCTOBER 2018 (FRIDAY)** |
| **VENUES** | **:** | **L50/N28** |

**INSTRUCTIONS :**

**(Please Write Your Lecture Name And Section In Your Answer Booklet)**

| | |
|---|---|
| **Name** | |
| **I/C No.** | |
| **Year / Course** | |
| **Section** | |
| **Lecturer Name** | |

This questions paper consists of printed **four (4)** pages excluding this page.

**QUESTION 1** **[15 Marks]**

a) Draw a Venn diagram illustrating set $(A - B) \cap (C - B)$

Give that $A \cap B \cap C \neq \{\}$ (2 marks)

b) There are 20 people in your neighbourhood own pets. Five people own cats, rabbit and hamster. Three of them own only hamsters, five own only rabbit and another three own cat. How many total pets in your neighbourhood. (4 marks)

c) Let P and Q are set, prove that $((P \cup Q)' \cap Q')' = P \cup Q$ (4 marks)

d) Prove the following theorem using indirect proof method.

For all integers, if $a^2 - 3b$ is even then $a$ is even and $b$ is even (5 marks)

**QUESTION 2** **[20 Marks]**

a) Write the following statement using $p$, $q$, $r$ and logical connective

$p$ : I go to the beach

$q$ : it is a sunny summer day

$r$ : it is Sunday

i) I go to the beach whenever it is Sunday and sunny summer day (2 marks)

ii) If it is not either Sunday or sunny summer day, then I do not go to the beach (2 marks)

iii) If I do not go to the beach, then it is not either Sunday or sunny summer day (2 marks)

iv) Which of these (i), (ii) and (iii) are equivalence statement using truth table. (2 marks)

b) Write the negation of $\forall x(x^2 + 2x - 3 = 0)$ and determine the resulting proposition is TRUE or FALSE with the domain of discourse is integer. (5 marks)

c) Express the following statement using predicates, quantifier and logical connective with the domain of discourse consist of all students at your school (7 marks)

i) There is a student at your school who can speak Russian but does not know C++

ii) Every student at your school either can speak Russian or knows C++

iii) No student at your school can speak Russian or knows C++

**QUESTION 3** **[22 Marks]**

a) Let $A = \{1,3,5\}$. Define $R$ on $A$ by $xRy$ if $3x + y$ is a multiple of 6.

   i) Find the element of $R$. (3 marks)

   ii) Draw the corresponding digraph. (2 marks)

   iii) Determine the domain and range $R$ (2 marks)

   iv) Determine whether the relation $R$ is irreflexive? (2 marks)

b) Consider the relation $R$ on $B = \{a, b, c, d\}$ given by the matrix

$$M_R = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

   i) List in-degrees and out-degrees of all vertices. (4 marks)

   ii) Detemine whether the relation $R$ is reflexive, symmetric, antisymmetric and/ or transitive? Justify your answers. (4 marks)

   iii) Is $R$ a partial order? Why or why not? (2 marks)

c) Let $C = \{1, 2, 3, 4\}$. Find a relation $R$ on $C$ that has exactly 3 ordered pair members and is both symmetric and antisymmetric. (3 marks)

**QUESTION 4** **[13 Marks]**

a) For each of the following mappings indicate what type of function they are (if any). Use the following key:

1. Not a function
2. A function which is onto but not one-to-one
3. A function which is one-to-one but not onto
4. A function which is bijection

   i) The mapping $f$ from $\mathbb{Z}$ *to* $\mathbb{Z}$ defined by $f(n) = -2(n)$ (2 marks)

   ii) The mapping $f$ from $\mathbb{Z}$ *to* $\mathbb{Z}$ defined by $f(n) = |n|$ (2 marks)

b) Find the inverse of $f(x) = \dfrac{x-5}{2x+1}$ (5 marks)

c) Functions $f$, $g$ and $h$ are given by

$f(x) = 2x + 3$ and $g(x) = -x^2 + 1$, $h(x) = 1/x$

Find the composite function defined by $h \circ (g \circ f)$ (4 marks)

**QUESTION 5** **[15 Marks]**

a) Suppose that the number of bacteria in a colony triples every hour.

i) Set up a recurrence relation for the number of bacteria after $n$ hours have elapsed.

(2 marks)

ii) If 10 bacteria are used to begin a new colony, how many bacteria will be in the colony in 10 hours? (3 marks)

b) Find a recurrence relation for the balance $B_k$ owed at the end of $k$ months on a loan of RM9000 at rate of 8% if a payment of RM150 is made each month.

[Hint: Express $B_k$ in terms of $B_{k-1}$; the monthly interest is (0.08/12) $B_{k-1}$]

(5 marks)

c) Write a recursive algorithm for computing $n^2$ where $n$ is a nonnegative integer using the fact that $(n+1)^2 = n^2 + 2n + 1$. (5 marks)

**Properties of Set**

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|---|---|
| Commutative laws | $A \cap B = B \cap A$ |
| | $A \cup B = B \cup A$ |
| Associative laws | $A \cap (B \cap C) = (A \cap B) \cap C$ |
| | $A \cup (B \cup C) = (A \cup B) \cup C$ |
| Distributive laws | $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ |
| | $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ |
| Absorption laws | $A \cup (A \cap B) = A$ |
| | $A \cap (A \cup B) = A$ |
| Idempotent laws | $A \cap A = A$ |
| | $A \cup A = A$ |
| Complement laws | $A \cap A' = \varnothing$ |
| | $A \cup A' = \mathrm{U}$ |
| Double complement laws | $(A')' = A$ |
| Complement of $U$ and $\varnothing$ | $\varnothing' = U$ |
| | $U' = \varnothing$ |
| De Morgan's laws | $(A \cap B)' = A' \cup B'$ |
| | $(A \cup B)' = A' \cap B'$ |
| Properties of universal set | $A \cup U = U$ |
| | $A \cap U = A$ |
| Identity laws | $A \cup \varnothing = A$ |
| | $A \cap U = A$ |
| Properties of empty set | $A \cup \varnothing = A$ |
| | $A \cap \varnothing = \varnothing$ |